Vertical Integration and Strategic Trade Policies

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Abstract

This paper examines the use of strategic trade policies such as export subsidies by a country to encourage the domestic production of an intermediate input and a final product in a model with international rivalry between firms in two countries. The choice of subsidies or taxes in several cases are examined. Whether subsidies are welfare improving depends on whether the firms in each country are vertically integrated. We showed that as long as the firms in at least one countries are vertically integrated, the optimal subsidy on the final-good production is positive.

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1 Introduction

The appropriate use of strategic export subsidies has long been one of the important topics for trade theorists and government policy planners. On the theoretic side, Brander and Spencer (1985) describe a framework in which a government can use export (or production) subsidy to encourage a domestic firm to produce more but a competing foreign firm to produce less, resulting in a rise in the profit of the domestic firm and the national welfare at the expense of the foreign firm's profit and foreign economy's welfare. It has also been pointed out by other economists that the results in Brander and Spencer (1985) are sensitive to some of the assumptions in their framework, and once these assumptions are relaxed, the optimal policy for a government could be an export tax.[1] In the real world, governments have recognized that widespread use of export subsidies could be disruptive and could hurt their economies. As a result, members of the World Trade Organization signed agreements to prohibit the use of government subsidies to promote the trade performance of domestic industries.

Despite the concerns in the theory and the prohibition of the use of trade-promoting subsidies, the discussion about strategic export subsidies is still very alive. This is partly due to the existence of many policies that are under disguised names but have trade-promoting effects, and partly due to the existence of many trade disputes concerning the use of strategic export subsidies.[2] There are still quite a number of issues involving possible use of export subsidies that have not been fully addressed.

One issue is about the existence of intermediate inputs. In the Brander-Spencer model, only a final product is considered, and only primary factors are used in the production process. This simplification is a convenience in the theoretical work, but not too close to the real world, in which many industries do use not only primary factors but also intermediate inputs. This fact of course has long been recognized, and recently there has been a rising number of papers analyzing various issues involving intermediate inputs and trade policies.[3]

[1]See, for example, Wong (1995) and Brander (1995) for two recent surveys of some of the issues.

[2]As Liao and Wong (2003) show, some policies such as minimum quality standard, which appear to be for the local economy and which do not involve any government budget, can have trade promoting effects.

[3]More recent work includes Spencer and Jones (1991, 1992), Ishikawa and Lee (1996),

To see the importance of intermediate inputs, let us consider the computer industry.[4] Computer is a final product, using primary factors (such as labor and capital) and intermediate inputs (such as computer chips). Both computers and computer chips are tradable, with major firms in some countries producing computer chips, such as Samsung in Korea, Micron Technology in the United States, and Infineo in Germany, and important producers in some countries producing computers; for example, Samsung in Korea and Hewlett Packer in the United States.[5]

Consider a country like Korea, and suppose that her government attempts to promote trade to improve her national welfare. A subsidy on the production of computers most likely will have the type of profit-shifting effect in the final-product industry suggested by Brander and Spencer (1985). However, the subsidy will increase the demand for computer chips, and that could benefit foreign computer chip producers and hurt the local ones. Thus whether the optimal subsidy is positive is not so sure. Similarly, if a subsidy is imposed on the local production of computer chips, while the local chip producers may benefit because of the profit-shifting effect, the drop in the chip price may simultaneously benefit the foreign computer producers possibly at the expense of local computer producers. Then what is the optimal subsidy?[6]

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and Rodrik and Yoon (1995).

[4]The computer and computer chip industries are interesting because there are only big producers concentrating in a few countries. They are oligopolies in the world markets. In many cases, government interventions in the industries are obvious, and there are always claims that foreign governments are illegally subsidizing the industries. In June 2002, Infineon, a German computer chip maker, filed petition to European Union against Samsung Electronics (Korea), and Hynix, claiming they received illegal government subsidy. In November 2002, Micron Technology Inc., a U.S. chip maker filed complaints against Hynix with the U.S. Commerce Department and International Trade Commission. It claimed that in fact bail out funds were illegal Korean government subsidies. See Yang (2000) for more discussion.

[5]A relatively small number of international firms share the world semiconductor market. Hong (1997, 76–77) shows that 15 international firms' world semiconductor market share was 67.1% in 1995. Specifically, three Korean memory chip producers, Samsung, LG, and Hyundai had 26.6% of world market share in 1995. In 1998, Hyundai took LG as part of a restructuring effort after Asian Crisis. It changed its name into Hynix. See Yang (2000, 123–123) for discussion about merging as a restructuring strategy in Korean semiconductor industry.

[6]Hong (1997, 113) shows that in 1996, 68% of Korean demand for semiconductor (an intermediate good for electronics) is met by imports. According to Irwin (1996), IBM

The purpose of the present paper is to determine the optimal subsidies on the final-good (such as computers) and intermediate-input (such as computer chips) production imposed simultaneously, and investigate the linkage between the final-good production and intermediate-input production. We argue that such linkage and thus the optimal subsidies depend on whether the final-good and intermediate-input industries in the same country are vertically integrated (as in the case of Samsung in Korea) or not (as in the case of Micron Technology and Hewlett Packard in the United States). As a result, the present paper considers four different cases, depending on whether the local final-good and intermediate-input industries and the foreign two industries are integrated. In each of these cases, we derive the optimal subsidies for the home government and examine whether they are positive.[7] An interesting paper by Rodrik and Yoon (1995) show that an optimal subsidies may not be positive in the presence of economies of scale. The present paper argues that optimal subsidies may not be positive even if economies of scale are absent.

The rest of the paper is organized as follows. Section 2 describes the features of a basic model, in which there are two countries, each with a local firm producing an intermediate input and a local firm producing a final good. These two industries are vertically integrated if the two firms within a country merge together and produce both intermediate input and final good. Sections 3 to 6 consider four separate cases: one with no vertical integration in either countries, one with vertical integration in the home country only, one with vertical integration in the foreign country only, and one with vertical integration in both countries. The last section concludes.

2 The Basic Model

We consider two countries, labeled home and foreign, and two industries in each country: one for a final good for consumption, and another one for an immediate input, which is used exclusively in the production of the final good. In each country, there is one firm producing the final good and one

started to buy Japanese semiconductor from 1977 even though it was one of the major semiconductor producers in the world.

[7] As mentioned, there are several papers in the literature that examine the use of subsidies in the presence of intermediate inputs, but none of them consider subsidies in both the final-good and intermediate-input industries with or without vertical integration.

firm producing the intermediate input. Let us label the home and foreign intermediate-input firms by firms d and f, respectively, and the home and foreign final-good firms by firms D and F, respectively. In some of the cases considered below, the two industries are vertically integrated, with the same firm producing the immediate input and the final good. Trade between the two countries in the intermediate product is allowed, while outputs of the final good are sold in the rest of the world. No transportation costs are assumed.

Production of the final good requires the intermediate input and other inputs. For simplicity, the input-output ratio for the intermediate input is assumed to be fixed at one to one. Two possible specific production subsidies are considered for the home government, which are denoted by s for the intermediate input and v for the final good. The foreign government remains passive in policy setting.

Denote the outputs of the home and foreign intermediate-input firms by x and x^*, respectively, and the outputs of the home and foreign final-good firms by y and y^*, respectively. The demand for the final good exists in the rest of the world only, with the demand represented by $p = p(Y)$, where Y is the demand and p the market price. It is assumed that $p' < 0$ and $p'' < \sigma$, where a prime denotes a derivative and σ is a sufficiently small, positive number.[8] In equilibrium, we have

$$Y = y + y^*. \tag{1}$$

If X is the total demand for the intermediate inputs by the final-good firms D and F, then in equilibrium $X = x + x^*$, and because of the one-to-one ratio between the intermediate input and final output, we have

$$Y = y + y^* = X = x + x^*. \tag{2}$$

3 Case 1: No Vertical Integration

We begin with the case in which there is no vertical integration in either country. There are therefore four separate firms in both countries. We consider the following three-stage game: In stage one, the home government sets

[8]The latter condition guarantees that the marginal revenue is falling with quantity and that the second-order condition is satisfied. It is satisfied if, for example, the demand is linear.

the export subsidies. In stage two, the intermediate-input firms choose their outputs in a non-cooperative, Cournot way.[9] In stage three, the final-good firms choose their outputs in a Cournot way. To simplify our analysis, we assume that the final-good firms take the prices of inputs as given, although they are duopolists in the output market.

3.1 Stage 3: Final-Good Production

We first analyze how the final-good firms choose their outputs. These two firms are facing constant marginal (non-intermediate-input) costs of w and w^*, respectively, while the home and foreign intermediate-input firms are facing constant marginal costs of c and c^*, respectively. To simplify our notation, we neglect the fixed costs. Denote the market price of the intermediate input by r. Taking into consideration the production subsidy v imposed by the home government, the profits of the domestic final-product firm, D, and the foreign final producer, F, are respectively given by

$$\Pi^D = [p - r - w + v]y \tag{3a}$$

$$\Pi^F = [p - r - w^*]y^*. \tag{3b}$$

Recall that the firms take the input prices and each other's output as given. Simple derivation easily gives the outputs of the firms, which can be expressed as functions of the intermediate-input prices and the production subsidy,

[9] We consider Cournot competition, not Bertrand competition, so that we can compare our results directly with those in Brander and Spencer (1985).

