6. More Loops, Control Structures, and Bootstrapping

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In this session

We will introduce additional looping procedures as well as control structures that are useful in R. We also provide applications to bootstrapping.

- Repeat and While loops,
- If-Then and If-Then-Else structures
- Introduction to the bootstrap, with examples

Repeat loops

The repeat loop is an infinite loop that is often used in conjunction with a **break** statement that terminates the loop when a specified condition is satisfied. The basic structure of the repeat loop is:

```
repeat{
   expression
   expression
   expression
   if(condition) break
}
```

Repeat loops

Below is a repeat loop for printing the square of integers from 1 to 10.

```
i <- 1
repeat {
   print(i^2)
   i <- i+1
   if(i > 10) break
}
```

While loops

The while loop is often used for executing a set of commands or statements repeatedly until a specific condition is satisfied.

The structure of a while loop consists of a boolean condition and statements that are written inside while loop brackets, for which repetitive execution is to be carried out until the condition of interest is satisfied:

```
while (condition) {
    expression
    expression
    expression
}
```

It is important to note that the while loop will first check that the condition is satisfied prior to executing a first iteration of the commands.

While loops

Below is a while loop for printing out the first few Fibonacci numbers: 0, 1, 1, 2, 3, 5, 8, 13,..., where each number is the sum of the previous two numbers in the sequence.

```
a = 0
b = 1
print(a)
while (b < 50) {
    print(b)
    temp = a + b
    a = b
    b = temp
}
```

While loops

Below is a while loop that creates a vector containing the first 20 numbers in the Fibonacci sequence

```
x = c(0,1)
n=20
while (length(x) < n) {
    position = length(x)
    new = x[position] + x[position-1]
    x = c(x,new)
}
```

If-Then and If-Then-Else structures

Sometimes a block of code in a program should only be executed if a certain condition is satisfied. For these situations, *if-then* and *if-then-else* structures can be used:

The *if-then* structure has the following general form:

```
if (condition) {
   expression
   expression
}
```

The *if-then-else* structure extends the same idea:

```
if (condition) {
   expression
   expression
}
else {
   expression
   expression
}
```

If-Then and If-Then-Else structures

An example: an *if-then-else* statement that takes the square root of the product of two numbers x and y, if the product is positive:

```
x <- 3
y <- 7
if( (x<0 & y<0) | (x>0 & y>0) ){
   myval <- sqrt(x*y)
}
else{
   myval <- NA
}
```

And the value of myval when x=3 and y=7 is:

```
> myval
[1] 4.582576
```

What is myval if x=2 and y=-10?

```
> myval
[1] NA
```

Introduction to bootstrapping

Bootstrapping is a very useful tool when the distribution of a statistic is unknown or very complex.

Bootstrapping is a *non-parametric* (i.e. assumption-lite) re-sampling method for estimaing standard errors, computing confidence intervals, and hypothesis testing.

The method is often used when sample sizes are small and *asymptotic* (i.e. large-n) approximations, may be difficult to apply.

“The bootstrap is a computer-based method for assigning measures of accuracy to sample estimates.” [B. Efron and R. J. Tibshirani, An Introduction to the Bootstrap, Boca Raton, FL: CRC Press, 1994.]

Introduction to bootstrapping

Bootstrapping uses three steps:

- Resample a given data set **with replacement** a specified number of time, where each *bootstrap sample* is the same size as the original sample
- Calculate a statistic of interest for each of the bootstrap samples.
- The distribution of the statistic from the bootstrap samples can then be used to estimate standard errors, create confidence intervals, and to perform hypothesis testing with the statistic.

Example: bootstrapping the median

We can bootstrap in in R by going round a loop, using the `sample(x, size, replace, prob)` function at each iteration:

