Measurement of ortho-positronium lifetime and $2\gamma$ to $3\gamma$ branching ratio of positronium

Semester Thesis

by

Fukuda Yasutaka, Ikeda Tatsuya, Khaw Kim Siang
Shimozawa Masaaki, Tanaka Shinichiro
Science Faculty, Kyoto University

August 16, 2010

Abstract

The goals of our experiments are to measure the lifetime of ortho-positronium and the ratio of $2\gamma$ to $3\gamma$ of positronium in the air. Parapositronium decays into $2\gamma$ while ortho-positronium decays into $3\gamma$ mainly. We used 5 NaI scintillators to surround the positronium source in order to achieve high acceptance. Difference in acceptance for $2\gamma$ and $3\gamma$ are taken into account by applying correction using Monte Carlo simulation. Our final result is $2\gamma : 3\gamma = 2.26 : 1$ for ratio measurement and $86.21 \pm 3.64$ ns for ortho-positronium lifetime measurement.
List of Figures

1. Two states of positronium .................................. 5
2. Feynman Diagram for para-positronium decaying to $2\gamma$ .... 7
3. Transformation from CM frame(left) to laboratory frame(right) 10
4. Feynman Diagrams for $3\gamma$ annihilation .................... 11
5. Schematic diagram of NaI scintillator ....................... 14
6. Schematic diagram(top) and picture(bottom) of the experimental setup: Radiative Source and Silica Stopper .... 15
7. Using 5 scintillators to surround the silica powder ........ 16
8. Discrete distribution of the signals at Oscilloscope .......... 17
9. Detailed diagram of signal dividing. To prevent the reflection of signals, 50Ω resistors are used. ....................... 18
10. Circuit for measuring the $3\gamma$ and $2\gamma$ decay rate .......... 19
11. The actual experimental setup: front view .................. 20
12. The actual experimental setup: rear view ................... 20
13. Picture of Photo-multiplier tube used with plastic scintillator. 22
14. Schematic diagram of the experiment. Note that a plastic scintillator is inserted in between the lead blocks and silica powder. ................................................................. 22
15. Circuit of the ortho-positronium lifetime measurement ... 24
16. Actual experimental setup for ortho-positronium lifetime measurement .................................................. 25
17. Energy distribution of gamma rays in $2\gamma$ events after loose energy cut .............................................. 26
18. $2\gamma$ events(energy cut of $476keV < e_1 < 534keV, 473keV < e_2 < 535keV$ and $962keV < e_1 + e_2 < 1062keV)$ ........................ 27
19. Energy distribution of gamma rays in $3\gamma$ events (without energy cut) ................................................. 28
20. $3\gamma$ events(energy cut of $e_i, e_j < 475keV$ and $e_i + e_j > 500keV$) ................................................ 29
21. Correlation between arrival time and energy ................ 30
22. Output of the signals ........................................ 31
23. Correlation between arrival time and energy after walk correction .................................................. 32
24. Energy spectrum of each channel ............................. 33
25. Arrival time of each channel ................................ 34
26. Arrival time of each channel after time cut ................. 35
27. Time difference of each channel .............................. 36
28. Fitting of lifetime for each channel .......................... 37
29. Decay of positronium in vacuum .............................. 39
30. Decay of ortho-Ps by through conversion to para-Ps ....... 39
31. Decay of positronium in the air .............................. 39
List of Tables

1 Dimensions of NaI scintillators.......................... 14
2 Setting values for all the apparatus. All attenuators have the same values............................................. 21
3 The setting values for ortho-positronium lifetime measurement 23
4 Decay rate of each channel .................................. 37
1 Introduction

Positronium(Ps), the bound state of an electron ($e^-$) and a positron ($e^+$), is the lightest hydrogen-like atom bound only by the electromagnetic interaction. Since the Ps atom is purely leptonic system and free from the hadronic and weak interaction, it can be described completely by quantum electrodynamics(QED). Therefore, Ps atom is an useful tool for testing QED.

As the electron and positron are equally massive, they rotate around the mutual center of mass. The reduced mass of Ps is $m_e/2$, and causes the Bohr radius to be 106pm and the energy levels to also roughly be half of what they are for the hydrogen atom, which is 6.8eV.

There are mainly two ground states of Ps, namely singlet ($^1S_0$, para-Ps, $S=0$) and triplet ($^3S_1$, ortho-Ps, $S=1$).

![Diagram of Positronium States](image)

Figure 1: Two states of positronium

The difference in energy level of para-Ps and ortho-Ps is $E_{para} - E_{ortho} = 8.4 \times 10^{-4}eV$. Under normal circumstances, this value can be ignored as the formation of Ps atom occurs at a energy higher than this value, and the formation probability is dominated by number of states, $2S+1$. In vacuum, the formation ratio para-Ps:ortho-Ps is 1:3, but in favor of para-Ps when it is exposed to the air due to the pick-off and the spin exchange interaction. Ps annihilates in the material to gamma rays. Their lifetimes are respectively 125 ps(para-Ps) and 142 ns(ortho-Ps) in vacuum. However, when ortho-Ps is exposed to the air, the value changes as the interactions stated will reduced its lifetime.

In this experiment, We investigate the formation ratio of para-Ps and ortho-Ps, and also the lifetime of ortho-Ps in the air.

2 Theory

2.1 Selection Rule

In this section, the mechanism on why para-Ps decays into $2\gamma$, $4\gamma$..., and ortho-Ps decays into $3\gamma$, $5\gamma$... is described. ($c = h = 1$ natural units are
used unless stated.)

