Godfrey's Island Rule: a generalized Sverdrup balance

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Billy Kessler
NOAA/PMEL
Seattle USA

Sverdrup balance is simple and remarkably robust

It has proven to be a realistic picture of the vertically-integrated, steady ocean circulation. Places where it does not hold are thus useful pointers to the activity of nonlinearity and other processes.

It has no dependence on poorly-known parameters (eddy viscosity, bottom drag coefficients, diapycnal exchanges).

But:

- Sverdrup balance does not tell us about WBCs, or islands.
- Only single basins can be considered.

Its utility for interpreting the circulation made these omissions very unsatisfying.

40 years after Sverdrup, Godfrey (1989) surmounted these limitations in a rather simple way.

Sverdrup Balance

Steady, linear, inviscid, vertically-integrated, wind-forced MOM* eqns:

$$\left.\begin{aligned} -fV &= -gP_x + \tau^x \\ fU &= -gP_y + \tau^y \end{aligned}\right\}$$

can rewrite:

$$\left.\begin{aligned} V &= V_g + V_{Ek} \\ U &= U_g + U_{Ek} \end{aligned}\right\}$$

Flow is Ekman + geostrophic

$$U_x + V_y = 0$$

$\Leftarrow$ Integrated top to bottom: non-divergent

Curl(MOM), use non-divergence, to get the Sverdrup balance: $\boxed{\beta V = Curl(\tau)}$

Use non-divergence again to find U_{Sv}:

$$U_x = -V_y = -\frac{1}{\beta}\frac{\partial}{\partial y}Curl(\tau) \;\Rightarrow\; U = -\frac{1}{\beta}\int_{x_e}^{x}\frac{\partial}{\partial y}Curl(\tau)dx$$

where x_e is the eastern boundary.

Since the fluid is non-divergent, a streamfunction exists:

$$\psi = \frac{1}{\beta}\int_{x_e}^{x} Curl(\tau)dx\;, \quad \text{where} \;\; V = \psi_x,\; U = -\psi_y,\; \psi(x_e) = 0$$

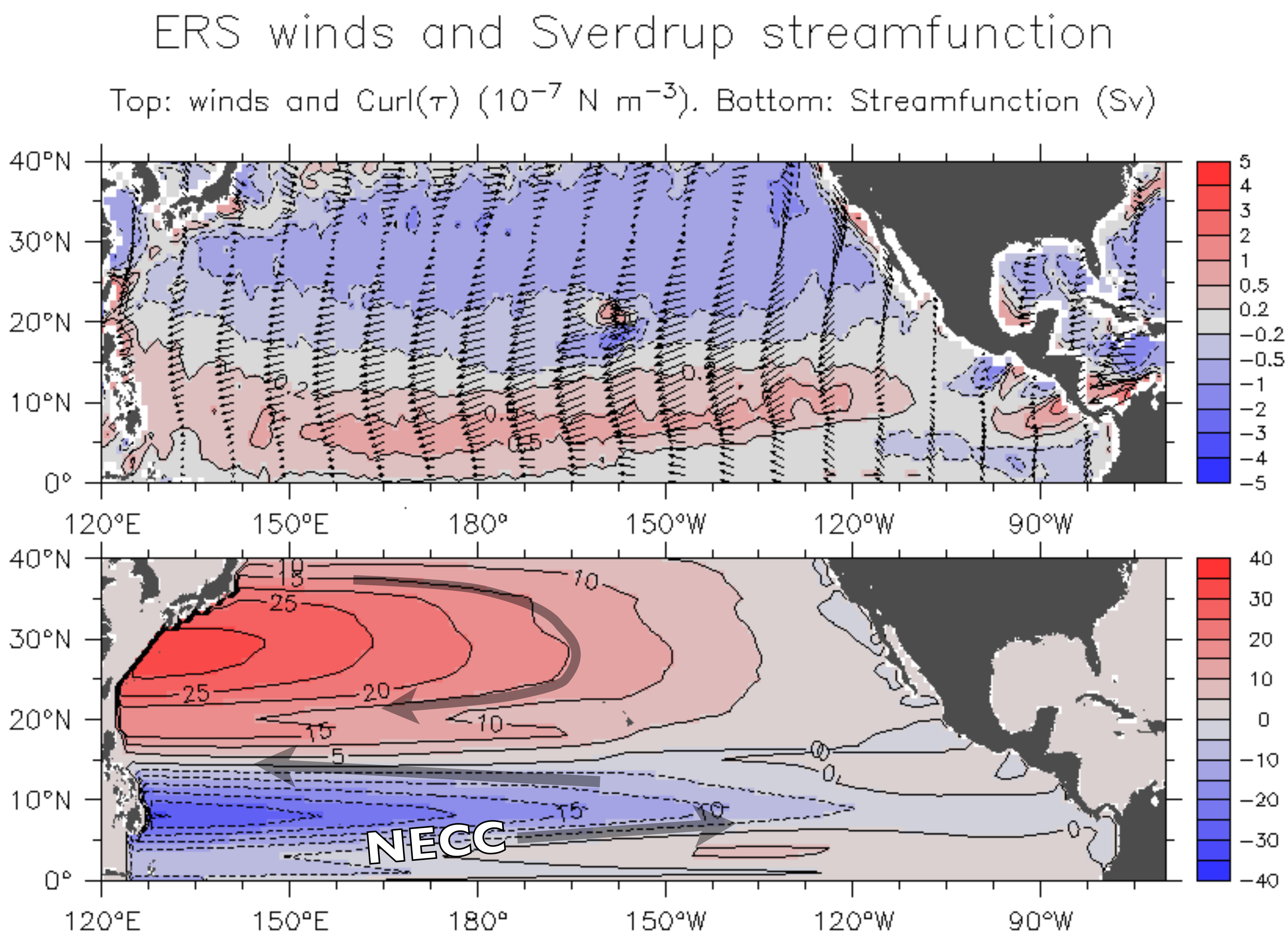
The rest of the lecture will explore consequences, limitations, and extensions of the Sverdrup balance.

* MOM = "momentum". The MOM equations are the mother of all we do.
Note the five underlined assumptions in the first line. These are crucial to what the Sverdrup balance does and does not say.
(Also note that I omit factors of ρ! It makes the equations messy, and it's easy to figure out where it goes at the end.)

Sverdrup's triumph: the NECC flows against the wind

Winds and Curl

Stream-function

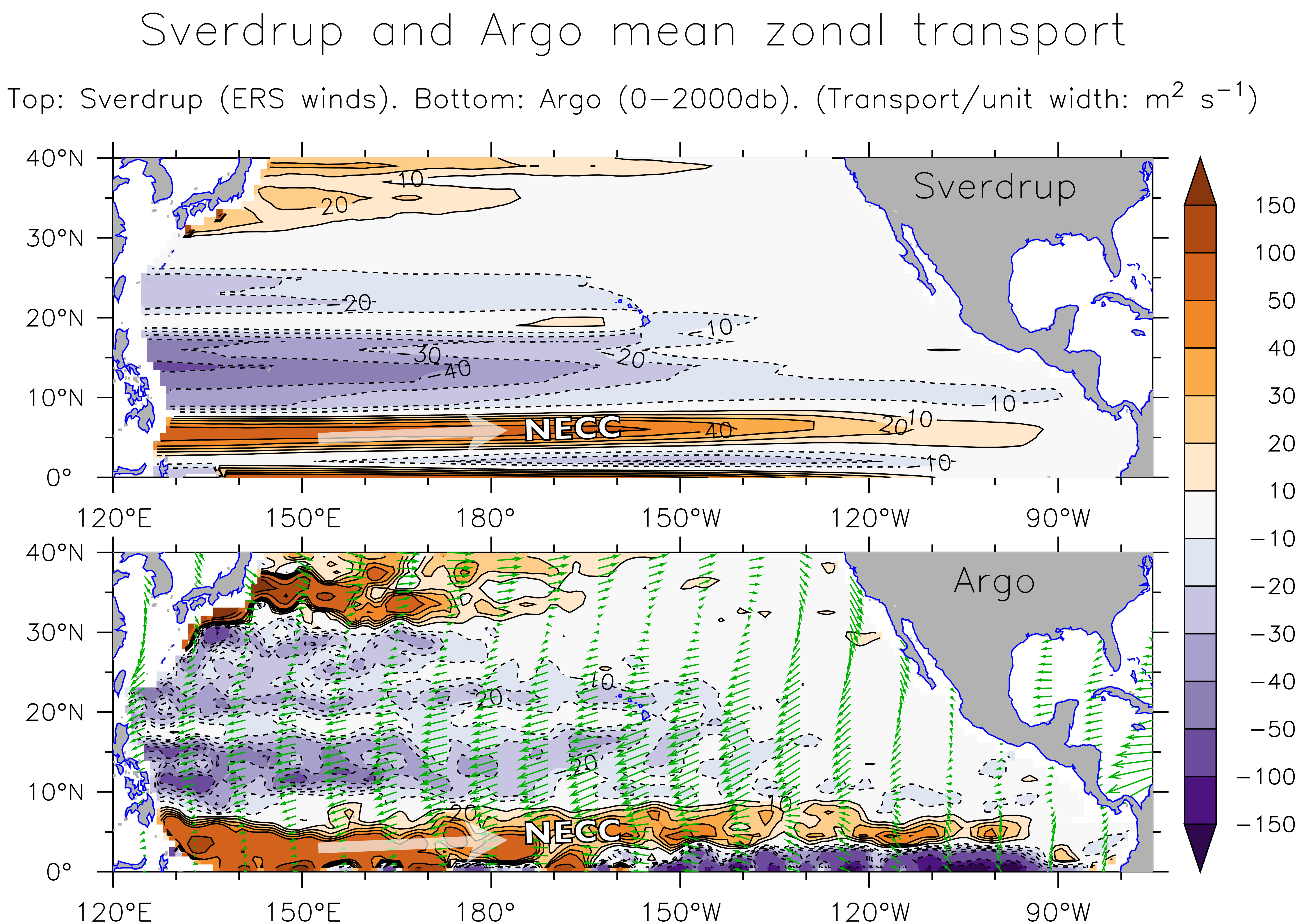


Positive Curl(τ) along the north flank of the ITCZ (similar in Atlantic)

Sverdrup's triumph: the NECC flows against the wind

$U_{Sverdrup}$

In situ U_g (Argo)



The NECC flows east where easterlies decrease (not at their minimum: it's the curl!)

Momentum vs vorticity: 4 advantages

- In a practical sense, taking the Curl is convenient because it removes the pressure gradient terms.
  (those need ocean observations, and are hard to measure ... (LNM etc))
  The equations now involve only the wind and β terms that can be measured externally, e.g., satellites.

- We now have β but no f: can get solutions at the equator.

- The MOM eqns are Newton's $F\!=\!ma$: they are appropriate to find the acceleration at a point. Can integrate in time.
  But the MOM eqns do not provide a boundary condition to find a steady basin solution. The vorticity eqn does:

$$\psi = \frac{1}{\beta}\int_{x_e}^{x} Curl(\tau)dx \ , \quad \text{where} \ \ V = \psi_x, \ U = -\psi_y, \ \boxed{\psi(x_e) = 0}$$

Eastern boundary

- A vorticity eqn expresses what the wind does to the ocean on the large scale: stretching and squashing a column of water.

Stretching vorticity

Rewrite the Sverdrup balance to separate the geostrophic and Ekman parts:

$$\beta V = \beta(V_g + V_{Ek}) = \beta V_g - \frac{\beta}{f}\tau^x = Curl(\tau)$$

$$\underbrace{\frac{\beta V_g}{f}}_{\text{From hydro data}} = \frac{1}{f}Curl(\tau) + \frac{\beta}{f^2}\tau^x = \underbrace{Curl(\frac{\tau}{f})}_{\text{From wind data}} \equiv w_e$$

From hydro data ← Independent → From wind data

w_e is the "Ekman Pumping velocity":

- divergence of the Ekman transport
- stretching imposed on the ocean by the wind

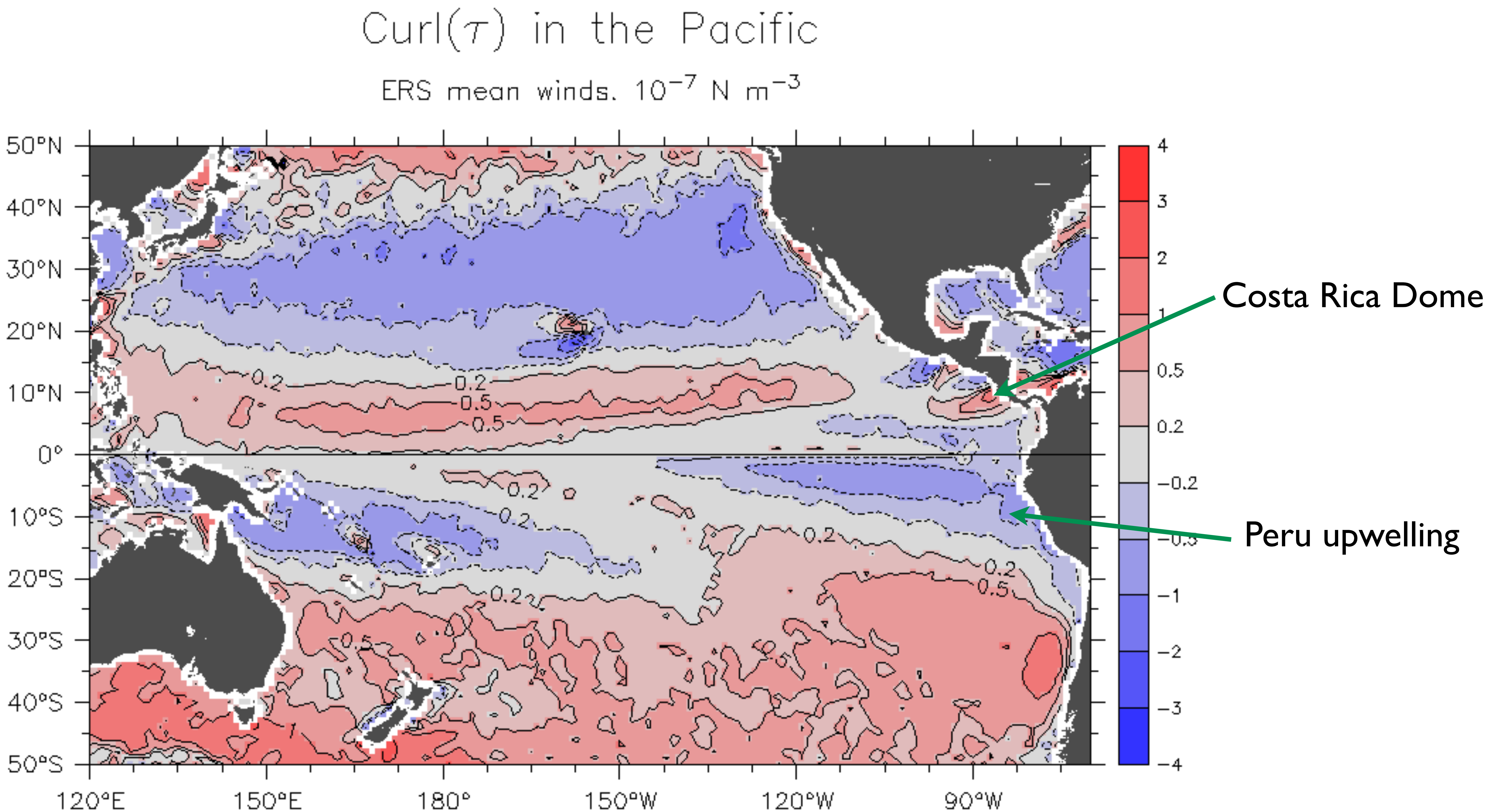
A water column in equilibrium near the equator is rotating slower than one close to the pole.

If the column is stretched (e.g., by Ekman upwelling), and its vorticity is conserved, it must either spin faster or move poleward:

$$D\left[(\zeta + f)/H\right]/Dt = 0$$

If its motion remains steady as we assume, then it must move poleward to a latitude where the change in f balances the effect of stretching: $V_g = w_e f/\beta$

Consequences of vorticity imposed by the wind

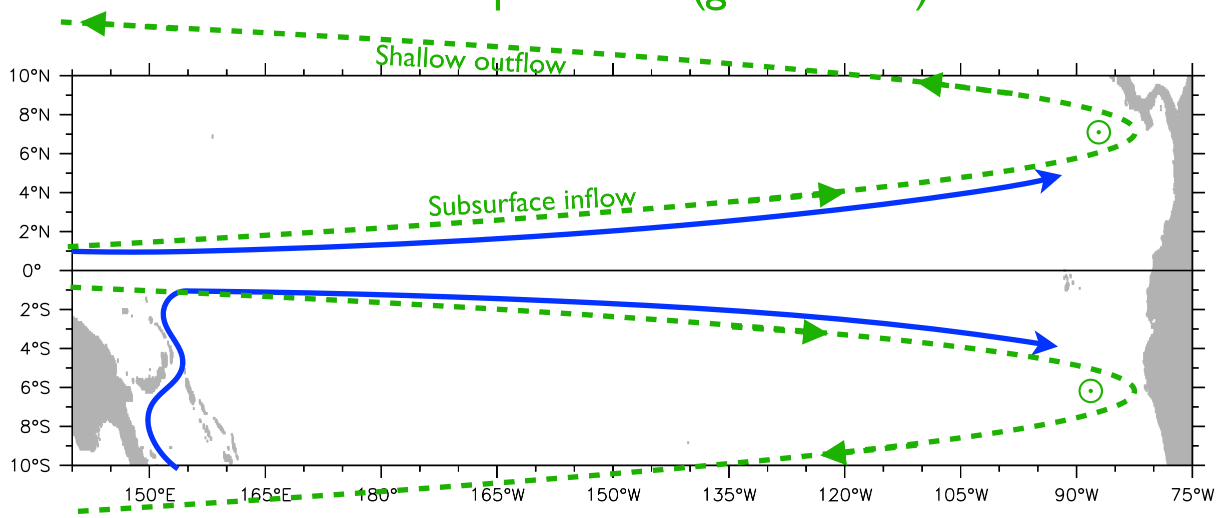


Because of upwelling curl (Peru and Costa Rica Dome), there must be subsurface poleward currents in the eastern tropical Pacific (predicted (!) by Roden 1948).
$\Longrightarrow$ Tsuchiya jets turn away from the equator in the east (next page).

Tsuchiya Jets (background ... digression)

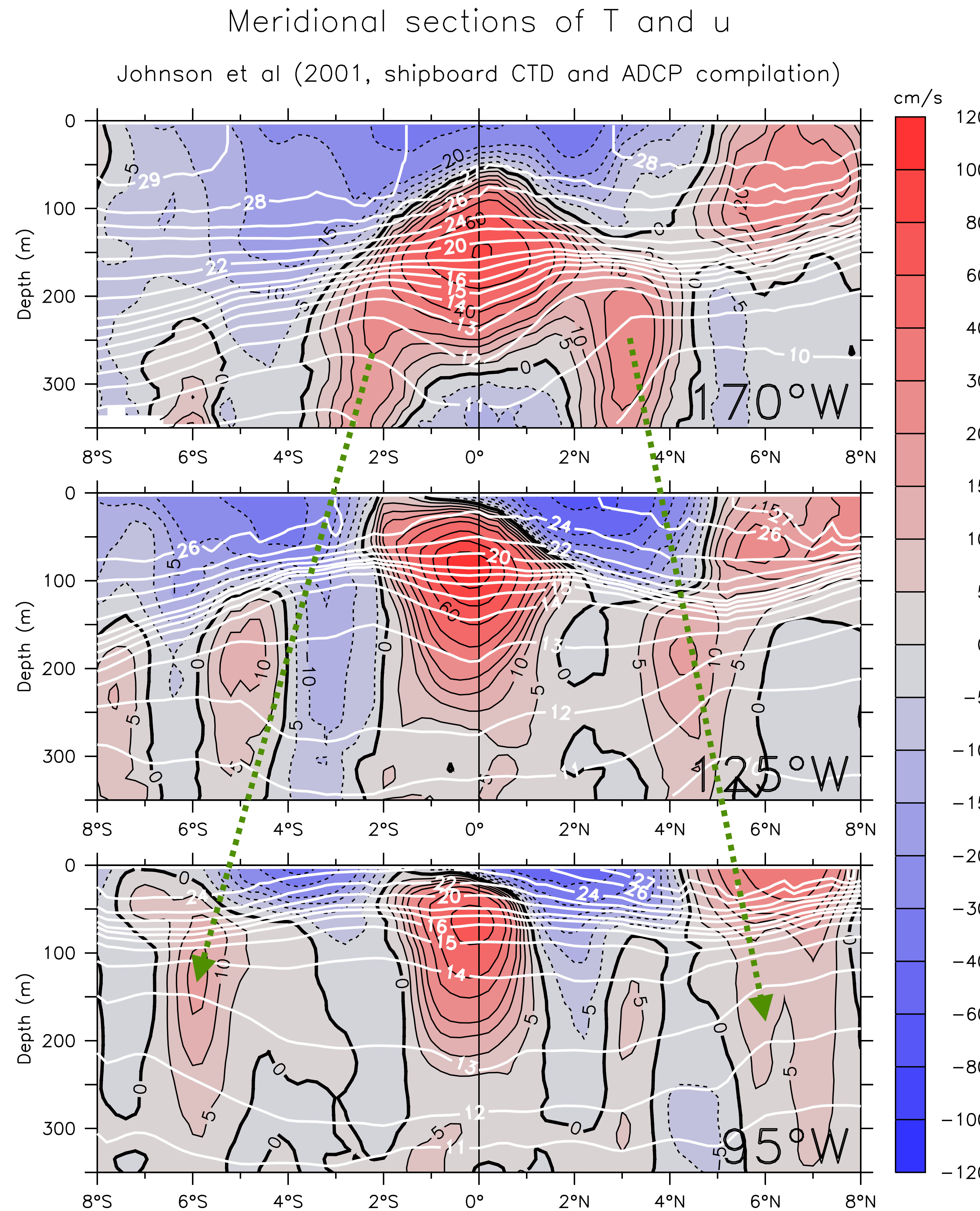
Schematic Tsuchiya Jets (~200m)

Circulation driven by point sources of vorticity (upwelling =⊙) at focus of parabolas (green dash)



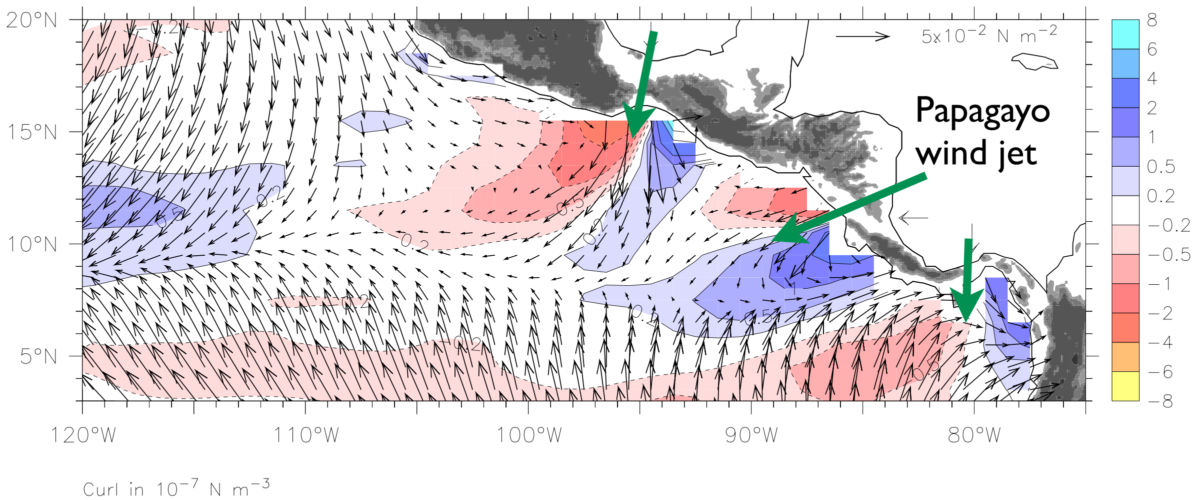
The Tsuchiya Jets are hypothesized as driven by the upwelling curl of the two upwelling regions near the American coast.

