AA 598B Special Topics

Decision-Making & Control for Safe Interactive Autonomy

Instructor: Prof. Karen Leung

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https://faculty.washington.edu/kymleung/aa598/

Announcements

- Guest lecture by Prof. David Fridovich-Keil from UT Austin today
- Proposal feedback
- Homework 2 due (recommended)
- Start your project!

Last time

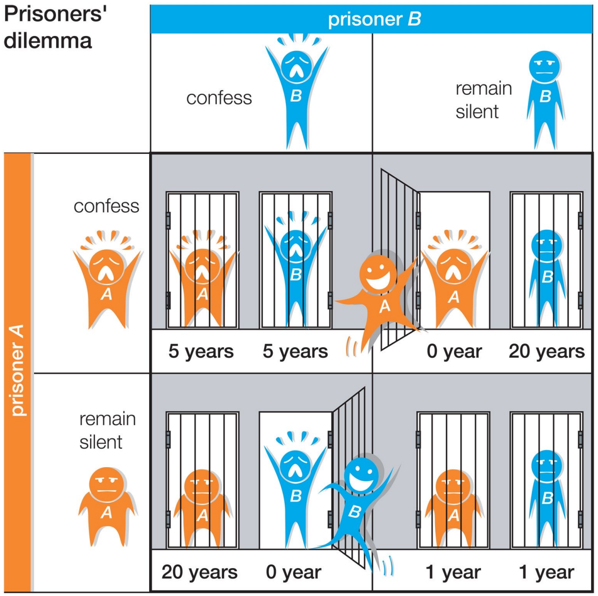
- Socially-aware planning approaches
  - Including the human agent's reward in the robot's reward
  - Social Value Orientation
  - Courtesy / counterfactual reasoning
  - Proactive and legibility
- Methods to solve an optimal control problem
  - Sequential quadratic programming
    - Assumes ability to get (good) gradient information

Today

- Brief introduction to game theory
- Sampling-based methods for planning

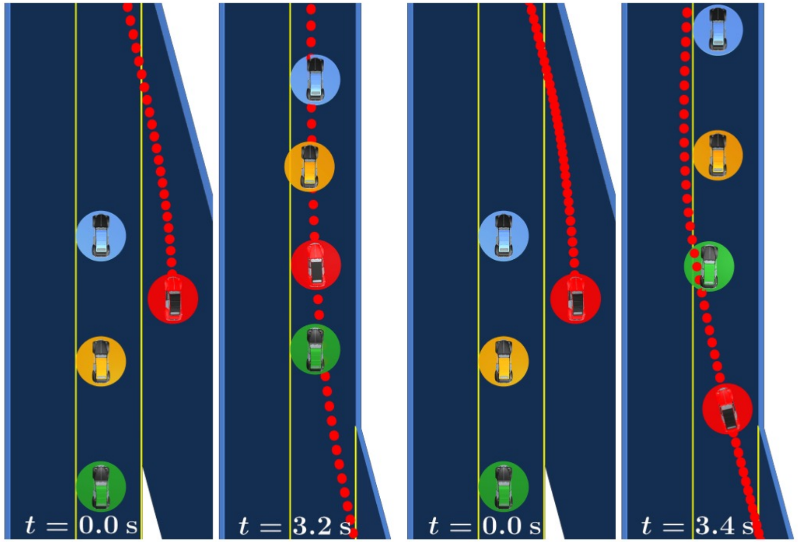
Game theory

"No dynamics"



https://www.britannica.com/science/game-theory/The-von-Neumann-Morgenstern-theory

Dynamic games



ALGAMES: A Fast Augmented Lagrangian Solver for Constrained Dynamic Games
Autonomous Robots (AuRo 2021),
***S. Le Cleac'h**, M. Schwager, Z. Manchester*

Game theory

Definition: A mathematical framework for modeling scenarios in which multiple decision-makers (agents) interact, with each agent's outcome depending not only on its own actions but also on the actions of others.

Relevance: In human-robot interaction, game theory helps model how robots can make decisions while considering the possible actions of human agents.

Problem formulation

$$\begin{aligned} \min_{X,U^1} \quad & J^1(X,U^1) \\ \text{s.t.} \quad & D(X,U) = 0, \\ & C(X,U) \leq 0, \end{aligned} \qquad \cdots \qquad \begin{aligned} \min_{X,U^M} \quad & J^M(X,U^M) \\ \text{s.t.} \quad & D(X,U) = 0, \\ & C(X,U) \leq 0, \end{aligned}$$

Problem formulation

$$\forall i \in [N] \begin{cases} \min_{X^i, U^i} & J^i(\mathbf{X}, U^i; \theta^i) \\ \text{s.t.} & x^i_{t+1} = f^i(x^i_t, u^i_t), \forall t \in [T-1] \\ & x^i_1 = \hat{x}^i_1 \\ & {}^p g^i(X^i, U^i) \geq 0 \\ & {}^s g(\mathbf{X}, \mathbf{U}) \geq 0. \end{cases}$$