By applying the change conjugate operator $C$ on the current vector, we have:

$$ (\hat{p}, \hat{j}) \rightarrow (-\hat{p}, -\hat{j}) $$

(1)

On the other hand, the interaction of charged particle and electromagnetic (EM) field is given by the product of $\hat{A}$ and $\hat{j}$:

$$ \Delta H = \int \frac{d^3 p}{2\pi^3} e A^\mu j_\mu $$

(2)

Since the EM interaction is invariant under the change conjugation transformation, it follows that:

$$ (\hat{\Phi}, \hat{A}) \rightarrow (-\hat{\Phi}, -\hat{A}) $$

(3)

Therefore, we derive for $n$-photons:

$$ C|\gamma\rangle = (-1)^n|\gamma\rangle $$

(4)

When the Ps atom has total orbital momentum $L$ and total spin $S$, it follows that:

$$ Spin \rightarrow (-1)^{S+1} $$

$$ \gamma^m_L \rightarrow (-1)^L $$

(5)

(6)

$$ anti - commutation \rightarrow (-1)^1 $$

(7)

when the change conjugate transformation is applied. From above, we have:

$$ C|Ps\rangle = (-1)^{L+S}|Ps\rangle $$

(8)

Since the system is invariant under change conjugate transformation,

$$ (-1)^n = (-1)^{L+S}|Ps\rangle $$

(9)

For para-Ps, $L = 0, S = 0$, we have even number of photons, and for ortho-Ps, $L = 0, S = 1$, we have odd number of photons. Therefore, we can differentiate ortho-Ps from para-Ps experimentally by looking at the number of photons detected.
2.2 Branching Ratio

In this experiment, we will compare the values of the formation ratio para-Ps:ortho-Ps and the lifetime of ortho-Ps in the air to those of in vacuum. When both electron and positron are not polarized, by taking the products of spin vectors, there are four possibilities:

\[
|S = 1, S_z = 1\rangle = |\uparrow\rangle |\uparrow\rangle, \quad (10)
\]
\[
|S = 1, S_z = 0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle |\downarrow\rangle), \quad (11)
\]
\[
|S = 1, S_z = -1\rangle = |\uparrow\rangle |\uparrow\rangle, \quad (12)
\]
\[
|S = 0, S_z = 0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle |\downarrow\rangle), \quad (13)
\]

By calculating the self product of the spin vectors, we get the probability of each state and thus

\[
\text{para} - \text{Ps} : \text{ortho} - \text{Ps} = 1 : 3 \quad (14)
\]

2.3 Lifetime

Lifetime of the positronium is given by,

\[
\text{Lifetime} = (\text{DecayRate})^{-1} = (\text{ScatteringCrossSection} \times \text{FluxDensity})^{-1} \quad (15)
\]

Now let us calculate the scattering cross section. Figure 2 is the Feynman Diagram of the decay of para positronium to \(2\gamma\). Its M matrix is given by

\[
\begin{align*}
 iM &= \varepsilon_\mu^\ast (-ie\gamma^\mu) \bar{\psi}(p_2) \frac{-i(p_1 - k_1 + m)}{(p_1 - k_1)^2 - m^2} \varepsilon_\nu(-ie\gamma^\nu)u(p_1) \\
 &\quad + \varepsilon_\mu(-ie\gamma^\mu) \bar{\psi}(p_2) \frac{-i(p_1 - k_2 + m)}{(p_1 - k_2)^2 - m^2} \varepsilon_\nu^\ast(-ie\gamma^\nu)u(p_1) \\
 &= ie^2 \varepsilon_\mu \varepsilon_\nu \bar{\psi}(p_2) \left( \gamma^\nu \frac{k_2\gamma^\mu - 2\gamma^\mu \gamma^\nu}{2p_1 k_1} + \gamma^\nu \frac{k_1\gamma^\mu - 2\gamma^\mu \gamma^\nu}{2p_1 k_2} \right) u(p_1) \quad (16)
\end{align*}
\]

Figure 2: Feynman Diagram for para-positronium decaying to \(2\gamma\)
In the case of non-polarization, the cross section can be calculated by taking the spin average of \(|M|^2\), then by sum of every helicity state:

\[ \frac{1}{4} \sum_{e^e} \sum_{e^e} |M|^2 \]

\[ = \frac{e^2}{4} \sum_{e^e} \sum_{e^e} \bar{e}_i e_i e_\rho e_\rho \delta(p_2)(\frac{\gamma^\nu \cdot k_2 y^\mu - 2\gamma^\mu p^\nu}{2p_1 k_1} + \frac{\gamma^\nu \cdot k_2 y^\mu - 2\gamma^\mu p^\nu}{2p_1 k_2})u(p_1) \]

\[ \bar{u}(p_1)(\frac{\gamma^\rho \cdot k_2 y^\rho - 2\gamma^\rho p^\rho}{2p_1 k_1} + \frac{\gamma^\rho \cdot k_2 y^\rho - 2\gamma^\rho p^\rho}{2p_1 k_2})v(p_2) \]  

(17)

Using this transformation,

\[ \sum_{\text{polarization}} e_i e_\rho \rightarrow g_{\mu \rho} \]  

(18)

it becomes,

\[ = \frac{e^2}{4} g_{\mu \rho} \bar{g}_{\nu \sigma} \sum_{p_1} \sum_{p_1} \bar{u}(p_2)\left(\frac{\gamma^\nu \cdot k_2 y^\mu - 2\gamma^\mu p^\nu}{2p_1 k_1} + \frac{\gamma^\nu \cdot k_2 y^\mu - 2\gamma^\mu p^\nu}{2p_1 k_2}\right)u(p_1) \]

\[ \bar{u}(p_1)(\frac{\gamma^\rho \cdot k_2 y^\rho - 2\gamma^\rho p^\rho}{2p_1 k_1} + \frac{\gamma^\rho \cdot k_2 y^\rho - 2\gamma^\rho p^\rho}{2p_1 k_2})v(p_2) \]  

