AMATH 584 Autumn Quarter 2020

Homework 2
DUE: Friday, October 16, 2020

Yale Faces B: Download two data sets (ORIGINAL IMAGE and CROPPED IMAGES)

Your job is to perform an SVD analysis of these data sets. Please start with the cropped images and perform the following analysis.

1. Do an SVD analysis of the images (where each image is reshaped into a column vector and each column is a new image).
2. What is the interpretation of the $\mathbf{U}$, $\mathbf{\Sigma}$ and $\mathbf{V}$ matrices? (Plot the first few reshaped columns of $\mathbf{U}$)
3. What does the singular value spectrum look like and how many modes are necessary for good image reconstructions using the PCA basis? (i.e. what is the rank r of the face space?)
4. compare the difference between the cropped (and aligned) versus uncropped images in terms of singular value decay and reconstruction capabilities.

This is an exploratory homework. So play around with the data and make sure to plot the different things like the modes and singular value spectrum. Good luck, and have fun.

Theorems: Show that for a matrix $\mathbf{A}$

- The nonzero singular values of $\mathbf{A}$ are the square roots of the nonzero eigenvalue of $\mathbf{AA}^*$ or $\mathbf{A}^*\mathbf{A}$
- If $\mathbf{A} = \mathbf{A}^*$, then the singular values are the absolute values of the eigenvalues of $\mathbf{A}$
- Given that the determinant of a matrix $\mathbf{U}$ is unity, show $|\det(\mathbf{A})| = \Pi_{j=1}^{m}\sigma_j$