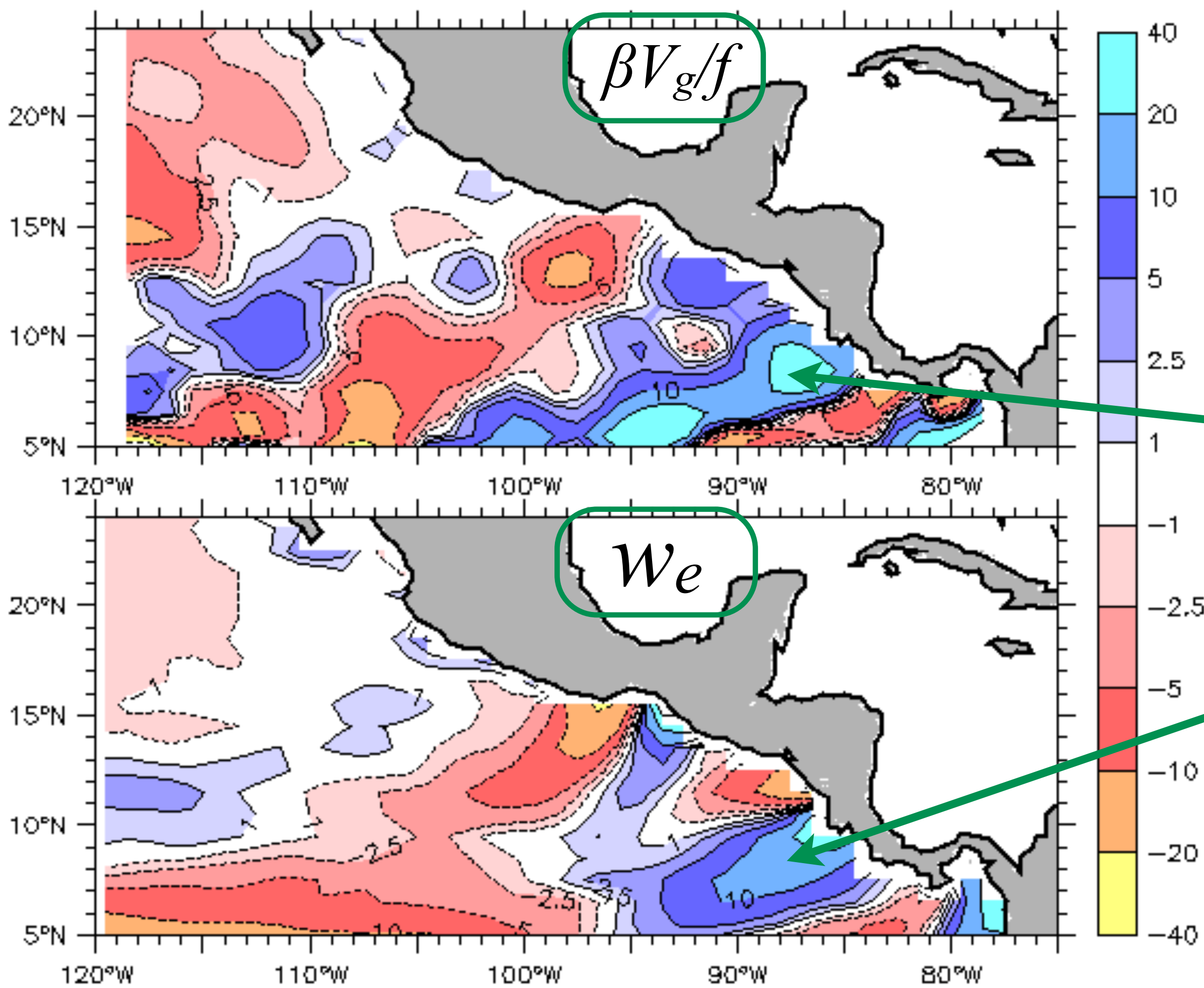
In theory, a point source of vorticity should produce a parabolic circulation with subsurface inflow to the upwelling (Tsuchiya Jets), then a shallow outflow (part of the NEC/SEC?).



The northern Tsuchiya Jet turns away from the equator

Mean ERS winds and Curl





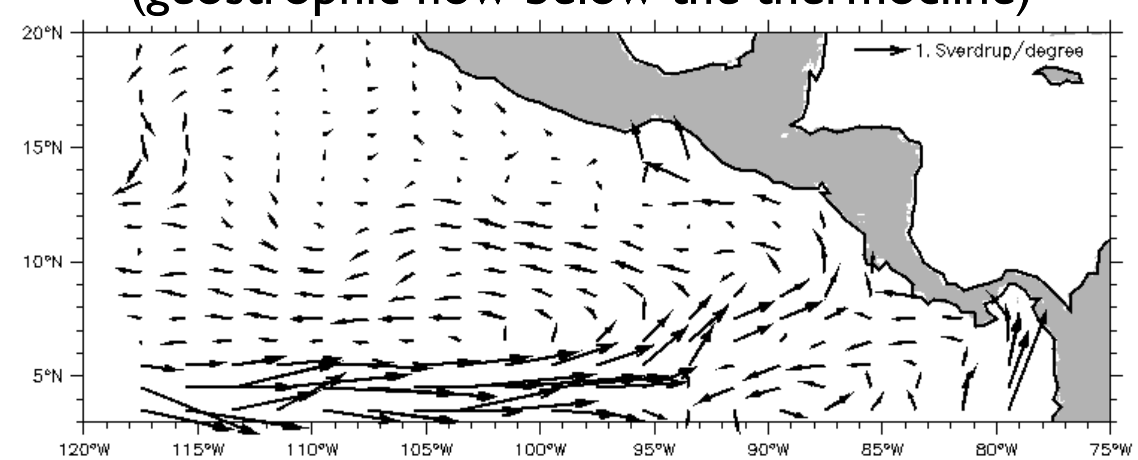
Northward V_g (from ocean data)

Upwelling Curl (from wind data)

Sverdrup balance (works!)

$$\underbrace{\frac{\beta V_g}{f}}_{\text{From hydro data}} = \frac{1}{f} Curl(\tau) + \frac{\beta}{f^2}\tau^x = \underbrace{Curl(\frac{\tau}{f})}_{\text{From wind data}} \equiv w_e$$

The northern Tsuchiya Jet turns away from the equator (geostrophic flow below the thermocline)



Even though the surface current is southward in this region, one can deduce a subsurface northward current.

Kessler (2002)

Godfrey: Sverdrup balance in terms of pressure* (1)

$$-fV = -gPx + \tau^x \qquad (1)$$

$$fU = -gP_y + \tau^y \qquad (2)$$

$$U_x + V_y = 0 \qquad (3)$$

$$P \equiv \frac{-1}{g}\int_{-P_o}^{0} \alpha p\, dp = \int_{-D}^{\eta} p\, dz \qquad (4)$$

$\Longleftarrow$ Integrated pressure "P"

Rewrite (1) and (2):

$$P_x = (fV + \tau^x)/g$$

$$P_y = (-fU + \tau^y)/g$$

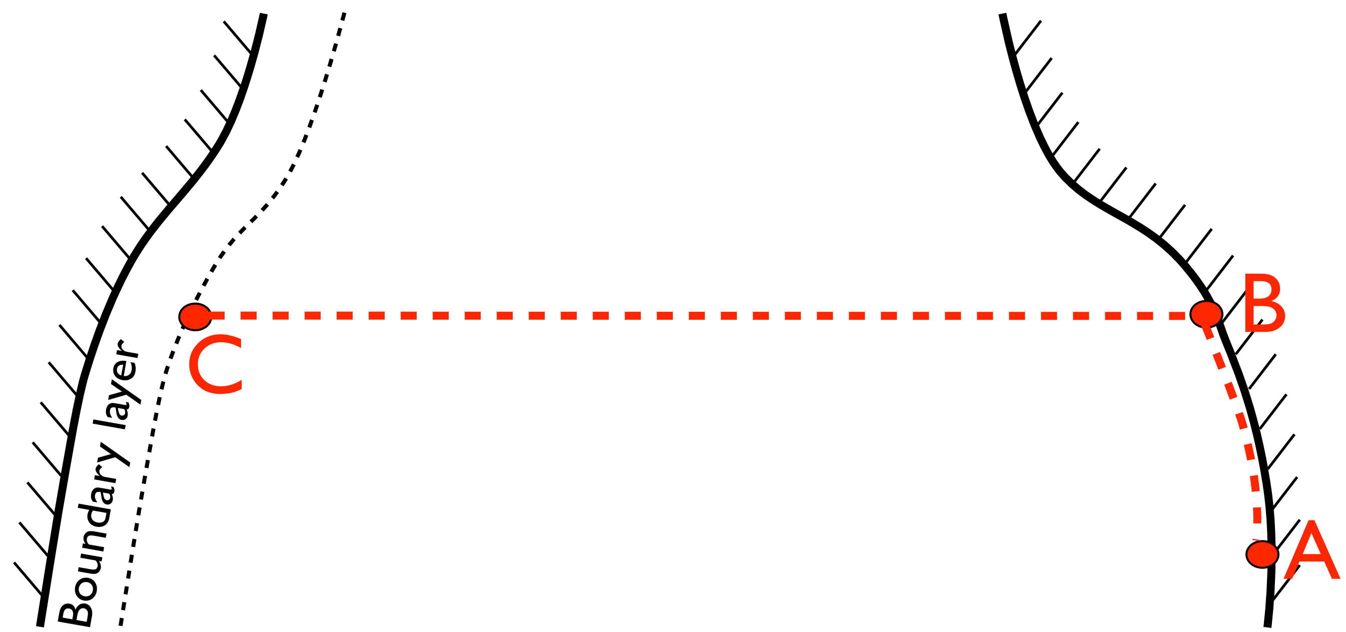
Solve from wind data:

Choose a southeastern point (A) to begin the integration.
(Entire solution is relative to A)

Work northward along the coast, then westward at each latitude.

Use winds and facts about U and V.

Find the pressure from the wind (next page).



* Godfrey (uniquely) thought in terms of pressure. This let him break one of the five Sverdrup assumptions (p3): that the ocean is inviscid. Then he could treat the western boundary layer consistently with the Sverdrup interior.

Sverdrup balance in terms of pressure (2)

Along an eastern boundary, $U=0$, so: $(2) \Longrightarrow \quad \dfrac{\partial P}{\partial l} = \dfrac{\tau^l}{g}$ ($l=$alongshore)

thus: $$P_B - P_A = \frac{1}{g}\int_A^B \tau^l dl \qquad (5)$$

An alongshore wind implies an alongshore pressure gradient.

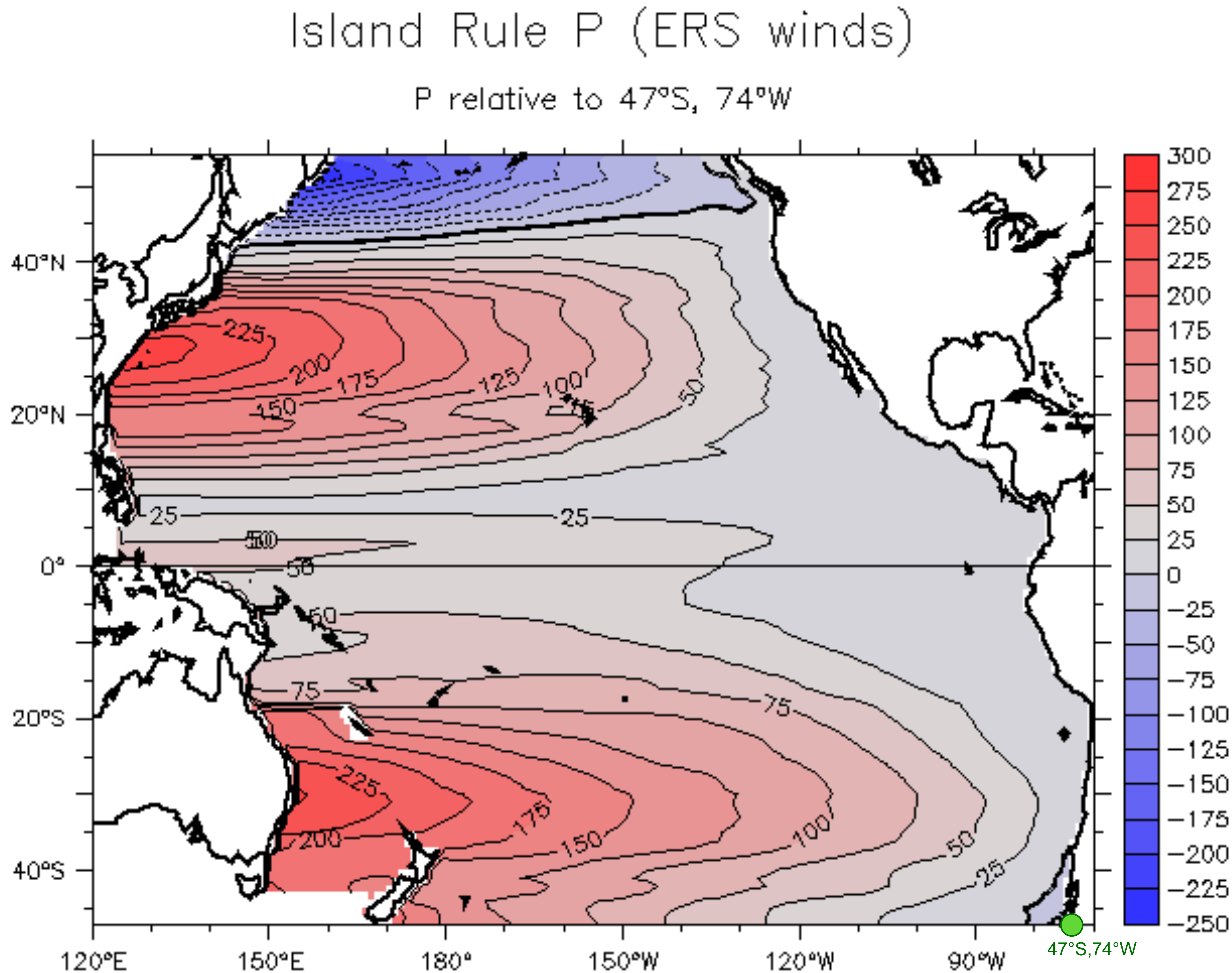
In the interior: $\beta V = Curl(\tau) \quad (6)$ and: $P_x = (fV + \tau^x)/g \quad (1)$

(1) and (6) $\Longrightarrow$ $$P_B - P_C = \frac{1}{g}\int_C^B \left(\tau^x + \frac{f}{\beta}Curl(\tau)\right) dx \qquad (7)$$

(7) is: $fV_g = -fV_E + fV_{Sv}$

Build a solution for the entire basin entirely from wind data.

A tedious calculation: Work up the coast using (5), then across the interior at each latitude using (7).



The pressure field consistent with Sverdrup transport minus Ekman transport

Godfrey's derivation of the Island Rule (1)
(First step: no islands, but include a WBC)

$$-fV = -gP_x + \tau^x + X \quad (1)$$
$$fU = -gP_y + \tau^y + Y \quad (2)$$
$$U_x + V_y = 0 \quad (3)$$

Two new terms:

(X, Y) are the stresses due (vaguely!) to lateral and bottom friction.

(X, Y) are assumed to be active only in strong currents (WBCs), and are a simple way to extend the MOM eqns to incorporate western boundary dynamics.

However ...
East of the boundary layer the situation is exactly the same as before, because the new frictional stresses are taken to be small except in the strong western boundary current.

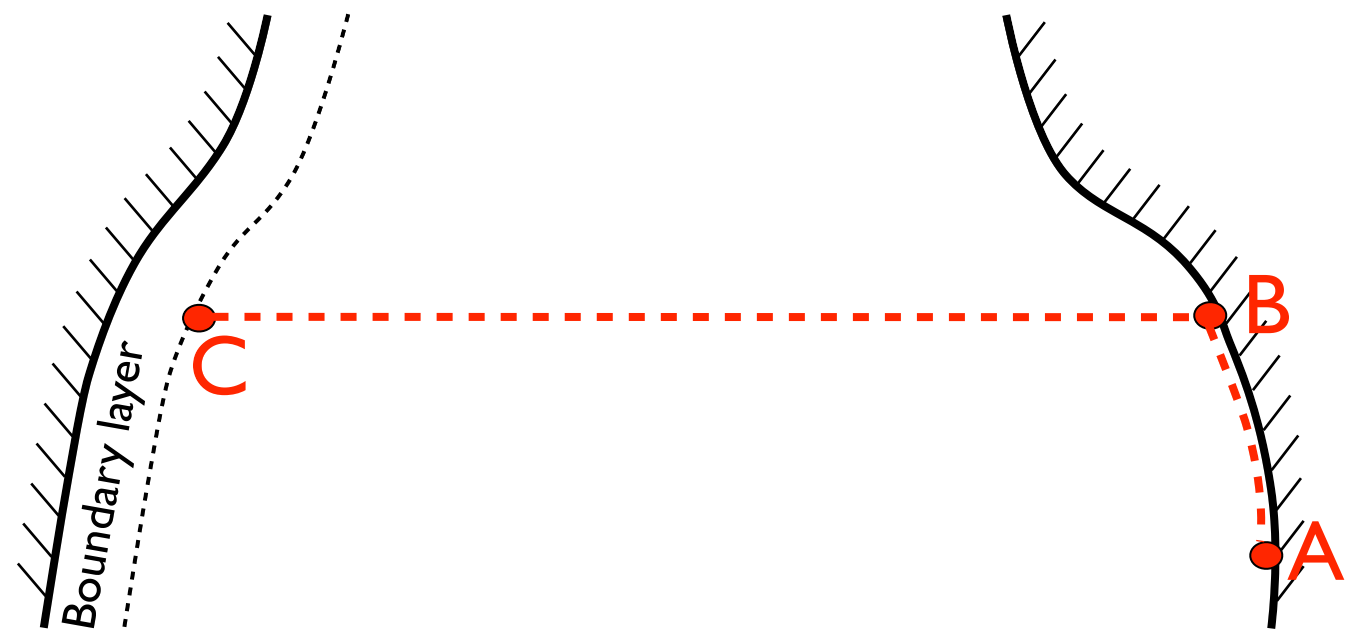
Godfrey's derivation of the Island Rule (2)
(First step: no islands, but include a WBC)

As before (p12), along the eastern coast, zonal (cross-shore) flow is zero, and in the interior, Sverdrup balance still holds (X=Y=0 in (1) and (2)), so:

$$\Rightarrow P_C - P_A = \frac{1}{g}\left[\int_B^C \left(\tau^x + \frac{f}{\beta}Curl(\tau)\right)dx + \int_A^B \tau^l dl\right] \quad (5)+(7)$$

(from p12)

Get P everywhere east of the western boundary region, relative to P_A (plot p13).



Consider the path FED crossing the entire ocean, including the western boundary (1)

The boundary layer (BL) is thin relative to the width of the ocean, so winds within it can be ignored.

Cross-shore flow is zero, so (in the case of a meridional boundary), (1) and (2) reduce to:

$$-fV = -gP_x \qquad (1')$$ (no cross-shore friction X)

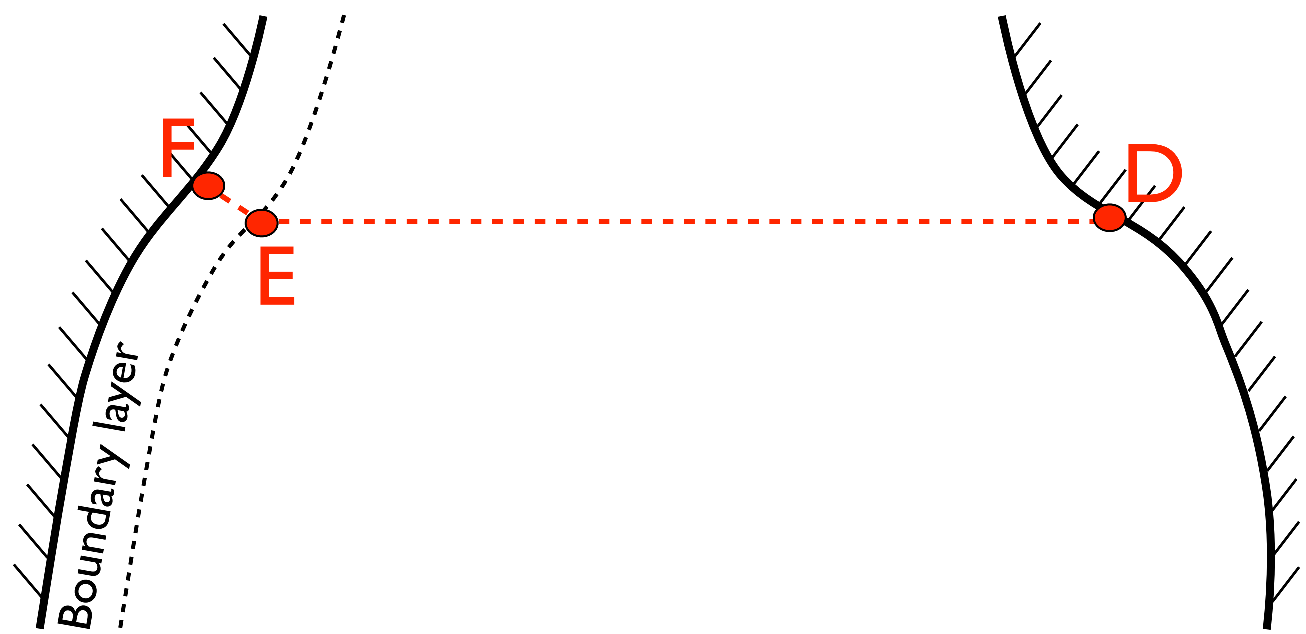
$$0 = -gP_y + Y \qquad (2')$$ (must keep the alongshore friction term Y!)