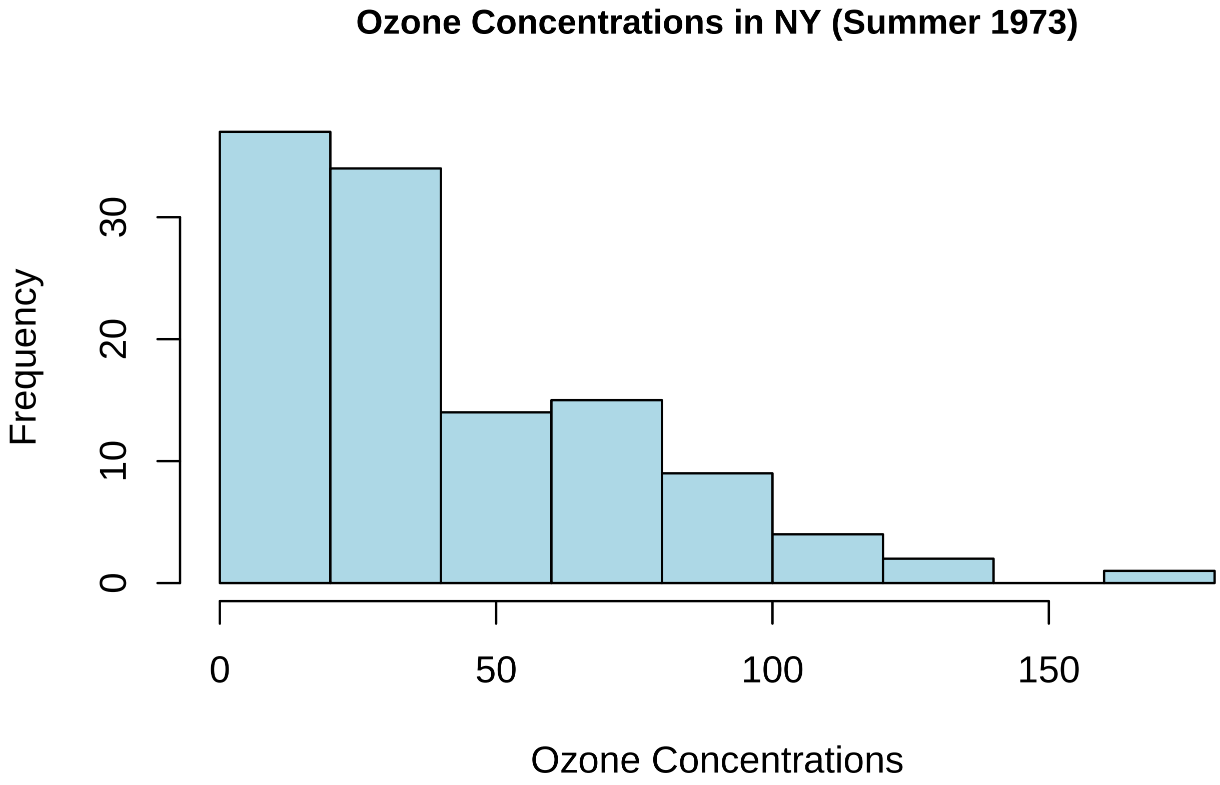
- `x` is a vector containing the items to be resampled.
- `size` specifies how many resamples to take: the default is the length of `x`
- `replace` determines if the sample will be drawn with or without replacement. The default value, FALSE i.e. sampling is performed without replacement
- `prob` lets us specify unequal probabilities of resampling each element of `x` – not needed here

Bootstrapping uses resamples of the same size as the original data, sampling with replacement – so `sample(x, replace=TRUE)`

Example: bootstrapping the median

Let’s consider the *airquality* dataset again. Below is a histogram of the daily ozone concentrations in New York, summer 1973.

```
hist(airquality$Ozone,col="lightblue",xlab="Ozone Concentrations",
main="Ozone Concentrations in NY (Summer 1973)")
```



What’s the median ozone level?

How accurately do we know the median?

Example: bootstrapping the median

First, let's work out the median;

```
> median(airquality$Ozone)
[1] NA
```

Several ozone concentration values are missing, but if we take the median of the 116 observed values;

```
> median(airquality$Ozone,na.rm=TRUE)
[1] 31.5
```

How might this value differ, in other similar experiments? We will use the bootstrap to estimate its distribution, and to provide a 95% confidence interval for the median.

Example: bootstrapping the median

To make the code easier to read, make a vector of the ozone concentrations with missing values excluded:

```
ozone <- airquality$Ozone[ !is.na(airquality$Ozone) ]
```