(19)

Additionally, from the relations below,

\[ \sum_s u(p, s)u(\bar{p}, s) = \not{p} + m \]  

(20)

\[ \sum_s v(p, s)v(\bar{p}, s) = \not{p} - m \]  

(21)

put them into equation(19),

\[ = \frac{e^2}{4} g_{\mu \rho} \bar{g}_{\nu \sigma}[(\not{p}_2 - m)_a(\frac{\gamma^\nu \cdot k_1 y^\mu - 2\gamma^\mu p^\nu}{2p_1 k_1} + \frac{\gamma^\nu \cdot k_2 y^\mu - 2\gamma^\mu p^\nu}{2p_1 k_2})_a b \]

\[ (\not{p}_1 + m)_{bc}(\frac{\gamma^\mu \cdot k_1 y^\mu - 2\gamma^\mu p^\mu}{2p_1 k_1} + \frac{\gamma^\mu \cdot k_2 y^\mu - 2\gamma^\mu p^\mu}{2p_1 k_2})_{cd} \]

\[ = \frac{e^2}{4} g_{\mu \rho} \bar{g}_{\nu \sigma} tr[(\not{p}_2 - m)(\frac{\gamma^\nu \cdot k_1 y^\mu - 2\gamma^\mu p^\nu}{2p_1 k_1} + \frac{\gamma^\nu \cdot k_2 y^\mu - 2\gamma^\mu p^\nu}{2p_1 k_2})(\not{p}_1 + m) \]

\[ (\frac{\gamma^\rho \cdot k_1 y^\rho - 2\gamma^\rho p^\rho}{2p_1 k_1} + \frac{\gamma^\rho \cdot k_2 y^\rho - 2\gamma^\rho p^\rho}{2p_1 k_2})] \]

\[ = \frac{e^2}{4} \left[ \frac{(A)}{4(p_1 k_1)^2} + \frac{(B)}{4(p_1 k_1)(p_1 k_2)} + \frac{(C)}{4(p_1 k_2)(p_1 k_1)} + \frac{(D)}{4(p_1 k_2)^2} \right] \]  

(22)
Expanding the terms, we have

\[
(A) = \text{tr} \left[ (p_2 - m)(\gamma^\mu k_1 \gamma^\nu - 2\gamma^\mu p_1^\nu)(p_1 + m)(\gamma_\mu k_1 \gamma_\nu - 2\gamma_\mu p_1^\nu) \right] = \text{tr}[6 \gamma^\mu p_1^\nu p_1 \gamma_\mu k_1 \gamma_\nu - 2 \gamma^\mu p_1^\nu p_1 \gamma_\mu k_1 \gamma_\nu - 6 \gamma^\mu p_1^\nu p_1 \gamma_\mu k_1 \gamma_\nu - 2 \gamma^\mu p_1^\nu p_1 \gamma_\mu k_1 \gamma_\nu]
\]

Similarly,

\[
(B) = \text{tr} \left[ (p_2 - m)(\gamma^\mu k_1 \gamma^\nu - 2\gamma^\mu p_1^\nu)(p_1 + m)(\gamma_\mu k_1 \gamma_\nu - 2\gamma_\mu p_1^\nu) \right] = \text{tr}[6 \gamma^\mu p_1^\nu p_1 \gamma_\mu k_1 \gamma_\nu - 2 \gamma^\mu p_1^\nu p_1 \gamma_\mu k_1 \gamma_\nu - 6 \gamma^\mu p_1^\nu p_1 \gamma_\mu k_1 \gamma_\nu - 2 \gamma^\mu p_1^\nu p_1 \gamma_\mu k_1 \gamma_\nu]
\]

By introducing the Mandelstam variables here,

\[
s = (p_1 + p_2)^2 = (k_1 + k_2)^2 = 2m^2 + 2p_1 \cdot p_2 = 2k_1 \cdot k
\]

\[
t = (p_1 - k_1)^2 = (p_2 - k_2)^2 = m^2 - 2p_1 \cdot k_1 = m^2 - 2p_2 \cdot k_2
\]

\[
u = (p_1 - k_2)^2 = (p_2 - k_1)^2 = m^2 - 2p_1 \cdot k_2 = m^2 - 2p_2 \cdot k_1
\]

equation (22) can be simplify further,

\[
(A) = 16 \left[ -2m^4 + m^2(m^2 - t) + \frac{1}{2}(m^2 - t)(m^2 - u) \right]
\]

\[
(B) = 8 \left[ -4m^4 + m^2(m^2 - t) + m^2(m^2 - u) \right] = (C)
\]

\[
(D) = 16 \left[ -2m^4 + m^2(m^2 - u) + \frac{1}{2}(m^2 - u)(m^2 - t) \right]
\]

Put everything together,

\[
\frac{1}{4} \sum_{e,e'} \sum_{\text{polarizations}} |M|^2 = \frac{2e^2}{p_1 k_2} \left[ \frac{p_1 k_1}{p_1 k_2} + \frac{p_1 k_1}{p_1 k_2} + 2m^2 \left( \frac{1}{p_1 k_1} + \frac{1}{p_1 k_2} \right) - m^4 \left( \frac{1}{p_1 k_1} + \frac{1}{p_1 k_2} \right)^2 \right]
\]

The next step will be calculating the differential scattering cross section and scattering cross section at CM frame and then transform them into laboratory frame.

