Relativistic hydro for quark-gluon plasma: applicability, initial conditions & limits of validity

Laurence Yaffe
University of Washington

Hydrodynamics at all scales, Nordita, September 13, 2023

outline

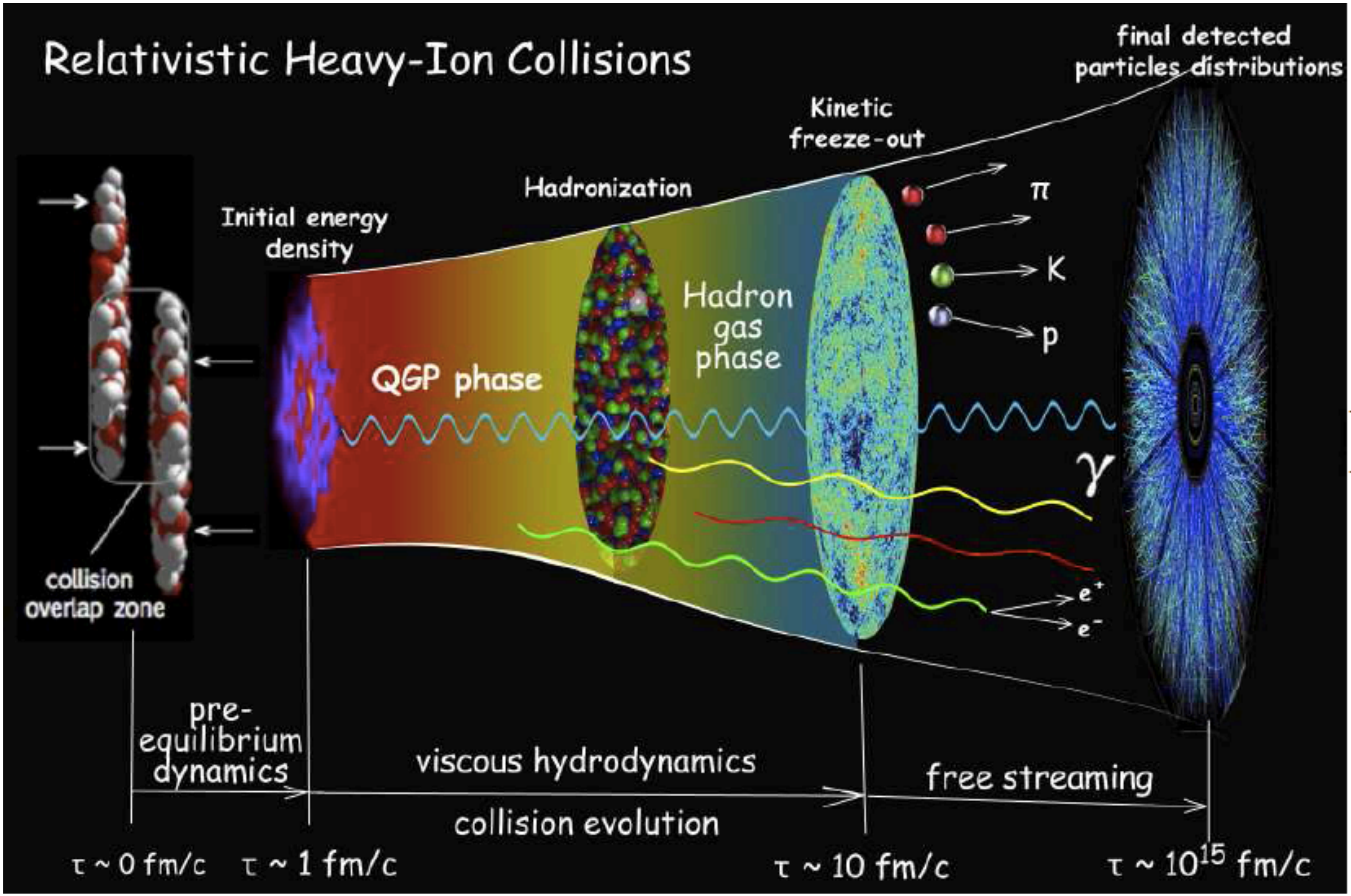
- Heavy ion collisions & quark-gluon plasma
- Relativistic hydrodynamics
  - assumptions & applicability
  - initial data
- Pre-hydro evolution
  - weak coupling vs. strong coupling
  - $\mathcal{N} = 4$ SYM plasma vs. QCD plasma
- Holographic modeling of pre-hydro evolution
  - granular initial data
  - transverse expansion
  - results
- Next steps

} w. Sebastian Waeber

Heavy-ion collisions

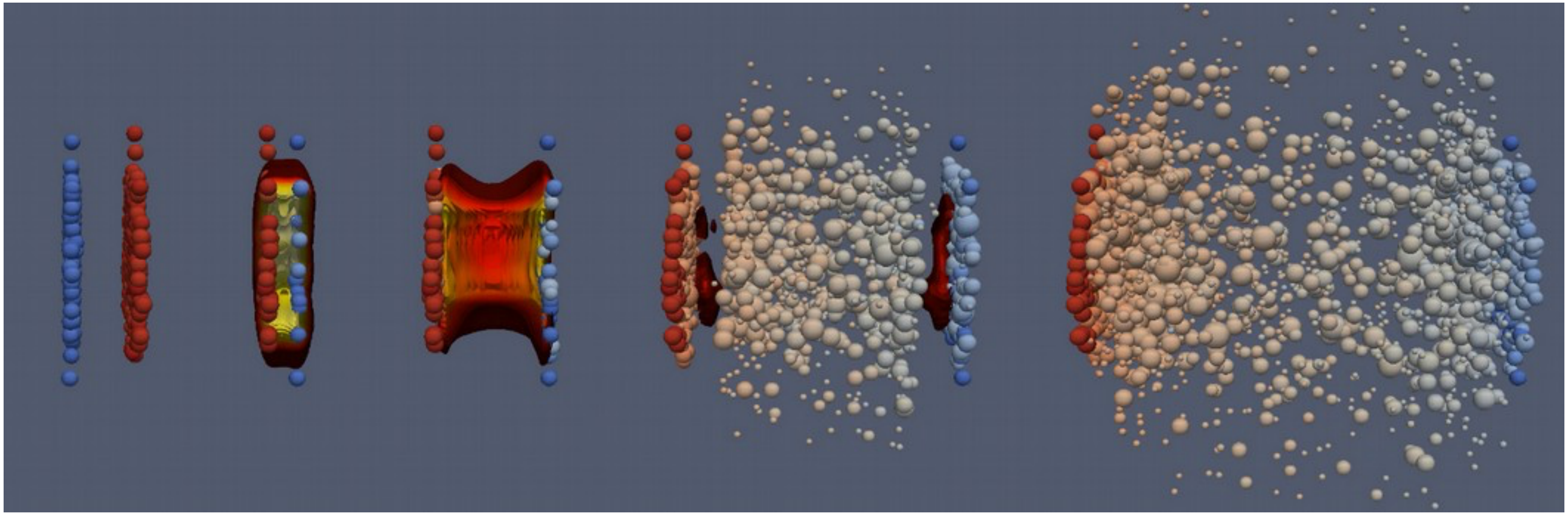
- Ultra-relativistic: energy/nucleon $\gg$ 1 GeV, Lorentz contraction factor $\gamma \gg 1$.
- Collision disrupts nucleus and constituent nucleons
- Liberated quarks & gluons form non-equilibrium quark-gluon plasma (QGP)
- Plasma expands & cools
- At sufficiently low temperature, hadrons re-form, system becomes expanding hadron gas
- Continuing expansion ➡ decreasing re-scattering and eventual free-streaming
- ``Hermetic" particle detectors individually measure energy, momentum & tag species of thousands of produced particles