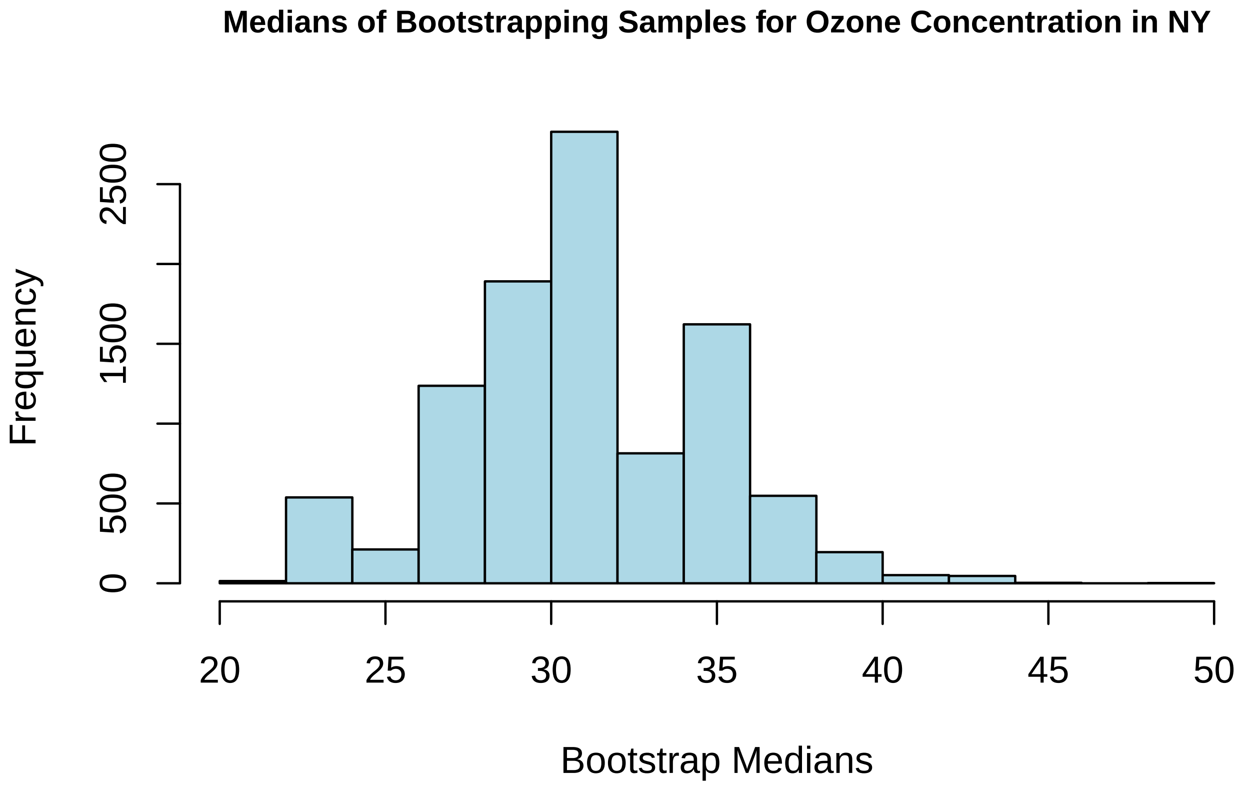
Using a `for()` loop, we can create 10,000 bootstrap samples and calculate the median for each sample:

```
nboot <- 10000    # number of bootstrap samples
bootstrap.medians <- rep(NA, nboot)
set.seed(10)
for(i in 1:nboot){
   bootstrap.medians[i] <- median(sample(ozone,replace=TRUE))
}
```

Example: bootstrapping the median

What do the medians look like? How do they compare with original ‘raw’ data?

```
hist(bootstrap.medians,col="lightblue",xlab="Bootstrap Medians",
main="Bootstrap Medians for Ozone Concentrations in NY",cex.main=.8)
```



10,000 of them, not 116

Much less skewed than raw data

Much less variable than raw data

Example: bootstrapping the median

The 95% confidence interval is given by the .025 and .975 quantiles of those boostrap medians;

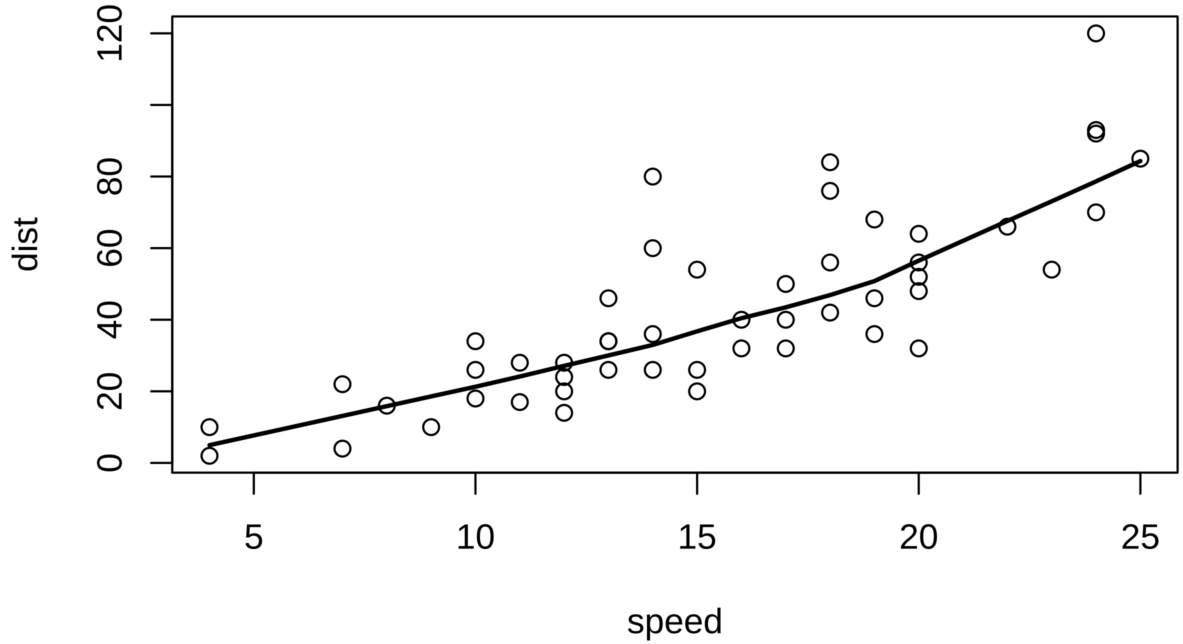
```
> quantile(bootstrap.medians, c(0.025, 0.975) )
 2.5% 97.5%
 23.5  39.0
```