⇨ Longshore flow in the BL is geostrophic $(1')$, but the longshore momentum eqn $(2')$ is a <u>balance between the longshore pressure gradient and longshore friction</u>.

For a non-meridional coast, with l = alongshore and c = cross-shore:

$$-fV^l = -g\frac{\partial P}{\partial c} \qquad (1'')$$

$$0 = -g\frac{\partial P}{\partial l} + L \qquad (2'')$$



Sorry, location ID letters are changed here!
... and continuing below.
The reason becomes clear on p18 and onward.

Consider the path FED crossing the entire ocean, including the western boundary (2)

$$P_D - P_F = \frac{1}{g}\left[\int_{FED} \tau^l \, dl + f\,T_0\right] dx \quad (8)$$

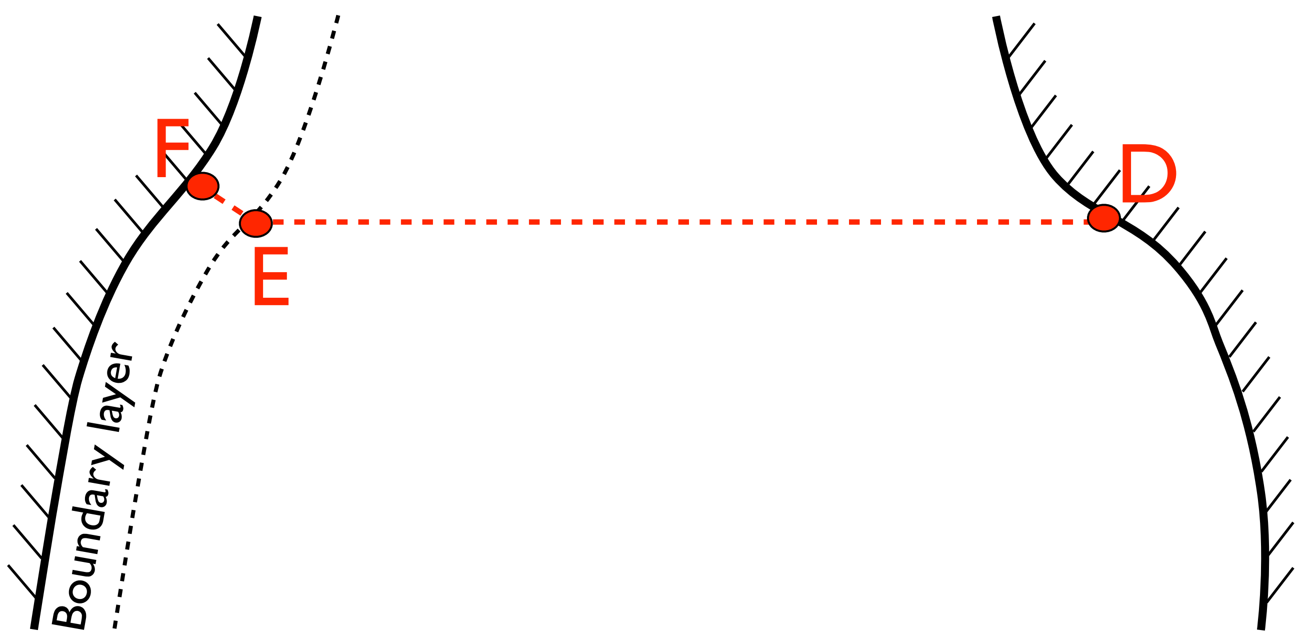
$T_0 \equiv \int V\,dx$, the total flow across FED (Sverdrup+WBC).

(fV_g) $\quad (-fV_{Ek}) \quad (fV_{Total})$

Compare (7) (p12), which does not include the western BL (i.e. from E to D only):

$$fV_g = -fV_{Ek} + fV_{Sv} \quad (7)$$

As we expect, WBC transport from F-E is the difference between Total and Sverdrup transport.



In a closed basin like the North Pacific ...

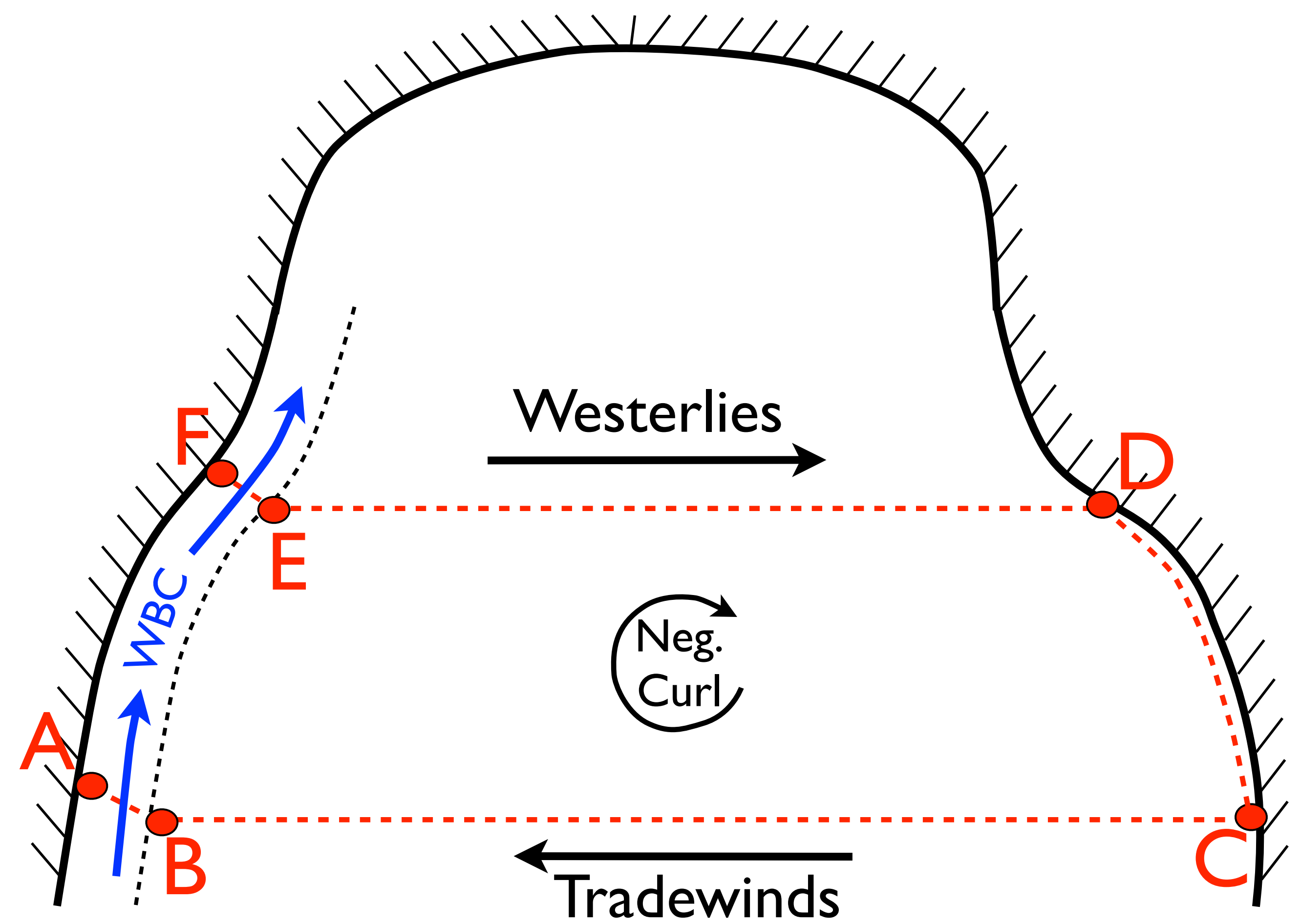
In a closed basin in steady state,
there can be no net transport across a section: $T_0 = 0$ in (8),
which reduces to:

$$P_F - P_A = \frac{1}{g} \int_{ABCDEF} \tau^l dl \qquad (9) \qquad \textbf{(!)}$$

Why should the path-integral of the wind have anything to do with Sverdrup balance?
And there is no f!

$$\left(\oint_C F \cdot dl = \iint_D Curl(F) dx\, dy \right)$$

Green's theorem
(we will come back to this)



In a closed basin like the North Pacific ...

$$P_F - P_A = \frac{1}{g} \int_{ABCDEF} \tau^l dl \qquad (9)$$

(9) is: $V_g = -V_{Ek}$

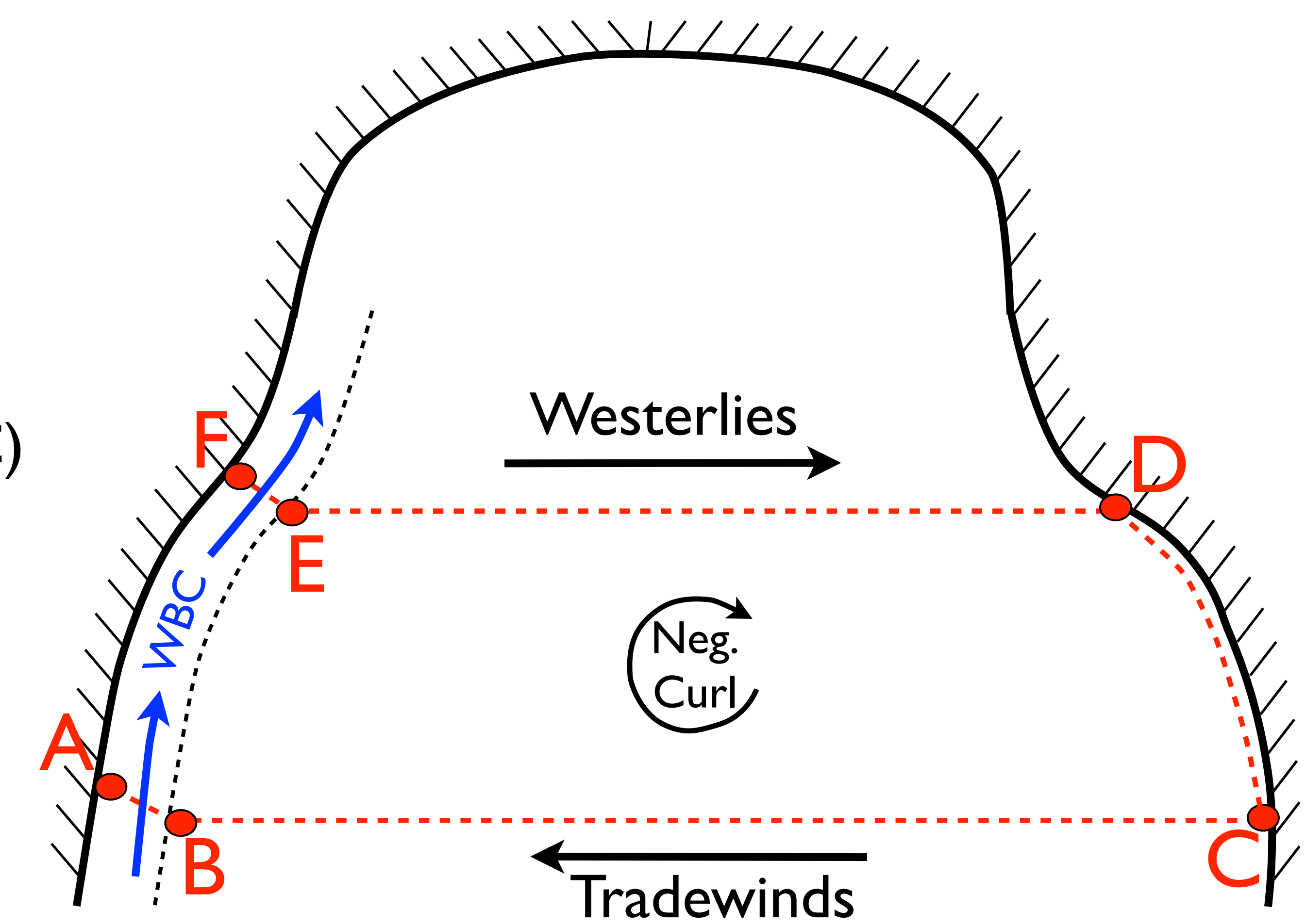
(zero transport in a closed basin)

Since this integral opposes both the mid-latitude westerlies and the subtropical tradewinds, $P_A - P_F > 0$, or $P_A > P_F$.

⇨ But, pressure is higher at A than at F

(ΔP balances alongshore friction in WBC)

$$0 = -gP_y + Y \qquad (2')$$



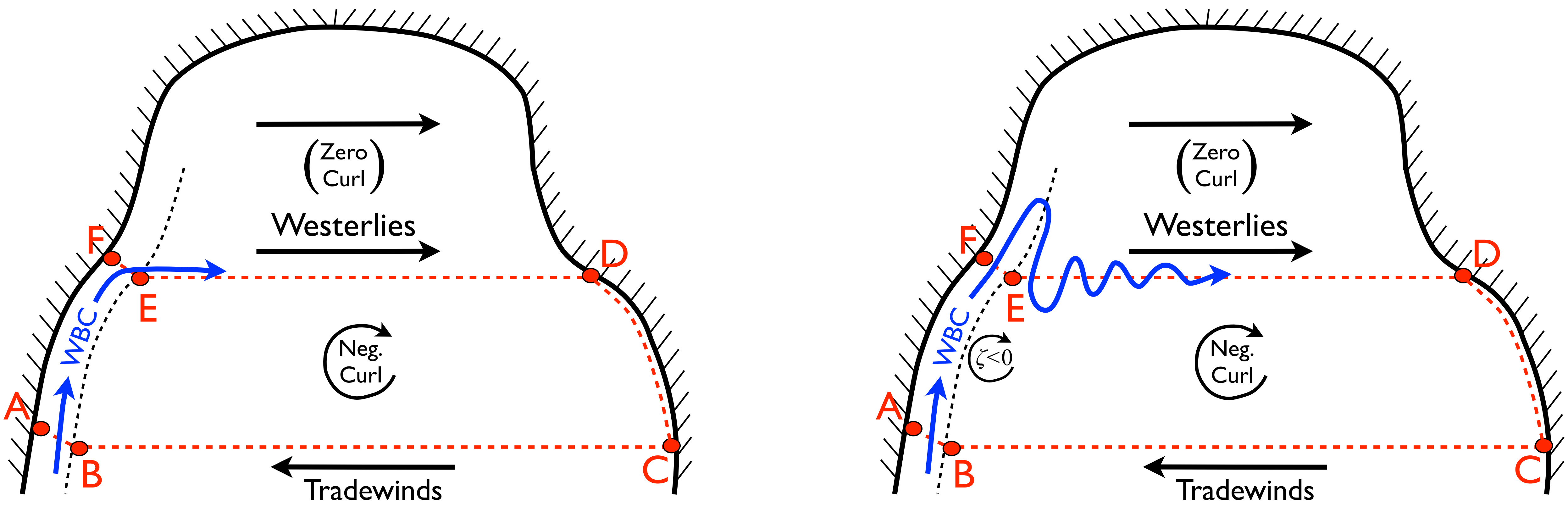
In a closed basin like the North Pacific ...

In Godfrey's linear system, the WBC is geostrophic (1′), but its alongshore friction Y balances the alongshore pressure gradient (2′).

$$-fV = -gP_x \qquad (1')$$

$$0 = -gP_y + Y \qquad (2')$$

Compare a nonlinear WBC (with zero Curl north of the "subtropical gyre"):



Godfrey: WBC turns due East (No pressure change north of F, vorticity dissipated at its creation latitude)

Nonlinear: "Inertial overshoot" (Offshore relative vorticity advected by WBC, but inshore is dissipated by coastal friction)

Summary list of equations for an ocean without islands

(1)-(3) are MOM and continuity from p10

$$P_B - P_A = \frac{1}{g}\int_A^B \tau^l dl \qquad (5)$$ (along the coast)

$$\beta V = Curl(\tau) \qquad (6)$$

$$P_B - P_C = \frac{1}{g}\int_C^B \left(\tau^x + \frac{f}{\beta} Curl(\tau)\right) dx \qquad (7)$$

Eqs (1)-(3), (5), (6), (7) describe the circulation and pressure in the interior ocean east of the western boundary.

Eqs (1′), (2′), and either (8) or (9) [for a closed basin] add the western boundary.

$$-fV = -gP_x \qquad (1')$$

$$0 = -gP_y + Y \qquad (2')$$

$$P_D - P_F = \frac{1}{g}\left[\int_{FED} \tau^l \, dl + f\, T_0\right] \qquad (8)$$

$$P_F - P_A = \frac{1}{g}\int_{ABCDEF} \tau^l dl \qquad (9)$$

(Position letters are vague here! Eqns are schematic!)

The Island Rule, finally (1)

$$P_{Q,T} - P_{R,S} = -\frac{1}{g}\left[\int_{Q,T}^{R,S} \tau^l \, dl + f\,T_0\right] dx \quad (8)$$

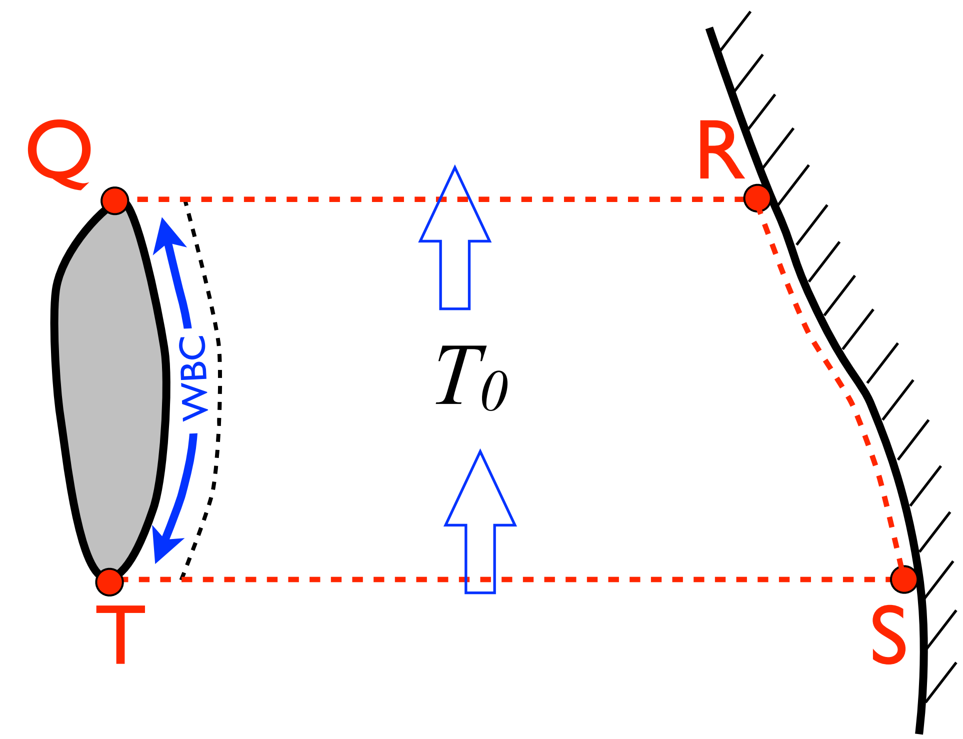
$$P_R - P_S = \frac{1}{g}\int_S^R \tau^l dl \quad (5)$$

East of an island, the flow T_0 in (8) is not necessarily zero.

(But T_0 is equal through both Q-R and T-S)

(5) and (8) combine to give an expression for the pressure difference along the path TSRQ:

$$P_Q - P_T = \frac{1}{g}\int_{TSRQ} \tau^l dl + (f_T - f_Q)T_0 \quad (11)$$



Note that although T_0 is the same across both T-S and Q-R, f is not.

The Island Rule, finally (2)

$$P_Q - P_T = \frac{1}{g} \int_{TSRQ} \tau^l dl + (f_T - f_Q) T_0 \qquad (11)$$

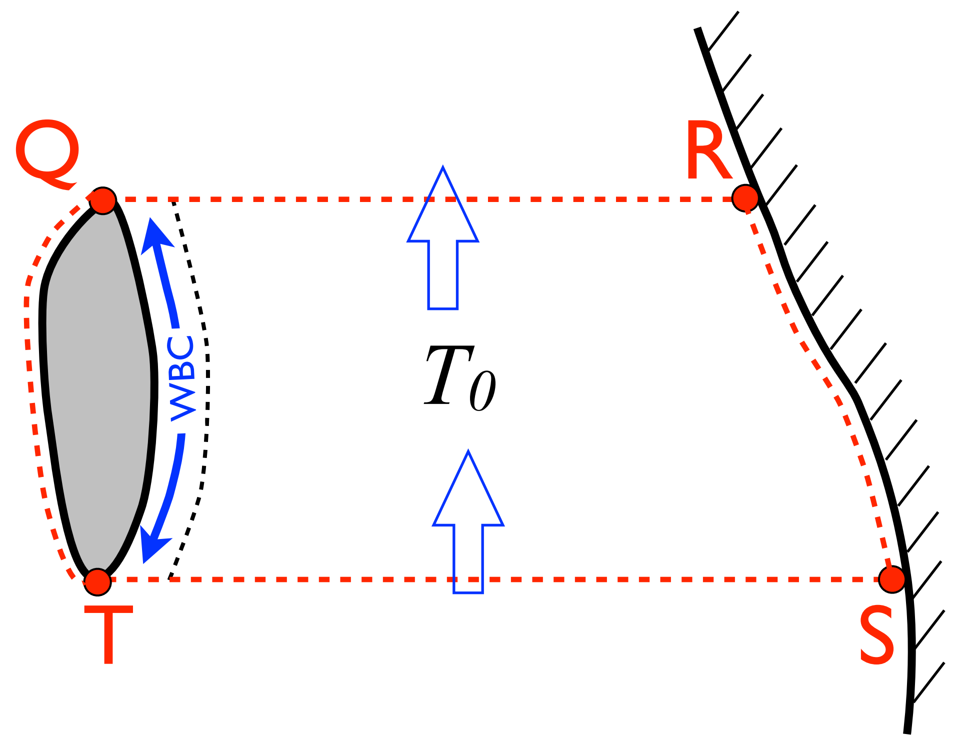
From previous page:
Pressure difference along TSRQ

Write another expression, based on (5),
for the alongshore pressure difference TQ on the west side of the island:

$$P_Q - P_T = \frac{1}{g} \int_T^Q \tau^l dl \qquad (5)$$

Equate (5) and (11), solve for T_0,
to get the Island Rule:

$$T_0 = \frac{1}{f_Q - f_T} \oint_{TSRQT} \tau^l \, dl \qquad (12)$$



The integral is the complete path, including the west side of the island.