\[
\left( \frac{d\sigma}{d\Omega} \right)_{CM} = \frac{1}{2E_A2E_B|v_A - v_B| (2\pi)^24E_{CM}} |M(p_A,p_B \rightarrow p_1,p_2)|^2
\]
By letting
\[ p_1 = (E, 0, 0, p^z) \]  
\[ p_2 = (E, 0, 0, -p^z) \]  
\[ k_1 = (E, E \sin \theta, 0, E \cos \theta) \]  
\[ k_2 = (E, -E \sin \theta, 0, -E \cos \theta) \]
and substitute them into equation (30),
\[
\left( \frac{d\sigma}{d \cos \theta} \right)_{CM} = \frac{2\pi\alpha^2}{s} \left( \frac{E}{p} \right) \left[ \frac{E^2 + p^2 \cos^2 \theta}{m^2 + p^2 \sin^2 \theta} + \frac{2m^2}{m^2 + p^2 \sin^2 \theta} - \frac{2m^4}{(m^2 + p^2 \sin^2 \theta)^2} \right]
\]  
\[
\sigma_{CM} = \int_0^1 d(\cos \theta) \frac{d\sigma}{d \cos \theta}
\]
\[
= \frac{2\pi\alpha^2}{s} \left[ -1 + E^2 + \frac{m^2}{E^2} + \frac{2m^2}{E^2} \left( \frac{E}{p} + \frac{E^2 + 2m^2}{Ep} + \frac{m^4}{E^3p} \right) \log \left( \frac{E + p}{E - p} \right) \right]
\]  
By looking it at the stationary frame of $e^-$ (laboratory frame), From the diagrams above, the 4-momenta of positron and electron are given by
\[ p_{e^+} = (E, p) \]  
\[ p_{e^-} = (E, 0) \]
and the cross section at laboratory frame is
\[
\sigma_{LAB} = \pi \left( \frac{\alpha}{m} \right)^2 \frac{1}{\gamma + 1} \left[ -\frac{\gamma + 3}{\sqrt{\gamma^2 - 1}} + \frac{\gamma^2 + 4\gamma + 1}{\gamma^2 - 1} \log \left( \gamma + \sqrt{\gamma^2 - 1} \right) \right]
\]
\[ \gamma = \sqrt{1 - \left(\frac{v}{c}\right)^2} \] 
As the positron is slowed down to non-relativistic velocity inside the silica powder, we put \( \gamma \approx 1 \),

\[ \sigma_{2\gamma} = \sigma_{\text{LAB}} \approx \frac{\pi}{v} \left( \frac{\alpha}{m} \right)^2 \quad (40) \]

Cross section of the annihilation into \( 3\gamma \) can also be calculated using the similar way. Feynman Diagrams. M matrix and cross section are as below.

Figure 4: Feynman Diagrams for \( 3\gamma \) annihilation

\[
iM = \partial(p_2)\epsilon^\mu_{\nu}(\gamma_{\mu}) - i(\hat{p}_1 - \hat{k}_1 + m)(p_1 - k_1)^2 - m^2 e^\nu_{\nu}(\gamma_{\nu}) - i(\hat{p}_2 - \hat{k}_3 + m)(p_2 - k_3)^2 - m^2 e^\nu_{\nu}(\gamma_{\nu})u(p_1) \\
+ \partial(p_2)\epsilon^\mu_{\nu}(\gamma_{\mu}) - i(\hat{p}_1 - \hat{k}_1 + m)(p_1 - k_1)^2 - m^2 e^\nu_{\nu}(\gamma_{\nu}) - i(\hat{p}_2 - \hat{k}_2 + m)(p_2 - k_2)^2 - m^2 e^\nu_{\nu}(\gamma_{\nu})u(p_1) \\
+ \partial(p_2)\epsilon^\mu_{\nu}(\gamma_{\mu}) - i(\hat{p}_1 - \hat{k}_3 + m)(p_1 - k_3)^2 - m^2 e^\nu_{\nu}(\gamma_{\nu}) - i(\hat{p}_2 - \hat{k}_1 + m)(p_2 - k_1)^2 - m^2 e^\nu_{\nu}(\gamma_{\nu})u(p_1) \\
+ \partial(p_2)\epsilon^\mu_{\nu}(\gamma_{\mu}) - i(\hat{p}_1 - \hat{k}_3 + m)(p_1 - k_3)^2 - m^2 e^\nu_{\nu}(\gamma_{\nu}) - i(\hat{p}_2 - \hat{k}_2 + m)(p_2 - k_2)^2 - m^2 e^\nu_{\nu}(\gamma_{\nu})u(p_1) \\
+ \partial(p_2)\epsilon^\mu_{\nu}(\gamma_{\mu}) - i(\hat{p}_1 - \hat{k}_2 + m)(p_1 - k_2)^2 - m^2 e^\nu_{\nu}(\gamma_{\nu}) - i(\hat{p}_2 - \hat{k}_1 + m)(p_2 - k_1)^2 - m^2 e^\nu_{\nu}(\gamma_{\nu})u(p_1) \\
+ \partial(p_2)\epsilon^\mu_{\nu}(\gamma_{\mu}) - i(\hat{p}_1 - \hat{k}_2 + m)(p_1 - k_2)^2 - m^2 e^\nu_{\nu}(\gamma_{\nu}) - i(\hat{p}_2 - \hat{k}_1 + m)(p_2 - k_1)^2 - m^2 e^\nu_{\nu}(\gamma_{\nu})u(p_1) \\
(41)\]
Even though the cases for $4\gamma$, $5\gamma$ or above can be calculated using the same way, the increasing of each vertex means the reduction of cross section by a factor of $\alpha^2$. Therefore, higher orders are neglected in our study.