$y = y(r, v)$ and $y^* = y^*(r, v)$.[10] The partial derivatives are

$$\frac{\partial y}{\partial r} = \frac{\partial y}{\partial w} = \frac{\Pi^F_{y^*y^*} - \Pi^D_{yy^*}}{D_y} = \frac{p' + p''(y^* - y)}{D_y} < 0 \tag{4a}$$

$$\frac{\partial y^*}{\partial r} = \frac{\partial y^*}{\partial w^*} = \frac{\Pi^D_{yy} - \Pi^F_{y^*y}}{D_y} = \frac{p' + p''(y - y^*)}{D_y} < 0 \tag{4b}$$

$$\frac{\partial Y}{\partial r} = \frac{2p'}{D_y} < 0 \tag{4c}$$

$$\frac{\partial y}{\partial v} = -\frac{\Pi^F_{y^*y^*}}{D_y} = -\frac{2p' + p''y^*}{D_y} > 0 \tag{4d}$$

$$\frac{\partial y^*}{\partial v} = \frac{\Pi^F_{y^*y}}{D_y} = \frac{p' + p''y^*}{D_y} < 0 \tag{4e}$$

$$\frac{\partial Y}{\partial v} = -\frac{p'}{D_y} > 0, \tag{4f}$$

where $D_y = \Pi^D_{yy}\Pi^F_{y^*y^*} - \Pi^D_{yy^*}\Pi^F_{y^*y} = p''p'(y + y^*) + 3(p')^2 > 0$. The total output function of the final product is $Y = Y(r, v) \equiv y(r, v) + y^*(r, v)$, which can be inverted to yield the demand for input: $r = r(Y, v) = r(X, v)$, where the assumed input-output ratio is used.

3.2 Stage 2: Intermediate-Input Production

Let us define $r'(X, v) \equiv \partial r/\partial X$, where $r'(X, v) = (\partial Y/\partial r)^{-1} < 0$ for any given value of v. As usual, it is assumed that this demand function is not too convex to the origin so that the marginal revenue is downward sloping.

It is important to note that the price of input rises with the home subsidy on the final product production: Total differentiation of (2) gives

$$\mathrm{d}X = \mathrm{d}Y = \frac{\partial Y}{\partial r}\mathrm{d}r + \frac{\partial Y}{\partial v}\mathrm{d}v. \tag{5}$$

With given X, condition (5) gives

$$\frac{\partial r}{\partial v} = -\frac{\partial Y/\partial v}{\partial Y/\partial r} = \frac{1}{2}. \tag{6}$$

[10]The marginal costs are taken as parameters.

Lemma 1 *$r' < 0$, and $\partial r/\partial v = 1/2$.*

Condition (6) is interesting because it suggests that an increase in the home final-good subsidy leads to a 50-percent rise in the intermediate-input price at a given total output of intermediate input. To determine how the two intermediate-input firms compete, we first state their profit functions as follows:

$$\Pi^d = (r - c + s)x \tag{7a}$$

$$\Pi^f = (r - c^*)x^*. \tag{7b}$$

Taking the home production subsidies and each other's output as given, with full consideration of the final-good firms' output choice, they have the following first-order conditions:

$$\frac{\partial \Pi^d}{\partial x} = r(X, v) - c + s + r'x = 0 \tag{8a}$$

$$\frac{\partial \Pi^f}{\partial x^*} = r(X, v) - c^* + r'x^* = 0, \tag{8b}$$

which give the reaction functions of the two firms: $x = h(x^*; s, v)$ and $x^* = h^*(x; s, v)$. These two functions are shown graphically by curves HH and FF, respectively, in Figure 1. With the usual properties of the demand function, the curves are negatively sloped, with curve HH steeper than curve FF.

Conditions (8) are solved for the Nash equilibrium output: $x = x(v, s)$ and $x^* = x^*(v, s)$. Differentiate conditions (8) and rearrange terms to give

$$\frac{\partial x}{\partial s} = -\frac{\Pi^f_{x^*x^*}}{D_x} = -\frac{2r' + r''x^*}{D_x} > 0 \tag{9a}$$

$$\frac{\partial x^*}{\partial s} = \frac{\Pi^f_{x^*x}}{D_x} = \frac{r' + r''x^*}{D_x} < 0 \tag{9b}$$

$$\frac{\partial X}{\partial s} = -\frac{r'}{D_x} > 0 \tag{9c}$$

$$\frac{\partial x}{\partial v} = -\frac{1}{D_x}\left(\frac{\partial r}{\partial v}\right)\left(\Pi^f_{x^*x^*} - \Pi^d_{xx^*}\right) = -\frac{r' + r''(x^* - x)}{2D_x} > 0 \tag{9d}$$

$$\frac{\partial x^*}{\partial v} = -\frac{1}{D_x}\left(\frac{\partial r}{\partial v}\right)\left(\Pi^d_{xx} - \Pi^f_{x^*x}\right) = -\frac{r' + r''(x - x^*)}{2D_x} > 0 \tag{9e}$$

$$\frac{\partial X}{\partial v} = -\frac{r'}{D_x} > 0, \tag{9f}$$

where $D_x = \Pi^d_{xx}\Pi^f_{x^*x^*} - \Pi^d_{xx^*}\Pi^f_{x^*x} = r''r'(x + x^*) + 3r'^2 > 0$.[11] Note that the effects of export subsidy on home and foreign intermediate input outputs as given by (9a) and (9b) are consistent with the corresponding results in Brander and Spencer (1985).

Graphically the effects of the final-good subsidy v can be shown in Figure 1. An increase in v will shift curves HH and FF away from the origin, leading to an increase in both outputs, x and x^*. In the diagram, x_0 and x_0^* are the initial outputs while x_1 and x_1^* are the final outputs.

The above analysis allows us to define the following reduced-form functions: $r = \tilde{r}(s, v) \equiv r(X(s, v), v)$, $y = \tilde{y}(s, v) \equiv y(\tilde{r}(s, v), v)$, and $y^* = \tilde{y}^*(s, v) \equiv y^*(\tilde{r}(s, v), v)$. The effects of the subsidies on the input price are

$$\frac{\partial \tilde{r}}{\partial s} = \frac{\partial r}{\partial X}\frac{\partial X}{\partial s} = -\frac{r'^2}{D_x} < 0 \tag{10a}$$

$$\frac{\partial \tilde{r}}{\partial v} = \frac{\partial r}{\partial X}\frac{\partial X}{\partial v} + \frac{\partial r}{\partial v} = -\frac{r'^2}{D_x} + \frac{1}{2} > 0. \tag{10b}$$

[11] In determining the signs of the expressions in (9), the assumption that the demand functions are not too convex has been used.

Similarly, the effects of the subsidies on the final-good outputs are

$$\frac{\partial \tilde{y}}{\partial s} = \frac{\partial y}{\partial r}\frac{\partial \tilde{r}}{\partial s} > 0 \tag{11a}$$

$$\frac{\partial \tilde{y}^*}{\partial s} = \frac{\partial y^*}{\partial r}\frac{\partial \tilde{r}}{\partial s} > 0 \tag{11b}$$

$$\begin{aligned}\frac{\partial \tilde{y}}{\partial v} &= \frac{\partial y}{\partial r}\frac{\partial \tilde{r}}{\partial v} + \frac{\partial y}{\partial v} \\ &= -\frac{1}{2D_x D_y}\left\{11p'r'^2 + 3p'r'r''(x+x^*) + D_x p''(y+3y^*)\right. \\ &\quad \left. + 2p''r'^2(y^*-y)\right\} > 0\end{aligned} \tag{11c}$$

$$\begin{aligned}\frac{d\tilde{y}^*}{dv} &= \frac{\partial y^*}{\partial r}\left(\frac{\partial r}{\partial v} + \frac{\partial r}{\partial X}\frac{\partial X}{\partial v}\right) + \frac{\partial y^*}{\partial v} \\ &= \frac{1}{2D_x D_y}\left\{7p'r'^2 + 3p'r'r''(x+x^*) + D_x p''(y+y^*)\right. \\ &\quad \left. + 2p''r'^2(y^*-y)\right\} < 0.\end{aligned} \tag{11d}$$

The following proposition and table summarize the effects of home government subsidies on the intermediate-input production.

Table 1: The Effects of Government Subsidies Without Vertical Integration in Both Countries

	x	x^*	y	y^*	Π^D	Π^d	Π^F	Π^f
Subsidy on intermediate input ($s\uparrow$)	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$
Subsidy on final product ($v\uparrow$)	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$

Proposition 1 *Suppose that both home and foreign industries are not vertically integrated. (i) A rise in the home intermediate-input subsidy increases the home intermediate-input output, decreases that of foreign firm, and increases both home and foreign final-good firms' production. (ii) A rise in the home final-good subsidy increases the home final-good firm's output, decreases that of foreign final-good firm, and increases both home and foreign intermediate-input outputs.*

3.3 Stage 1: Home Subsidies

After deriving the effects of the subsidies on the firms' output decision, we now determine the optimal subsidies for the home economy. Its national welfare is defined as:

$$W = \Pi^D + \Pi^d - sx - vy. \tag{12}$$

The home government chooses the optimal subsidies on the domestic intermediate-input and final-good production to maximize national welfare. The first-order conditions are,

$$\frac{\partial W}{\partial s} = \Pi_y^D \frac{\partial \tilde{y}}{\partial s} + \Pi_{y^*}^D \frac{\partial \tilde{y}^*}{\partial s} + \Pi_x^d \frac{\partial x}{\partial s} + \Pi_{x^*}^d \frac{\partial x^*}{\partial s} + \Pi_s^d - x - s\frac{\partial x}{\partial s} = 0 \tag{13a}$$

$$\frac{\partial W}{\partial v} = \Pi_y^D \frac{\partial \tilde{y}}{\partial v} + \Pi_{y^*}^D \frac{\partial \tilde{y}^*}{\partial v} + \Pi_x^d \frac{\partial x}{\partial v} + \Pi_{x^*}^d \frac{\partial x^*}{\partial v} + \Pi_v^D - y - v\frac{\partial \tilde{y}}{\partial s} = 0. \tag{13b}$$