Heavy ion collisions



Shen & Heinz, 1507.01558

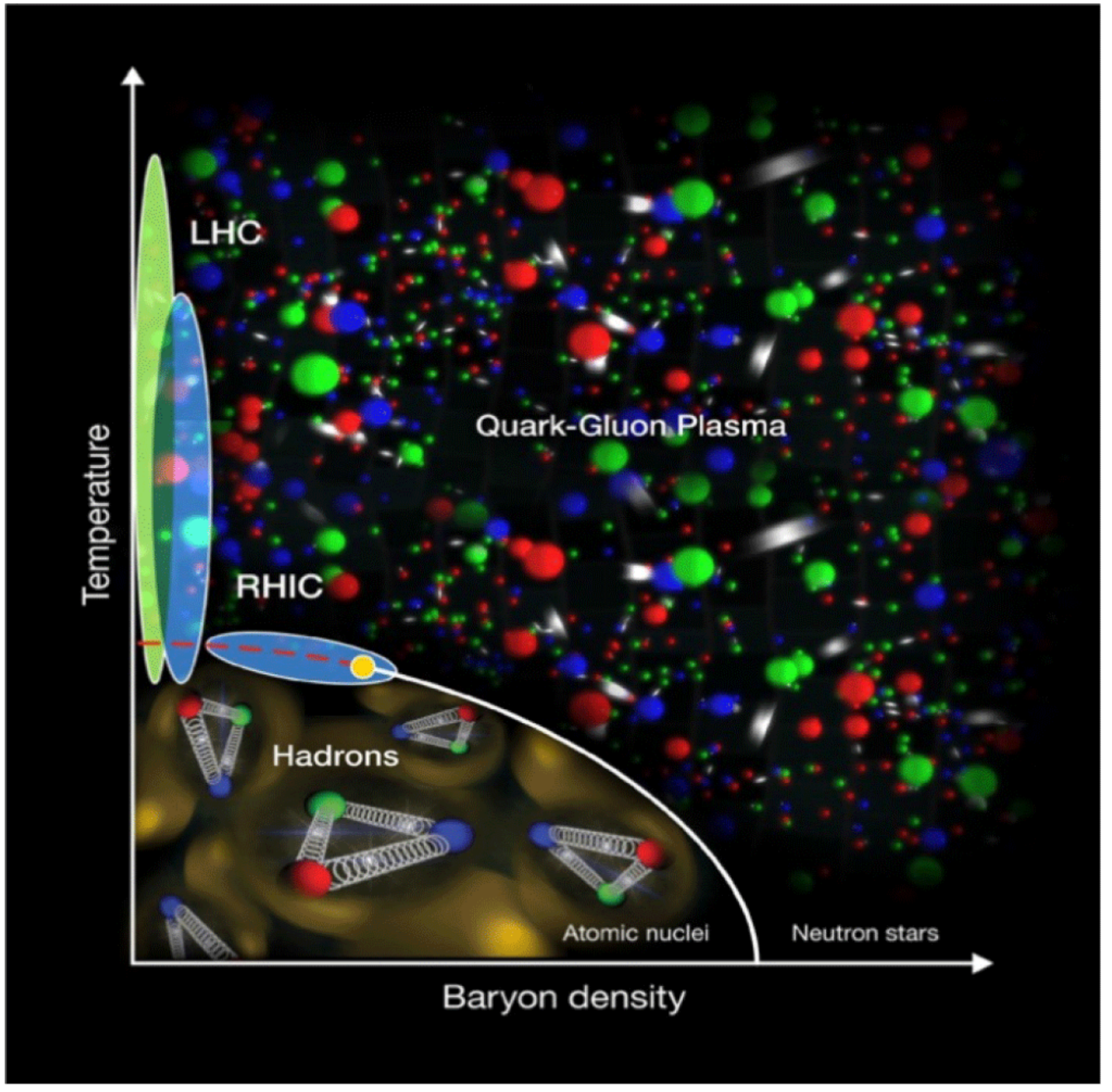
Heavy ion collisions



MADAI collaboration, Hannah Petersen and Jonah Bernhard

Why bother?

- Infer thermodynamic properties & explore QCD phase diagram:
- Study thermalization, entropy production, etc. in strongly coupled, strongly correlated system.
- Playground for studying interplay between perturbative & non-perturbative quantum field dynamics, thermodynamics, hydrodynamics, kinetics.



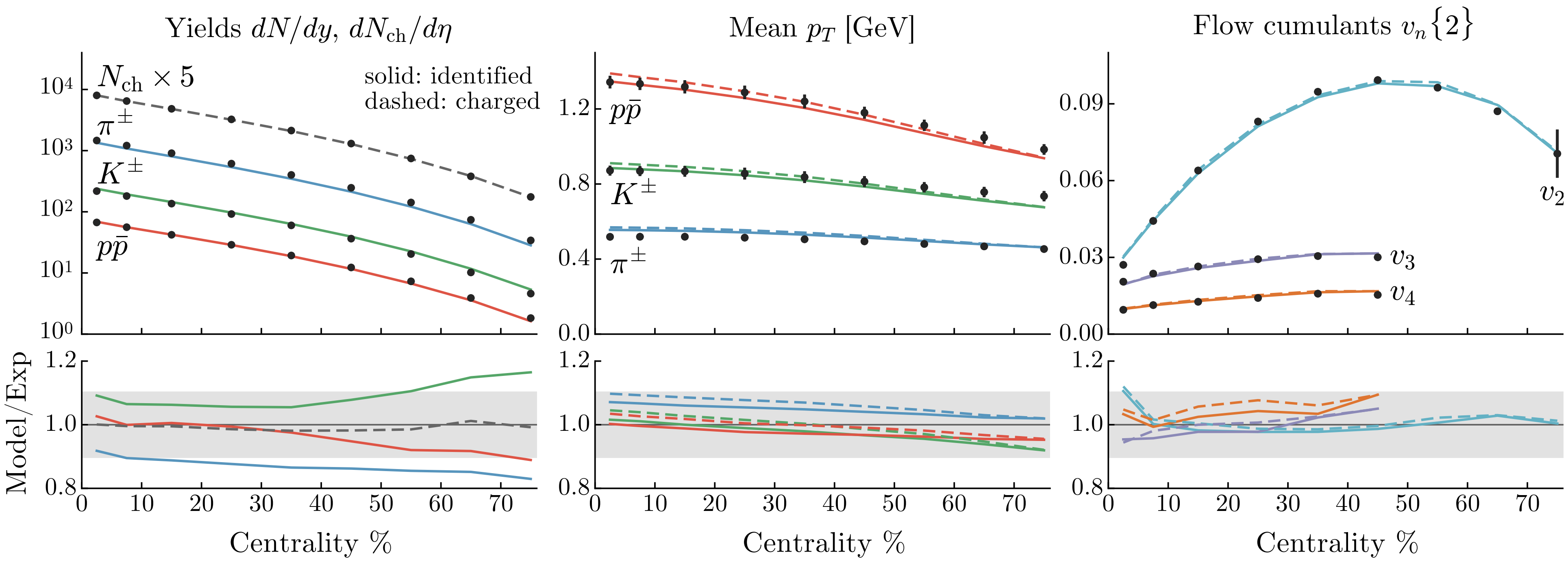
https://gdrqcd.in2p3.fr/working-group-2/