The flux density can be calculated from the ground state wave function of positronium which is

$$\Psi(r) = \frac{1}{\sqrt{\pi a^3}} \exp^{-\frac{r}{a}}$$

(43)

Here, $a = \frac{2}{m a^2} = 2a_0$ is the Bohr radius that we have derived above. As the ratio of $\text{para} – Ps : \text{ortho} – Ps$ is $1 : 3$, we can get

$$\Gamma_{\text{para} – Ps} \approx 4\Gamma_{2\gamma} = 4\sigma_{2\gamma}v|\Psi(0)|^2 = \frac{1}{2} m a^5 = 0.805 \times 10^{10} \text{[s}^{-1}]$$

(44)

$$\tau_{\text{para} – Ps} = 1.23 \times 10^{-10} \text{[s]}$$

(45)

Similarly,

$$\Gamma_{\text{ortho} – Ps} \approx \frac{4}{3}\Gamma_{3\gamma} = \frac{2(\pi^2 - 9)}{9\pi} m a^6 = 0.723 \times 10^7 \text{[s}^{-1}]$$

(46)

$$\tau_{\text{ortho} – Ps} = 1.4 \times 10^{-7} \text{[s]}$$

(47)

When the positronium atom is exposed to the air, as we have discussed, some of the $\text{ortho} – Ps$ will changed into $\text{para} – Ps$ by processes such as spin exchange. Thus, the $1 : 3$ ratio will be changed as well as the lifetime of $\text{ortho} – Ps$. 

---

12
3 Methods

3.1 Measurement of the 2γ and 3γ annihilation rates of Positronium

3.1.1 Brief explanation of the experiment

In this experiment, we use $^{22}$Na as the positron source and silica powder as the stopper. Positronium will be created by the low velocity positron and the free electron inside the powder. In a very short time, the positronium will annihilate and gamma rays will be released. Our aim is to measure the ratio of $3\gamma$ to $2\gamma$. Five NaI scintillators are used to detect the gamma rays. The candidate for $3\gamma$ event is when there are signals coming out from 3 of the scintillators, and $2\gamma$ event is when there are signals from 2 of the scintillators. Monte Carlo simulation is used to calculate the acceptance of $3\gamma$ and $2\gamma$ events and then applied to our result.

3.1.2 Experimental setup

Apparatus used in this experiment are as below:

- $^{22}$Na Radioactive source of $\beta^+$.  
- NaI scintillator For the detection of gamma rays. Consists of NaI crystal and Photo-multiplier tube (PMT).
- Silica powder Material with small value of work function which ease the electron to be free. Used as the positron stopper.
- Vacuum container As the container for $^{22}$Na, lead and silica powder. This experiment was supposed to be done under vacuum condition but was changed due to some technical problems.
- High voltage Voltage supply for the PMT. Abbreviate as HV from now on.
- Amplifier To amplify the signals from PMT. Abbreviate as AMP.
- Discriminator Output a digital signal if the input analog signal from PMT has value greater than a certain threshold. Abbreviate as Disc.
- Coincidence To decide whether the signals are coming out at the same time, at certain time window. Abbreviate as Coin.
- Divider Can be used to divide or sum up the signals. Abbreviate as Div.
- Fixed delay Used to delay the timing of the signals. Abbreviate as Delay.
- Analog to digital converter Measure the total charge of a signal which is proportional to the gamma ray energy. Abbreviate as ADC.
- Time to digital converter Measure the time difference between 2 signals. Abbreviate as TDC.
**Gate generator** Used to generate the time gate or time window for the ADC.

**Attenuator** Used to modify the amplitude of the signal. Abbreviate as Atten.

**Scaler** Counting the frequency of the digital signals.

**Oscilloscope** Used to confirm the analog signal coming from PMT.

**Lead block** Shielding from the other radiations.

![Figure 5: Schematic diagram of NaI scintillator](image)

<table>
<thead>
<tr>
<th></th>
<th>a [cm]</th>
<th>b [cm]</th>
<th>c [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>scintillator 1</td>
<td>5.60</td>
<td>5.55</td>
<td>16.93</td>
</tr>
<tr>
<td>scintillator 2</td>
<td>5.60</td>
<td>5.55</td>
<td>16.89</td>
</tr>
<tr>
<td>scintillator 3</td>
<td>5.58</td>
<td>5.55</td>
<td>16.94</td>
</tr>
<tr>
<td>scintillator 4</td>
<td>5.59</td>
<td>5.55</td>
<td>16.95</td>
</tr>
<tr>
<td>scintillator 5</td>
<td>5.55</td>
<td>5.49</td>
<td>16.94</td>
</tr>
</tbody>
</table>

Table 1: Dimensions of NaI scintillators.

### 3.1.3 Concept of the measurement for the $3\gamma$ to $2\gamma$ ratio experiment

In this section, the trigger of the event is discussed.
Figure 6: Schematic diagram(top) and picture(bottom) of the experimental setup: Radiative Source and Silica Stopper
Figure 7: Using 5 scintillators to surround the silica powder
Signals from the 5 scintillators are converted into digital signals by using the discriminator module. Then, divider is used to get the sum of these signals. By looking at the sum by using oscilloscope (figure 8), we can see that the distribution of the height is discrete, i.e. 1 small box represents 1 gamma ray signal. Hence, we can select > 2 gamma, > 3 gamma, > 4 gamma or > 5 gamma event by discriminating it once more using certain threshold value. However, there is a voltage limit on the signal divider module. So we used attenuators to reduce the height of the signals before input them into divider (figure 9). The complete circuit is shown in figure 10.

3.1.4 Experimental setup

In this section, we will discuss how the value of each apparatus is set.

High Voltage  By looking at the raw signal coming from PMT, the HV is set so that 1275keV gamma rays from $^{22}$Na are visible near the full ADC range. This is for calibration use.