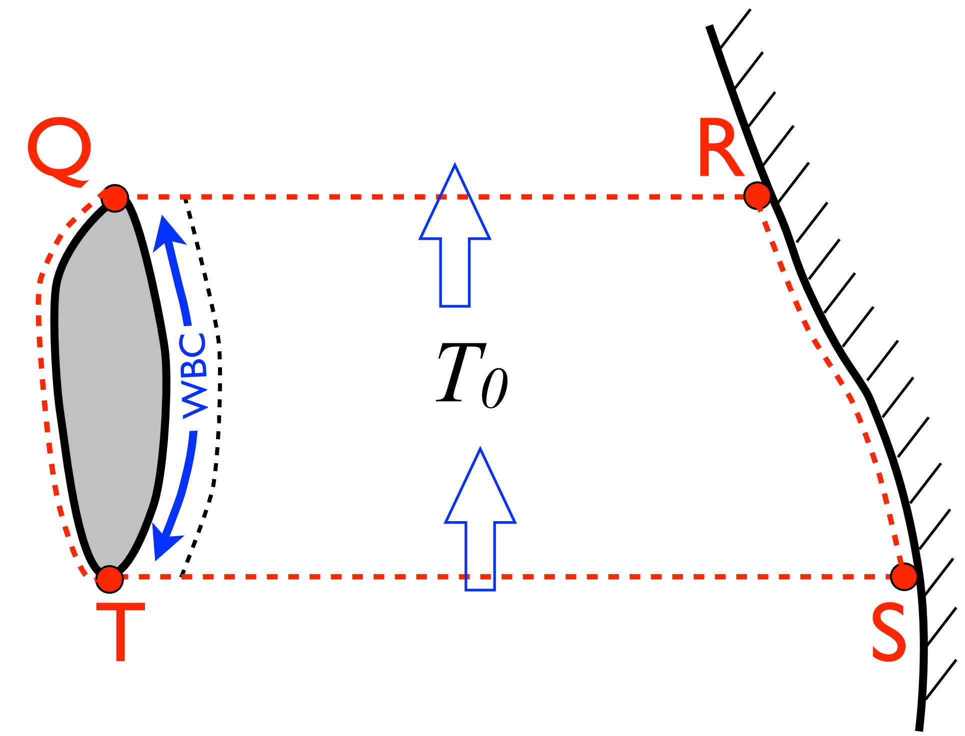
(12) gives the transport T_0 between the island and the coast to its east,
or equivalently, the streamfunction value on the coast of the island.

(To be precise: $\psi = -T_0$ since $V = \psi_x$)

The Island Rule, finally (3)

The Island Rule (previous page):

$$T_0 = \frac{1}{f_Q - f_T} \oint_{TSRQT} \tau^l \, dl \qquad (12)$$



(12) gives the transport T_0 between the island and the coast to its east.

Western boundary transport along the island's coast is the difference between total transport T_0, (constant over the latitude range of the island) and the Sverdrup transport at each latitude $T_{Sv}(y) = Curl(\tau(y))/\beta$

$T_{WBC}(y) = T_0 - T_{Sv}(y)$

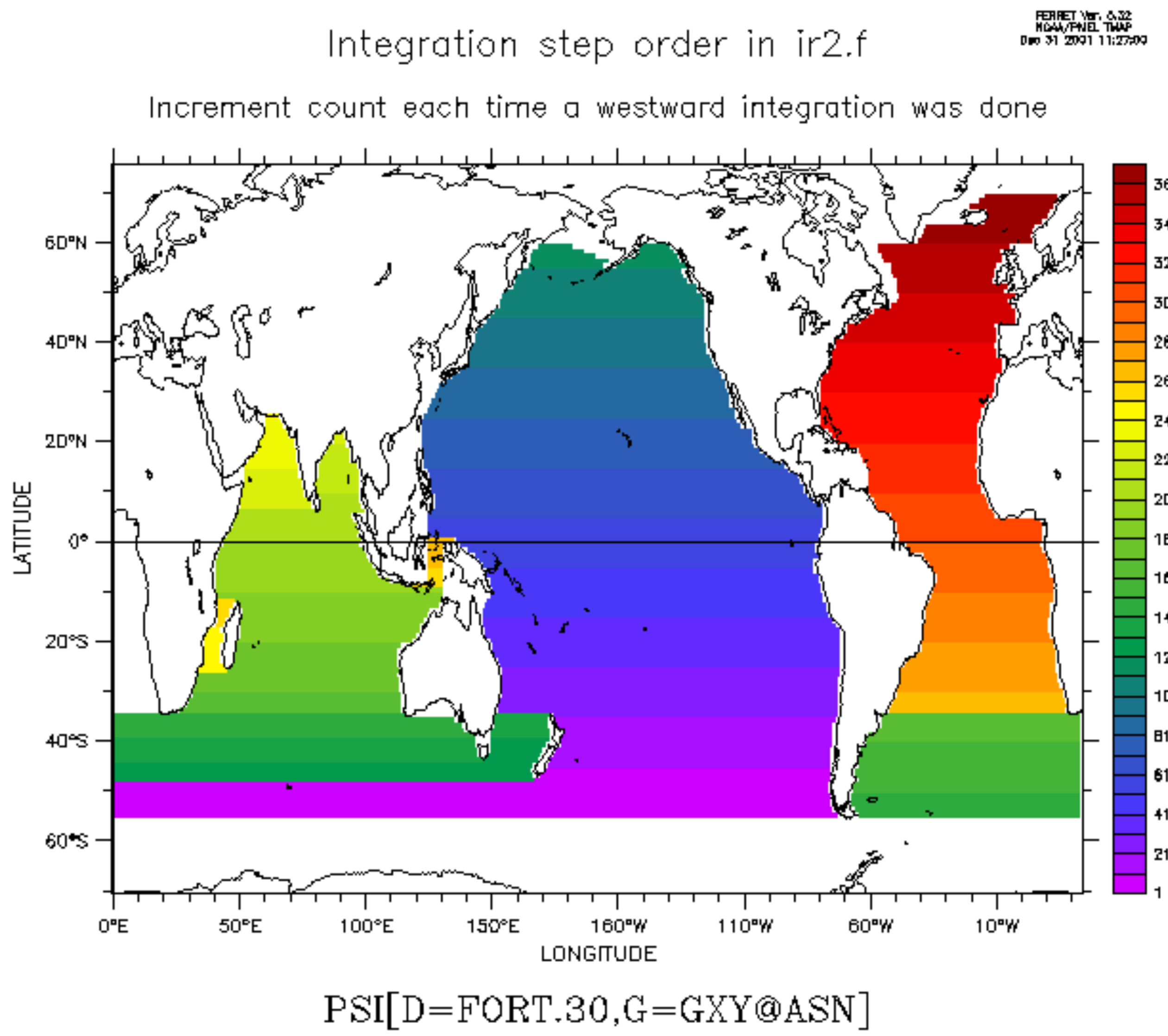
$\Rightarrow$ First find the Island Rule T_0, then find the Sverdrup transport T_{Sv}.

Subtract to find the WBC at each latitude.

A tedious calculation!

DO 100, n=1, (seems like ∞)
- Start at Cape Horn.
- Work up each west coast in turn.
- Integrate the alongshore wind.
- At each latitude work west until an island or continent is reached. Save the value.
- Move north one latitude, repeat.
- Return to each island, work west again ...

100 CONTINUE

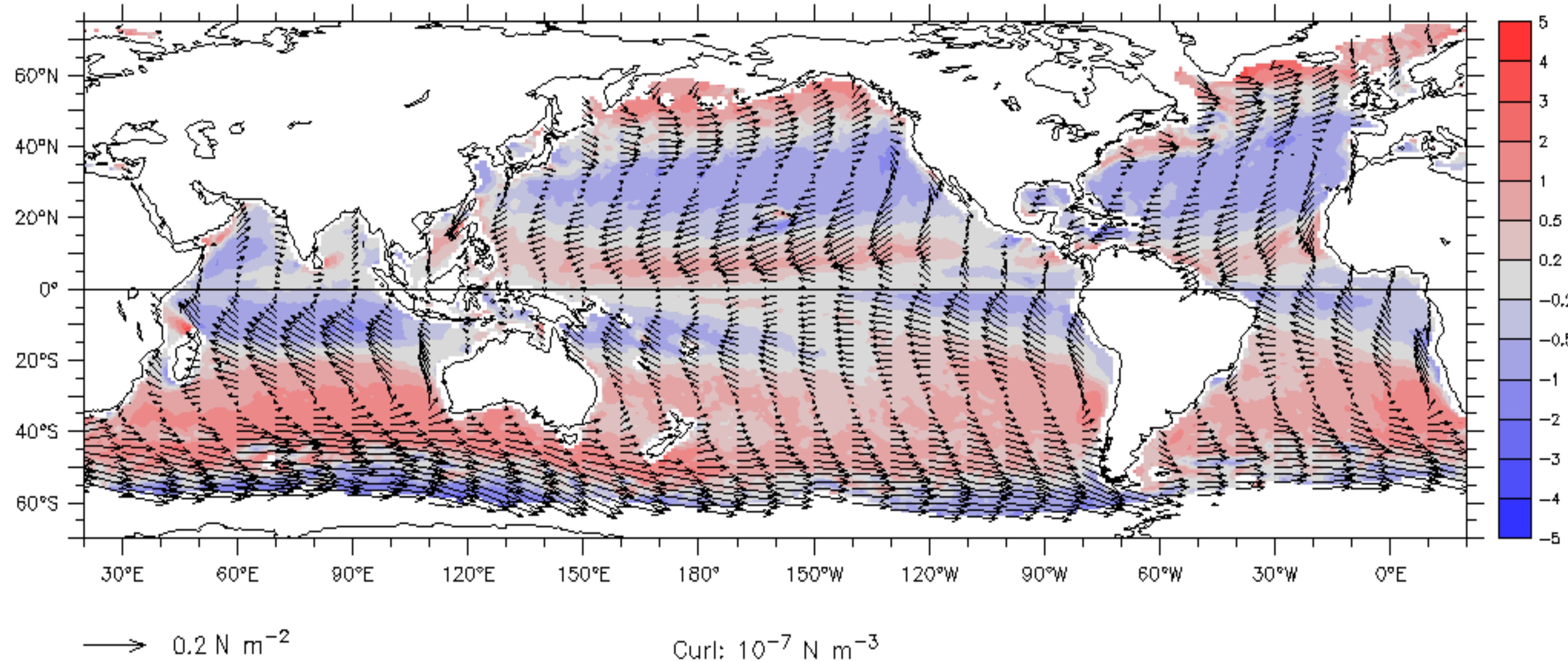


This example calculation took 370 steps (and did not include N.Caledonia or the other small Pacific islands).

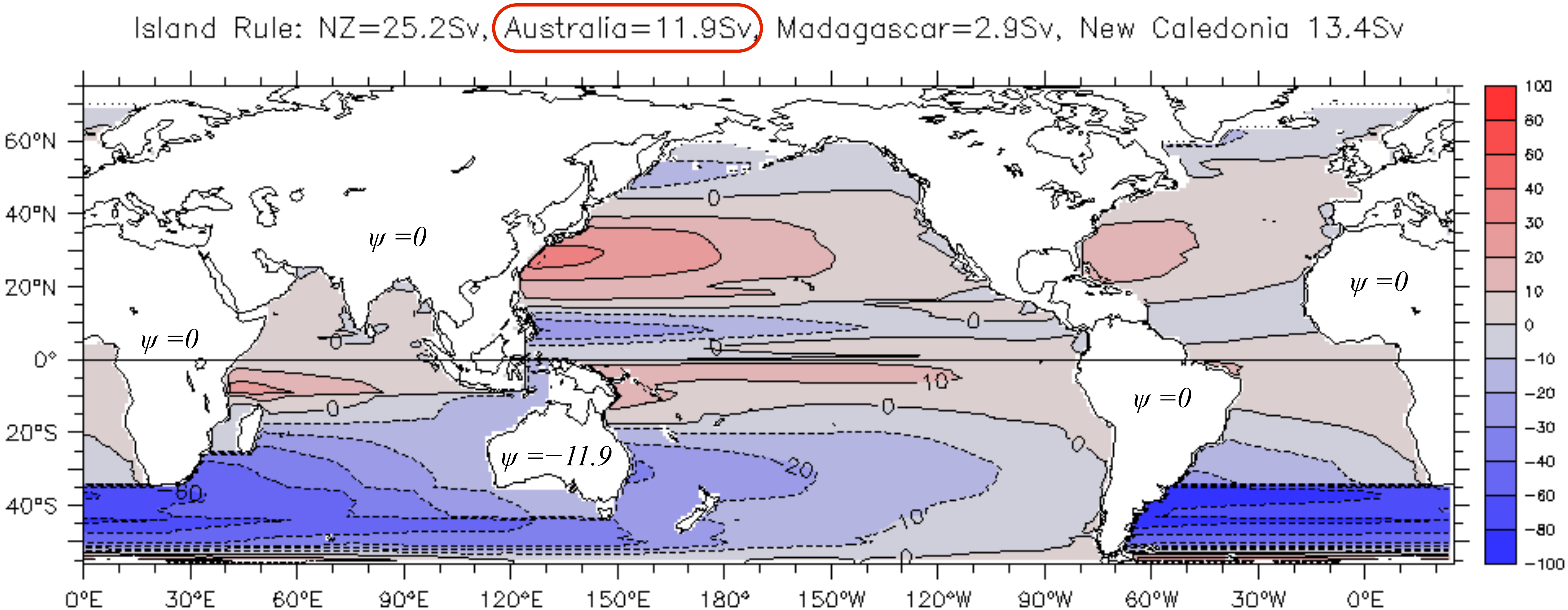
A further problem is the difficulty of defining the "coastal wind" from data (see extra slides below).

Winds and Curl for the calculation

Mean ERS winds (1990s, relatively crude)



Island Rule streamfunction (ERS winds [1990s])



Godfrey's triumph: estimating the transport of the Indonesian Throughflow from wind data alone*. (Direct observational estimates are 12-16Sv)

Since the streamfunction is constant around the entire coastline of the Americas and Asia/Europe/Africa, the value at Australia (here 11.9Sv) is the transport through the S.Pacific and thus the ITF.

But the method clearly fails in the Antarctic Circumpolar Current.

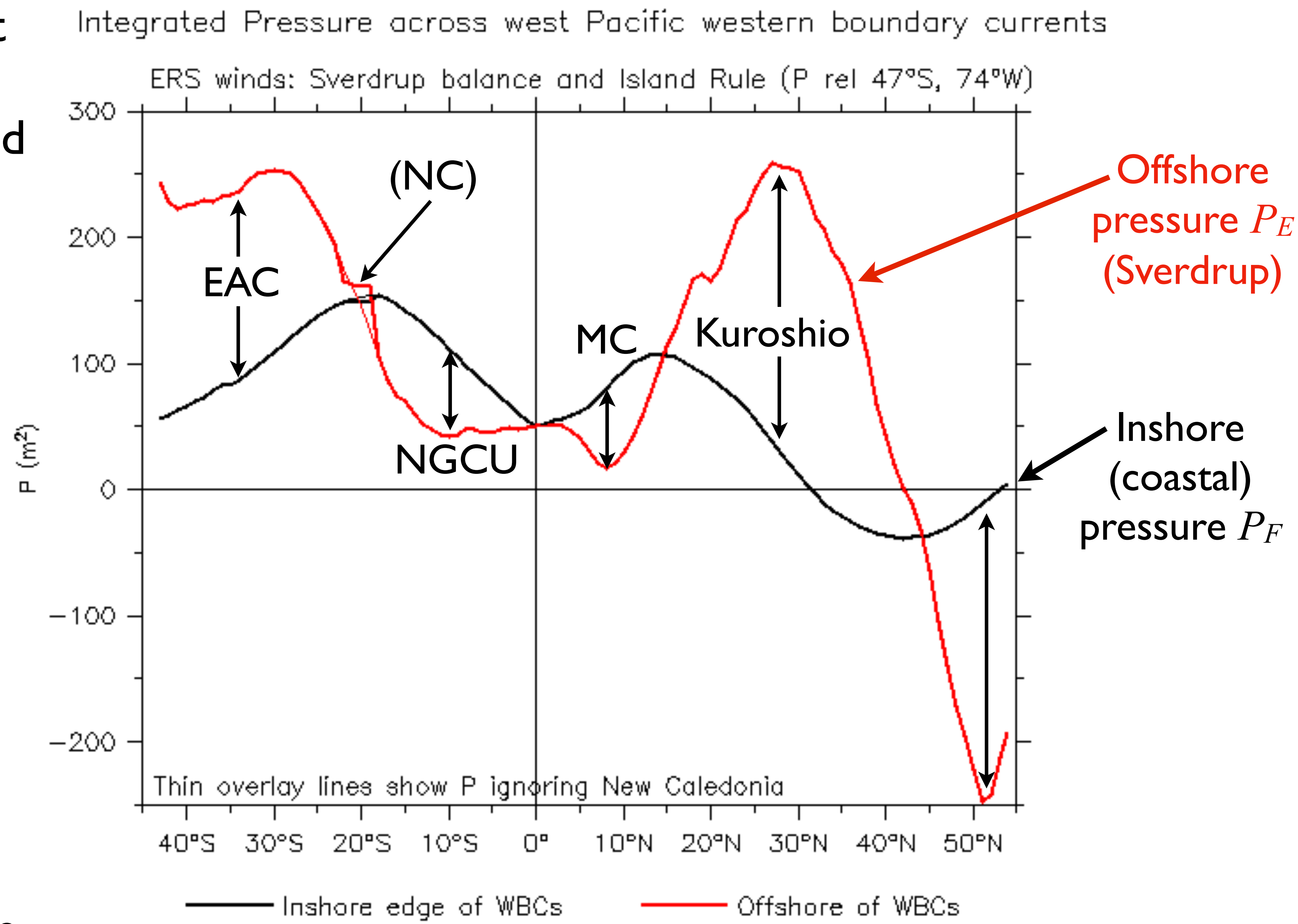
(* May be compensating errors in the ITF, whose narrow channels are not truly "inviscid")

Island Rule pressure gradients across Pacific WBCs

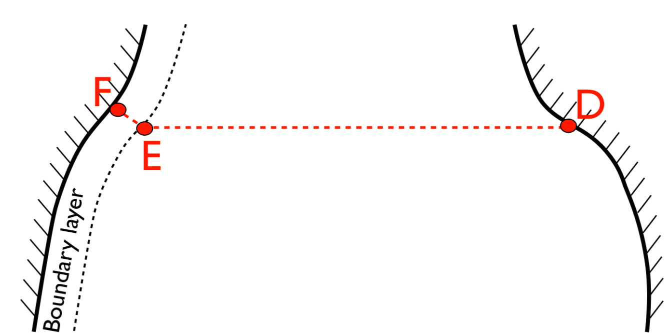
With T_0 known, the pressure gradient across WBCs can be derived from the wind using (8).

Where offshore (P_E) is higher, the WBC is poleward; where inshore (P_F) is higher, the WBC is equatorward.

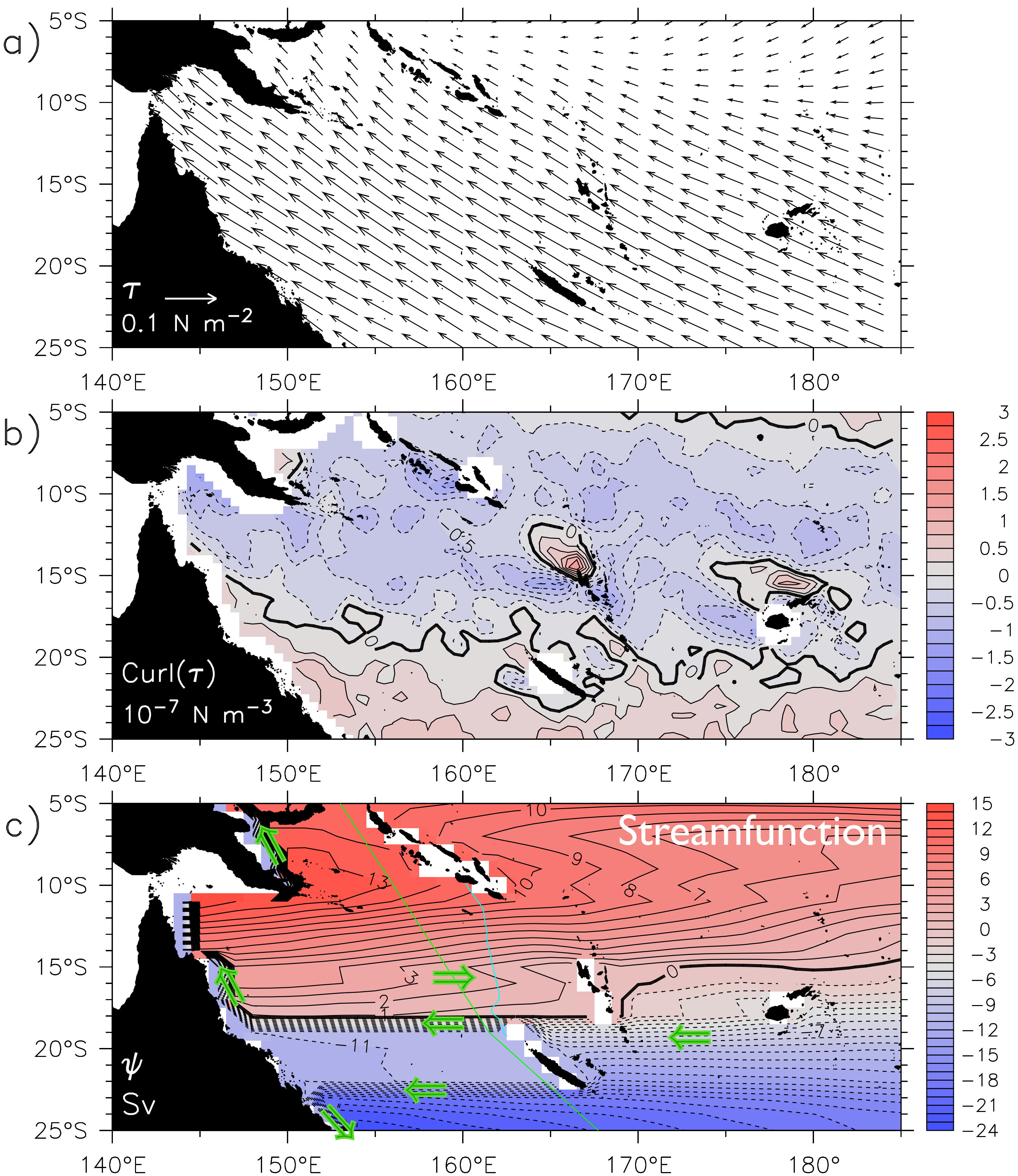
Example of N.Caled. shows flattening of pressure behind an island: dP/dy on the west coast is due entirely to alongshore wind (small).



$$P_D - P_F = \frac{1}{g}\left[\int_{FED} \tau^l \, dl + f\, T_0\right] dx \quad (8)$$