Conditions (13) can be solved for the optimal subsidies,[12]

$$\hat{s} = \underset{(+)}{\left(\frac{\partial x}{\partial s}\right)^{-1}} \left(\underset{(-)}{\Pi_{y^*}^D} \underset{(+)}{\frac{\partial \tilde{y}^*}{\partial s}} + \underset{(-)}{\Pi_{x^*}^d} \underset{(-)}{\frac{\partial x^*}{\partial s}}\right) \tag{14a}$$

$$\hat{v} = \underset{(+)}{\left(\frac{\partial \tilde{y}}{\partial v}\right)^{-1}} \left(\underset{(-)}{\Pi_{y^*}^D} \underset{(-)}{\frac{\partial \tilde{y}^*}{\partial v}} + \underset{(-)}{\Pi_{x^*}^d} \underset{(+)}{\frac{\partial x^*}{\partial v}}\right). \tag{14b}$$

To determine the signs of $\hat{s}$ or $\hat{v}$ in (14), note that

$$\begin{aligned} &\Pi_{y^*}^D \frac{\partial \tilde{y}^*}{\partial s} + \Pi_{x^*}^d \frac{\partial x^*}{\partial s} \\ &= \frac{1}{D_y D_x}\left\{r'^2\left[-p'^2 y + D_y x\right] - p'' p' r'^2 y(y - y^*) + r'' r' x x^* D_y\right\} \end{aligned} \tag{15a}$$

$$\begin{aligned} &\Pi_{y^*}^D \frac{\partial \tilde{y}^*}{\partial v} + \Pi_{x^*}^d \frac{\partial x^*}{\partial v} \\ &= \frac{1}{2D_y D_x}\{(r')^2[7p'^2 y - D_y x] - 2p'' p' r'^2 y(y - y^*) \\ &\qquad + 3r'' r' p'^2 y(x + x^*) + p'' p' y(y + y^*) D_x - r'' r' x(x - x^*) D_y\}. \end{aligned} \tag{15b}$$

[12] Note that $\Pi_{y^*}^D = p'y < 0$, $\Pi_{x^*}^d = r'x < 0$.

The signs of the terms in (15) are ambiguous, and so are the signs of $\hat{s}$ or $\hat{v}$. Let us first examine the optimal subsidy on the final-good production, as given by (14b). The term $(\partial\tilde{y}/\partial v)^{-1}\Pi^D_{y^*}(\partial\tilde{y}^*/\partial v)$ is the usual *profit-shifting effect*, and $(\partial\tilde{y}/\partial v)^{-1}\Pi^d_{x^*}(\partial x^*/\partial v)$ is the *linkage effect* (indirect effect) through the foreign intermediate-input production. The former is positive, but the latter is negative because of an increase in foreign intermediate-input output at a higher level of subsidy v, causing a fall in the home intermediate-input firm's profit. In Brander and Spencer (1985), there is no intermediate input and thus no linkage effect: With only the profit-shifting effect, the optimal subsidy on the final good is necessarily positive. Depending on the magnitudes of the profit-shifting and linkage effects, the optimal v may be positive or negative.

In equation (14a), $(\partial x/\partial s)^{-1}\Pi^d_{x^*}(\partial x^*/\partial s)$ is the positive profit-shifting effect. The term $(\partial x/\partial s)^{-1}\Pi^D_{y^*}(\partial\tilde{y}^*/\partial s)$ represents the linkage effect through the final-good production. It is negative because a rise in the subsidy leads to an increase in the foreign final-good production, causing a drop in the home final-good firm's profit.

Even though the signs of the optimal subsidies are ambiguous, at least one of them must be positive. To see why, note that, by expanding the terms, we have

$$\Pi^D_{y^*}\frac{\partial\tilde{y}^*}{\partial s}+\Pi^d_{x^*}\frac{\partial x^*}{\partial s}+\Pi^D_{y^*}\frac{\partial\tilde{y}^*}{\partial v}+\Pi^d_{x^*}\frac{\partial x^*}{\partial v}=\frac{\Delta}{2D_xD_y}>0, \tag{16}$$

where $\Delta \equiv r'^2(5p'^2y+xD_y)+4p''p'r'^2y(y^*-y)+3r''r'p'^2y(x+x^*)+p''p'y(y+y^*)D_x+r''r'x(3x^*-x)D_y$. Condition (16) rules out the possibility that both $\hat{s}$ and $\hat{v}$ are negative. We now have the following proposition:

Proposition 2 *Suppose that both industries in the countries are not vertical integrated. (i) Either the optimal s or the optimal v, but not both, may be negative. (ii) The optimal s is positive if $-p'^2y+D_yx>0$. (iii) The optimal v is positive if $7p'^2y-D_yx>0$.*

4 Case 2: Vertical Integration in Home

To see the roles of vertical integration and fragmentation in production, we turn to the cases in which the industries in at least one of the countries are integrated. This section covers the case in which the home industries are integrated, i.e., both the intermediate input and the final product are produced by one single firm, which is now labeled I.

We maintain the same game analyzed above: In stage 1, the home government chooses the subsidies; in stage 2, home and foreign intermediate-input firms choose their outputs, and in stage 3, both home and foreign final-good firms decide their outputs. The new feature of this game is that firm I chooses intermediate-input output in stage 2 and final-good output in stage 3.

4.1 Stage 3: Final-Good Production

Our analysis again begins with stage 3. The final-good firms' profit functions are:

$$\Pi^I = \Pi^D + \Pi^d = (p - r - w + v)y + (r - c + s)x \tag{17a}$$

$$\Pi^F = (p - r - w^*)y^*. \tag{17b}$$

Note that firm I's overall profit includes what it can earn from the sale of intermediate input and final good. Since the focus of the present paper is on the roles of vertical integration, we assume that firm I behaves like firm F in choosing the final product output, i.e., taking the intermediate-input price r as given. As a result, maximizing the two profit functions in (17) gives the same first-order conditions and Nash equilibrium in terms of the final-good outputs. Let the equilibrium outputs be represented by $y = y(r, v)$, $y^* = y^*(r, v)$, and $Y = y + y^* = Y(r, v)$. It should be noted that because the two firms take the intermediate-input price and subsidy as given, $(r-c+s)x$ is regarded as a fixed cost to the home firm at this stage. Thus the final-good output functions are the same as those in the previous case. The derived demand for the intermediate input can be obtained by inverting function $Y = Y(r, v)$ to give $r = r(X, v)$.[13]

[13]Notation is being slightly abused here for the sake of simplifying the notation. Some of the functions in this case are similar to but not necessarily the same as those in the previous case, even if the same symbol is used.

4.2 Stage 2: Intermediate-Input Production

The foreign intermediate-input firm f chooses its output in a Cournot way to maximize its profit function given below:[14]

$$\Pi^f = (r - c^*)x^*. \tag{18}$$

Home firm I also chooses its output, but it will take into consideration the impacts of its intermediate-input output on its final-good sale and its overall profit. The first-order conditions for profit maximization of firms I and f are given by

$$\begin{aligned}\frac{\partial \Pi^I}{\partial x} &= \Pi_y^D \frac{\partial y}{\partial r}\frac{\partial r}{\partial X} + \Pi_{y^*}^D \frac{\partial y^*}{\partial r}\frac{\partial r}{\partial X} + \Pi_r^D \frac{\partial r}{\partial X} + \Pi_x^d \\ &= \Pi_{y^*}^D \frac{\partial y^*}{\partial r}\frac{\partial r}{\partial X} + \Pi_r^D \frac{\partial r}{\partial X} + \Pi_x^d \\ &= -p'y + \Pi_x^d = 0 \qquad (19a)\\ \frac{\partial \Pi^f}{\partial x^*} &= r - c^* + r'x^* = 0. \qquad (19b)\end{aligned}$$

The term $\Pi_{y^*}^D(\partial y^*/\partial r)(\partial r/\partial X) + \Pi_r^D(\partial r/\partial X) = -p'y > 0$ captures the profit change in the final-good production. This term implies a reaction curve of the home firm in terms of intermediate-input production more to the right as compared with the corresponding one in the previous case. The reaction function of the foreign intermediate-input firm, however, is the same as that in the previous case. Therefore, the integrated firm tends to produce more intermediate goods than non-integrated intermediate good firm does described in model 1.[15] The home and foreign reaction functions are illustrated by curves HH and FF, respectively, in Figure 2. Conditions (19) are solved for the Nash equilibrium of the intermediate-input market, when the subsidies are taken as given. In Figure 2, it is depicted by point N.

[14] The reader is reminded that to simplify the present notation, we choose to have the same symbols for these two functions as those in the previous case even though the functional forms are different. No confusion is likely in our analysis because we are not doing comparison of the cases.

[15] Because $\partial \Pi^I/\partial x = -p'y > 0$ and because of the second-order condition $\Pi_{xx}^I < 0$, the optimal x in this case is bigger than that in case 1.