Octal Discriminator There are 2 parameters, threshold and width. Threshold is chosen so that signals from 3 gamma decays are visible, but be able to reject low energy gamma rays. Width is chosen so that signals from different PMTs are able to overlap each other. This is to ensure coincidence works and can be used as the trigger.
Attenuator The amplitude of the digital signals from discriminators are reduced so that its sum value is below the working range of divider.

ADC and Gate generator The gate width of ADC is chosen by looking at the raw signal at oscilloscope so that the wave shape is contained inside the gate.

HD Discriminator The threshold is decided depending on ≥ 2gamma measurement or ≥ 3gamma measurement.
Figure 10: Circuit for measuring the $3\gamma$ and $2\gamma$ decay rate.
Figure 11: The actual experimental setup: front view

Figure 12: The actual experimental setup: rear view
### Table 2: Setting values for all the apparatus. All attenuators have the same values.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>HV ch1</td>
<td>-923 V</td>
</tr>
<tr>
<td>HV ch2</td>
<td>-859 V</td>
</tr>
<tr>
<td>HV ch3</td>
<td>-1030 V</td>
</tr>
<tr>
<td>HV ch4</td>
<td>-891 V</td>
</tr>
<tr>
<td>HV ch5</td>
<td>-989 V</td>
</tr>
<tr>
<td>threshold1</td>
<td>12.3 mV</td>
</tr>
<tr>
<td>width1</td>
<td>142 ns</td>
</tr>
<tr>
<td>threshold2</td>
<td>39.8 mV (for 3γ rate), 23.5 mV (for 2γ rate)</td>
</tr>
<tr>
<td>width2</td>
<td>123 ns</td>
</tr>
<tr>
<td>gate</td>
<td>500 ns</td>
</tr>
<tr>
<td>atten</td>
<td>15 dB</td>
</tr>
</tbody>
</table>

### 3.2 Measurement of ortho-positronium lifetime

#### 3.2.1 Brief explanation of the experiment

In this experiment, we measure the time difference between the injection of positron and 3 gamma decay, which is the lifetime of ortho – Ps. Four scintillators are used and additionally a plastic scintillator with PMT are used to tag the positron. We record the events which are tagged by the plastic scintillator and with at least 2 signals out of 4 scintillators. This is to measure the rate of 2 gamma decay from para – ps as well.

#### 3.2.2 Apparatus

Most of the apparatus are the same with the $3\gamma : 2\gamma$ ratio experiment.

**Plastic scintillator** Material that will release weak UV light when charged particle passes through it. Used to tag the positron in this experiment.

#### 3.2.3 Concept of the measurement

In this section, We will discuss on how to measure the lifetime of ortho – Ps.

As shown in the theoretical part, ortho-Ps can only decay into odd number of gamma rays. However, when it is exposed to the air, by interacting with the surrounding molecules(electrons), it is converted into para – Ps by
Figure 13: Picture of Photo-multiplier tube used with plastic scintillator.

Figure 14: Schematic diagram of the experiment. Note that a plastic scintillator is inserted in between the lead blocks and silica powder.
pick off and spin conversion processes. Since the *para* – *Ps* has very short lifetime (125 ps), promptly it decays into 2 gamma rays. Hence, the long lifetime component of *ortho* – *Ps* is disappearing and its lifetime looks shorten. In order to measure the lifetime of *ortho* – *Ps*, we use a plastic scintillator to tag the positron. Since the positron will be stopped inside the silica powder and form the positronium atom almost instantaneously, the tagged time is considered as the starting time. Then, the gamma rays are detected using NaI scintillators, where the arrival times corresponding to the lifetime of the positronium. By taking enough statistics, we can calculate the average lifetime from the gradient of number versus time plot. Although *ortho* – *Ps* decays only into 3γ, 2γ or above threshold is applied for this experiment. This is for calibration usage as 3γ event’s spectrum is continuous and there will no reference point for energy calibration. Electronic circuit of this experiment is shown in figure 15.

### 3.2.4 Setting values of the apparatus

<table>
<thead>
<tr>
<th>Setting</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>HV ch1</td>
<td>-914 V</td>
</tr>
<tr>
<td>HV ch2</td>
<td>-859 V</td>
</tr>
<tr>
<td>HV ch3</td>
<td>-891 V</td>
</tr>
<tr>
<td>HV ch4</td>
<td>-989 V</td>
</tr>
<tr>
<td>HV plastic</td>
<td>-1000 V</td>
</tr>
<tr>
<td>threshold 1</td>
<td>12.3 mV</td>
</tr>
<tr>
<td>width 1</td>
<td>142 ns</td>
</tr>
<tr>
<td>threshold 2</td>
<td>49.9 mV</td>
</tr>
<tr>
<td>width 2</td>
<td>123 ns</td>
</tr>
<tr>
<td>gate</td>
<td>500 ns</td>
</tr>
<tr>
<td>atten</td>
<td>15 dB</td>
</tr>
</tbody>
</table>

Table 3: The setting values for ortho-positronium lifetime measurement
Figure 15: Circuit of the ortho-positronium lifetime measurement
Figure 16: Actual experimental setup for ortho-positronium lifetime measurement
4 Results

4.1 Measurement of $3\gamma$ to $2\gamma$ ratio

A brief description of notation is given here.

$e_1$ Energy of the 1st gamma ray after calibration
$e_2$ Energy of the 2nd gamma ray after calibration
$e_3$ Energy of the 3rd gamma ray after calibration
$t_1$ Arrival time of 1st gamma ray after calibration
$t_2$ Arrival time of 2nd gamma ray after calibration
$t_3$ Arrival time of 3rd gamma ray after calibration

Entries Number of total events

4.1.1 Result of Run 1: At least 2 gamma rays

The energy distribution of each gamma ray after loose energy cut($400\text{keV} < e_1, e_2 < 600\text{keV}$) is shown at figure 17. Total initial event is 100,000, reduced to 40201 after loose energy cut.