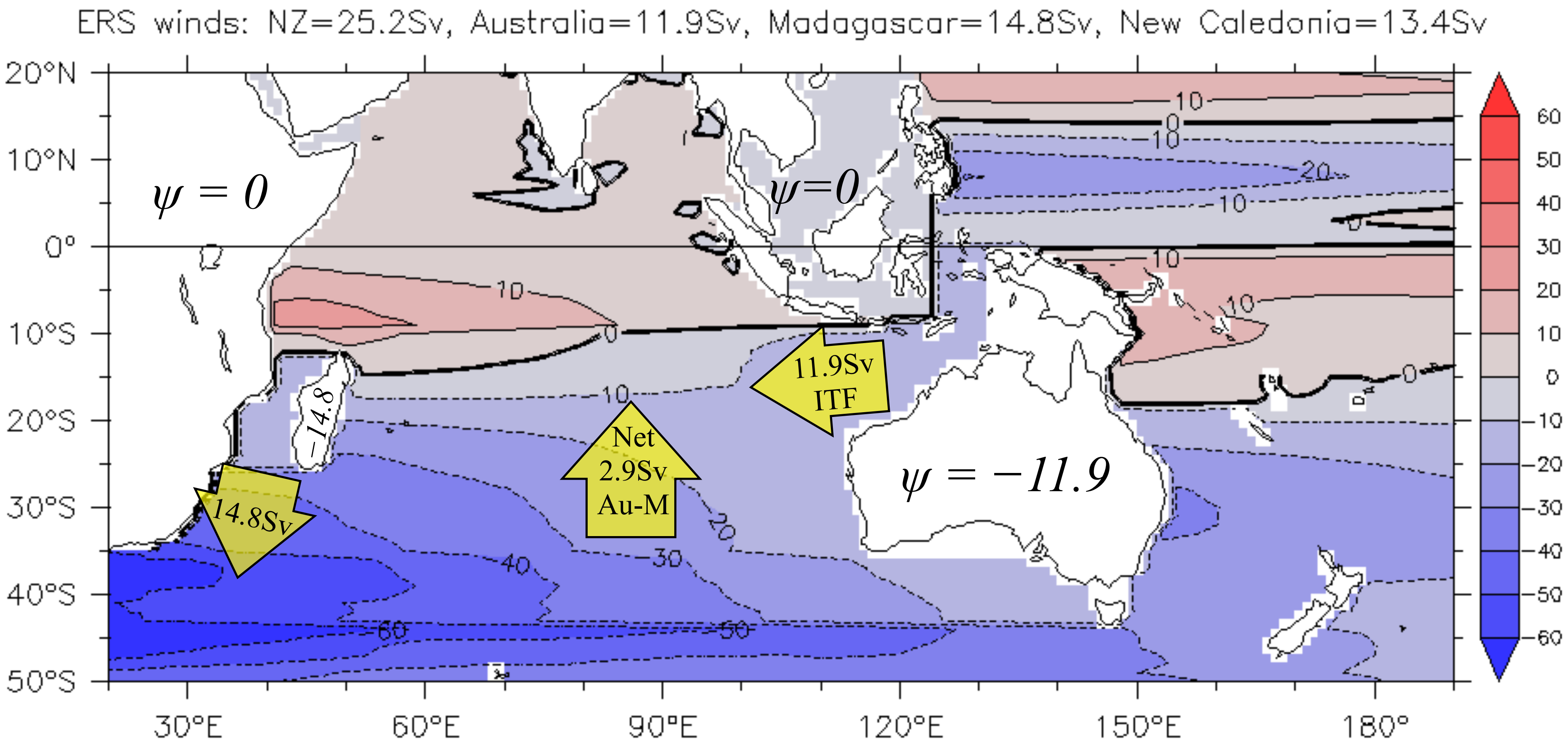
Island Rule streamfunction details in the SW Pacific

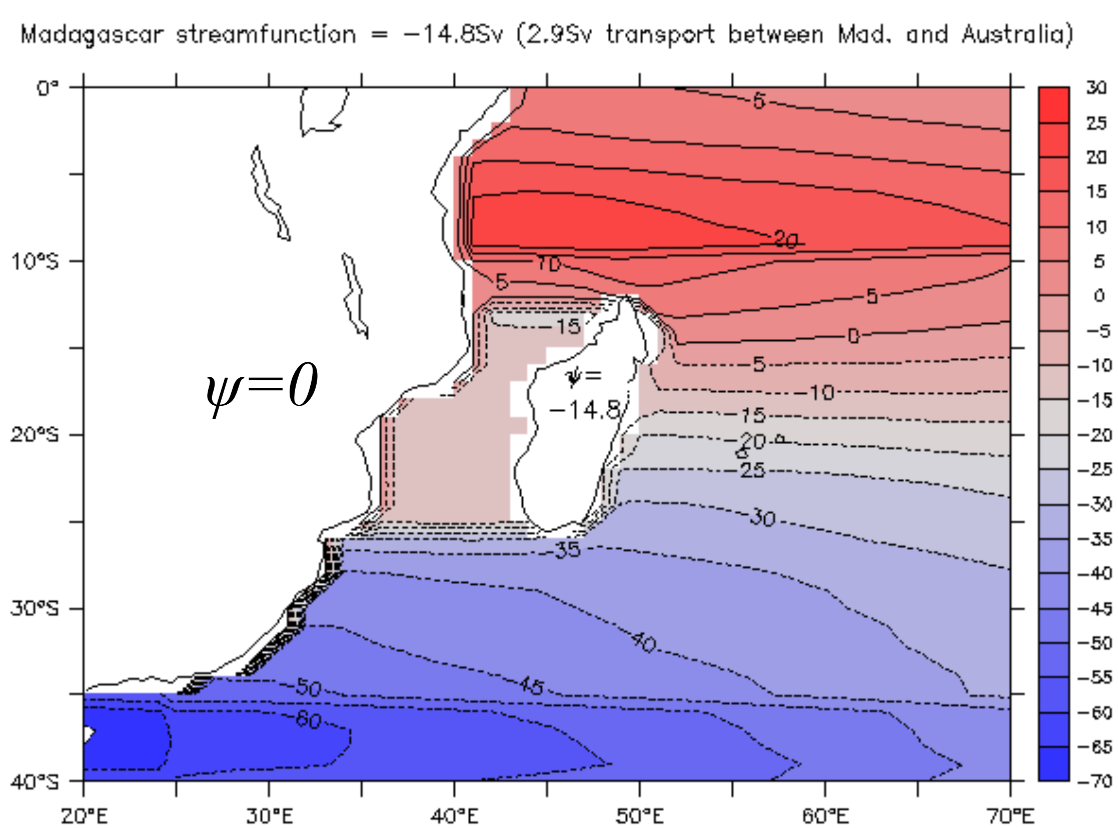


Qiu et al (2009)

- NQC/NGCU ~ 25Sv at 10°S
- Flattening ψ west of New Caled.
  North Caledonian Jet
- Wind dipoles behind Fiji
  and especially Vanuatu:
  IR resets ψ to constant:
  Coral Sea Countercurrent
- Similar situation west of Hawaii
  (Hawaiian Lee CounterCurrent)
- WBC east of NC ~ 10Sv
- Solomon Islands "island term"
  (see below) is <u>not</u> so small.
  Because winds are much
  larger inside the Solomon Sea,
  the "island term" predicts
  about 1.3Sv (clockwise),
  for a net southward WBC
  along the east coast. (real?)

Island Rule streamfunction details in the Indian Ocean





Flat streamfunction behind Madagascar: all the incoming zonal transport is distributed into two zonal jets (mostly turns to the south).

A more physically-intuitive derivation (1)

Although simple, the connection between the pressure difference across long ocean sections and the Sverdrup (vorticity) balance is not obvious. We usually think of WBCs as redistributing the interior (Sverdrup) zonal transport.
A more physically-intuitive derivation makes this connection explicit (also by Godfrey (1989)).

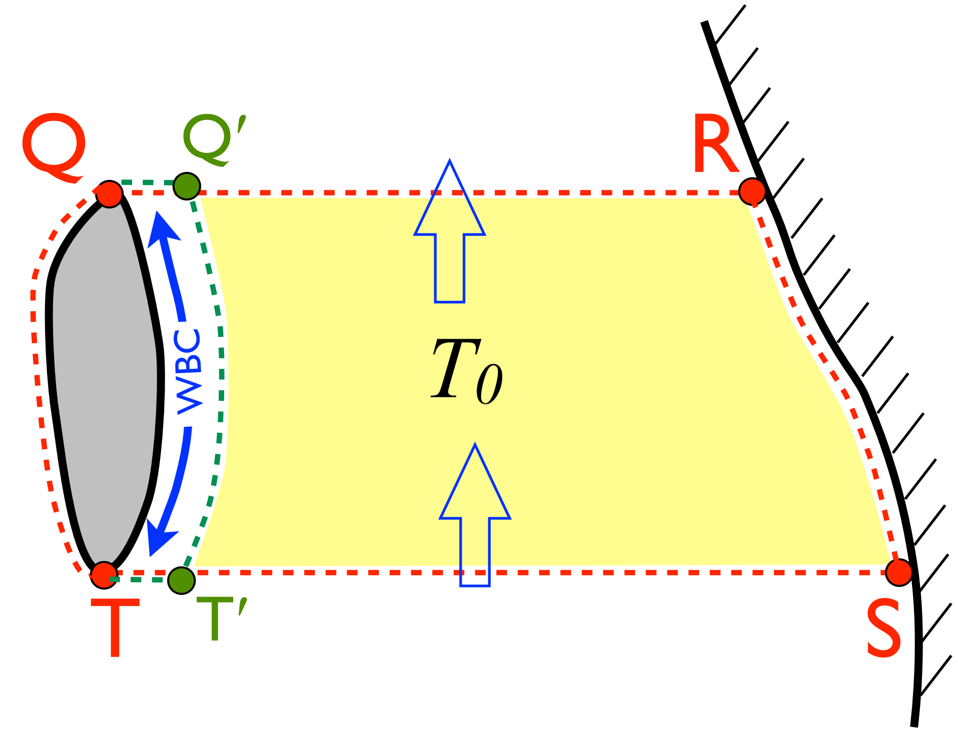
$$T_0 = \frac{1}{f_Q - f_T} \oint_{TSRQT} \tau^l dl \qquad (12)$$ (the Island Rule)

Add and subtract the integral $\tau^l \cdot dl$ along the inner path $Q'T'$ (green line just east of the boundary layer):

$$T_0 = \frac{1}{f_Q - f_T} \left[\oint_{\substack{TSRQ \\ outer}} \tau^l dl + \int_{\substack{Q'T' \\ inner}} \tau^l dl - \int_{\substack{Q'T' \\ inner}} \tau^l dl \right]$$

$$= \frac{1}{f_Q - f_T} \left[\oint_{\substack{T'SRQ'T' \\ inner}} \tau^l dl + \oint_{Island} \tau^l dl \right] \qquad (13)$$

Simplify (next page) (Yellow region) — Wind integral around the island ($TT'Q'QT$)



A more physically-intuitive derivation (2)

Green's Theorem: $\oint\limits_{T'SRQ'T'} \tau^l dl = \iint\limits_{T'SRQ'T'} Curl(\tau)\, dx\, dy$, and: $\beta \equiv \frac{df}{dy} \approx \frac{f_Q - f_T}{\Delta y}$

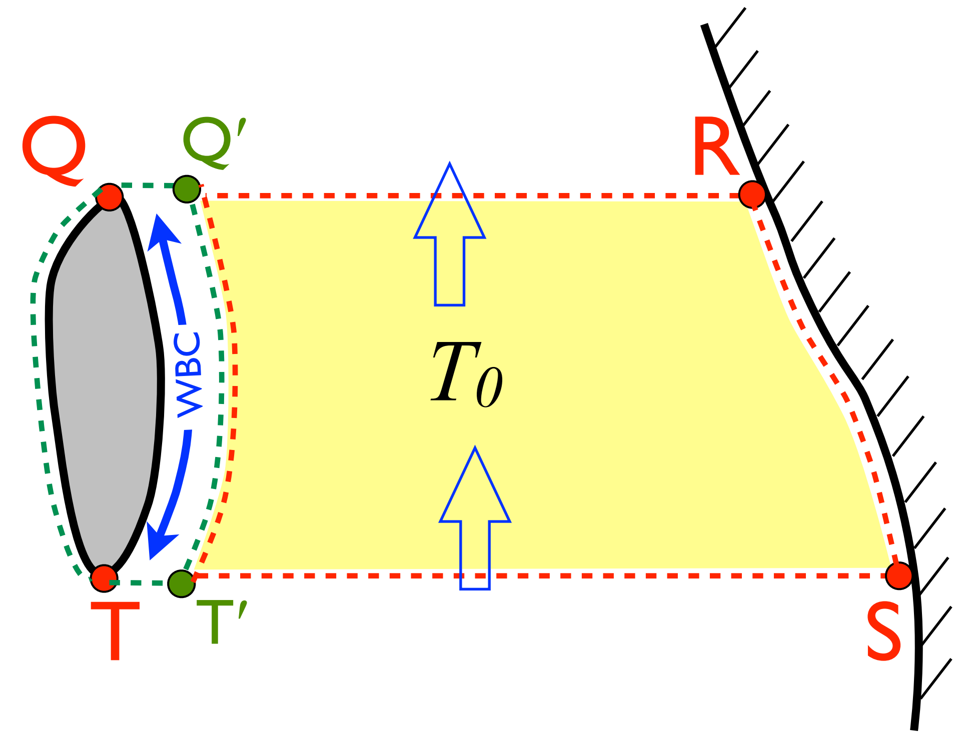
The inner path integral in (13) can be simplified:

$$\frac{1}{f_Q - f_T} \oint\limits_{\substack{T'SRQ'T' \\ inner}} \tau^l dl = \frac{1}{\beta \Delta y} \iint\limits_{T'SRQ'T'} Curl(\tau)\, dy\, dx$$

$$= \int_{x=T'Q'}^{x=RS} \underbrace{\frac{1}{\Delta y} \int_{y=TS}^{y=QR} \frac{Curl\tau}{\beta} dy}\, dx$$

$$= \int_{x=T'Q'}^{x=RS} \overline{V_{Sv}}\, dx = \overline{T_{Sv}}$$

where the overbar is the meridional average



$\overline{T_{Sv}}$ is the meridional-average Sverdrup transport in the yellow box east of the island. (Note T_0 includes the WBC)

A more physically-intuitive derivation (3)

Thus get an alternate Island Rule:

$$T_0 = \overline{T_{Sv}} + \frac{1}{f_Q - f_T} \oint_{Island} \tau^l \, dl \qquad (14)$$

Simple Sverdrup solution east of the island plus a usually-small island term.

Compare the original:

$$T_0 = \frac{1}{f_Q - f_T} \oint_{TSRQT} \tau^l dl \qquad (12)$$

Long path-integral of wind

In addition to being more intuitive, it is much easier to use (14) than (12):

- In practice, it is easy to find the Curl of the wind, thus the Sverdrup transport, but computing the along-path wind integrals is tedious (slide 25). And we need T_{Sv} anyway to get T_{WBC} (slide 24).
- The circumisland term in (14) is usually small and can often be ignored, but the around-basin wind integral in (12) is large (it’s the whole solution).

(See slides describing calculation methods in Extra Slides below)

A more physically-intuitive derivation (4)

The alternate Island Rule:

$$T_0 = \overline{T_{Sv}} + \frac{1}{f_Q - f_T} \oint_{Island} \tau^l \, dl \qquad (14)$$

Simple Sverdrup solution east of the island plus a usually-small island term.

Implications:

The island circulation (2nd term on RHS of (14)) is generally small, so (14) states that:

- The WBC against the island is due only to the <u>variation</u> of T_{Sv} with latitude. (if T_{Sv} is constant in y, there will be no WBC: recall $T_{WBC}(y) = T_0 - T_{Sv}(y)$)
- There must be a zero WBC at some point along the island (WBC bifurcation).
- Unlike the path-integral of wind east of a continent, which can support a large alongshore pressure gradient (p16-19), the WBC against an island will generally provide only a small part of the <u>net</u> transport east of the island.
- Since $U_{Sv} = -dT_{Sv}/dy$, (14) emphasizes the role of the Sverdrup <u>zonal</u> transport feeding mass to the western boundary region (the intuitive picture).

Conclusion A: Problems and limitations

The extremely simple Ekman+geostrophic dynamics here explain a lot, but ...

- Linear: no WBC overshoot/recirculation ....
- No mixing!
- No information about vertical structure
- How to define "coastal wind"?
- Eastern boundary condition fails at a zonal coast (Central America and West Africa)
- Does not work in the Southern Ocean (but no linear solution does)
- Wind-forced $\Longrightarrow$ no thermohaline circulation:
- Some islands might be within the frictional western boundary layer: Violate the assumption that the island is independent.
  $\Longrightarrow$ Japan, Solomon Islands (?), Taiwan (Papers by Wajsowicz, J. Yang)

Slides below show how these ideas can be expanded to time-dependent flows.

Extensions: Time-dependent Island Rules

Time-dependent Island Rule (1)

Three possibilities have been considered:

1) A succession of steady states (e.g., series of time averages)
2) A "small" island (KW quickly adjust pressure) (Firing et al)
3) A "large" island (WBC adjustment ~ as if continent)

→ Only (2) is a "time-dependent Island Rule".

(1) and (3) are work-arounds.

(2) and (3) take the incoming zonal transport as key
("A more physically-intuitive derivation" above),
and use a Rossby wave model to estimate the incoming U_g.
Where the steady I.R. finds the WBC transport as the difference between interior Sverdrup transport T_{Sv} and the Island Rule streamfunction T_0,
(2) and (3) get the transport by an integral over incoming U_g.

(3) is not an "Island Rule"! It requires external information.
But it can be useful in interpreting time-dependent circulation along the east coast of Australia.

Time-dependent Island Rule (2)

1) The simplest: Assume that a Sverdrup balance is established in a short time compared to the timescale of interest. Use the slowly-varying time-average wind and find an succession of "steady" I.R. solutions.
→ A poor assumption, except for very long timescales.

2) Firing et al. derive a rule for a "small" island (Hawaii), so Kelvin waves pass quickly around the island, and the adjustment is complete in a timescale much shorter than the frequency of interest.
→ A complete solution is possible from the wind alone, like the I.R. (discussion next pages)

3) Kessler and Gourdeau consider a "large" island (Australia-New Guinea) so that the island pressure takes much <u>longer</u> to adjust than the timescale of interest.
The WBC can be deduced from the incoming zonal transport (from a Rossby model), <u>IF</u> a boundary condition at the poleward edge of the western boundary can also be specified.
→ It is not self-contained.

The Firing et al (1999) Time-dependent Island Rule (TDIR)

The TDIR applies to a "small" island.
"small" = coastal Kelvin waves travel quickly around the island (a much shorter time than the timescale of interest).
(Firing et al consider Hawaii, at timescales longer than seasonal; and I have done this for the Solomon Islands)

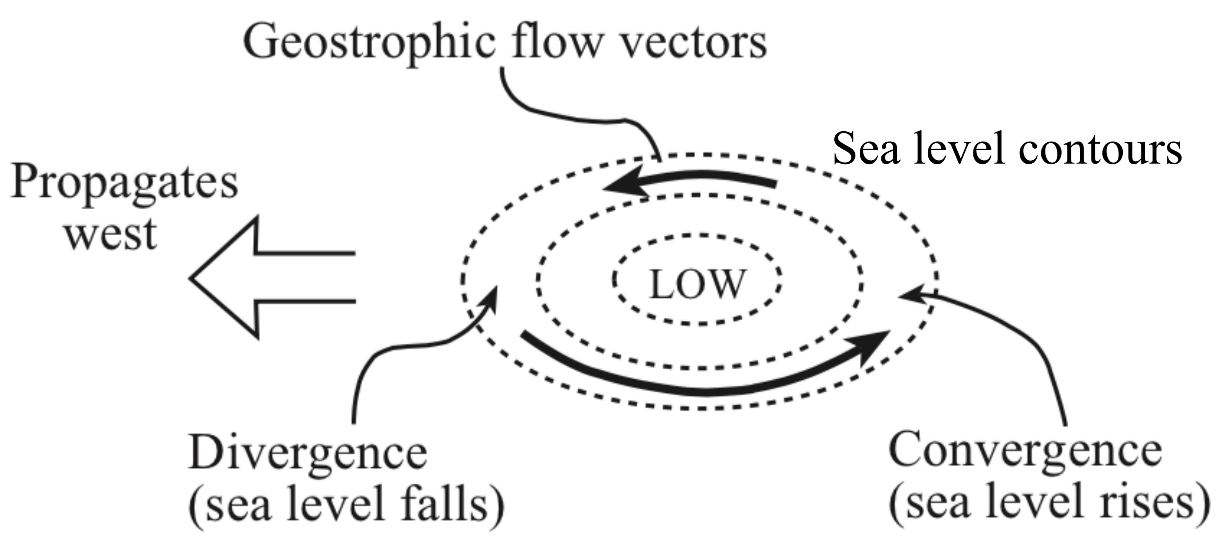
The short adjustment time means that the streamfunction around the island can be considered steady, like the mean Island Rule.

The TDIR then works by balancing mass in the western boundary layer.

The TDIR requires a means to specify the incoming zonal transport, usually from a wind-driven Rossby wave model.
First, we will consider Rossby waves and what happens when they arrive at a land boundary.

Rossby wave propagation

The mechanism of Rossby propagation
Example: Low sea level in the Northern Hemisphere



One way to understand Rossby waves
is to consider the <u>mass balance</u> of geostrophic flow on a β-plane.

On an f-plane, geostrophic flow is non-divergent, but on a β-plane,
for a given pressure gradient, u_g is larger closer to the equator (diagram).

Flow around an anomaly on a β-plane is unbalanced:

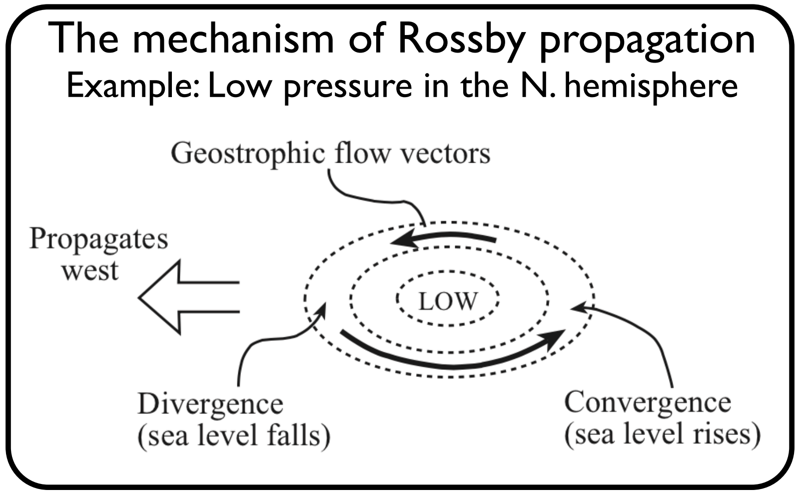
Currents around a low diverge in the west, converge in the east (diagram).

→ Sea level falls in the west, rises in the east:

Δt later, the low has "moved itself" west.

