AMATH 584 Autumn Quarter 2020

Homework 5
DUE: Monday, December 7, 2020

I. Eigenvalues and Power Iterations:

(a) Generate a random, symmetric matrix $\mathbf{A}$ which is m by m where $m = 10$. Use the EIGS command in MATLAB (or the equivalent in Python) to give you the *ground truth* eigenvalues and eigenvectors .

(b) Find the largest eigenvalue with the power iteration method. Compare the accuracy of the method as a function of iterations.

(c) Find all ten eigenvalues by Rayleigh Quotient iteration and guessing initial "eigenvectors". Compare the accuracy of the method as a function of iterations and discuss your initial guesses to find all eigenvalue/eigenvector pairs.

(d) Repeat (b) and (d) with a random matrix that is not symmetric. Be sure to plot the eigenvalue in the complex plane.

II. Back to Yale Faces:

Download the data set for CROPPED IMAGES.

(a) Power iterate on the matrix of images to find the dominant eigenvector and eigenvalue. Compare it to the leading order SVD mode.

(b) Use randomized sampling to reproduce the SVD matrices: $\mathbf{U}$, $\mathbf{\Sigma}$ and $\mathbf{V}$.

(c) Compare the randomized modes to the true modes along with the singular value decay as a function of the number of randomized samples.