

## Formalized Theories

- 1 A *theory* is a set whose members are just the sentences in some language that follow from the set.
- 2 A sentence  $\phi$  is a *thesis* of a theory  $T$ :
  - a iff  $\phi \in T$ .
  - b iff  $\phi$  is a consequence of  $T$ .
- 3 A *model* of a theory  $T$  is an interpretation of the language of  $T$  under which all the theses of  $T$  come out true.
- 4 A theory  $T$  is *consistent*:
  - a. iff  $T$  has a model.
  - b iff no sentence and its negation are both theses of  $T$ .
  - c iff some sentence (in the language of  $T$ ) is not a thesis of  $T$ .
- 5 A theory  $T$  is *complete* iff for any sentence  $\phi$  (in the language of  $T$ ) either  $\phi$  or the negation of  $\phi$  is a thesis of  $T$ .
- 6 A set of sentences  $\Gamma$  is *independent* iff for every element  $\phi$  of  $\Gamma$ , neither  $\phi$  nor the negation of  $\phi$  is a consequence of  $\Gamma \sim \{\phi\}$ .
- 7 A set of sentences  $\Gamma$  is *decidable* iff there is a step-by-step procedure which, applied to any arbitrary sentence  $\phi$ , will decide in a finite number of steps whether or not  $\phi$  belongs to  $\Gamma$ .
- 8 A theory  $T$  is *decidable* iff the set of theses of  $T$  is decidable.
- 9 A theory  $T$  is an *axiomatizable* (or *formal*) theory iff there is a decidable subset of  $T$  whose consequences (in the language of  $T$ ) are just the theses of  $T$ . Such a subset is a *set of axioms* for  $T$ .
- 10 A theory  $T$  is *finitely axiomatizable* iff  $T$  has a set of axioms that is finite.
- 11 A theory  $T$  is  $\omega$ -*categorical* iff any two models of the theory in the domain  $\omega$  are isomorphic.