The energy distribution of each gamma ray after tight energy cut($476\text{keV} < e_1 < 534\text{keV}$ and $473\text{keV} < e_2 < 535\text{keV}$) is shown at figure 18. Note that the
cut values are different due to the difference in resolutions of both scintillator and PMT. The number is now further reduced to 7524. Plot of $e_1 + e_2$ is also shown in the same figure. Furthermore, the time difference $t_1 - t_2$ is shown as well.

Distribution of gamma ray energy for 3$\gamma$ events before applying energy cuts. The total event is 75847.

However, we realised that energy cuts on 3$\gamma$ events will significantly reduce the statistics. So we run another 3$\gamma$ experiment and compare the number of 3$\gamma$ without energy cut and 3$\gamma$ with energy cut.

4.1.2 Result of Run 2: At least 3 gamma rays

As described earlier, due to the lack of statistics, we performed this experiment and recorded 1,500,000 new events. Energy cuts of $e_1, e_2, e_3 < 475$keV
Figure 19: Energy distribution of gamma rays in $3\gamma$ events (without energy cut)
$ei + ej > 500\text{keV}$ are applied and the energy of each gamma ray and total energy ($e_0 = e_1 + e_2 + e_3$) are plotted (figure 20).

By applying energy cut of $922\text{keV} < e_0 < 1122\text{keV}$, we get the number of $3\gamma$ events = 14230.

### 4.2 Ratio of 2\(\gamma\) to 3\(\gamma\) of positronium decay

From 1,500,000 events for at least 2\(\gamma\),
number of events with 2\(\gamma\) is 7,524.
number of events with 3\(\gamma\)(without energy cut) is 75,847.

From 1,500,000 events for at least 3\(\gamma\),
number of events with 3\(\gamma\)(without energy cut) is 1,412,464.
number of events with 3\(\gamma\)(energy cut of $922\text{keV} < e_0 < 1122\text{keV}$) is 14,230.

Using figures above, $\text{BR}(e+e\rightarrow 2\gamma/e+e\rightarrow 3\gamma) = \left(\frac{7524}{75847}\right)\left(\frac{1412464}{14230}\right) = 9.85$.
From the Monte Carlo Simulation, the acceptance of $e+e\rightarrow 2\gamma$ and $e+e\rightarrow 3\gamma$ is 33.489% and 7.769% respectively. Applying the correction factor from simulation, we get $\text{BR}(e+e\rightarrow 2\gamma/e+e\rightarrow 3\gamma) = \left(\frac{7524}{75847}\right)\left(\frac{1412464}{14230}\right)\left(\frac{7669}{33489}\right) = 2.26$. 

Figure 20: $3\gamma$ events(energy cut of $ei, ej < 475\text{keV}$ and $ei + ej > 500\text{keV}$)
Figure 21: Correlation between arrival time and energy

This value is as our expectation as the ortho – $Ps$ is converted to para – $Ps$ mostly through interactions and this enhanced the rate of 2gamma decay.

### 4.3 Lifetime of ortho positronium atom

Before measuring the lifetime, we need to do a correction which is called walk correction to the raw data. From figure 21, it is clear that there is a correlation between TDC channel and ADC channel. The main reason is the difference in arrival time for signal with higher energy and lower energy as shown in figure 22. Ideally, the arrival time should have no dependency on the energy.

By using fitting program, we can make the correction. As the result, by letting $T$ as the corrected time, $t$ as the original time and $e$ as the energy, we
have,

\[ T = t + \sqrt{\frac{e - 152.069}{2.382}} - 12.826 \]  \hspace{1cm} (48)

Correlation plot after walk correction is shown in figure 23.

Energy spectrum of each channel is shown in figure 24. Note that there is a peak around 511 keV. This is due to mainly para-Ps -> 2\gamma events and its scattering. Energy cut of 150keV < e1, e2, e3, e4 < 460keV is applied for determining the lifetime.

Time spectrum of each channel is shown in figure 25. Note that there is a peak around 1000 ns. This is due to the overflow of the signal, which means the arrival of stop signal to TDC is later than the maximum range of it. Arrival time \( \leq 500ns \) is applied and the spectrum is shown in figure 26.

By subtracting the arrival time of plastic scintillator, \( t_5 \) from each channel, \( t_1-t_4 \), we can calculate the lifetime of positronium. The plot is shown in figure 27.

Notice that there is a peak at each channel. This is due to the prompt annihilation of positronium. Fitting is done at the position where the gradient is nearly constant. We fixed the end time as 175 ns and varying the start time from 25 ns to 100 ns to get the average value of the decay rate. The inverse of decay rate is lifetime.
Figure 23: Correlation between arrival time and energy after walk correction
Figure 24: Energy spectrum of each channel
Figure 25: Arrival time of each channel
Figure 26: Arrival time of each channel after time cut
Figure 27: Time difference of each channel
Start time (ns), End time = 175 ns

<table>
<thead>
<tr>
<th></th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>CH1</td>
<td>$\Gamma(0.1ns^{-1})$</td>
<td>0.122±0.0106</td>
<td>0.116±0.0146</td>
<td>0.121±0.0263</td>
</tr>
<tr>
<td>CH2</td>
<td>$\Gamma(0.1ns^{-1})$</td>
<td>0.123±0.0187</td>
<td>0.131±0.0484</td>
<td>0.142±0.0520</td>
</tr>
<tr>
<td>CH3</td>
<td>$\Gamma(0.1ns^{-1})$</td>
<td>0.112±0.0110</td>
<td>0.108±0.0147</td>
<td>0.114±0.0264</td>
</tr>
<tr>
<td>CH4</td>
<td>$\Gamma(0.1ns^{-1})$</td>
<td>0.110±0.0152</td>
<td>0.110±0.0219</td>
<td>0.123±0.0511</td>
</tr>
</tbody>
</table>