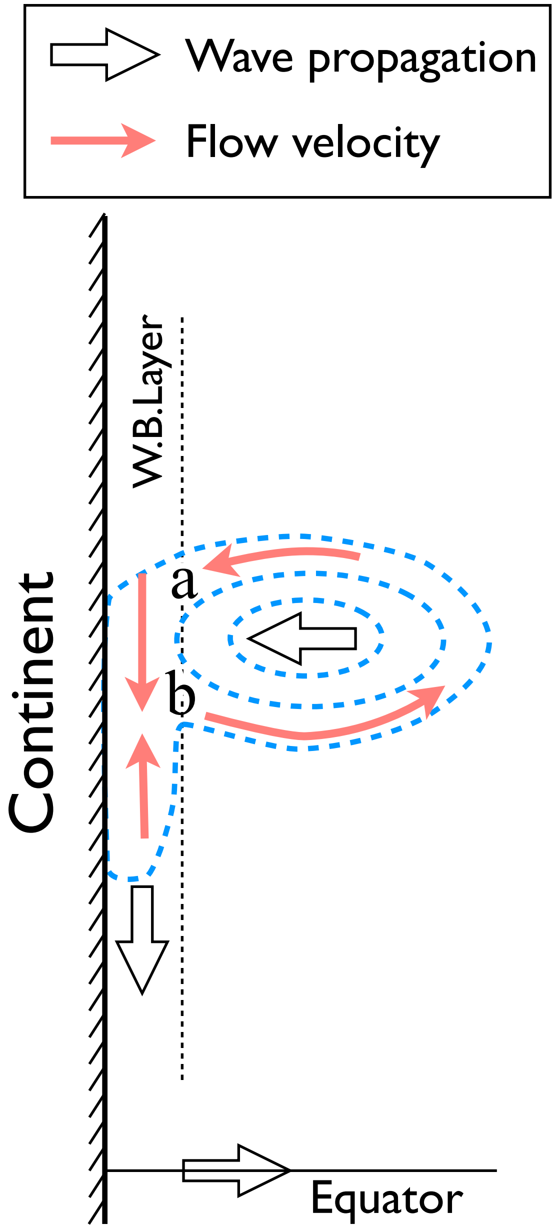
(The same thing works for a high [try it!])

What happens when the Rossby feature arrives at a continent?



Information only travels equatorward on a western boundary

(Because coastal Kelvin waves go equatorward on a western boundary)



East of the boundary layer, nothing has changed.

At the coast, again balance mass. Consider points a and b:

North of a, with no waves to carry a signal there, coastal pressure cannot change:
No flow exists. The anomaly can only turn south.

At b, the eastward outgoing current is larger than the arriving southward current:
There is a mass imbalance at b.

In the open ocean, this imbalance was resolved by the feature moving west,
but that cannot happen against a coast.

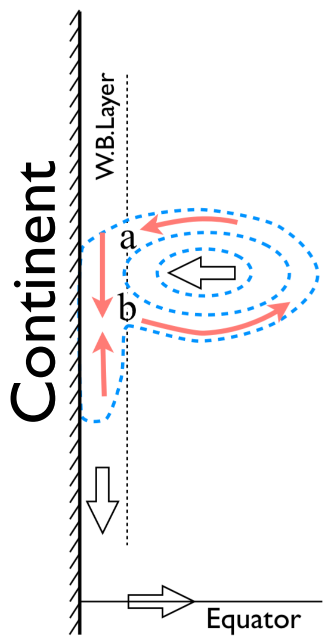
The needed inflow to balance mass at b must come from south along the coast!

Low pressure and northward flow extends equatorward in a coastal Kelvin wave.

Note that the pressure propagates as a wave,
but the velocity is due geostrophically to the pressure.
They do not always have the same direction!

The anomaly propagates to the equator, then eastward along the equator:
In this case (low pressure anomaly), we find
northward flow along the coast and eastward flow on the equator.

What happens when the Rossby wave arrives at a (small) island?

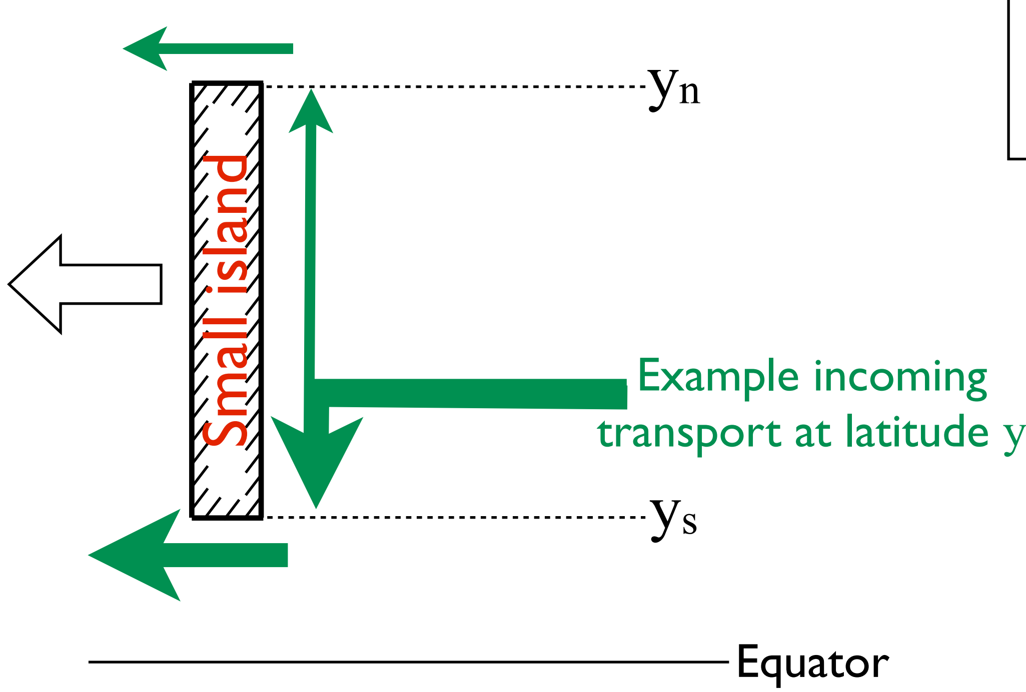


Information only travels equatorward on a western boundary,
but now the Kelvin wave can go <u>around</u> the island!
→ Coastal pressure adjusts quickly around the entire coast.

Unlike the continent, pressure *can* change north of a:
The imbalances can be satisfied by currents flowing either north or south.

Firing et al derive a simple rule, giving the WBC transport in terms of the incoming transport at each latitude:

> Against an <u>island</u>:
> "This vorticity constraint says that, in the absence of circumisland wind, an inflow to the boundary current at latitude y will split, with fraction $(y - y_s) / (y_n - y_s)$ going north and the remainder going south."
>
> Firing et al. (1999, JPO)



Thus:

Use a Rossby model to estimate the incoming transport at each latitude (green in example at left).

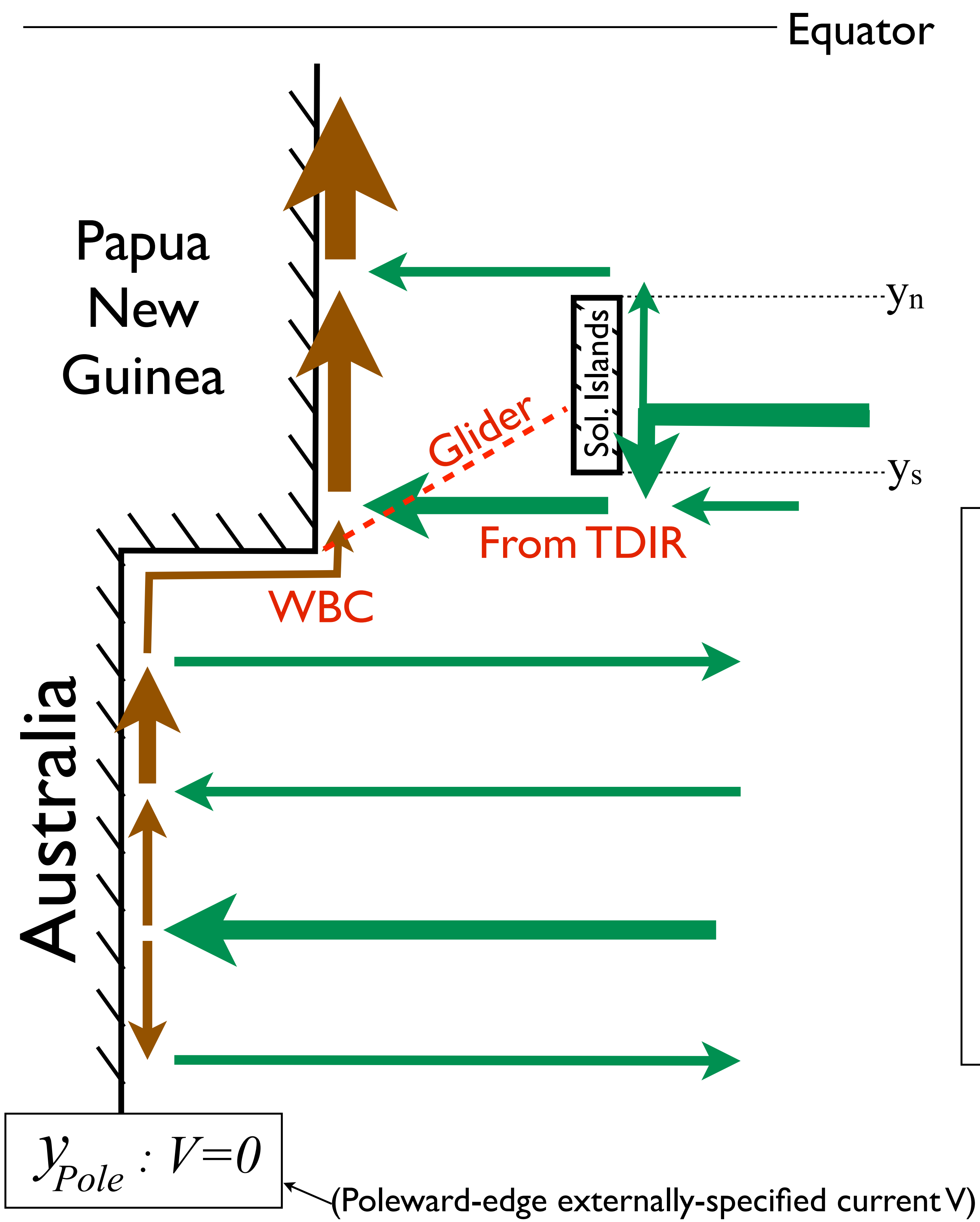
Split each element of transport according to the rule.
Sum the contributions to get the total WBC.

At the tips of the island, the transport flows west in jets.

West of the island, the adjusted pressure continues to propagate west.

Combine the large and small-island Island Rules

(Diagnosing the transport measured by gliders crossing the Solomon Sea)



Example currents!

(Would come from a Rossby model)

Against a continent,
western boundary transport
is the equatorward integral
of incoming zonal transport.
(Balance mass integrating equatorward)
(Must specify the WBC at the poleward end)

$$\int_{WB\ layer} V(y)dx = -\int_{y_{Pole}}^{y} U_{RW}\, dy$$

Extra slides below

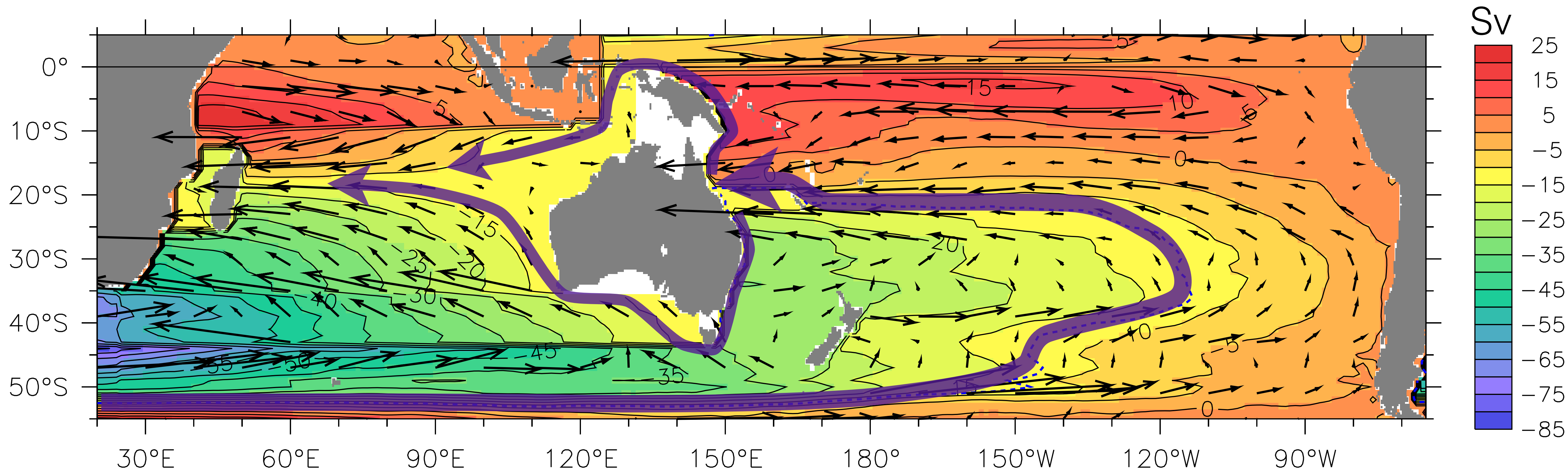
The linear wind-driven circulation: the ITF is open ... Godfrey Island Rule

Some flow exits the Pacific through the ITF: Increases the LLWBC of the tropical gyre

If the ITF were closed:

All the incoming transport would exit back to the south

Two gyres: Subtropical would be closed by a ~30 Sv southward WBC,
Tropical would be closed by a ~10 Sv northward WBC.



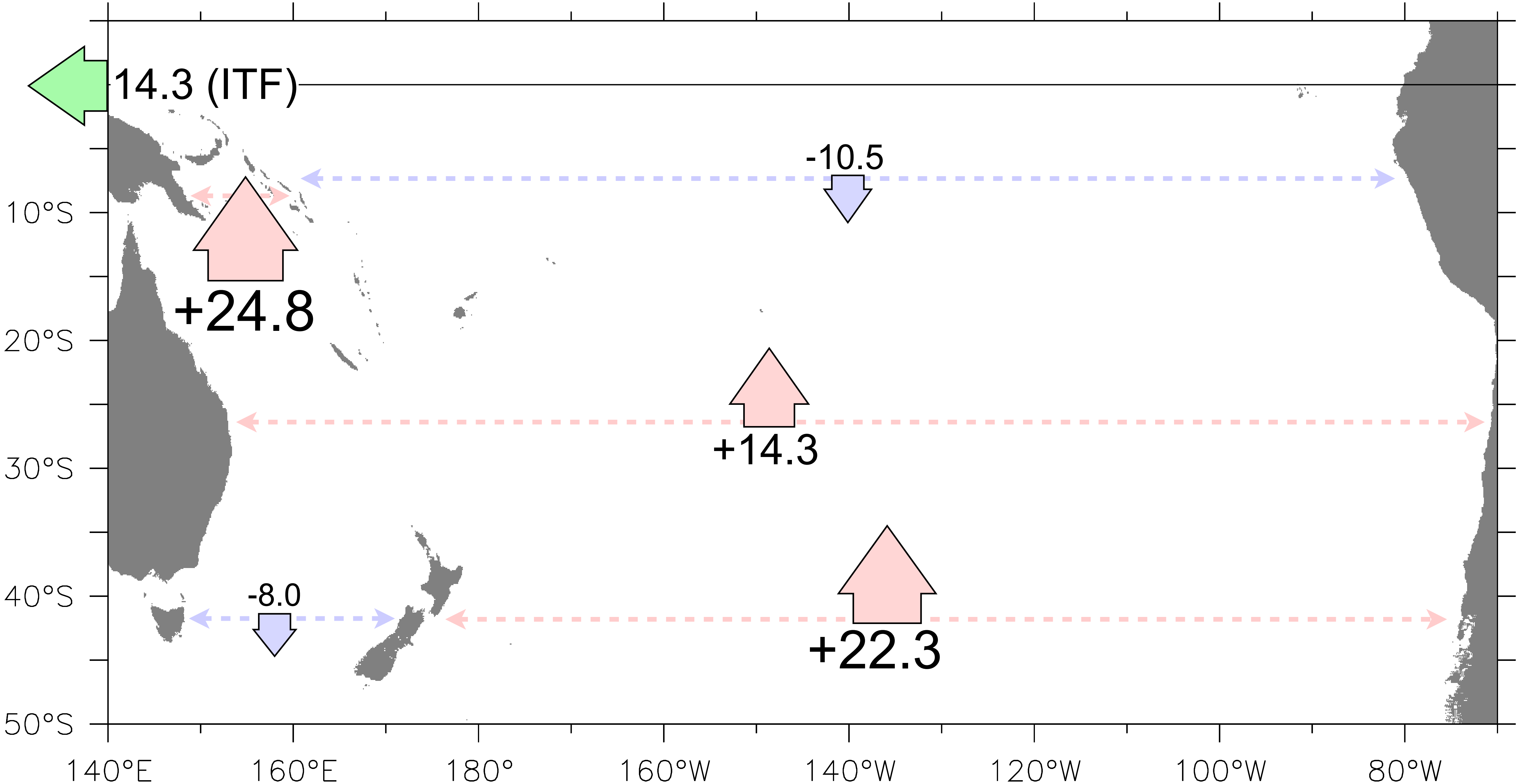
Island Rule circulation

2 great simplifications: Linear, vertically-integrated

S. Pacific Sverdrup (Island Rule) mean transport

CCMP-ASCAT winds 1990-2013 (deduction from wind data only)

Size of arrows and numerical values (Sv) varies as the (Transport)$^{1/2}$



The Island Rule predicts that ITF transport is ~14 Sv.

Thus: NGCU transport increases from ~11 Sv (no ITF) to ~25 Sv,

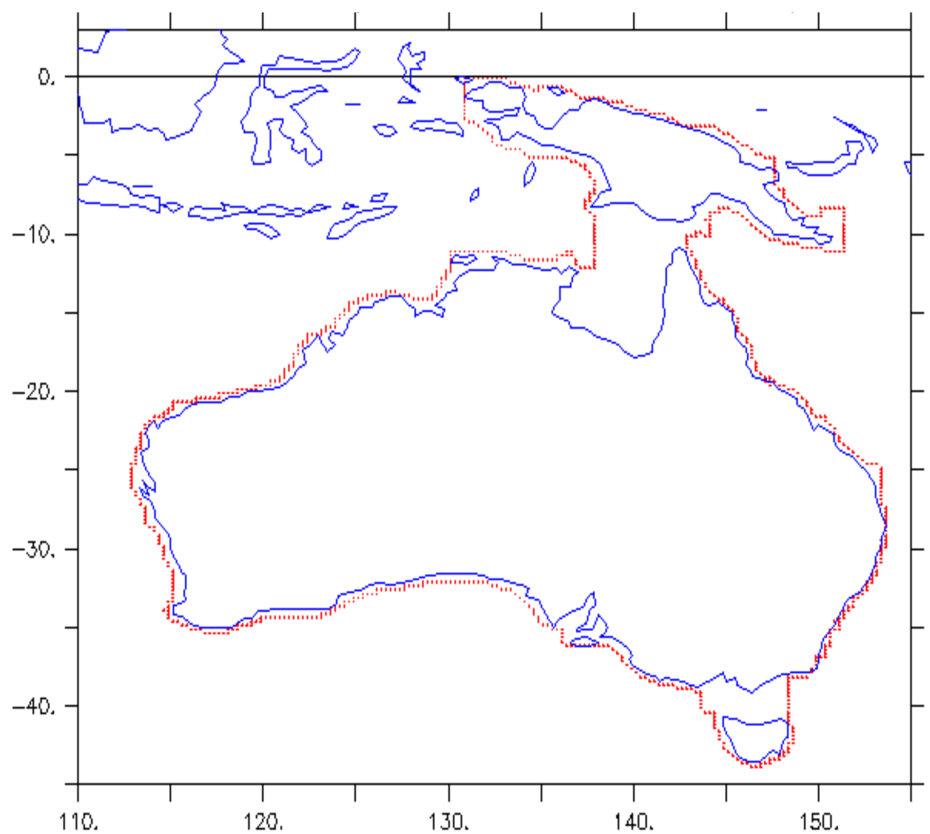
EAC transport decreases from ~30 Sv to ~16 Sv

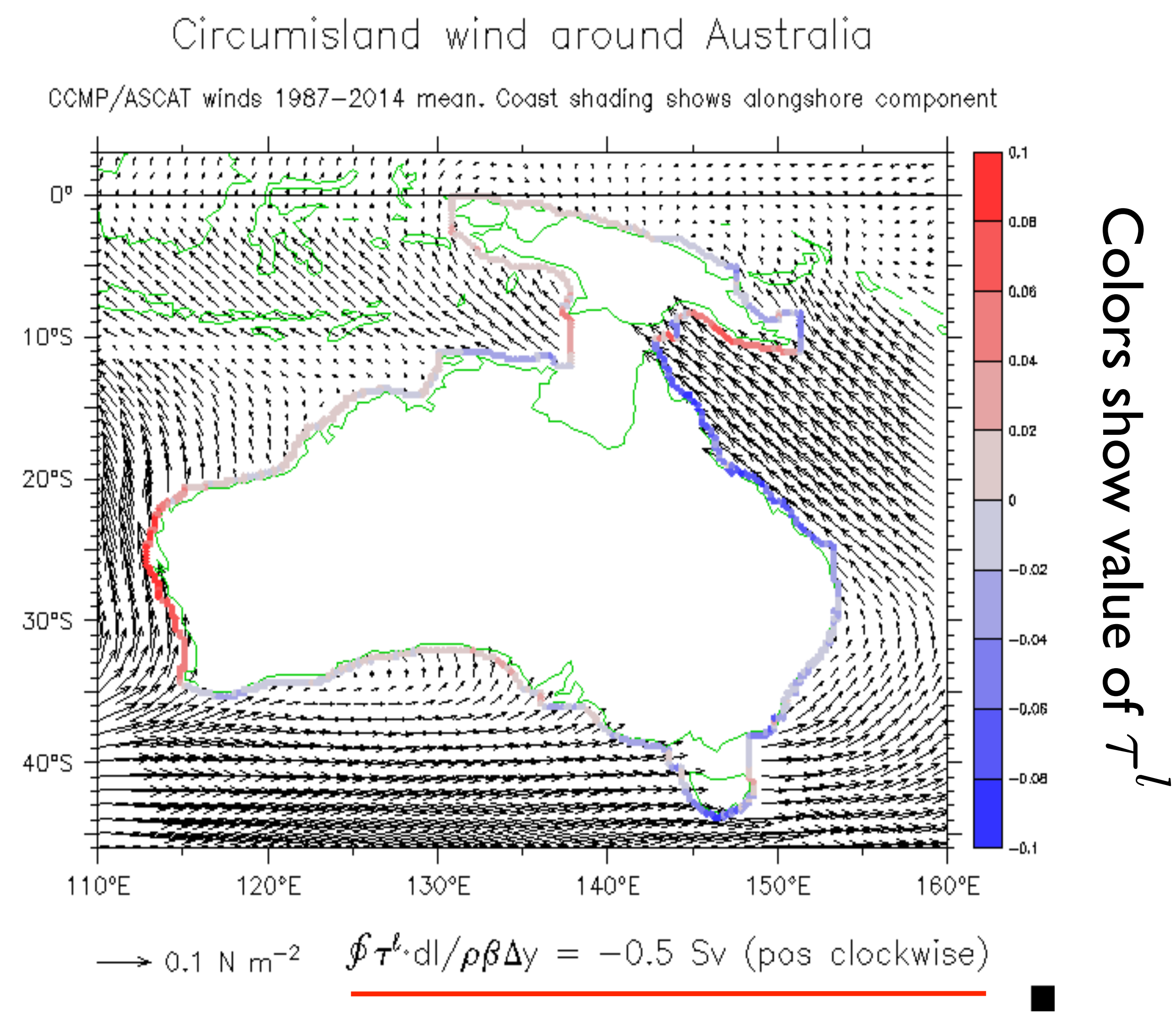
The circum-island term

The "alternate" Island Rule: $$T_0 = \overline{T_{Sv}} + \frac{1}{f_Q - f_T} \oint_{Island} \tau^l \, dl \qquad (14)$$

1) The circum-island term

750 points define the coast of Australia





1. Find the coastal points, in order!
2. Find the component of wind parallel to the coast at each point $\tau^l \cdot dl$.
3. Integrate (clockwise).