Conditions (19a) and (19b) are differentiated to solve for the following effects of subsidies:

$$\frac{\partial x}{\partial s} = -\frac{\Pi^f_{x^*x^*}}{D^i_x} = -\frac{2r' + r''x^*}{D^i_x} > 0 \tag{20a}$$

$$\frac{\partial x^*}{\partial s} = \frac{\Pi^f_{x^*x}}{D^i_x} = \frac{r' + r''x^*}{D^i_x} < 0 \tag{20b}$$

$$\frac{\partial X}{\partial s} = -\frac{r'}{D^i_x} > 0 \tag{20c}$$

$$\frac{\partial x}{\partial v} = \frac{1}{D^i_x}\left[\Pi^f_{x^*x^*}\left(\frac{\partial \tilde{y}}{\partial v}p' - \frac{\partial r}{\partial v}\right) + \frac{\partial r}{\partial v}\Pi^I_{xx^*}\right] > 0 \tag{20d}$$

$$\frac{\partial x^*}{\partial v} = \frac{1}{D^i_x}\left[-\Pi^f_{x^*x}\left(\frac{\partial \tilde{y}}{\partial v}p' - \frac{\partial r}{\partial v}\right) - \frac{\partial r}{\partial v}\Pi^I_{xx}\right] < 0, \tag{20e}$$

where $D^i_x = \Pi^I_{xx}\Pi^f_{x^*x^*} - \Pi^I_{xx^*}\Pi^f_{x^*x} > 0$.[16] The effects of the input subsidy s on x and x^*, as given by (20a) to (20c), are intuitive. The more interesting results are the effects of the final-good subsidy on the input production, as given by (20d) and (20e): An increase in v encourages the home firm to produce more input but the foreign firm to produce less. These effects can be explained intuitively by referring back to the first-order conditions (19). An increase in v creates two reinforcing effects on the home integrated firm's reaction curve: the direct effect, which was explained in the previous case, and the effect due to a rise in its final-good production. In the present case, when the home integrated firm produces more final good because of a higher subsidy on the final-good production, it tends to use more of its own intermediate-input. This is a very important feature of vertical integration. Condition (20d) shows that the home reaction curve in Figure 2 shifts so much to the right to create a Nash equilibrium at which the foreign intermediate-input output drops.

Because the subsidies affect the output and thus the price of the intermediate-input production, we can measure the total effects of the subsidies on the final-good outputs by defining the following functions: $y = \tilde{y}(v, s) \equiv y(r(X(s, v), v), v)$ and $y^* = \tilde{y}^*(s, v) \equiv y^*(r(X(s, v), v), v)$. The effects of the

[16] The signs of the effects of the subsidies and $D^i_x > 0$ depend on the assumption of not-too-convex demand functions.

subsidies on these outputs are

$$\frac{\partial \tilde{y}}{\partial s} = \frac{\partial y}{\partial r}\frac{\partial r}{\partial X}\frac{\partial X}{\partial s} > 0 \tag{21a}$$

$$\frac{\partial \tilde{y}^*}{\partial s} = \frac{\partial y^*}{\partial r}\frac{\partial r}{\partial X}\frac{\partial X}{\partial s} > 0 \tag{21b}$$

$$\frac{\partial \tilde{y}}{\partial v} = \frac{\partial y}{\partial r}\left(\frac{\partial r}{\partial v} + \frac{\partial r}{\partial X}\frac{\partial X}{\partial v}\right) + \frac{\partial y}{\partial v} > 0 \tag{21c}$$

$$\frac{\partial \tilde{y}^*}{\partial v} = \frac{\partial y^*}{\partial r}\left(\frac{\partial r}{\partial v} + \frac{\partial r}{\partial X}\frac{\partial X}{\partial v}\right) + \frac{\partial y^*}{\partial v} < 0. \tag{21d}$$

Note that the signs of these effects are the same as those in case 1 with no vertical integration. This suggests that vertical integration tends to affect mainly the effects of subsidies on the intermediate-input production. These results are summarized by the following proposition and table:

Proposition 3 *Suppose that home industries are vertically integrated but foreign industries are not. (i) The home input subsidy increases the home firm's intermediate-input production but decreases that of foreign firm. It also increases both home and foreign final-good production. (ii) The final-good subsidy increases the home firm's final good production but decreases that of foreign firm. It increases home firm's intermediate good production but decreases that of foreign firm.*

Table 2: The Effects of Government Subsidy With Vertical Integration in the Home Country

	x	x^*	y	y^*	Π^I	Π^F	Π^f
Subsidy on intermediate good ($s \uparrow$)	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$
Subsidy on final good ($v \uparrow$)	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$

4.3 Stage 1: Home Subsidies

With vertical integration, the domestic welfare is defined as:

$$W^I = \Pi^I - sx - vy. \tag{22}$$

The home government chooses the optimal subsidies s and v to maximize the welfare. The first-order conditions are given as:

$$\begin{aligned} \frac{\partial W^I}{\partial s} &= \Pi^I_y \frac{\partial \tilde{y}}{\partial s} + \Pi^I_{y^*} \frac{\partial \tilde{y}^*}{\partial s} + \Pi^I_x \frac{\partial x}{\partial s} + \Pi^I_{x^*} \frac{\partial x^*}{\partial s} + \Pi^I_s - x - s\frac{\partial x}{\partial s} \\ &= \Pi^I_{y^*} \frac{\partial \tilde{y}^*}{\partial s} + \Pi^I_{x^*} \frac{\partial x^*}{\partial s} - s\frac{\partial x}{\partial s} = 0 \qquad (23a) \\ \frac{\partial W^I}{\partial v} &= \Pi^I_y \frac{\partial \tilde{y}}{\partial v} + \Pi^I_{y^*} \frac{\partial \tilde{y}^*}{\partial v} + \Pi^I_x \frac{\partial x}{\partial v} + \Pi^I_{x^*} \frac{\partial x^*}{\partial v} + \Pi^I_v - y - v\frac{\partial y}{\partial s} \\ &= \Pi^I_{y^*} \frac{d\tilde{y}^*}{dv} + \Pi^I_{x^*} \frac{dx^*}{dv} - v\frac{d\tilde{y}}{ds} = 0 \qquad (23b) \end{aligned}$$

Conditions (23a) and (23b) are solved for the optimal subsidies:

$$\hat{s} = \underset{(+)}{\left(\frac{\partial x}{\partial s}\right)^{-1}} \left(\underset{(-)}{\Pi^I_{y^*}} \underset{(+)}{\frac{\partial \tilde{y}^*}{\partial s}} + \underset{(-)}{\Pi^I_{x^*}} \underset{(-)}{\frac{\partial \tilde{x}^*}{\partial s}}\right) \qquad (24a)$$

$$\hat{v} = \underset{(+)}{\left(\frac{\partial \tilde{y}}{\partial v}\right)^{-1}} \left(\underset{(-)}{\Pi^I_{y^*}} \underset{(-)}{\frac{\partial \tilde{y}^*}{\partial v}} + \underset{(-)}{\Pi^I_{x^*}} \underset{(-)}{\frac{\partial \tilde{x}^*}{\partial v}}\right) > 0. \qquad (24b)$$

In determining the sign of $\hat{v}$ in (24b), note that $\Pi^I_{y^*} = p'y < 0$ and $\Pi^I_{x^*} = \Pi^D_{y^*}(\partial y^*/\partial r)(\partial r/\partial X)(\partial X/\partial x^*) + \Pi^d_{x^*} < 0$. In (24b), $(\partial \tilde{y}/\partial v)^{-1}\Pi^I_{y^*}(\partial \tilde{y}^*/\partial v)$ is the profit-shifting effect, and $(\partial \tilde{y}/\partial v)^{-1}\Pi^I_{x^*}(\partial x^*/\partial v)$ is the linkage effect through a change in the foreign-input production. Both of these two effects are positive, guaranteeing a positive optimal final-good subsidy. Note that in the present case, a final-good subsidy discourages the foreign production of intermediate input, as shown in Figure 2, whereas in the previous case such a subsidy encourages the foreign intermediate-input production. This explains why in the present case the optimal final-good subsidy is necessarily positive.

In (24a), $(\partial x/\partial s)^{-1}\Pi^I_{x^*}(\partial x^*/\partial s)$ is the positive profit-shifting effect, and $(\partial x/\partial s)^{-1}\Pi^I_{y^*}(\partial \tilde{y}^*/\partial s)$ is the negative linkage effect: A rise in s encourages the foreign final-good production, but hurts the part of the home firm's profit that comes from the final-good production. The total effect is,

$$\begin{aligned}
&\Pi^I_{y^*}\frac{\partial \tilde{y}^*}{\partial s}+\Pi^I_{x^*}\frac{\partial \tilde{x}^*}{\partial s}\\
&=\underset{(-)(+)}{\Pi^I_{y^*}\frac{\partial \tilde{y}^*}{\partial s}}+\left(\underset{(-)}{\Pi^D_{y^*}}\underset{(+)}{\frac{\partial y^*}{\partial r}}\underset{(-)}{\frac{\partial r}{\partial x^*}}+\underset{(-)}{\Pi^d_{x^*}}\right)\underset{(-)}{\frac{\partial \tilde{x}^*}{\partial s}}.
\end{aligned}\tag{25}$$

The sign in condition (25) seems to be ambiguous, but after expanding the terms, it is found to be positive:

$$\Pi^I_{y^*}\frac{\partial \tilde{y}^*}{\partial s}+\Pi^I_{x^*}\frac{\partial x^*}{\partial s}=\frac{1}{D^i_x}\left[r'x(r'+r''x^*)+\frac{p''p'r''yx^*(y-y^*)}{2}\right]>0. \tag{26}$$

Condition (26) implies that the optimal s is positive.

