AMATH 584 Autumn Quarter 2020

Homework 3
DUE: Monday, November 2, 2020

I. QR Decomposition: Download two data sets (ORIGINAL IMAGE and CROPPED IMAGES)

Develop a numerical algorithm that implements the modified Gram-Schmidt orthogonalization procedure. Compare your algorithm to (i) the **qrfactor.m** code that we built in class (you can download it from the third lecture on QR on the website), and (ii) MATLAB's QR algorithm on a variety of matrices to see how well your algorithm works. Be sure to try it on a matrix that is ill-conditions, i.e. $\mathbf{cond}(\mathbf{A}) \gg 1$.

II. Consider the polynomial

$$p(x) = (x-2)^9 = x^9 - 18x^8 + 144x^7 - 672x^6 + 2016x^5 - 4032x^4 + 5376x^3 - 4608x^2 + 2304x - 512$$

(a) Plot the polynomial $p(x)$ for $x \in [1.920, 2.080]$ for step-sizes of $\delta x = 0.001$ using the right-hand side of the expression above
(b) Plot the polynomial again over the same interval using the left-hand side of the expression, i.e. $(x-9)^9$.

III. Consider the conditioning of a matrix.

(a) Construct a random matrix of size $m \times n$ where $m > n$, i.e. use $\mathbf{A}$=$\mathbf{randn}(m, n)$. Study the condition number as a function of the size of the matrix (increase the m and n).
(b) For a fixed m and n, copy the first column of $\mathbf{A}$ and append it as the $(n+1)$th column of $\mathbf{A}$. What is the condition number and determinant of the matrix?
(c) Take the appended $(m+1)$th column and add noise to it, i.e. $\mathbf{a}_{n+1} = \mathbf{a}_{n+1} + \epsilon\ \mathbf{rand}(m, 1)$ and see what happens to the condition number as a function of ϵ.