Heavy ion collisions: lessons

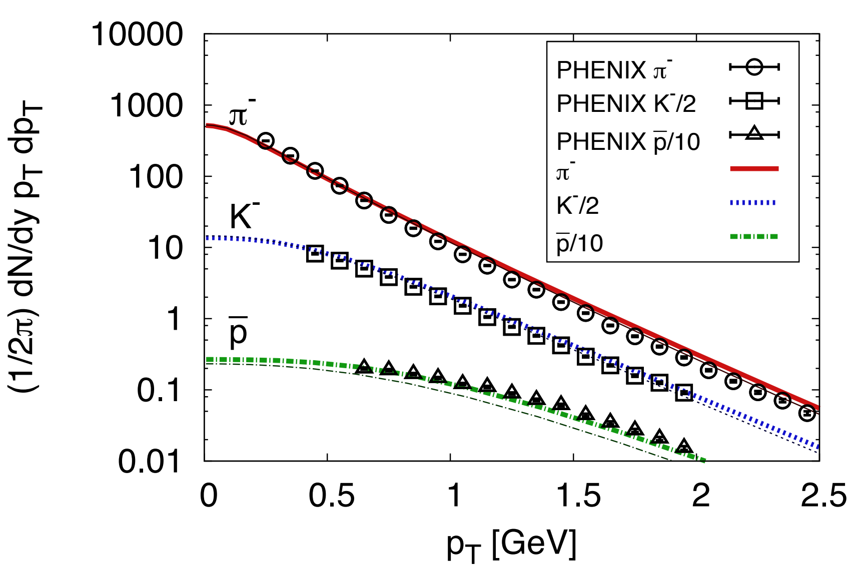
- Modeling particle production using near-ideal relativistic hydrodynamics & equilibrium thermodynamics (via lattice gauge theory) works remarkably well.
- Resulting flow is highly relativistic.
- Produced plasma has low viscosity, $4\pi\,\eta/s \approx 1$ ($\hbar \equiv k_B \equiv 1$).
  - Kinetic theory: $\eta \sim \rho \mathrm{v}\, \ell_{\mathrm{mfp}}$.
  - Weak coupling ➡ large $\ell_{\mathrm{mfp}} \gg \lambda_{\text{de Broglie}}$ ➡ large η.
  - Small η ➡ small $\ell_{\mathrm{mfp}} \sim \lambda_{\text{de Broglie}}$ ➡ produced QGP is strongly coupled system.
- Granularity of colliding nuclei is substantial, nuclei are ``lumpy".
  - Reflected in substantial odd-order azimuthal moments of produced particle distributions.

hydrodynamic modeling of QGP

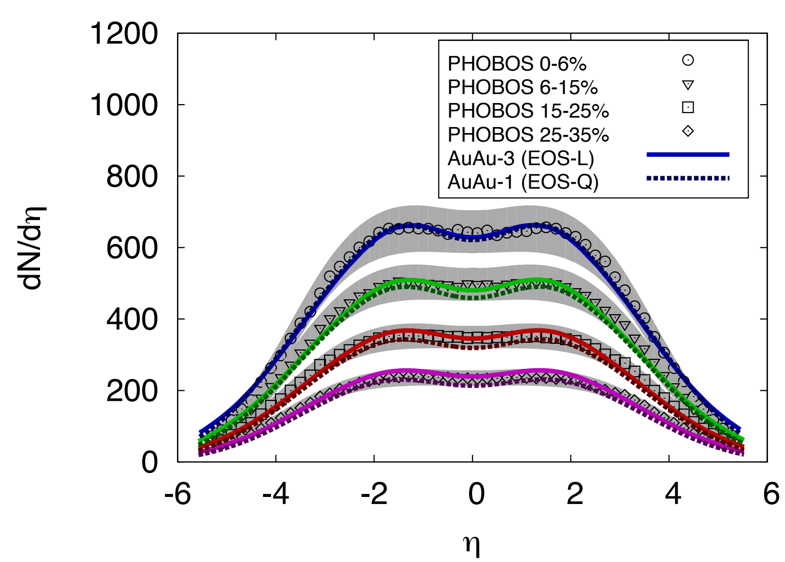
- low-viscosity hydro, suitably tuned (+ hadronic cascade) well-describes much data:



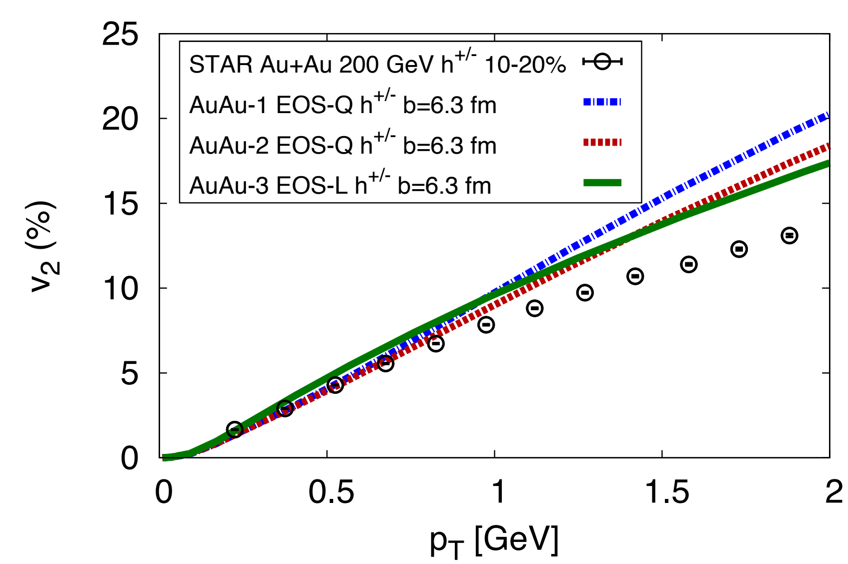
Bernhard, Moreland, Bass, Liu, Heinz 1605.03954



p_T spectra for central collisions



centrality dependence of pseudorapidity distribution



elliptic flow of charged hadrons

Schenke, Jeon, Gale 1004.1408

relativistic hydrodynamics

- Effective theory describing long-wavelength, low-frequency degrees of freedom
  - Assumes scale separation between relevant (hydro) & irrelevant (non-hydro) d.o.f.
  - Neutral fluid: relevant d.o.f = energy & momentum densities, constitutive relation for stress tensor
  - Inputs: equation of state & transport coefficients + initial data: energy & momentum densities on initial Cauchy surface.
- Relativistic hydro is not UV-complete:
  - Hiscock & Lindblom ('87): generic short-wavelength instabilities in "Eckart" theory
  - Perfectly normal for effective field theories: most EFTs need UV regularization
  - Multiple regularizations: Müller-Israel-Stewart (ad-hoc relaxation time), BDNK (special choice for fluid frame), BRSSS (2nd order conformal), …
  - No unique "best" or "physical" UV regulator, hydro ➡ neglect of non-hydro modes!

hydro applicability

- Well-behaved spatial gradient expansion?

$$
\begin{aligned}
T^{\mu\nu} &= O(1) + O(\partial) + O(\partial^2) + \cdots \\
&= [\mathcal{E}\, u^\mu u^\nu + \mathcal{P}\Delta^{\mu\nu}] + [\eta\, \sigma^{\mu\nu} + \zeta\, (\partial \cdot u)\, \Delta^{\mu\nu}] + O(\partial^2)
\end{aligned}
$$

- Relevant time scales ≫ non-hydro relaxation times?
  - "Extreme hydro": one *e*-folding of non-hydro relaxation sufficient.
- Well-understood initial data? From where?
- Well-understood transport coefficients (EFT parameters)?
- Proper matching at freeze-out?