Proposition 4 *When home firms are vertically integrated but foreign firms are not, the optimal subsidies on intermediate good and final good production are positive.*

5 Case 3: Vertical Integration in Foreign

We now examine another case, in which the two industries in foreign are integrated while the two home industries are not. We again consider the following three-stage game: In stage 1, the home government sets the subsidies. In stage 2, the home and foreign (integrated) intermediate-input firms choose their production, and in stage 3, the home and foreign final-good firms determine their production.

5.1 Stage 3: Final-Good Production

Our analysis again begins with the last stage. Denote the foreign integrated firm by I^*. The profit functions of the home final-good firm and the foreign integrated firm are

$$\Pi^D=(p-r-w+v)y, \tag{27a}$$

$$\Pi^{I^*}=\Pi^F+\Pi^f=(p-r-w^*)y^*+(r-c^*)x^*. \tag{27b}$$

In (27b), the profit of the foreign firm includes the profits it gets from the production of intermediate input and final product. Again, with the price of the intermediate input determined, and assuming that they compete in a Cournot way and take the subsidies as given, the Nash equilibrium is similar to that in the previous two cases. Denote the Nash equilibrium final-good output functions by $y = y(r, v)$, $y^* = y^*(r, v)$, and $Y = y + y^* = Y(r, v)$. Note again that the foreign firm will regard $(r - c^*)x^*$ as part of the fixed cost, and thus the output functions of the firms are the same as those in the previous two cases. The derived demand for the intermediate input is obtained as before.

5.2 Stage 2: Intermediate-Input Production

The profit function of the home intermediate-input firm is

$$\Pi^d = (r - c + s)x. \tag{28}$$

Both the home firm and the foreign integrated firm choose their outputs in a Cournot way, taking the subsidies as given, to maximize their profits. The first-order conditions are:

$$\frac{\partial \Pi^d}{\partial x} = r - c + s + r'x = 0 \tag{29a}$$

$$\begin{aligned}\frac{\partial \Pi^{I^*}}{\partial x^*} &= \Pi^F_y \frac{\partial y}{\partial r}\frac{\partial r}{\partial X} + \Pi^F_{y^*}\frac{\partial y^*}{\partial r}\frac{\partial r}{\partial X} + \Pi^F_r \frac{\partial r}{\partial X} + \Pi^f_{x^*} \\ &= \Pi^F_y \frac{\partial y}{\partial r}\frac{\partial r}{\partial X} + \Pi^F_r \frac{\partial r}{\partial X} + \Pi^f_{x^*} \\ &= -p'y^* + \Pi^f_{x^*} = 0. \end{aligned} \tag{29b}$$

Conditions (29) give the reaction functions of the two firms in terms of the intermediate input. The two reaction functions are represented by curves HH and FF, respectively, in Figure 3. They are both negatively sloped, with curve HH steeper than curve FF. They are solved to yield the Nash equilibrium outputs of the intermediate input, $x = x(s, v)$ and $x^* = x^*(s, v)$, which are depicted by the intersecting point of the two curves.

The Nash equilibrium depends on both subsidies. Differentiate equations (29) to yield the following effects of the subsidies:

$$\frac{\partial x}{\partial s} = -\frac{\Pi^{I^*}_{x^*x^*}}{D^{i^*}_x} = -\frac{1}{D^{i^*}_x}\left(\frac{5}{2}p' + \frac{p''(y+y^*)}{2} + r''x^*\right) > 0, \tag{30a}$$

$$\frac{\partial x^*}{\partial s} = \frac{\Pi^{I^*}_{x^*x}}{D^{i^*}_x} = \frac{1}{D^{i^*}_x}(p' + r''x^*) < 0, \tag{30b}$$

$$\frac{\partial X}{\partial s} = -\frac{1}{D^{i^*}_x}(\frac{3}{2}p' + \frac{p''(y+y^*)}{2p'}) > 0, \tag{30c}$$

$$\frac{\partial x}{\partial v} = \frac{1}{D^{i^*}_x}\left[-\Pi^d_{xx^*}\{\frac{dy^*}{dv}p' - \frac{\partial r}{\partial v}\} - \frac{\partial r}{\partial v}\Pi^{I^*}_{x^*x^*}\right] = -\frac{1}{D^{i^*}_x}\frac{\partial r}{\partial v}\Pi^{I^*}_{x^*x^*} > 0, \tag{30d}$$

$$\frac{\partial x^*}{\partial v} = \frac{1}{D^{i^*}_x}\left[\Pi^d_{xx}\{\frac{dy^*}{dv}p' - \frac{\partial r}{\partial v}\} + \frac{\partial r}{\partial v}\Pi^{I^*}_{x^*x}\right] = \frac{1}{D^{i^*}_x}\frac{\partial r}{\partial v}\Pi^{I^*}_{x^*x} < 0, \tag{30e}$$

where $D^{i^*}_x = \Pi^d_{xx}\Pi^{I^*}_{x^*x^*} - \Pi^d_{xx^*}\Pi^{I^*}_{x^*x} > 0$.[17] The interesting result is (30e): An increase in v lowers the foreign output x^*, This result can be explained in terms of the first-order conditions (29). It can be shown that an increase in v will shift the home intermediate-input firm's reaction curve to the right, just as in case 1. The corresponding change in the foreign integrated firm's reaction curve is not so certain: The direct effect will tend to shift it upward, as in case 1, but the home subsidy will discourage the foreign firm from producing more final good, and this effect tends to shift the foreign firm's reaction curve down. The net effect is not clear. Condition (30e) shows that the foreign firm's reaction curve will shift down sufficiently so that there is a drop in the foreign output. In Figure 3, the reaction curve of the home firm shifts to a position, as shown as H′H′, with a lower foreign production of the intermediate input.

Since the intermediate input production of each of the firms depends on the subsidies, we can define two reduced-form functions of the final product outputs, $y = \tilde{y}(s,v) \equiv y(r(X,v),v)$ and $y^* = \tilde{y}^*(s,v) \equiv y^*(r(X,v),v)$. The dependence of these outputs on the subsidies can be obtained by direct differentiation:

[17] Note $D^{i^*}_x > 0$ holds under the assumption of not-so-convex demand functions.

$$\frac{\partial \tilde{y}}{\partial s} = \frac{\partial y}{\partial r}\frac{\partial r}{\partial X}\frac{\partial X}{\partial s} > 0 \tag{31a}$$

$$\frac{\partial \tilde{y}^*}{\partial s} = \frac{\partial y^*}{\partial r}\frac{\partial r}{\partial X}\frac{\partial X}{\partial s} > 0 \tag{31b}$$

$$\frac{\partial \tilde{y}}{\partial v} = \frac{\partial y}{\partial r}\left(\frac{\partial r}{\partial v} + \frac{\partial r}{\partial X}\frac{\partial X}{\partial v}\right) + \frac{\partial y}{\partial v} > 0 \tag{31c}$$

$$\frac{\partial \tilde{y}^*}{\partial v} = \frac{\partial y^*}{\partial r}\left(\frac{\partial r}{\partial v} + \frac{\partial r}{\partial X}\frac{\partial X}{\partial v}\right) + \frac{\partial y^*}{\partial v} < 0. \tag{31d}$$

They results on the output effects of the home subsidies are summarizes by the following proposition and table:

Proposition 5 *Suppose that the home firms are not vertically integrated but foreign firms are. (i) The home government subsidy on intermediate-input production increases home firm's intermediate-input production and decreases that of the foreign firm. It increases both home and foreign final-good firms' outputs. (ii) The home government subsidy on final-good production increases the home firm's final-good production and decreases that of foreign firm. It increases the home firm's intermediate-input production but decreases that of the foreign intermediate-input firm.*

Table 3: Effects of Government Subsidies With Vertical Integration in the Foreign Country

	x	x^*	y	y^*	Π^D	Π^d	Π^{I^*}
Subsidy on intermediate good ($s \uparrow$)	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$
Subsidy on final good ($v \uparrow$)	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$

5.3 Stage 1: Home Subsidies

The national welfare is defined in the present model as:

$$W = \Pi^D + \Pi^d - sx - vy. \tag{32}$$

The home government chooses both subsidies to maximize its national welfare. The first-order conditions are given as:

$$\begin{aligned}\frac{\partial W}{\partial s} &= \Pi_y^D \frac{\partial \tilde{y}}{\partial s} + \Pi_{y^*}^D \frac{\partial \tilde{y}^*}{\partial s} + \Pi_x^d \frac{\partial x}{\partial s} + \Pi_{x^*}^d \frac{\partial x^*}{\partial s} + \Pi_s^d - x - s\frac{\partial x}{\partial s} \\ &= \Pi_{y^*}^D \frac{\partial \tilde{y}^*}{\partial s} + \Pi_{x^*}^d \frac{\partial x^*}{\partial s} - s\frac{\partial x}{\partial s} = 0 \qquad (33a)\\ \frac{\partial W}{\partial v} &= \Pi_y^D \frac{\partial \tilde{y}}{\partial v} + \Pi_{y^*}^D \frac{\partial \tilde{y}^*}{\partial v} + \Pi_x^d \frac{\partial x}{\partial v} + \Pi_{x^*}^d \frac{\partial x^*}{\partial v} + \Pi_v^D - y - v\frac{\partial \tilde{y}}{\partial s} \\ &= \Pi_{y^*}^D \frac{\partial \tilde{y}^*}{\partial v} + \Pi_{x^*}^d \frac{\partial x^*}{\partial v} - v\frac{\partial \tilde{y}}{\partial s} = 0. \qquad (33b)\end{aligned}$$

Conditions (33) are solved for the optimal subsidies,

$$\hat{s} = \underset{(+)}{\left(\frac{\partial x}{\partial s}\right)^{-1}} \left(\underset{(-)}{\Pi_{y^*}^D} \underset{(+)}{\frac{\partial \tilde{y}^*}{\partial s}} + \underset{(-)}{\Pi_{x^*}^d} \underset{(-)}{\frac{\partial x^*}{\partial s}}\right) \qquad (34a)$$

$$\hat{v} = \underset{(+)}{\left(\frac{\partial \tilde{y}}{\partial v}\right)^{-1}} \left(\underset{(-)}{\Pi_{y^*}^D} \underset{(-)}{\frac{\partial \tilde{y}^*}{\partial v}} + \underset{(-)}{\Pi_{x^*}^d} \underset{(-)}{\frac{\partial x^*}{\partial v}}\right) > 0. \qquad (34b)$$

The optimal subsidy on the final-good production $\hat{v}$ is positive. To explain this result, note that $(\partial\tilde{y}/\partial v)^{-1}\Pi_{y^*}^D(\partial\tilde{y}^*/\partial v)$ is the profit-shifting effect, while $(\partial\tilde{y}/\partial v)^{-1}\Pi_{x^*}^d(\partial x^*/\partial v)$ is the linkage effect through a change in the foreign intermediate-input production. The profit-shifting effect is positive because of a drop in the foreign final-good production, and the linkage effect is also positive because of a drop in the foreign intermediate-input production, as Figure 3 shows.