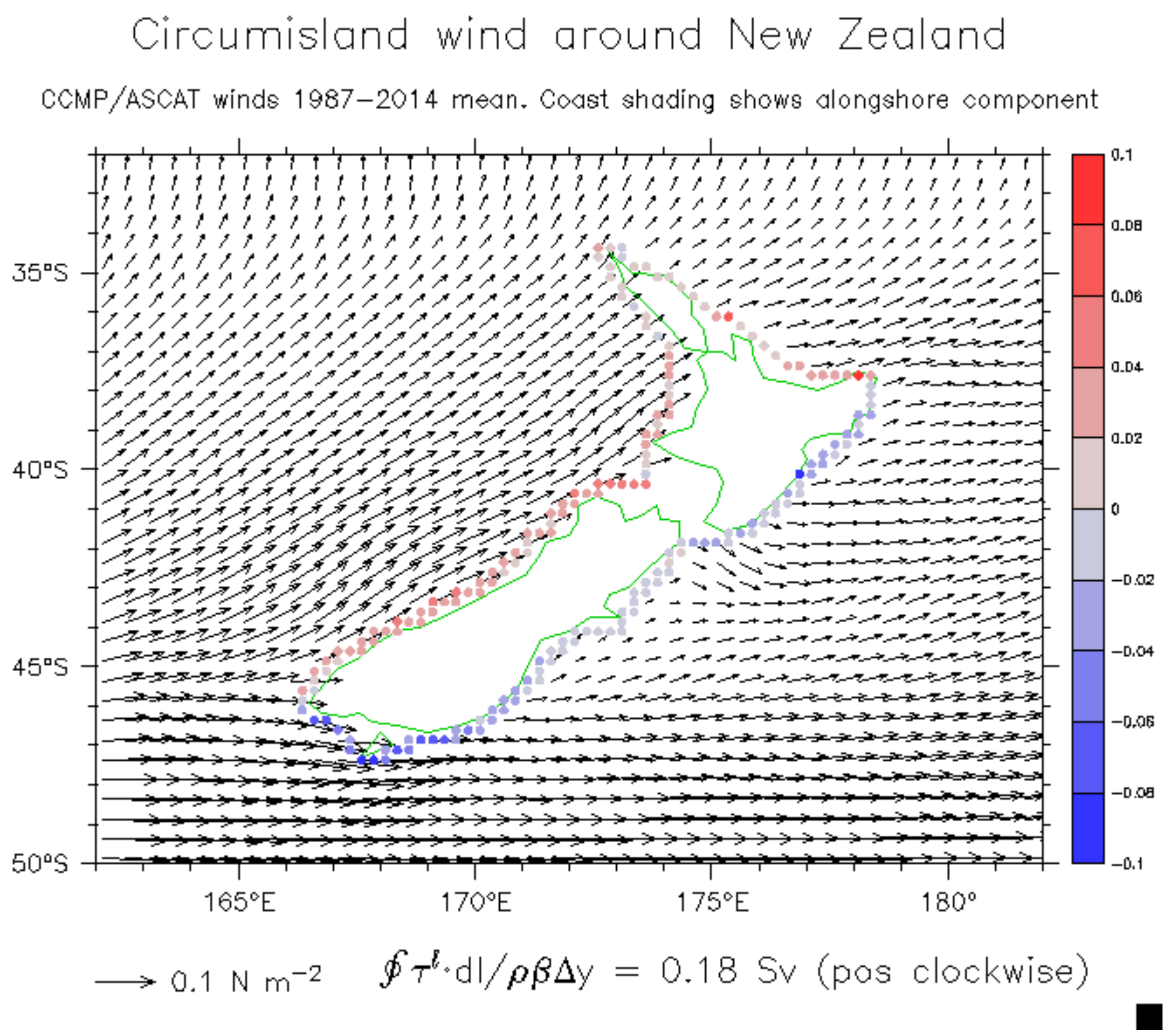
This is a very tedious calculation!
(And the result is usually small)

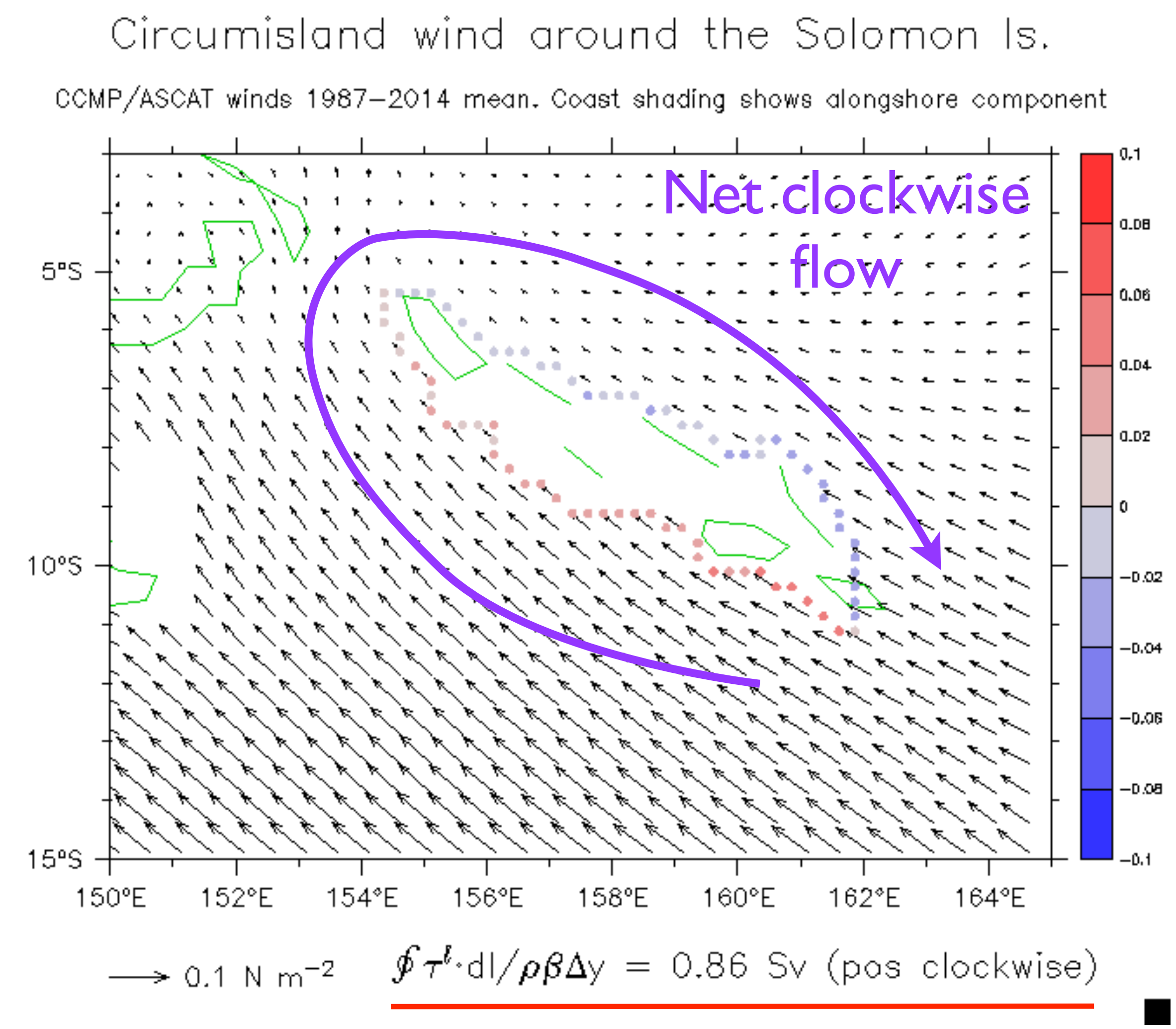
The integral is usually small because of cancellation.

(How far from the coast is the "coastal wind"?)

Examples of New Zealand and the Solomon Islands

Colors show value of τ^l at each coastal gridpoint





$$T_0 = \overline{T_{Sv}} + \underbrace{\frac{1}{f_Q - f_T} \oint_{Island} \tau^l \, dl}_{\text{The circum-island term}} \qquad (14)$$

The circum-island term

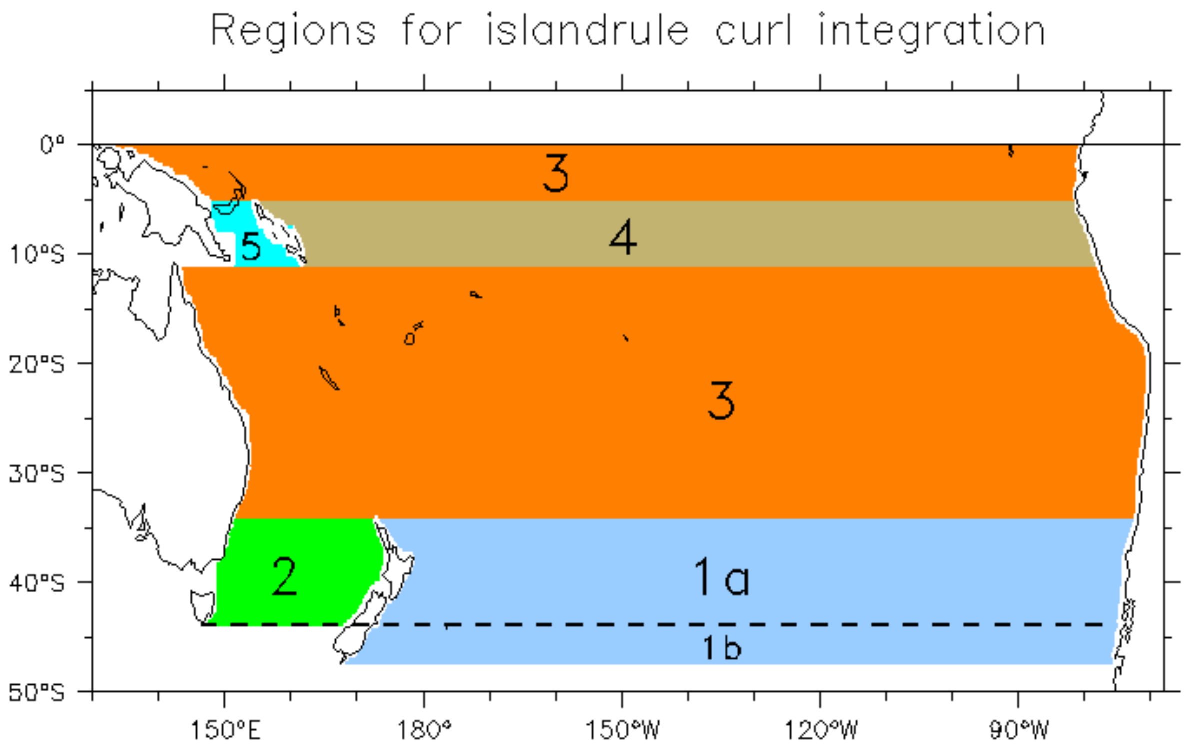
The Solomon Islands give a relatively large value because the winds inside the Solomon Sea are stronger than those in the Pacific to the east: Clockwise circulation due to the circum-island wind ... But still pretty small.

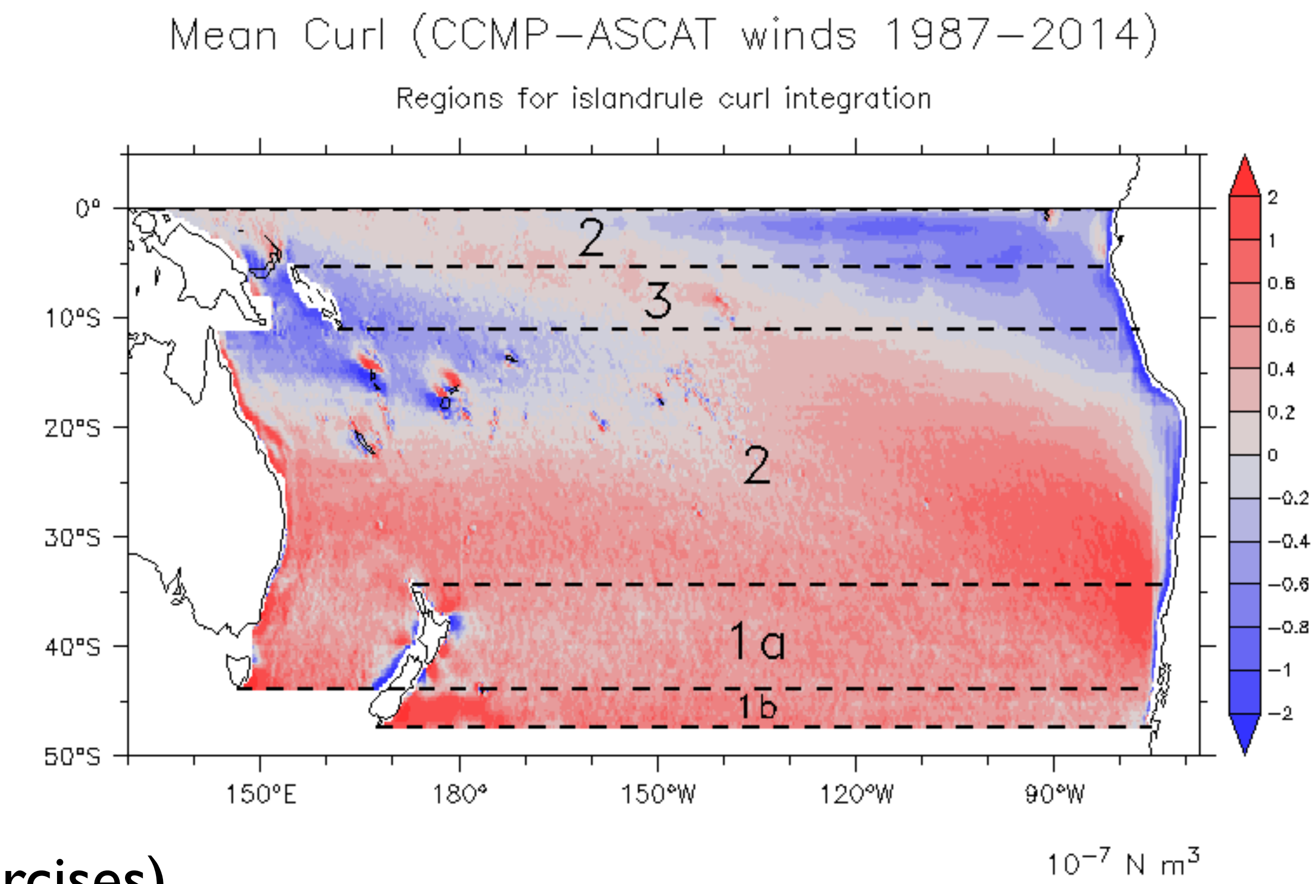
The interior Sverdrup term

2) The Sverdrup term

$$T_0 = \overline{T_{Sv}} + \frac{1}{f_Q - f_T} \oint_{Island} \tau^l \, dl \qquad \overline{T_{Sv}} = \frac{1}{\beta \Delta y} \iint_{TSRQ} Curl(\tau)\, dy\, dx$$

Calculate using earlier expression
(see derivation of (14))



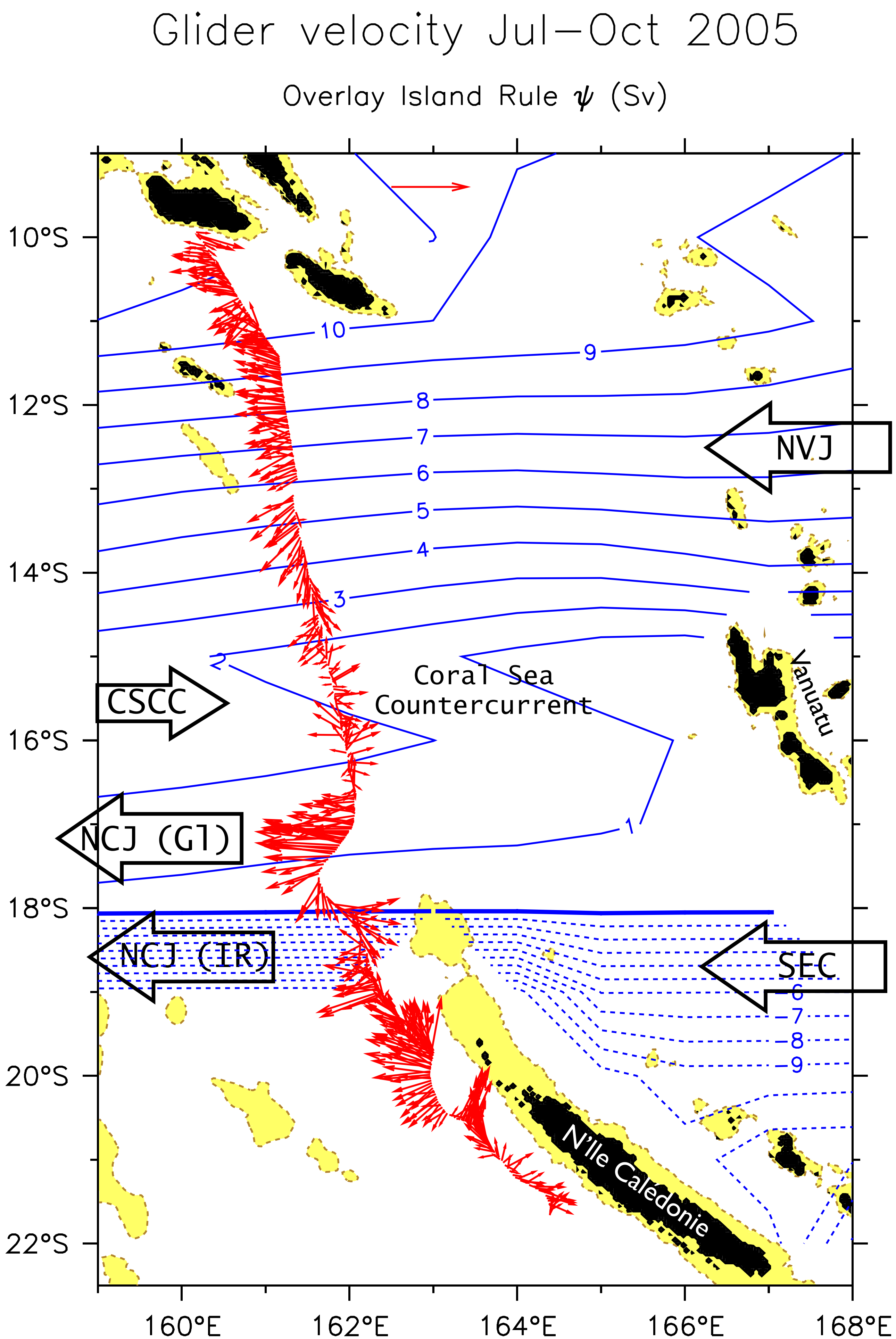


Must use the "multiple island rule" (see exercises).
Maps show the regions of integration for a Solomon Sea calculation.

The "multiple island rule" is needed because the interior region is not simple: NZ is partly in front of Australia.

1) integrate to find the Island Rule streamfunction for New Zealand (regions 1a,b).
2) restart the integration from the west side of New Zealand to Australia (region 2).
3) do regions 3-5 (can combine if omitting the Solomon Islands). Sum all the pieces (careful about Δy!)

(These steps are easy and straightforward to calculate).
It is essential to take NZ into account; NZ increases the streamfunction value at Australia by about 15%.



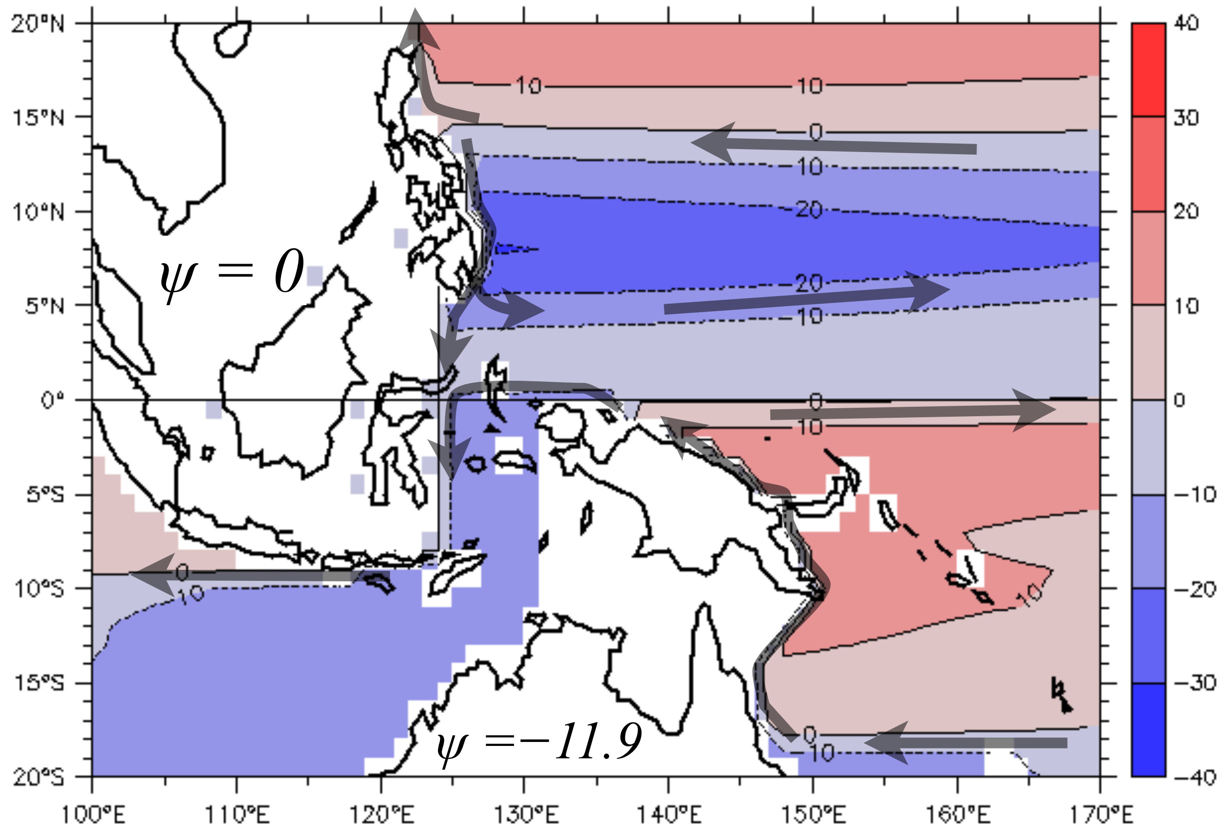
The Island Rule solution (blue contours) compared with observed currents in the western South Pacific.