Payoff structure

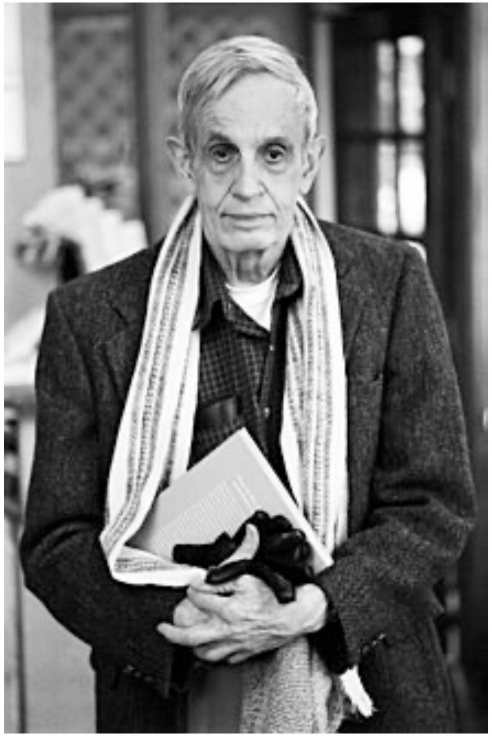
- Zero sum two player game
  - Total payoff always sums to zero
  - One player’s gain is exactly equal to the other player’s loss
  - E.g., tic-tac-toe
- General sum games
  - Payoff does not need to sum to zero
  - No strong sense of win or lose
  - More representative of human-robot interactions(?)

Types of solutions: Nash equilibrium

- At Nash equilibrium, every player is playing optimally given the choices of others,
  - No player has an incentive to deviate from their chosen strategy.

$$J_i(u_i^*, u_{-i}^*) \leq J_i(u_i, u_{-1}^*) \quad \forall\, u_i \in U_i$$

- Other types:
  - Subgame Perfect Equilibrium (Nash over multiple steps)
  - Correlated Equilibrium (follow recommendation from external source)
  - Bayesian Nash Equilibrium (Nash with incomplete information, have beliefs over others)



Wikipedia

Finding Nash equilibria

Generally difficult to find

- Find *local open-loop Nash equilibria* via KKT conditions

$$\begin{aligned} \min_x \quad & f(x) \\ \text{s.t.} \quad & \mathbf{g}(x) \leq 0 \\ & \mathbf{h}(x) = 0 \end{aligned}$$

$$\mathcal{L}(x, \mu, \lambda) = f(x) + \mu^T \mathbf{g}(x) + \lambda^T \mathbf{h}(x)$$

$$\begin{aligned} \nabla_x \mathcal{L}(x^\star, \mu^\star, \lambda^\star) &= 0 \\ \nabla_\mu \mathcal{L}(x^\star, \mu^\star, \lambda^\star) &= 0 \\ \nabla_\lambda \mathcal{L}(x^\star, \mu^\star, \lambda^\star) &= 0 \\ \mu^\star &\geq 0 \end{aligned}$$

KKT conditions

"First-order optimality conditions with constraints"

Inferring costs and objectives

$$\forall i \in [N] \begin{cases} \min_{\mathbf{x},\mathbf{u}^i} \; J^i(\mathbf{u}; x_1) & \text{(1a)} \\ \text{s.t. } x_{t+1} = f_t(x_t, u_t^1, \ldots, u_t^N), \forall t \in [T-1]. & \text{(1b)} \end{cases}$$

KKT conditions

$$\mathbf{G}(\mathbf{x}, \mathbf{u}, \boldsymbol{\lambda}) := \begin{bmatrix} \left. \begin{matrix} \nabla_{\mathbf{x}} J^i + \boldsymbol{\lambda}^{i\top} \nabla_{\mathbf{x}} \mathbf{F}(\mathbf{x}, \mathbf{u}) \\ \nabla_{\mathbf{u}^i} J^i + \boldsymbol{\lambda}^{i\top} \nabla_{\mathbf{u}^i} \mathbf{F}(\mathbf{x}, \mathbf{u}) \end{matrix} \right\} \forall i \in [N] \\ \mathbf{F}(\mathbf{x}, \mathbf{u}) \end{bmatrix} = \mathbf{0}. \quad (5)$$

have data on y.

$$\max_{\theta, \mathbf{x}, \mathbf{u}} \; p(\mathbf{y} \mid \mathbf{x}, \mathbf{u}) \quad \text{(4a)}$$

$$\text{s.t. } (\mathbf{x}, \mathbf{u}) \text{ is an OLNE of } \Gamma(\theta) \quad \text{(4b)}$$

$$(\mathbf{x}, \mathbf{u}) \text{ is dynamically feasible under } f, \quad \text{(4c)}$$

⇕ KKT

$$\max_{\theta, \mathbf{x}, \mathbf{u}, \boldsymbol{\lambda}} \; p(\mathbf{y} \mid \mathbf{x}, \mathbf{u})$$

$$\text{s.t. } \mathbf{G}(\mathbf{x}, \mathbf{u}, \boldsymbol{\lambda}; \theta) = \mathbf{0}.$$

Peters et al 2021