The sign of the optimal subsidy on input production $\hat{s}$ is ambiguous. In equation (34a), the term, $(\partial x/\partial s)^{-1}\Pi_{x^*}^d(\partial x^*/\partial s)$ is the profit-shifting effect and $(\partial x/\partial s)^{-1}\Pi_{y^*}^D(\partial\tilde{y}^*/\partial s)$ is the linkage effect. The former effect is positive but the latter effect is negative, because an increase in s will cause a drop in the foreign intermediate-input production but a rise in the foreign final-good production. The net effect of s is,

$$\Pi_{y^*}^D \frac{\partial y^*}{\partial s} + \Pi_{x^*}^d \frac{\partial x^*}{\partial s} = \frac{1}{D_x^{i^*}}\left[\left(-\frac{1}{2}y + x\right)p'r' - \frac{1}{2}p''y(y^* - y) + r''xx^*\right].$$

With a not-too-convex demand functions, $\hat{s}$ is positive if $-y/2 + x > 0$.

Proposition 6 *Suppose that home firms are not vertically integrated but foreign firms are. (i) The optimal subsidy on intermediate-input may not be positive. It is positive if* $-y/2+x>0$*. (ii) The optimal subsidy on final-good is positive.*

6 Case 4: Vertical Integration in both countries

We now turn to another case in which the firms in each country are vertically integrated. This means that each integrated firm is capable of producing both intermediate input and final product. As before, denote the homr and foreign firms by I and I*, respectively. We consider again a three-stage game. In stage 1, the home government chooses the optimal subsidies. In stage 2, they compete in the intermediate input production. Again, in order to allow comparison with other cases, both firms are assumed to trade freely in the international markets. In stage 3, both firms compete in a Cournot sense in the final good production, while taking the intermediate-input price and subsidies as given.

6.1 Stage 3: Final-Good Production

As before, the profit of each vertically integrated firm is equal to the sum of what it can earn from the sales of intermediate input and final good:

$$\Pi^I = \Pi^D + \Pi^d = (p - r - w + v)y + (r - c + s)x. \tag{35a}$$

$$\Pi^{I^*} = \Pi^F + \Pi^f = (p - r - w^*)y^* + (r - c^*)x^*. \tag{35b}$$

The first-order conditions in terms of the final products and the comparative static effects on the final product outputs remain the same as those in case 1. Denote again the Nash equilibrium outputs by $y = y(r, v)$, $y^* = y^*(r, v)$, and $Y = y + y^* = Y(r, v)$. The latter function can be inverted to given the demand function for the intermediate input, $r = r(X, v)$.

6.2 Stage 2: Intermediate-Input Production

The first-order conditions in terms of the intermediate inputs are:

$$\begin{aligned}
\frac{\partial \Pi^I}{\partial x} &= \Pi_y^D \frac{\partial y}{\partial r}\frac{\partial r}{\partial X} + \Pi_{y^*}^D \frac{\partial y^*}{\partial r}\frac{\partial r}{\partial X} + \Pi_r^D \frac{\partial r}{\partial X} + \Pi_x^d \\
&= \Pi_{y^*}^D \frac{\partial y^*}{\partial r}\frac{\partial r}{\partial X} + \Pi_r^D \frac{\partial r}{\partial X} + \Pi_x^d \\
&= -p'y + \Pi_x^d = 0 \qquad \text{(36a)} \\
\frac{\partial \Pi^{I^*}}{\partial x^*} &= \Pi_y^F \frac{\partial y}{\partial r}\frac{\partial r}{\partial X} + \Pi_{y^*}^F \frac{\partial y^*}{\partial r}\frac{\partial r}{\partial X} + \Pi_r^F \frac{\partial r}{\partial X} + \Pi_{x^*}^f \\
&= \Pi_y^F \frac{\partial y}{\partial r}\frac{\partial r}{\partial X} + \Pi_r^F \frac{\partial r}{\partial X} + \Pi_{x^*}^f \\
&= -p'y^* + \Pi_{x^*}^f = 0 \qquad \text{(36b)}
\end{aligned}$$

Conditions (36) give the reaction functions of the firm, and they can be represented by curves HH and FF for the home and foreign firms, respectively, in Figure 4. The conditions can be solved for the Nash equilibrium outputs, $x = x(s, v)$ and $x^* = x^*(s, v)$. Graphically, the equilibrium is depicted by point N, the intersecting point of the curves.

As in cases 2 and 3, the first-order conditions in (36) reflect two opposing factors. As the output of the intermediate input increases, its price falls. On the one hand, the decrease in the input price implies a lower cost for final good producer in the other country. Thus it shifts profit to the competitor. On the other hand, the integrated firm itself also gains profit due to a lower final-good production cost.

Differentiate conditions (36) and solve for the following comparative static results, with the signs of the terms dependent on a not-too-convex demand functions:

$$\frac{\partial x}{\partial s} = -\frac{\Pi^{I^*}_{x^*x^*}}{D^{i^*}_x} = -\frac{1}{D^{i^*}_x}\left[\frac{5}{2}p' + \frac{p''(y+y^*)}{2} + r''x^*\right] > 0 \tag{37a}$$

$$\frac{\partial x^*}{\partial s} = \frac{\Pi^{I^*}_{x^*x}}{D^{i^*}_x} = \frac{1}{D^{i^*}_x}(p' + r''x^*) < 0 \tag{37b}$$

$$\frac{\partial X}{\partial s} = -\frac{1}{D^{i^*}_x}\left(\frac{3}{2}p' + \frac{p''(y+y^*)}{2}\right) > 0 \tag{37c}$$

$$\begin{aligned}\frac{\partial x}{\partial v} &= \frac{1}{D^{i^*}_x}\left[\Pi^{I^*}_{x^*x^*}\left(\frac{\partial y}{\partial v}p' - \frac{\partial r}{\partial v}\right) - \Pi^{I}_{xx^*}\left(\frac{\partial y^*}{\partial v}p' - \frac{\partial r}{\partial v}\right)\right] \\ &= -\frac{\Pi^{I^*}_{x^*x^*}}{D^{i^*}_x} = -\frac{1}{D^{i^*}_x}\left[\frac{5}{2}p' + \frac{p''(y+y^*)}{2} + r''x^*\right] > 0 \end{aligned} \tag{37d}$$

$$\begin{aligned}\frac{\partial x^*}{\partial v} &= \frac{1}{D^{i^*}_x}\left[\Pi^{I}_{xx}\left(\frac{\partial y^*}{\partial v}p' - \frac{\partial r}{\partial v}\right) - \Pi^{I}_{x^*x^*}\left(\frac{\partial y}{\partial v}p' - \frac{\partial r}{\partial v}\right)\right] \\ &= \frac{\Pi^{I^*}_{x^*x}}{D^{i^*}_x} = \frac{1}{D^{i^*}_x}(p' + r''x^*) < 0, \end{aligned} \tag{37e}$$

where $D^{i^*}_x = \Pi^I_{xx}\Pi^{I^*}_{x^*x^*} - \Pi^I_{xx^*}\Pi^{I^*}_{x^*x} > 0$.[18] To understand these effects, we can refer to Figure 4. Let us focus on the subsidy on the intermediate input. Condition (36a) shows that an increase in v will shift curve HH to the right. The rise in the home final-good production will raise the demand for the integrated firm's intermediate-input production, causing a substantial shift of curve HH. By condition (36b), the shift in curve FF at a higher level of v is unknown: The direct effect is positive, as was explained in case 1, but the indirect effect is negative because of a drop in the foreign integrated firm's final-good production. Condition (37e) shows that curve HH will shift so much to the right that there is a drop in the equilibrium value of x^*, as Figure 4 shows.