hydro initial data

- Formulating hydro initial data is conceptually (& practically) problematic:
  - "Glauber" initial conditions:
    - Model nucleon density in nucleus (Woods-Saxon probability distribution, hard core repulsion)
    - Model "initial" fluid entropy density as linear combination of participant density (ρ_{part}) and binary collision density (ρ_{coll})
    - Treat result as hydro initial data at some starting time — no actual pre-hydro dynamical evolution.
  - "Color-glass condensate"/IP-Glasma:
    - Study asymptotia: arbitrarily high energy, asymptotically weak coupling
      - Beautiful picture: highly collinear gluon dynamics, elaborate hierarchy of scales, logarithmic evolution, …
      - Asymptopia is very, very far from accessible QGP!
      - Instantaneous switch from weak-coupling to strong coupling (fluid) description — inherently inconsistent.
- What are superior alternatives? Are there feasible alternatives?

accessible QGP

- low viscosity, $\eta/s \approx 0.1$
- effective temperatures T_{eff} = few $\times T_c$, not $\ggg T_c$
- effective coupling $\propto 1/\ln(T_{\text{eff}}/T_c)$ not at all small!
- substantial thermal masses, $m_{\text{th}}/T = O(1)$, not $\ll 1$
- near-conformal, $(\epsilon - 3p)/\epsilon$ small except very close to T_c

➡ accessible QGP = strongly coupled plasma, not weakly coupled!

holographic modeling

- complementary model:

  Early-stage QGP = strongly coupled, near-conformal non-Abelian plasma ≈ strongly coupled, maximally supersymmetric ($\mathcal{N} = 4$) Yang-Mills plasma

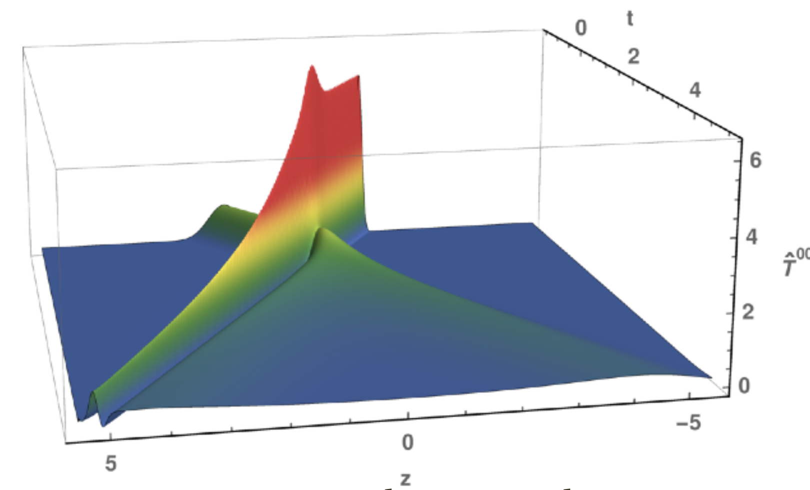
hot QCD	$\mathcal{N} = 4$ SYM
- non-Abelian plasma	- non-Abelian plasma
- neutral fluid hydro	- neutral fluid hydro
- weak dependence on N_c	- weak dependence on N_c
- strongly coupled	- fixed, arbitrary coupling
- near-conformal prior to hadronization	- conformal

- use gauge/gravity duality to solve (honestly) pre-hydrodynamic evolution of initial states in strongly coupled $\mathcal{N} = 4$ SYM which resemble real colliding nuclei

holographic collisions

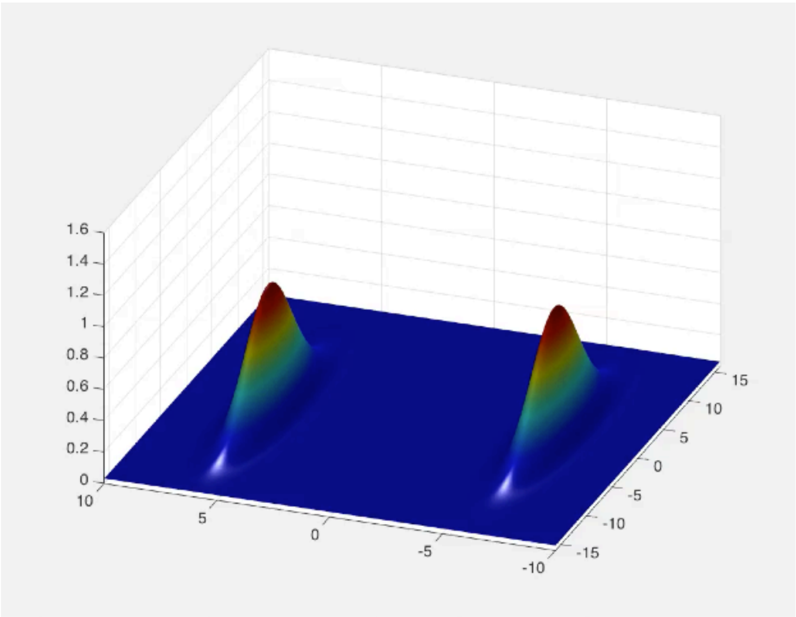
- Use numerical GR to solve dual gravitational initial value problem with characteristic formulation, spectral methods

- Warm-ups:
  - planar shocks (3D PDEs)
  - finite "nuclei" w. smooth Gaussian profiles (5D PDEs), Lorentz contraction $\gamma = 8$



S. Waeber, A. Rabenstein, A. Schäfer, LGY, 2019

- Needed:
  - "realistic" incoming projectiles: lumpy granular structure, required to generate observed triangular flow (v_3), Lorentz contraction $\gamma \geq 100$
  - computationally very demanding



P. Chesler, LGY, 2015

old results: lessons

- planar collisions:
  - near-universal Gaussian rapidity dependence
  - nearly boost-invariant flow
  - asymmetric collisions ≈ geometric mean of symmetric collisions
- localized collisions:
  - “pre-hydro” development of transverse flow
  - rapid hydrodynamization, $t_{\text{hydro}}\, T_{\text{eff}} \approx 0.3$
  - extreme hydrodynamics: huge anisotropy but well-behaved gradient expansion down to $R\, T_{\text{eff}} \approx 0.5 - 1$

old results: limitations

- 5D GR calculations: both time & memory constrained

  - spectral methods permit use of relatively spare spatial grid, ex. $32^2 \times 256 \times 96$. Nevertheless, $O(400)$ field components per grid point ⇒ 80 Gb (each time slice).

  - horizon fixing condition ⇒ linear elliptic PDE with 260K × 260K matrix.

  - parallelizes well on multi-core CPUs w. unified memory, but less well on clusters or distributed memory systems.

  - wall clock time (2015) = multiple months.

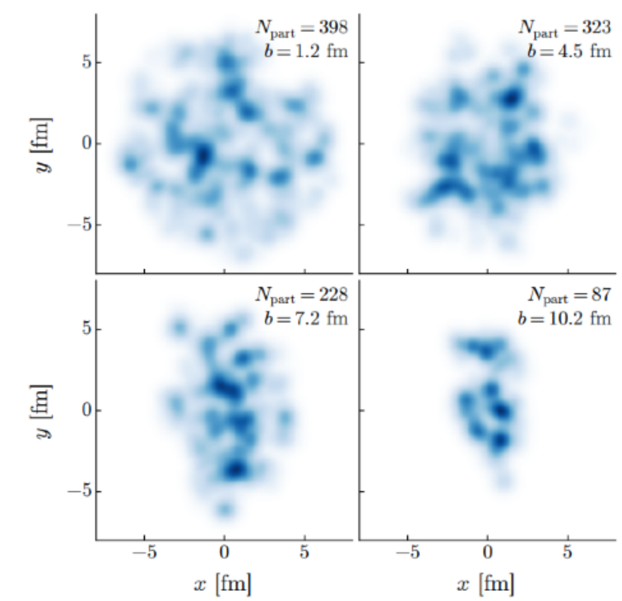
➡ simple Gaussian projectile profile, not realistic energy density distribution.