Both show a broad North Vanuatu Jet from 11°S-14°S, and a narrow North Caledonian Jet right at the tip of the New Caledonia reef (the coarse-grid IR solution had the reef too far south).

Between the two currents, in the lee of Vanuatu, the Island Rule predicts a wind-driven countercurrent (the CSCC). A shear-instability analysis (Qiu et al 2009) found that extensive eddy activity in this region was due to shear between the CSCC and the two westward jets. These eddies were observed by the glider in Sep 2005.

Gourdeau et al (2007)

Island Rule streamfunction details in the W. Pacific

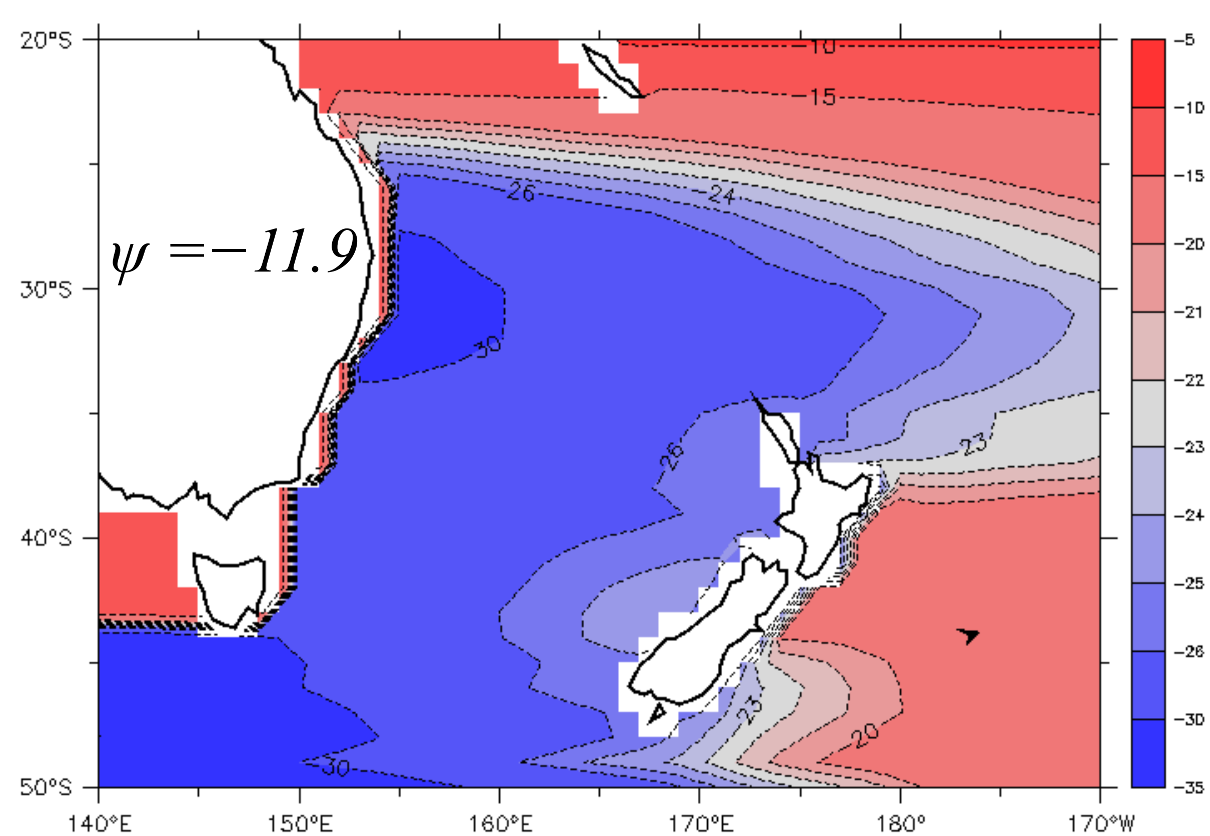


Assume the Indonesian Throughflow is wide open (frictionless).

Obviously unrealistic, but the Island Rule still provides a reasonable estimate of the transport.

The NGCU transports about 25 Sv, of which 6 Sv go through the ITF and the rest goes east along the Equator. The other 6Sv of the ITF come from the Mindanao Current.

Island Rule: NZ=25.2Sv, Australia=11.9Sv, New Caledonia 13.4Sv



Extra contours show circulation near New Zealand

WBC east of NZ is about 6Sv.

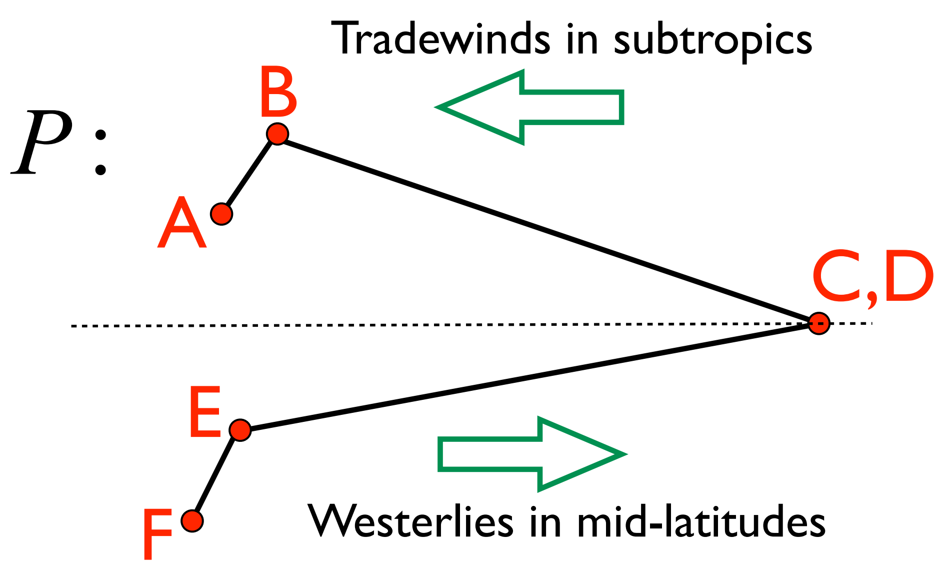
In a closed basin like the North Pacific ...

$$P_F - P_A = \frac{1}{g} \int_{ABCDEF} \tau^l \, dl \qquad (9)$$

Since this integral opposes both the mid-latitude westerlies and the subtropical tradewinds, $P_F - P_A < 0$, or $P_F < P_A$.

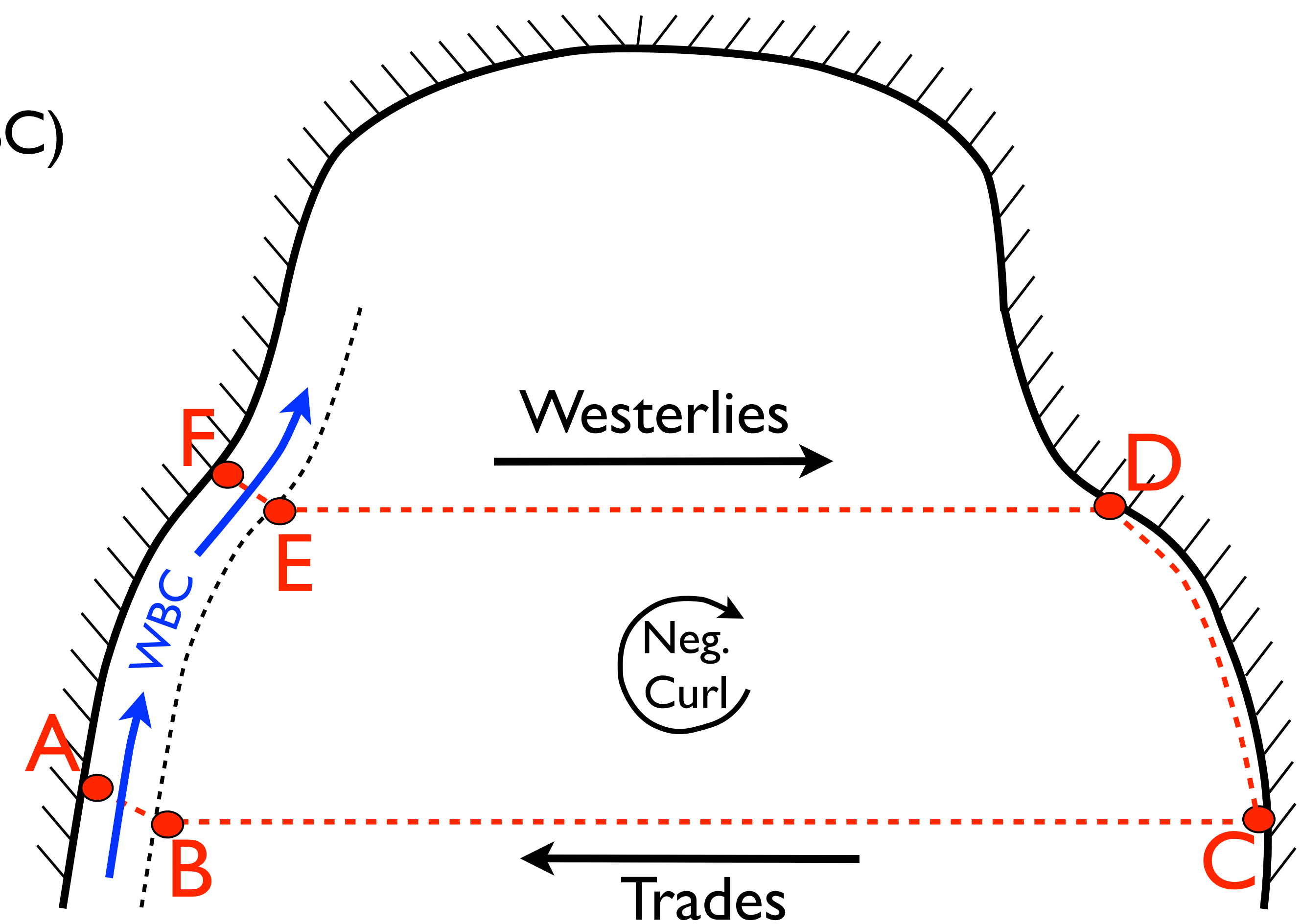
⇨ Pressure is higher at A than at F

(balance alongshore friction in WBC)

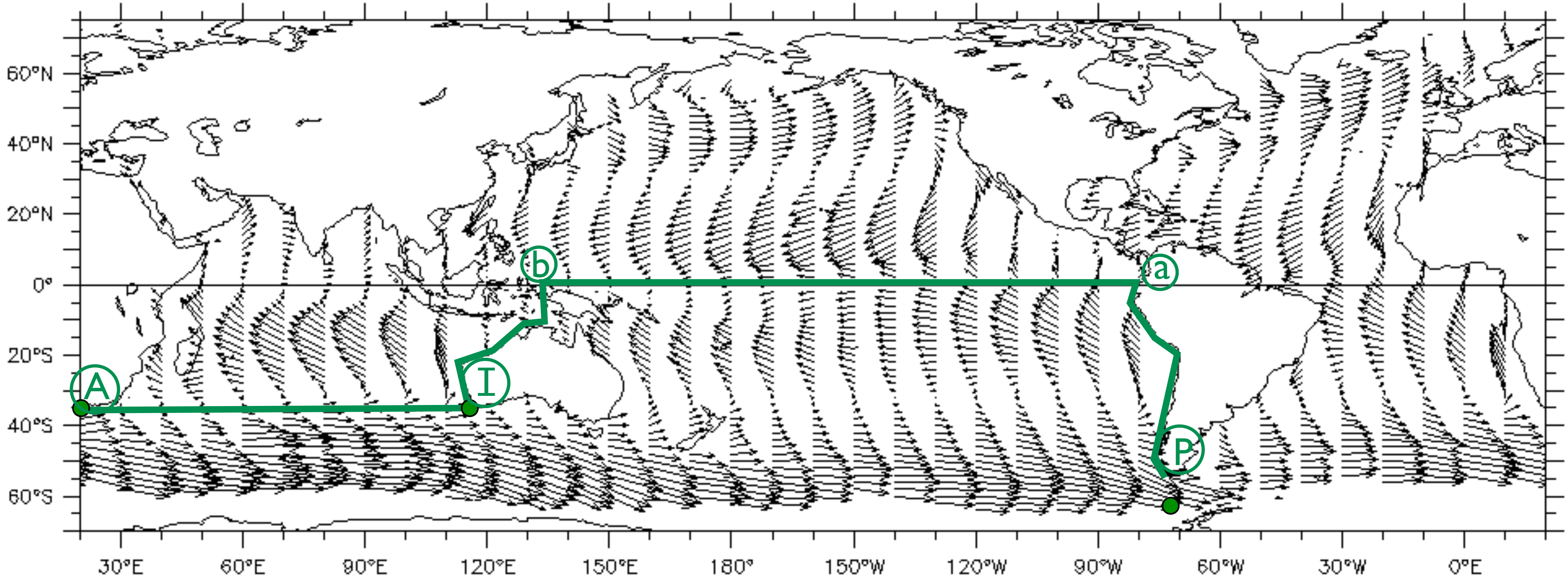


(9) is: $V_g = -V_{Ek}$

(zero transport)



Pressure in the different basins (1)



Find the coastal pressure difference between the basins by successive applications of (5) and (8).

$$P_B - P_A = \frac{1}{g}\int_A^B \tau^l dl \qquad (5)$$

Use (5) for P-a and b-I (west coast).

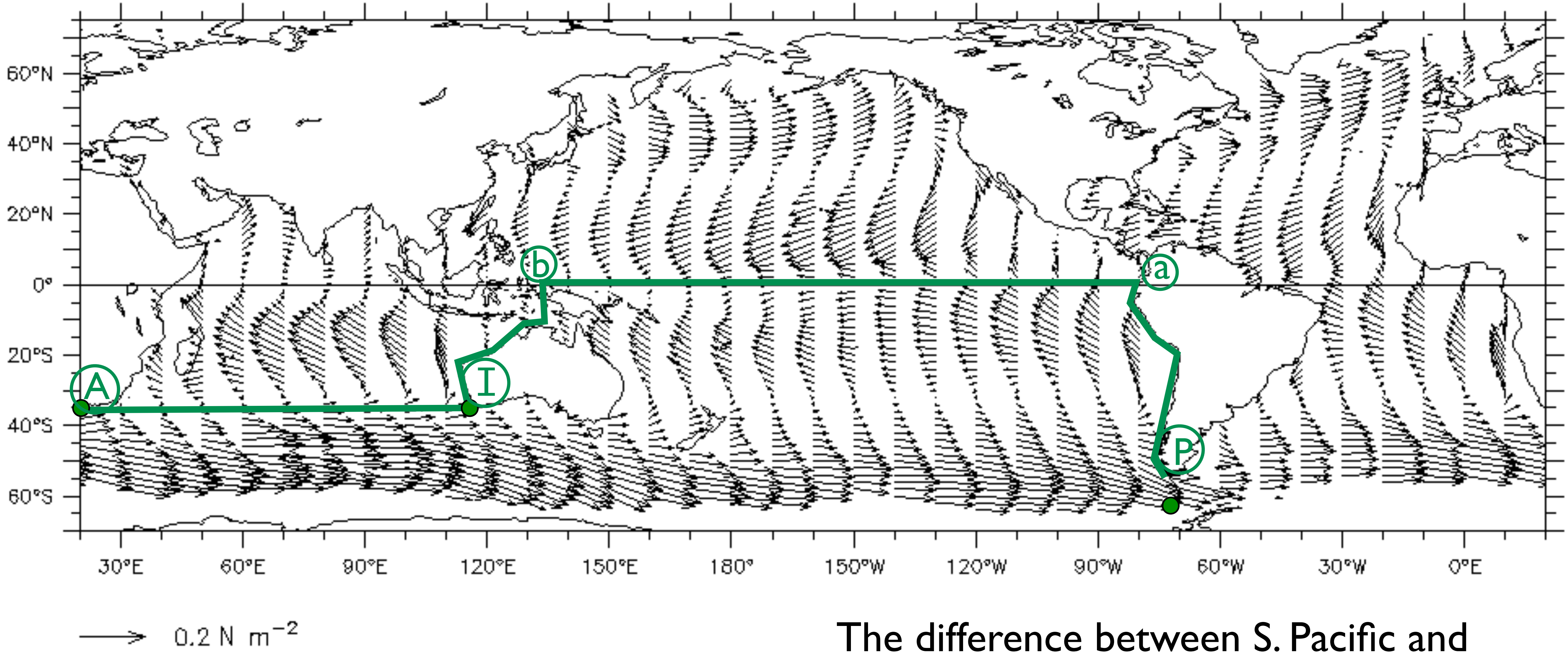
Use (8) for a-b and I-A (trans-ocean).

$$P_D - P_F = \frac{1}{g}\left[\int_{FED} \tau^l\, dl + f\, T_0\right] \qquad (8)$$

Note T_0 is the same for both, but f is different (and $f_{Eq} = 0$).

(fV_g) $(-fV_{Ek})$ (fV_{Total})

Pressure in the different basins (2)



$$P_B - P_A = \frac{1}{g}\int_A^B \tau^l dl \qquad (5)$$

$$P_D - P_F = \frac{1}{g}\left[\int_{FED} \tau^l \, dl + f\, T_0\right] \qquad (8)$$

(fV_g) $\quad (-fV_{Ek})$ $\quad (fV_{Total})$

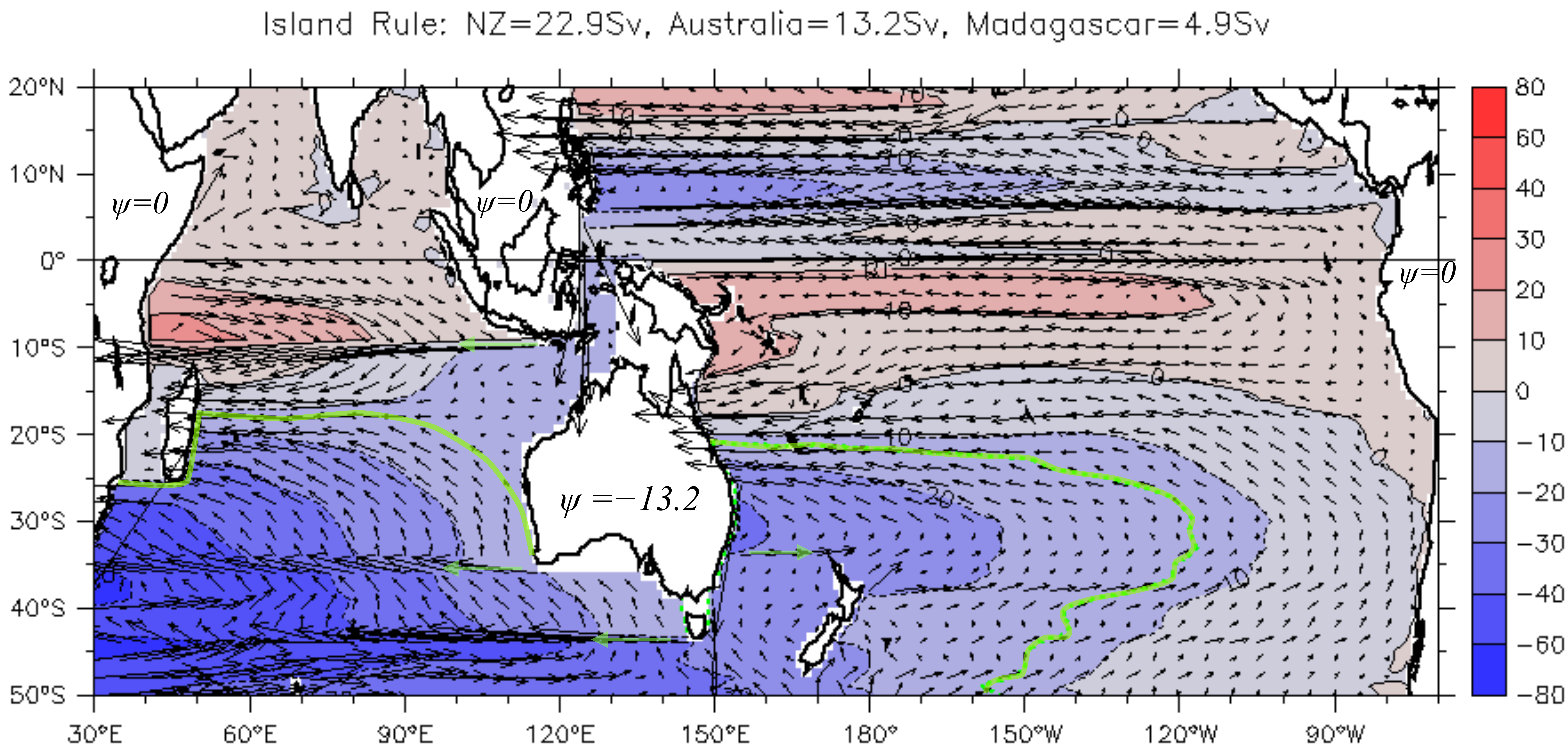
The difference between S. Pacific and S. Indian Ocean P is due only to the wind integral from P to I: Indian P higher ($V_g>0$ into S. Pacific).

The difference between the S. Indian and Atlantic is due mostly to the net southward transport across I-A ($V_g<0$ out of Indian):

(Godfrey 2010)

Southern hemisphere supergyre

Sverdrup streamfunction and velocity



Green line shows the bifurcation of the SEC: Split WBC along the coast of Australia

A consequence of an island is that the streamfunction behind the island is reset to a value constant in latitude, with all the zonal transport in the latitude band east of the island concentrated in two jets to its west: Tasmania, SW Australia, Java.