Table 4: Decay rate of each channel

Figure 28: Fitting of lifetime for each channel

### 4.4 Calculation of positronium lifetime

By fitting the decay rate with a constant value (figure 28), we get

$$\Gamma = 0.0116 \pm 0.00049 \text{ns}^{-1}$$  \hspace{1cm} (49)

Since the average lifetime $\tau = \frac{1}{\Gamma}$, we obtain

$$\tau = \frac{1}{\Gamma} = 86.21 \pm 3.64 \text{ns}$$  \hspace{1cm} (50)

The lifetime of ortho – Ps at vacuum is 142 ns. However, when it is exposed to the air, due to the pick off and spin exchange reactions, ortho – Ps is converted to para – Ps, and hence has a shorter lifetime. The value obtained in our experiment, 86 ns is a reasonable value of ortho – Ps lifetime.
5 Discussion

5.1 Effect from the air molecules

Let the number of ortho-Ps be $N_0$. In vacuum,

$$\frac{dN_0}{dt} = - \left( \frac{1}{\tau} \right) N_{0,v}$$

the above equation holds. Meanwhile in the air, the corresponding equation is,

$$\frac{dN_0}{dt} = - \left( \frac{1}{\tau} \right) N_{0,a} - \lambda N_{0,a}$$

We can solve both equations to get $N_{0,v}, N_{0,a}$.

$$N_{0,v} = N_{0,v,initial} \exp\left( -\left( \frac{1}{\tau} \right) t \right)$$

and

$$N_{0,a} = N_{0,a,initial} \exp\left[ -\left( \frac{1}{\tau} + \lambda \right) t \right]$$

If we relate the lifetime of ortho-Ps in vacuum (142 ns) and lifetime (86.21 ns) that we have measured using ($\frac{1}{\tau}$) and ($\frac{1}{\tau} + \lambda$), we get

$$\frac{1}{\tau} = \frac{1}{140}$$

and

$$\frac{1}{\tau} + \lambda = \frac{1}{87.15}$$

and $\lambda = \frac{1}{87.15} - \frac{1}{140} = 0.0043/\text{ns}$. 


外文缩写为英文翻译外文部分为英文

5 Discussion

5.1 Effect from the air molecules

Let the number of ortho-Ps be $N_0$. In vacuum,

$$\frac{dN_0}{dt} = - \left( \frac{1}{\tau} \right) N_{0,v}$$

the above equation holds. Meanwhile in the air, the corresponding equation is,

$$\frac{dN_0}{dt} = - \left( \frac{1}{\tau} \right) N_{0,a} - \lambda N_{0,a}$$

We can solve both equations to get $N_{0,v}, N_{0,a}$.

$$N_{0,v} = N_{0,v,initial} \exp\left( -\left( \frac{1}{\tau} \right) t \right)$$

and

$$N_{0,a} = N_{0,a,initial} \exp\left[ -\left( \frac{1}{\tau} + \lambda \right) t \right]$$

If we relate the lifetime of ortho-Ps in vacuum (142 ns) and lifetime (86.21 ns) that we have measured using ($\frac{1}{\tau}$) and ($\frac{1}{\tau} + \lambda$), we get

$$\frac{1}{\tau} = \frac{1}{140}$$

and

$$\frac{1}{\tau} + \lambda = \frac{1}{87.15}$$

and $\lambda = \frac{1}{87.15} - \frac{1}{140} = 0.0043/\text{ns}$. 


38
Figure 29: Decay of positronium in vacuum

Figure 30: Decay of ortho-Ps by through conversion to para-Ps

Figure 31: Decay of positronium in the air
6 Conclusion

In this section, summary of our experiment is given. We have used $^{22}\text{Na}$ as the positron source and silica powder as the stopper to create the positronium atom. Two experiments are performed, and the results are;
1. The ratio of $2\gamma : 3\gamma$ is 2.26 : 1
2. The average lifetime of ortho-positronium is $86.21 \pm 3.64$ ns
7 Appendix

7.1 Notation

\[ x^\mu = (x^0, x) \]

\[ \lambda = \gamma \mu p_\mu \]

\[ \alpha = \frac{e^2}{4\pi} = \frac{e^2}{4\pi \hbar c} \approx \frac{1}{137} \]

7.2 Calculation of lifetime

The decay of an unstable nucleus is entirely random and it is impossible to predict when a particular atom will decay. However, it is equally likely to decay at any time. Therefore, given a sample of a particular radioisotope, the number of decay events \( -dN \) expected to occur in a small interval of time \( dt \) is proportional to the number of atoms present. If \( N(t) \) is the number of atoms at time \( t \), then the decay rate \( \frac{dN}{dt} \) is proportional to \( N(t) \):

\[
\frac{dN(t)}{dt} = -\lambda N(t) \quad (51)
\]

The solution of the above differential equation is simply

\[
N(t) = N(0) \exp(-\lambda t) \quad (52)
\]

Where \( N(0) \) is the value of \( N \) at time 0 (\( t = 0 \)). Since the average lifetime, \( \tau \) is defined as the time needed for the radioisotope to become \( 1/e \) of the initial atoms, we get the following relation:

\[
\tau = \frac{1}{\lambda} \quad (53)
\]

By taking logarithm of both sides,

\[
\log N(t) = -\lambda t + \log N(0) \quad (54)
\]

Therefore, we can calculate the lifetime of the atom by plotting a number of atom versus time graph and measure the gradient.
References