AMATH 584 Autumn Quarter 2020

Homework 4

DUE: Monday, November 16, 2020

I. Eigenvalues and multiplicity:

(a) Consider the matrix

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Determine its eigenvalues and eigenvectors and the algebraic and geometric multiplicity of each.

(b) Consider the matrix

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

Determine its eigenvalues and eigenvectors and the algebraic and geometric multiplicity of each.

II. Eigen Decomposition:

For each of the following statements, prove that it is true or give an example to show it is false. Here $\mathbf{A} \in \mathbb{C}^{m \times m}$ unless otherwise indicated.

(a) If λ is an eigenvalue of $\mathbf{A}$ and $\mu \in \mathbb{C}$, then $\lambda - \mu$ is an eigenvalue of of $\mathbf{A} - \mu\mathbf{I}$

(b) If $\mathbf{A}$ is real and λ is an eigenvalue of $\mathbf{A}$, then so is $-\lambda$.

(c) If $\mathbf{A}$ is real and λ is an eigenvalue of $\mathbf{A}$, then so is $\bar{\lambda}$ (bar denotes complex conjugate).

(d) If λ is an eigenvalue of $\mathbf{A}$ and $\mathbf{A}$ is nonsingular, then λ^{-1} is an eigenvalue of $\mathbf{A}^{-1}$.

(e) If all the eigenvalues of $\mathbf{A}$ are zero, then $\mathbf{A} = 0$.

(f) If $\mathbf{A}$ is Hermitian and λ is an eigenvalue of $\mathbf{A}$, then $|\lambda|$ is a singular value of $\mathbf{A}$.

(g) If $\mathbf{A}$ is diagonalizable and all its eigenvaleus are equal, then $\mathbf{A}$ is diagonal.

III. Specialty Matrices

Let $\mathbf{A} \in \mathbb{C}^{m \times m}$ by tridiagonal and Hermitian, with all its sub- and super-diagonal entries nonzero. Prove that the eigenvalues of $\mathbf{A}$ are distinct (Hint: Show that for any $\lambda \in \mathbb{C}$, $\mathbf{A} - \lambda\mathbf{I}$ has rank at least $m - 1$).