AMATH 584 Autumn Quarter 2020

Homework 1
DUE: Friday, October 9, 2020

1. Show that if matrix $\mathbf{A}$ is triangular and unitary, then it is diagonal
2. Consider that the matrices $\mathbf{A} \in \mathbb{C}^{n\times n}$ and $\mathbf{B} \in \mathbb{C}^{n\times n}$ are Hermitian (self-adjoint)
    - Prove that all eigenvalues λ_k of $\mathbf{A}$ are real
    - Prove that if $\mathbf{x}_k$ is the kth eigenvector, then eigenvectors with distinct eigenvalues are orthogonal
    - Prove the sum of two Hermitian matrices is Hermitian
    - Prove the inverse of an invertible Hermitian matrix is Hermitian as well
    - Prove the product of two Hermitian matrices is Hermitian if and only if $\mathbf{AB} = \mathbf{BA}$.
3. Consider the matrix $\mathbf{U} \in \mathbb{C}^{n\times m}$ which is unitary
    - Prove that the matrix is diagonalizable
    - Prove that the inverse $\mathbf{U}^{-1} = \mathbf{U}^*$
    - Prove it is isometric with respect to the ℓ_2 norm, i.e. $\|\mathbf{Ux}\| = \|\mathbf{x}\|$.
    - Prove that all eigenvalues have modulus unity