➡ limited $O(4-10)$ projectile aspect ratio vs. $O(100's - 1000)$ of real experiments.

transverse derivative expansion

w. Sebastian Waeber, arXiv:2206.01819

- Real nuclear projectiles:
  - huge aspect ratio, $\gamma \geq$ 100-1000
  - transverse gradients $\ll$ longitudinal gradients
  - initial state fluctuations very relevant



$\therefore$ Take advantage of slow transverse variation:

- let $\tilde{\mathbf{x}}^\perp = \epsilon\, \mathbf{x}^\perp$, write $G_{\mu\nu}(x^0, x^\parallel, \mathbf{x}^\perp) = \tilde{G}_{\mu\nu}(x^0, x^\parallel, \tilde{\mathbf{x}}^\perp)$
- net effect: $\partial_{\mathbf{x}^\perp} \Rightarrow \epsilon\, \partial_{\tilde{\mathbf{x}}^\perp}$
- expand in ϵ = formal parameter counting transverse derivatives
- return to original $\mathbf{x}^\perp$ for actual calculation

transverse derivative expansion

- Disadvantages:
  - systematic truncation error
- Advantages (at low orders):
  - much simpler equations, faster to evaluate by $O(10)$
  - much smaller memory requirements
  - 3D horizon fixing condition ⇒ decoupled 1D conditions
  - at $O(\epsilon)$, decoupling of fields into relevant and negligible
  - surprisingly small truncation error already at first order
  - Feasible! (Although still challenging)

holographic initial data

- single projectile: $$ds^2_{FG} = \frac{1}{\rho^2}\big(-dt^2 + d\rho^2 + (d\mathbf{x}^\perp)^2 + dz^2 + \rho^4 h_\pm(\mathbf{x}^\perp, z^\mp, \rho)(dz^\pm)^2\big)$$

$$\langle T^{00}\rangle = \langle T^{zz}\rangle = \frac{N_c^2}{2\pi^2}\, h_\pm\Big|_{\rho=0}$$

$$\langle T^{0z}\rangle = \pm\frac{N_c^2}{2\pi^2}\, h_\pm\Big|_{\rho=0}.$$

$$\frac{N_A \times 200\,\text{GeV}}{2} = E_{\text{RHIC}} = \int d^2\mathbf{x}_\perp\, dz\, \langle T^{00}\rangle = \frac{N_c^2}{2\pi^2}\int d^2\mathbf{x}_\perp\, dz\, h_\pm\Big|_{\rho=0}$$

$$\Big(\frac{d^2}{d\rho^2} - \frac{3}{\rho}\frac{d}{d\rho} + \nabla_\perp^2\Big)\rho^4\, h_\pm = 0$$

⇒ bulk geometry determined by boundary $T^{\mu\nu}$

$$h_\pm(\mathbf{x}^\perp, z^\mp) = \sum_{i=0}^{196} G_\pm(\mathbf{x}^\perp, z^\mp, \mathbf{x}_i^\perp, z_i^\mp)$$

nuclear energy density = sum of nucleon energy densities

$$G_\pm(\mathbf{x}^\perp, z^\mp, \mathbf{x}_0^\perp, z_0^\mp) = \frac{\mu^3}{\sqrt{2\pi w^2/\gamma^2}}\exp\big(-\frac{\gamma^2}{2}(z^\mp - z_0^\mp)^2/w^2\big)\exp\big(-\frac{1}{2}(\mathbf{x}^\perp - \mathbf{x}_0^\perp)^2/w^2\big)$$

Gaussian single nucleon density

$$P(\mathbf{x}^\perp, z^\mp) = \frac{n}{1 + \exp\Big(\big(\sqrt{(x^\perp)^2 + \gamma^2(z^\mp)^2} - R\big)/a\Big)}$$

Woods-Saxon probability distribution for nucleon positions, with d_{min} hard core enforced

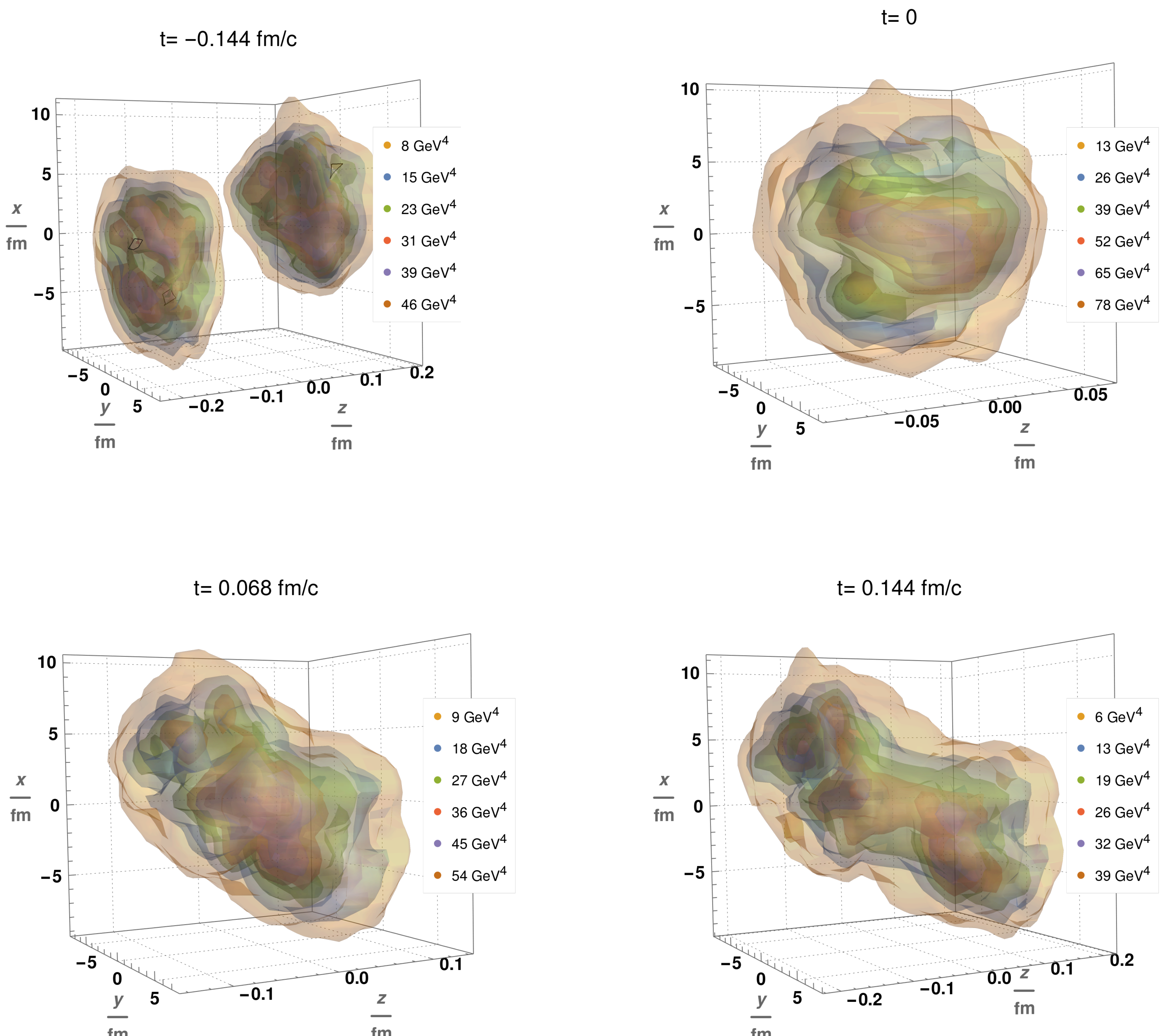
- superpose well-separated projectiles, transform to infalling coordinates