Using the fact that the intermediate input productions are affected by the home subsidies, we can define two new reduced-form functions for the final-product productions: $y = \tilde{y}(s,v) \equiv y(r(s,v),s,v)$ and $y^* = \tilde{y}^*(s,v) \equiv y^*(r(s,v),s,v)$. The dependence of the final-product outputs on the subsidies is

[18] Note that $D^{i^*}_x > 0$ holds under the not-too-convex demand assumption.

$$\frac{\partial \tilde{y}}{\partial s} = \frac{\partial y}{\partial r}\frac{\partial r}{\partial X}\frac{\partial X}{\partial s} > 0 \tag{38a}$$

$$\frac{\partial \tilde{y}^*}{\partial s} = \frac{\partial y^*}{\partial r}\frac{\partial r}{\partial X}\frac{\partial X}{\partial s} > 0 \tag{38b}$$

$$\frac{\partial \tilde{y}}{\partial v} = \frac{\partial y}{\partial r}\left(\frac{\partial r}{\partial v} + \frac{\partial r}{\partial X}\frac{\partial X}{\partial v}\right) + \frac{\partial y}{\partial v} > 0 \tag{38c}$$

$$\frac{\partial \tilde{y}^*}{\partial v} = \frac{\partial y^*}{\partial r}\left(\frac{\partial r}{\partial v} + \frac{\partial r}{\partial X}\frac{\partial X}{\partial v}\right) + \frac{\partial y^*}{\partial v} < 0. \tag{38d}$$

These effects are summarized by the following proposition:

Proposition 7 *When firms in both countries are vertically integrated, the directions of the effect of government subsidy is same as in proposition 6.*

6.3 Stage 1: Home Subsidies

The national welfare is defined as:

$$W = \Pi^I - sx - vy. \tag{39}$$

The government chooses the subsidies to maximize the national welfare. The first-order conditions are:

$$\begin{aligned}\frac{\partial W}{\partial s} &= \Pi^I_y \frac{\partial \tilde{y}}{\partial s} + \Pi^I_{y^*}\frac{\partial \tilde{y}^*}{\partial s} + \Pi^I_x \frac{\partial x}{\partial s} + \Pi^I_{x^*}\frac{\partial x^*}{\partial s} + \Pi^I_s - x - s\frac{\partial x}{\partial s} \\ &= \Pi^I_{y^*}\frac{\partial \tilde{y}^*}{\partial s} + \Pi^I_{x^*}\frac{\partial x^*}{\partial s} - s\frac{\partial x}{\partial s} = 0 \qquad (40a)\\ \frac{\partial W}{\partial v} &= \Pi^I_y \frac{\partial \tilde{y}}{\partial v} + \Pi^I_{y^*}\frac{\partial \tilde{y}^*}{\partial v} + \Pi^I_x \frac{\partial x}{\partial v} + \Pi^I_{x^*}\frac{\partial x^*}{\partial v} + \Pi^I_v - y - v\frac{\partial \tilde{y}}{\partial s} \\ &= \Pi^I_{y^*}\frac{\partial \tilde{y}^*}{\partial v} + \Pi^I_{x^*}\frac{\partial x^*}{\partial v} - v\frac{\partial \tilde{y}}{\partial s} = 0. \qquad (40b)\end{aligned}$$

Conditions (40) are solved for the optimal subsidies:

$$\hat{s} = \underset{(+)}{\left(\frac{\partial x}{\partial s}\right)^{-1}} \left(\underset{(-)}{\Pi^I_{y^*}} \underset{(+)}{\frac{\partial \tilde{y}^*}{\partial s}} + \underset{(-)}{\Pi^I_{x^*}} \underset{(-)}{\frac{\partial x^*}{\partial s}}\right) \tag{41a}$$

$$\hat{v} = \underset{(+)}{\left(\frac{\partial y}{\partial v}\right)^{-1}} \left(\underset{(-)}{\Pi^I_{y^*}} \underset{(-)}{\frac{\partial \tilde{y}^*}{\partial v}} + \underset{(-)}{\Pi^I_{x^*}} \underset{(-)}{\frac{\partial x^*}{\partial v}}\right) > 0. \tag{41b}$$

According to (41b), the optimal subsidy on the final product $\hat{v}$ is positive. The intuition behind this result is similar to those in cases 2 and 3: On top of the positive profit-shifting effect, there is a positive linkage effect due to a drop in the foreign intermediate-input production. The sign of the optimal subsidy on the intermediate input $\hat{s}$ is ambiguous. In equation (41b), the $(\partial x/\partial s)^{-1}\Pi^I_{x^*}(\partial x^*/\partial s)$ is the profit-shifting effect while $(\partial x/\partial s)^{-1}\Pi^I_{y^*}(\partial \tilde{y}^*/\partial s)$ is the linkage effect. Although the profit-shifting effect is positive, the linkage effect is negative because of a rise in the foreign final-good production at a higher level of s. The total effect is

$$\begin{aligned} &\Pi^D_{y^*}\frac{\partial \tilde{y}^*}{\partial s} + \Pi^d_{x^*}\frac{\partial x^*}{\partial s} \\ &= \frac{1}{D^{i*}_x}\left\{\frac{y}{2}\left[-\frac{p'^2}{2} - \frac{p'p''}{2} + r''x^*(p'+p'') - p''(y-y^*)(p'-r')\right]\right. \\ &\quad \left. + \ \ r'p'x + r'r''xx^*\right\}. \end{aligned} \tag{42}$$

Consider the following condition:

$$-\frac{p'^2 y}{2} + r'p'x > 0. \tag{43}$$

Proposition 8 *Suppose that firms in each of the countries are vertically integrated. (i) The sign of the optimal subsidy on the intermediate input is in general ambiguous. It is positive if the demand is not too convex and if condition (43) holds. (ii) The optimal subsidy on the final product is positive.*

Proof. This proposition follows immediately conditions (41) to (43). ■

The table indicates the optimal home subsidies that are positive in these four cases. The first quadrant shows case 1, the second quadrant shows case 2, and so on.

Table 4: Positive Optimal Home Subsidies

		Home Firms	
		Non-Integrated	Integrated
Foreign Firms	Non-Integrated	-	$\hat{s}$, $\hat{v}$
	Integrated	$\hat{v}$	$\hat{v}$

7 Concluding Remarks

By first examining the linkage between local and foreign intermediate-input and final-good industries and the choice of optimal subsidies on intermediate input and final good production, this paper derives the optimal export subsidies in the two industries for a government, assuming that the foreign government is passive in policy setting. It was demonstrated that the linkage and the subsidies depend crucially on whether the industries are integrated. If the industries in both countries are not integrated, then the optimal subsidies may not be positive although at least one of them has to be positive. If the industries in either or both countries are integrated, then the optimal subsidy on the final good will be positive, but the optimal subsidy on the intermediate input may not be, except in the case in which the home firms are integrated, in which the optimal subsidy on the intermediate input will also be positive.

One interesting implication of the results in this paper is that vertical integration in either country will always lead to a positive optimal subsidy on the final product. To understand the intuition behind this result, we can note that the profit-shifting effect well known in the literature still exists in the presence of intermediate inputs, and vertical integration prevents the linkage effect from becoming negative and dominating. That is because the integrated firm always takes into consideration the effects of a final-product subsidy on the intermediate-input production. Thus if the home industries are integrated, the home firm will tend to produce more intermediate input when the production of the final good is encouraged by the subsidy. If the foreign industries are integrated, the foreign firm will tend to lower its output of intermediate input when its production of the final product is being discouraged by the home subsidy.

The model considered in this paper is a simple extension of a well-known international duopoly model considered in the literature to study the use of export subsidies. It is based on some simplifing assumptions, some of which could be far away from the reality. We do not want to claim wide applicability of the results derived. Rather, the main objective of tahis paper is to show that the neglect of the existence of intermediate inputs in the analysis in the literature could give misleading results. Relaxation of some of the assumptions in this paper to yield more realistic results and stronger policy implicaitons will be the topics of future research.