- Could read off from the previous graph
- (23.5, 39.0) is a range of median values we might expect to see (i.e. the uncertainty in the medians) if repeating the experiment many times
- This method does assume that the ozone measurement on different days is independent so probably understates uncertainty, here!

Example: bootstrap for lowess curve

The bootstrap is a very powerful idea. For a more sophisticated example, recall the `cars` data, and the line we put through it;

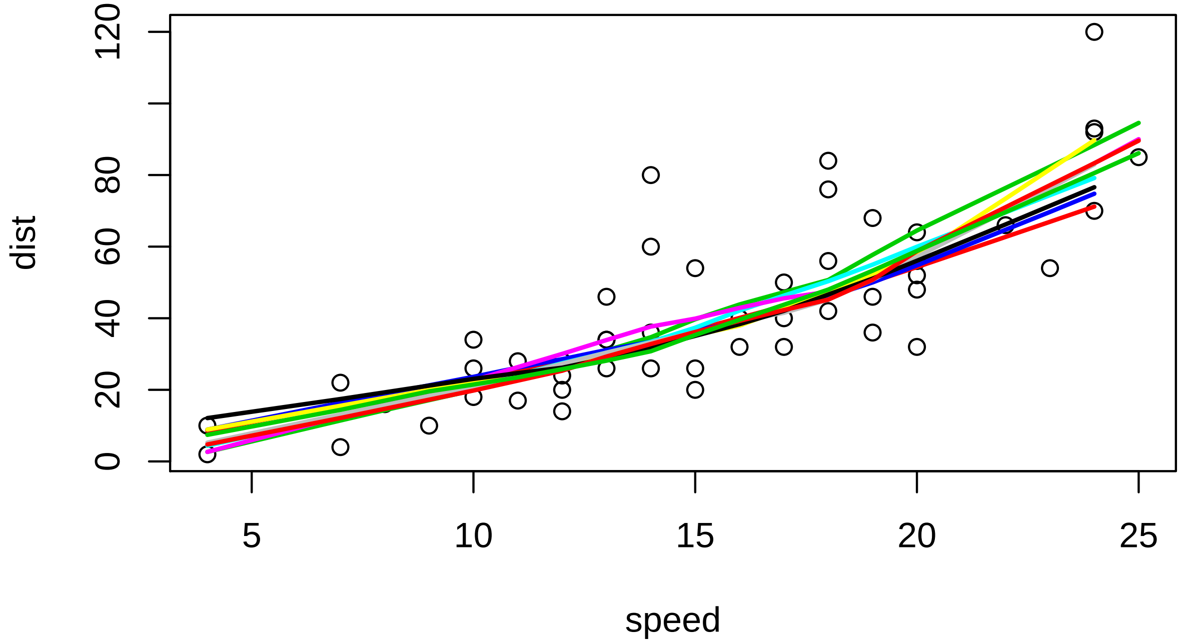
```
data(cars)
plot(dist~speed,data=cars)
with(cars, lines(lowess(speed, dist), col="tomato", lwd=2))
```



Example: bootstrap for lowess curve

To bootstrap the *curve*, we resample entire observations;

```
m <- dim(cars)[1]     # obtain the sample size
nboot <- 20
for(i in 1:nboot){
   mysample <- sample(1:m, replace=T) # i.e. which rows are resampled?
   with(cars[mysample,],
     lines(lowess(speed, dist), col=(i+1), lwd=2)
   )}
```