$$ds^2_{EF} = u^{-2}\Big(g^{EF}_{\mu\nu}(x,r)\, dx^\mu dx^\nu - 2\, dr du\Big)$$

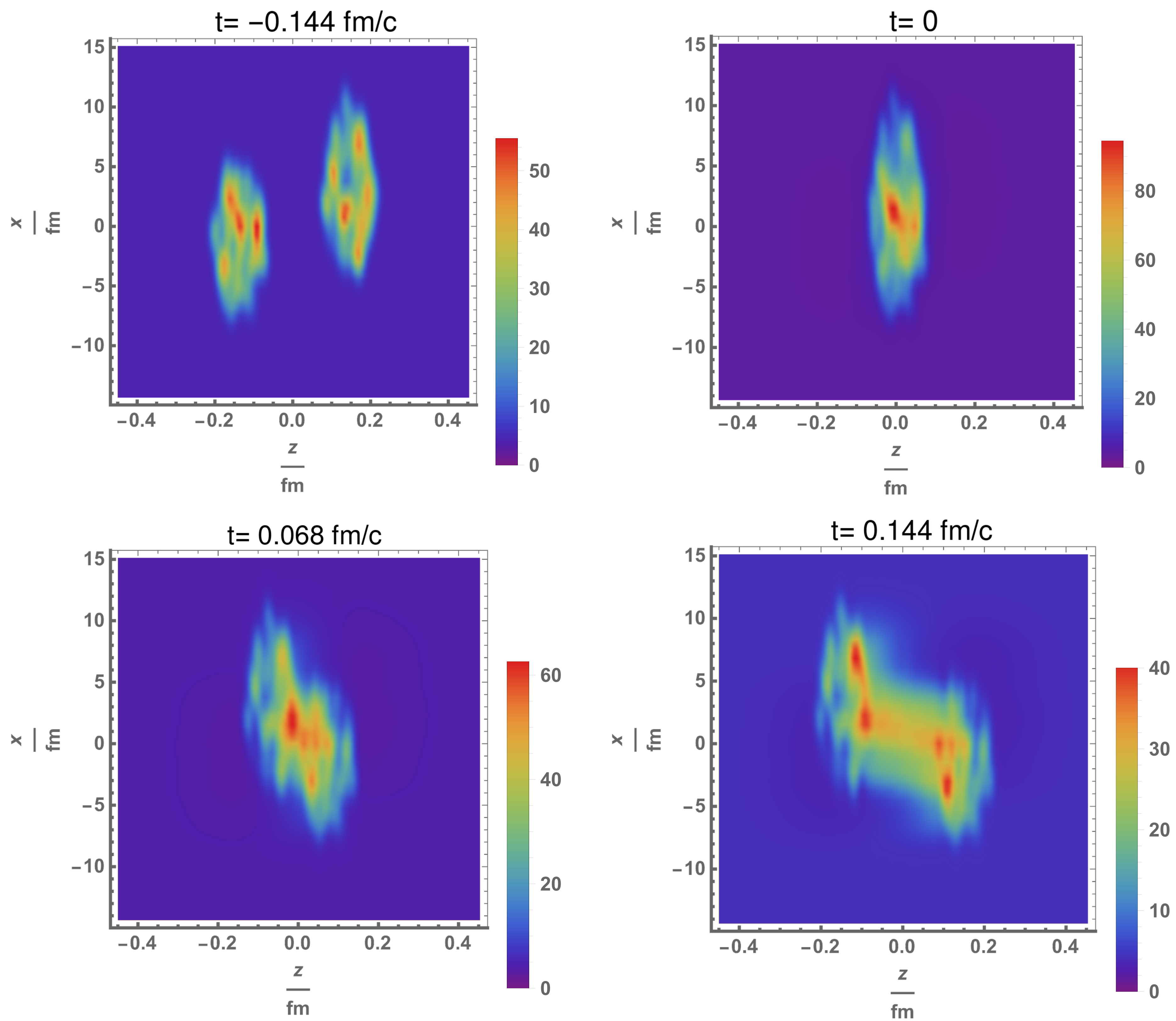
Results

w. Sebastian Waeber, arXiv:2211.09190

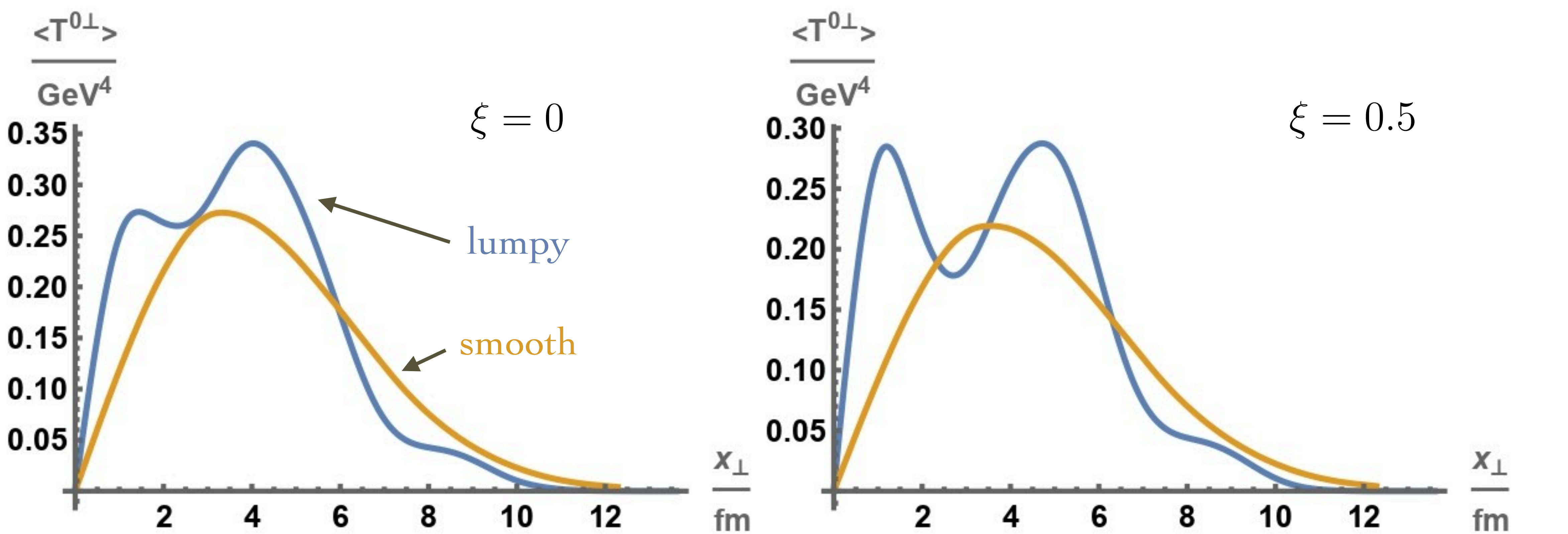
