David Keyt

The set of real numbers is not enumerable.

1) Every set S of natural numbers can be associated with a real number in the interval [0, 1/9]:

$S \leftrightarrow .a_0\ a_1\ a_2\ a_3 \quad \ldots a_n \ldots$ where $a_n = 1$ if $n \in S$ and $a_n = 0$ if $n \notin S$.

Thus: $.010101010101 \ldots \leftrightarrow$ the set of odd natural numbers

$.1010101010 \ldots \leftrightarrow$ the set of even natural numbers

$.1111111111 \ldots \leftrightarrow$ the set of all natural numbers

$.1011 \leftrightarrow \{0, 2, 3\}$

2) Consequently, an enumeration of the real numbers will give rise immediately to an enumeration of the set of all sets of natural numbers.

3) But the set of all sets of natural numbers is not enumerable.

4) Hence, the set of real numbers is not enumerable.