Vertical velocity as a function of depth

The steady, linear vorticity equation was found to be

$$f w_z = \beta v - Curl(\tau)/H_E \tag{2}$$

Below the Ekman layer, $\tau = 0$ and $v = v_g$, so

$$w(z) = \frac{\beta}{f}\int_{-D}^{z} v_g \, dz \tag{4}$$

which can be evaluated from the observed ocean data, assuming that $w = v = 0$ at $z = -D$, which here is taken to be 450m.

Above $z = -H_E$, the Ekman component contributes to $w(z)$, and the full expression is

$$w(z) = \frac{\beta}{f}\int_{-D}^{z} v_g \, dz - Curl(\tau/f)(z + H_E/H_E), \quad z \geq -H_E \tag{5}$$

If the Sverdrup balance balances, then at $z = 0$, the two terms on the RHS of (5) cancel. Or, this can be viewed as $w_{geostrophic}$ matches w_{Ekman} at the base of the Ekman layer.