Complex demodulation

• The original data (X(t)) is taken to be the periodic signal of interest plus everything else (Z(t)). The amplitude A and phase P of the periodic signal are assumed to be time-dependent but to vary "slowly" (compared to frequency ω).

$$X(t) = A(t)\cos(\omega t+P(t)) + Z(t)$$

$$= \tfrac{1}{2}A(t)[\exp\{i(\omega t+P(t))\} - \exp\{-i(\omega t+P(t))\}] + Z(t)$$

• "Demodulate" by multiplying by $\exp(-i\omega t)$:

$$Y(t) = X(t)\exp\{-i\omega t\}$$

which can be written (note that $\exp(-i\omega t)$ cancels in 1st term, adds in 2nd)

$$Y(t) = \underbrace{\tfrac{1}{2}A(t)\exp\{iP(t)\}}_{(a)} + \underbrace{\tfrac{1}{2}A(t)\exp\{-i(2\omega t+P(t))\}}_{(b)} + \underbrace{Z(t)\exp\{-i\omega t\}}_{(c)}$$

Term (a) varies slowly, with no power at or above frequency ω.
Term (b) varies at frequency 2ω.
Term (c) varies at frequency ω. (Note that Z(t) has no power at frequency ω, so the shifted term (c) has no power at zero frequency).

• Low-pass filter to remove frequencies at or above frequency ω. This removes terms (b) and (c), and smooths (a). The result is:

$$Y'(t) = \tfrac{1}{2}A'(t)\exp\{iP'(t)\},$$ where the prime indicates smoothed.

• Extract A′ and P′: $A' = 2|Y'|$, $\exp\{iP'(t)\} = 2Y'/A'$.

The smoothing does two things. First, it removes the unwanted terms (b) and (c). The choice of smoother determines the width of the frequency band of variability retained. I smooth with a triangle of length (2T−1), where $T = 2\pi/\omega$ is the demodulation period. This results in a half-power bandwidth from T/(1+0.44295) to T/(1−0.44295). Second, the smoothing smooths the amplitude and phase time series. The half-power of the smoothed A′ and P′ is T/0.44295. (If a second triangle smoothing is done then use 0.262 instead of 0.44295 in these expressions).