energy density



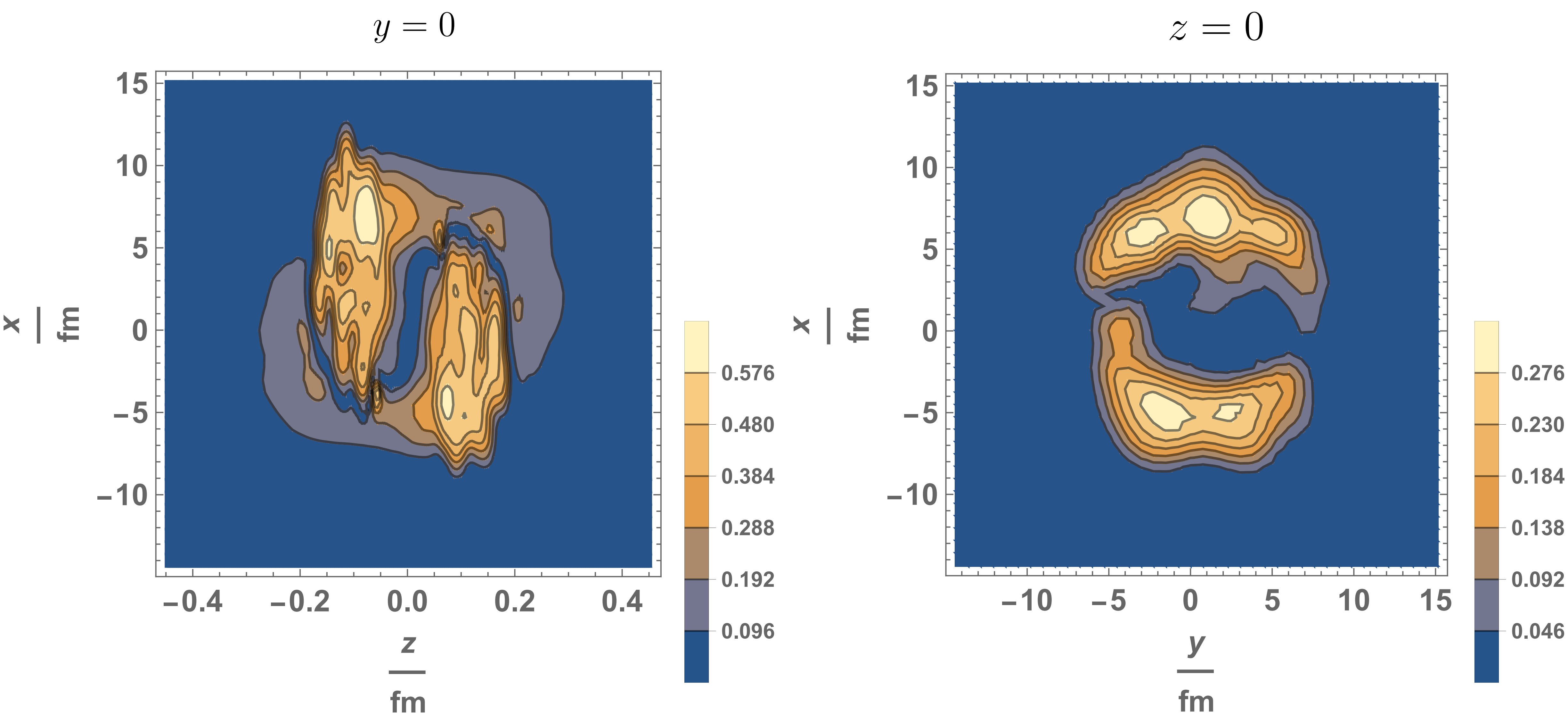
energy density, $y = 0$



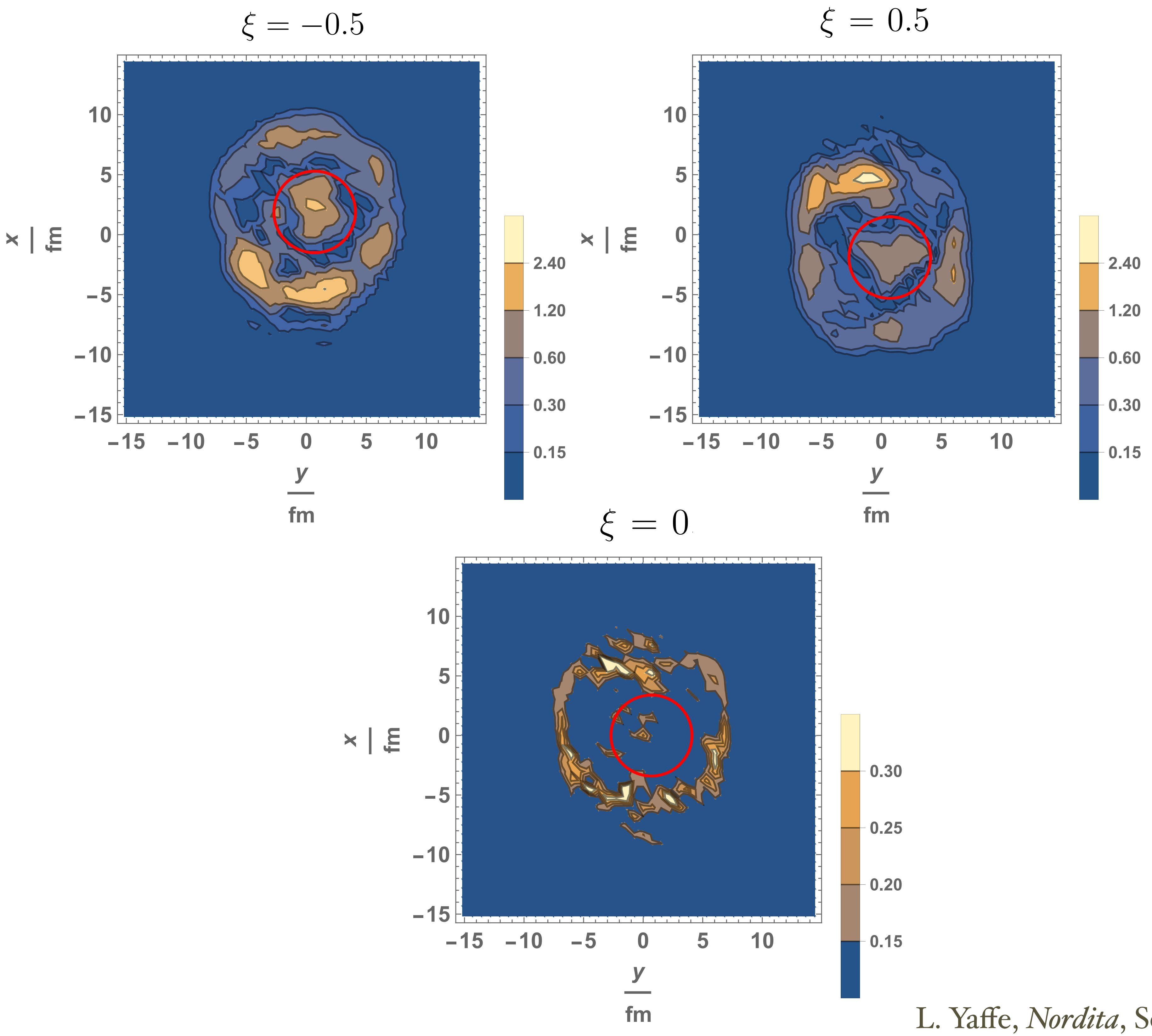
transverse momentum density, angle-averaged, $\tau = 0.1$ fm/c