Example: bootstrap for lowess curve

For a smoother version, note `lowess()` only produces output at the *sampled* points – so we extrapolate to the others using `approx()`;

```
nboot <- 1000
boot.speed <- matrix(NA, 1000, m) # for storing the curve's value at all m points
set.seed(1314) # Battle of Bannockburn
for(i in 1:nboot){
   mysample <- sample(1:m,replace=T)
   low1 <- with(cars, lowess(speed[mysample], dist[mysample]))
   low.all <- approx(low1$x, low1$y, xout=cars$speed, rule=2)
   boot.speed[i,] <- low.all$y
}
```

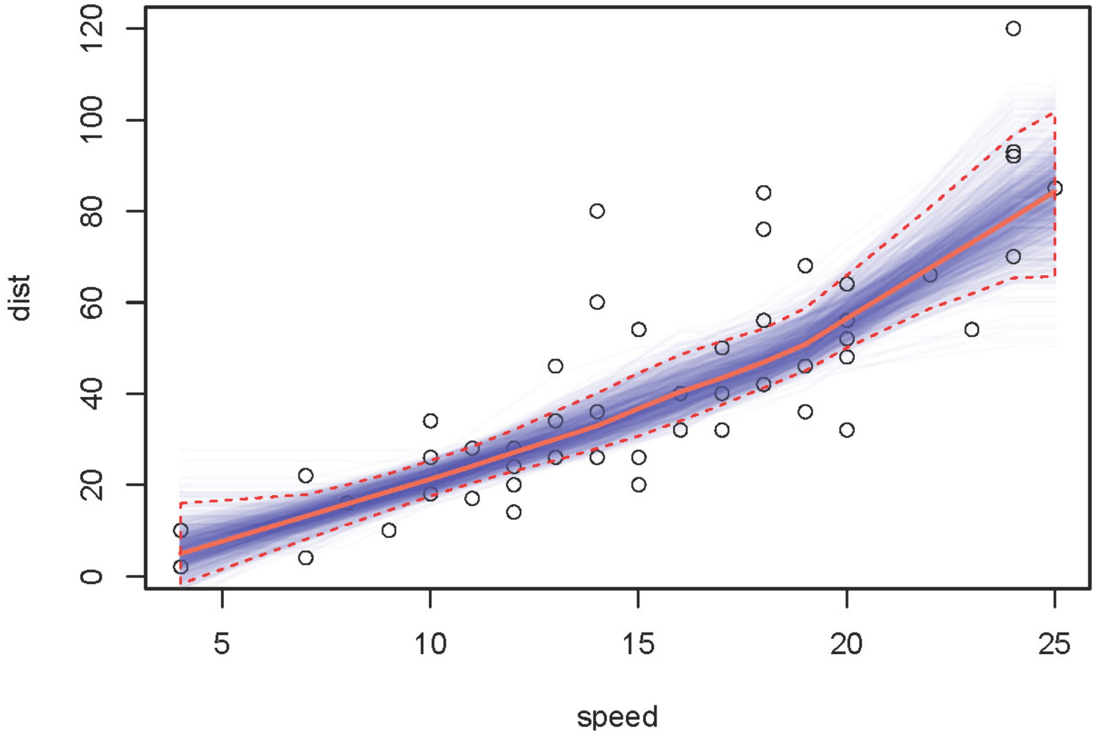
Now work out the lower and upper ranges of the lines, at all m values of `speed`;

```
upper <- rep(NA, m)
lower <- rep(NA, m)
for(j in 1:m){
upper[j] <- quantile(boot.speed[,j], 0.975)
lower[j] <- quantile(boot.speed[,j], 0.025)}
```

Example: bootstrap for lowess curve

Finally, make a cool blue picture, using transparency;

```
plot(dist~speed,data=cars)
for(i in 1:nboot){
   lines(x=cars$speed, y=boot.speed[i,], col="#0000FF05") }
with(cars, lines(lowess(speed, dist), col="tomato", lwd=2)) # raw data lowess
polygon(x=c(cars$speed, rev(cars$speed)), y=c(upper, rev(lower)),
        density=0, col="red", lty=2) # pointwise 95% conf ints
```



Summary

- `while{}` and `repeat{}` are useful tools for looping until a condition is satisfied

- *if-then* and *if-then-else* structures allow blocks of code to be executed under different specified conditions

- Bootstrapping is a powerful statistical technique for expressing accuracy/inaccuracy. (*Almost* all other methods used for this can be thought of as approximations to some form of bootstrap)

- Bootstrapping can be implemented in a few lines of R, using loops and the `sample()` function