fluid 3-velocity $|\vec{u}/u^0|$
$t = 0.1$ fm/c



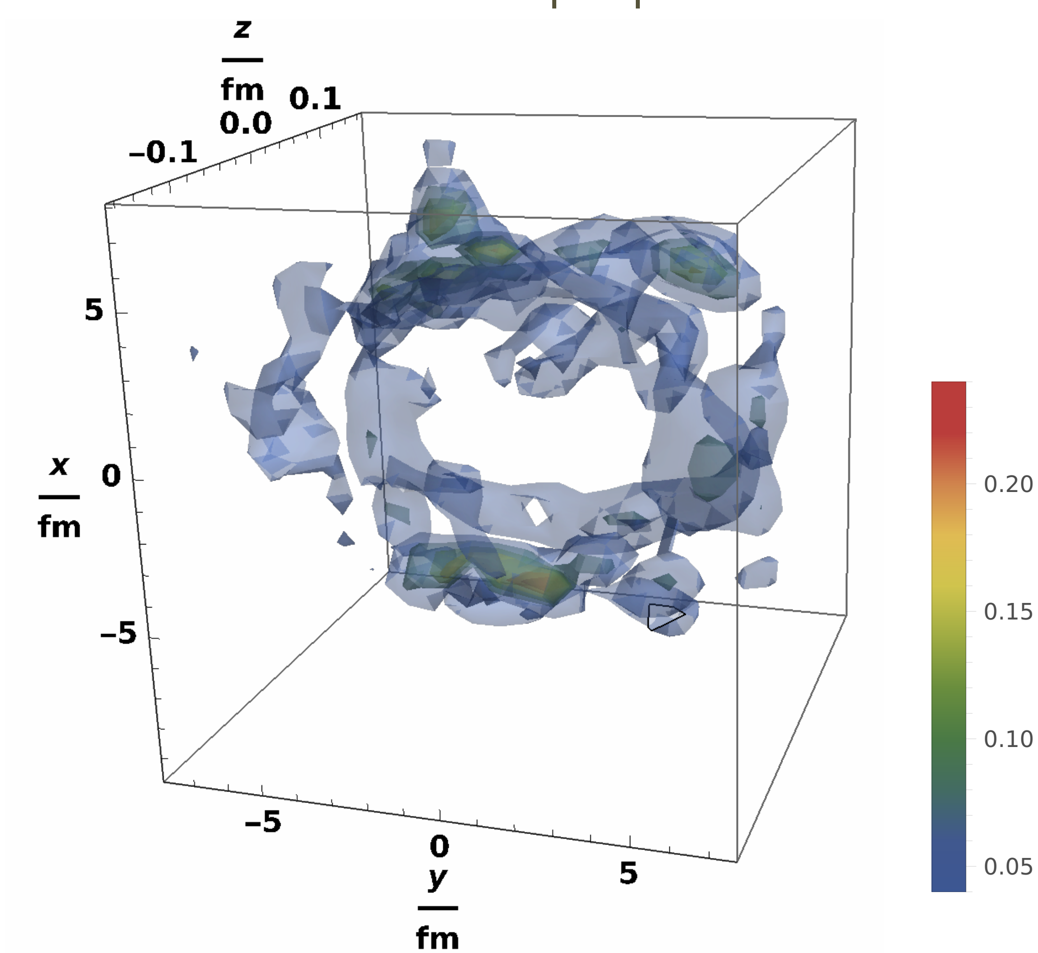
hydro residual, $\tau = 0.106$ fm/c



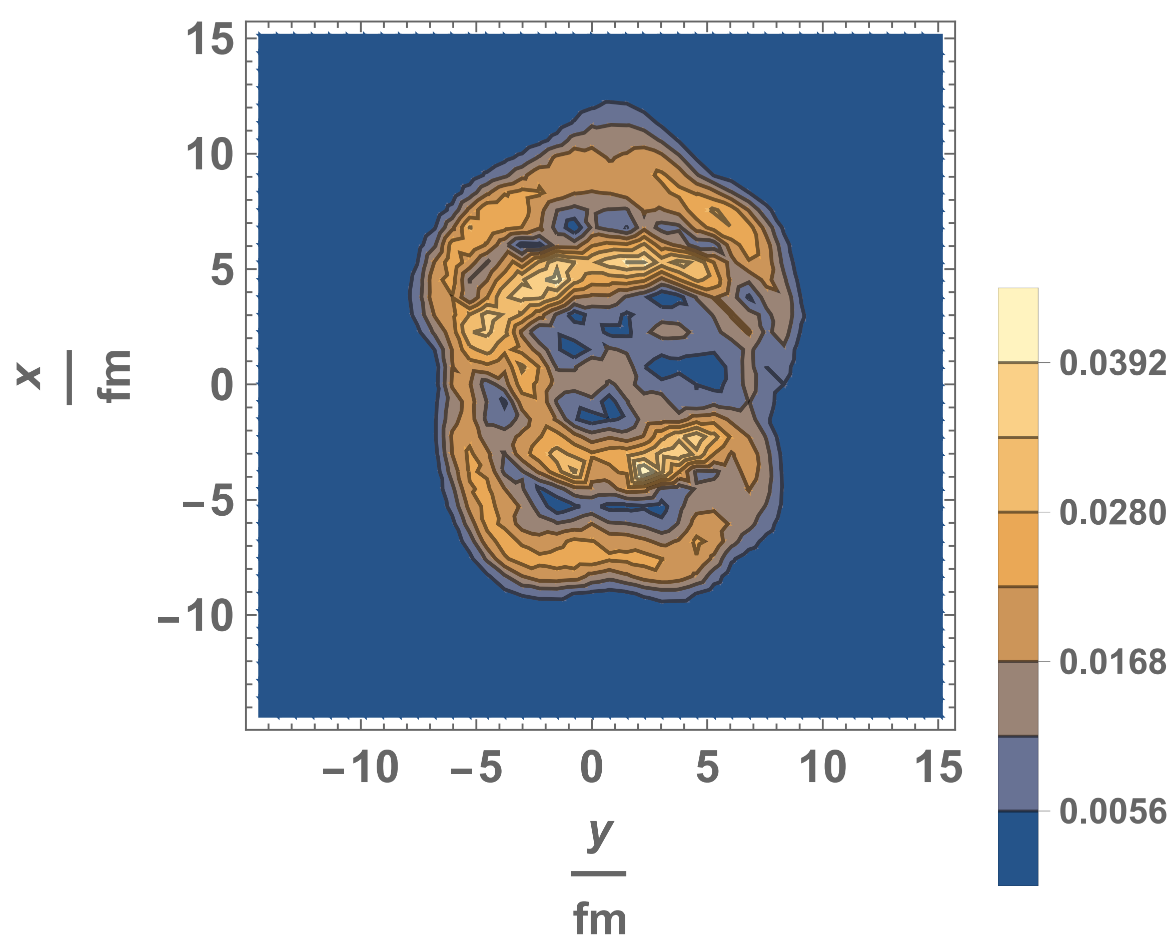
vorticity $|\vec{\omega}|$, $t = 0.1$ fm/c

$$\omega^\alpha \equiv -\frac{1}{2}\,\epsilon^{\alpha\beta\gamma\delta}\,u_\delta\,\partial_\beta u_\gamma$$

$|\vec{\omega}|$



$\xi = 0$



Conclusions & next steps

- Holographic modeling of pre-hydro evolution in heavy ion collisions, with realistic model of nuclear energy density distribution, is feasible.
- Significant transverse flow develops in pre-hydro phase.
- Granularity causes major variations in onset of hydrodynamic behavior.
- Very little angular momentum imparted to hydrodynamized fluid.

- Need to run longer, do multiple realizations, different impact parameters, …
- Would like to add baryon & electric charge density, dynamics E&M.
- Need more people to get involved…