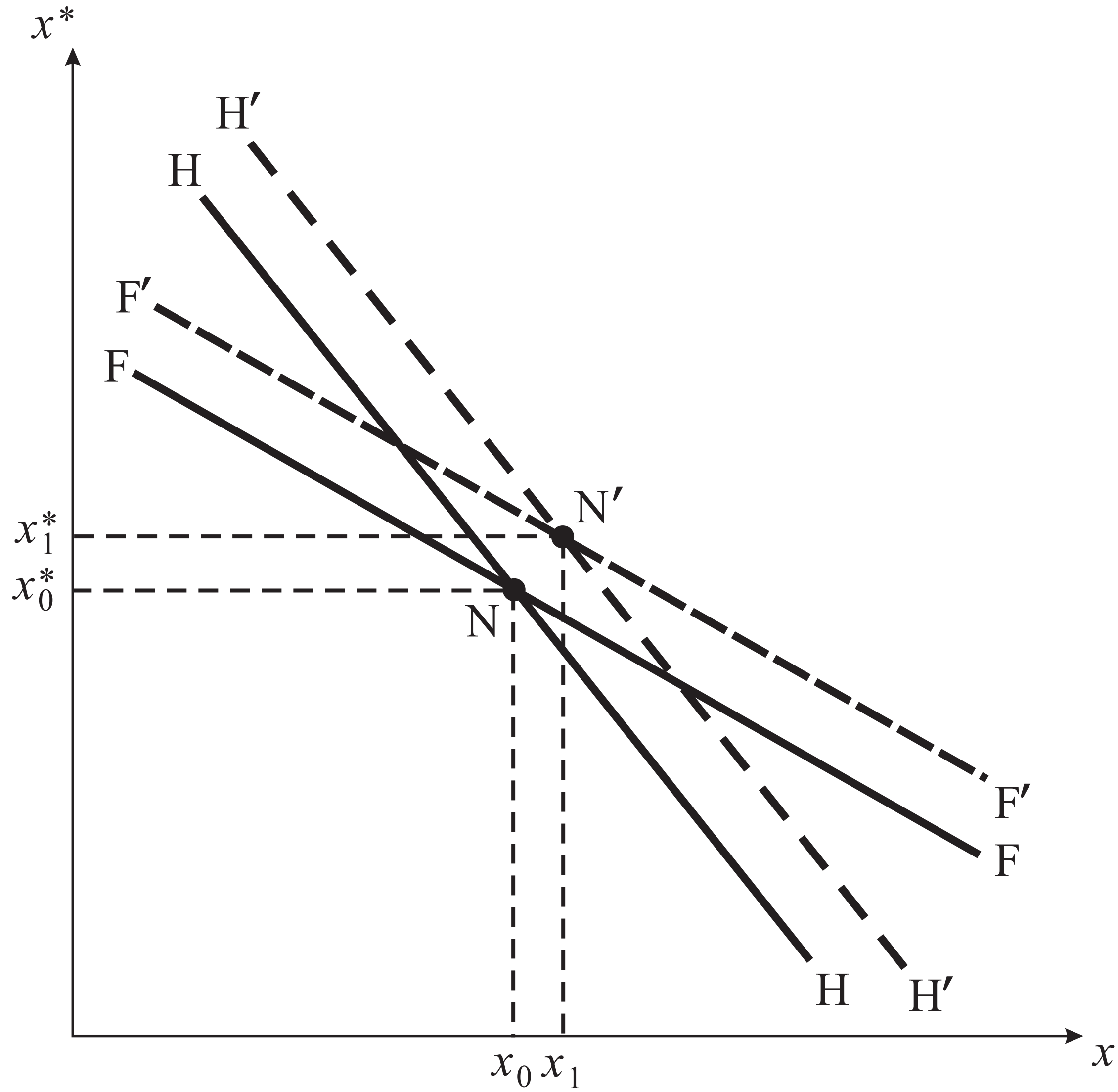

Figure 1:
The Intermediate-Input Market in Case 1

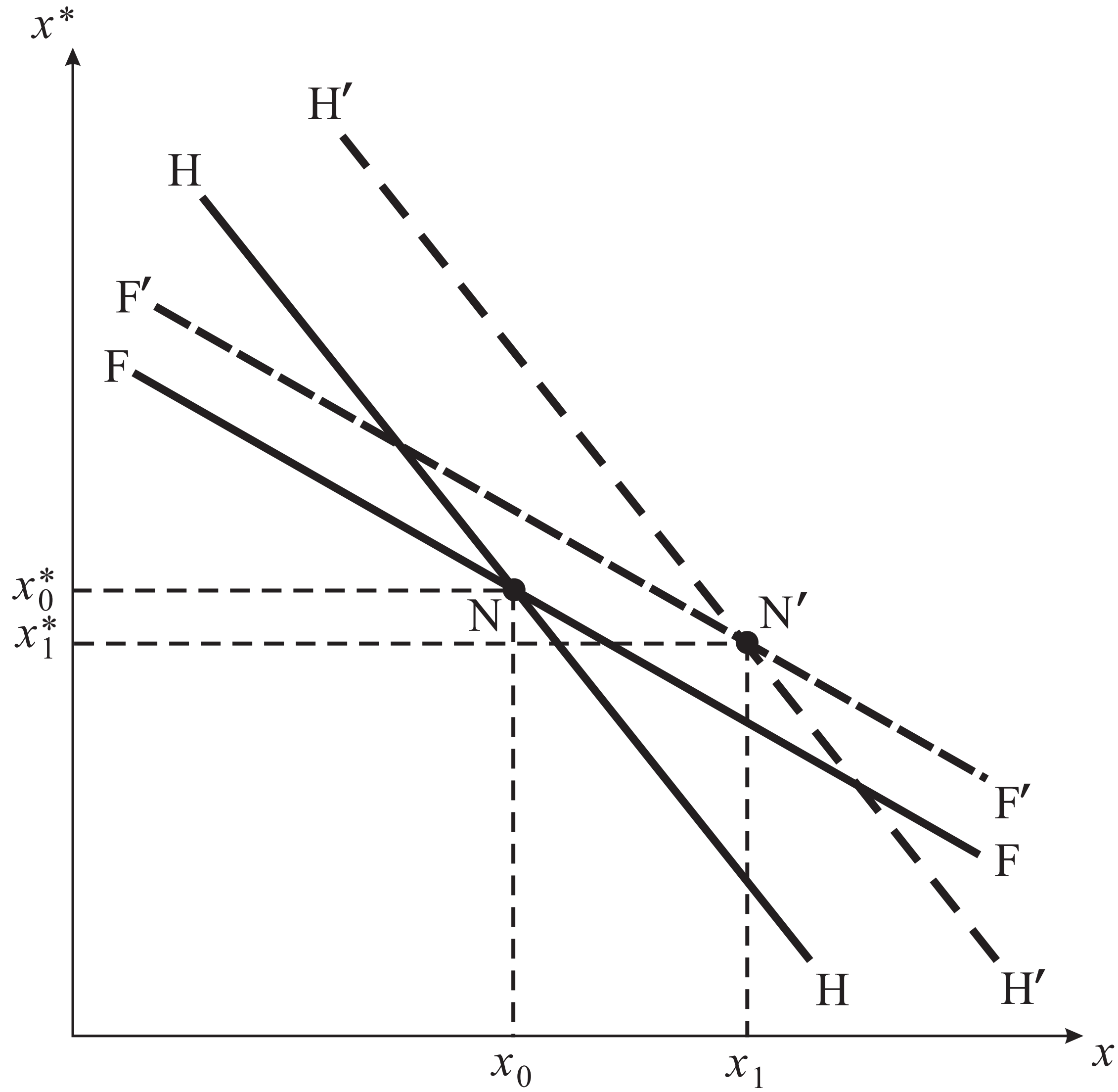

Figure 2:
The Intermediate-Input Market in Case 2

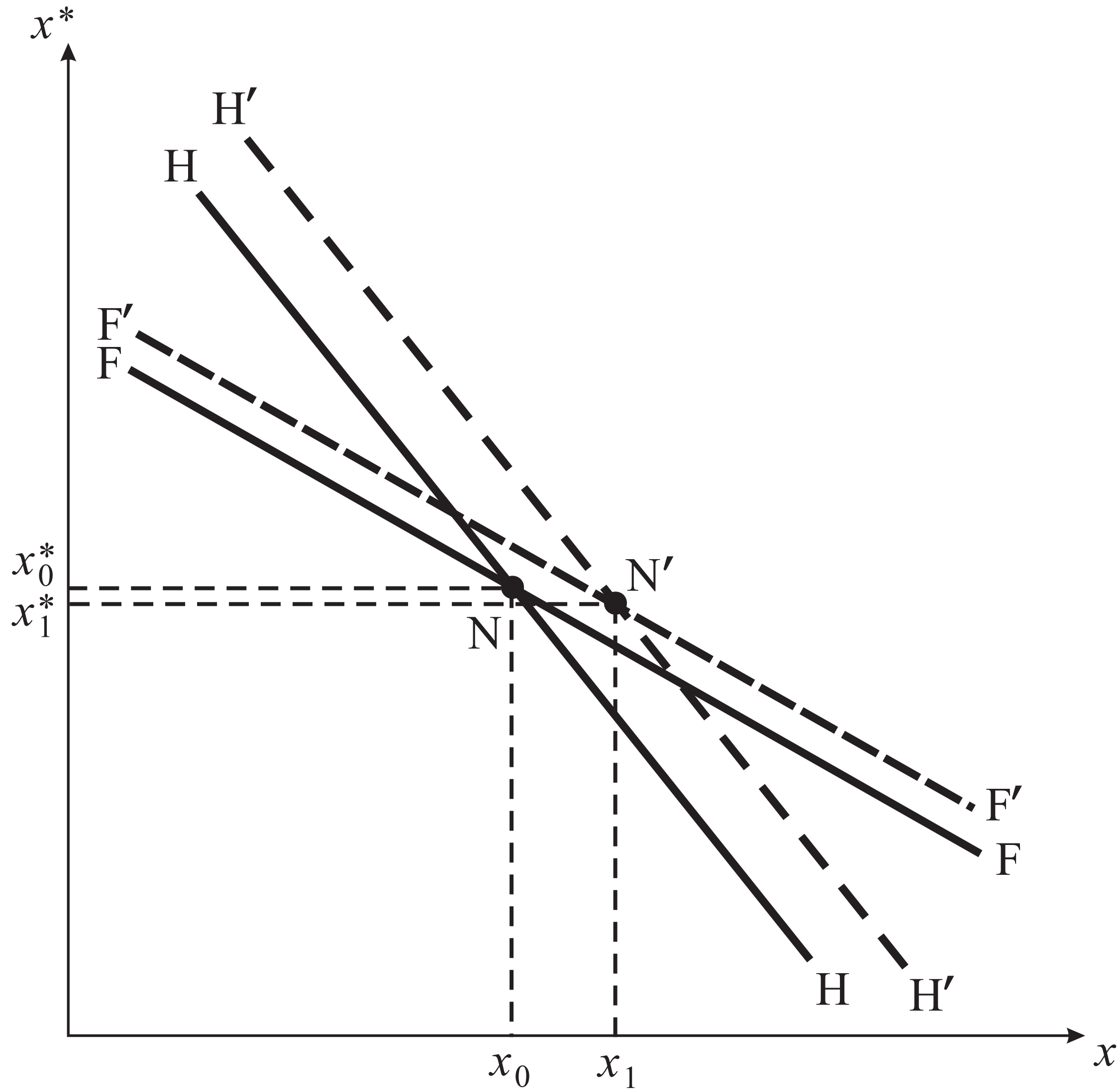

Figure 3:
The Intermediate-Input Market in Case 3

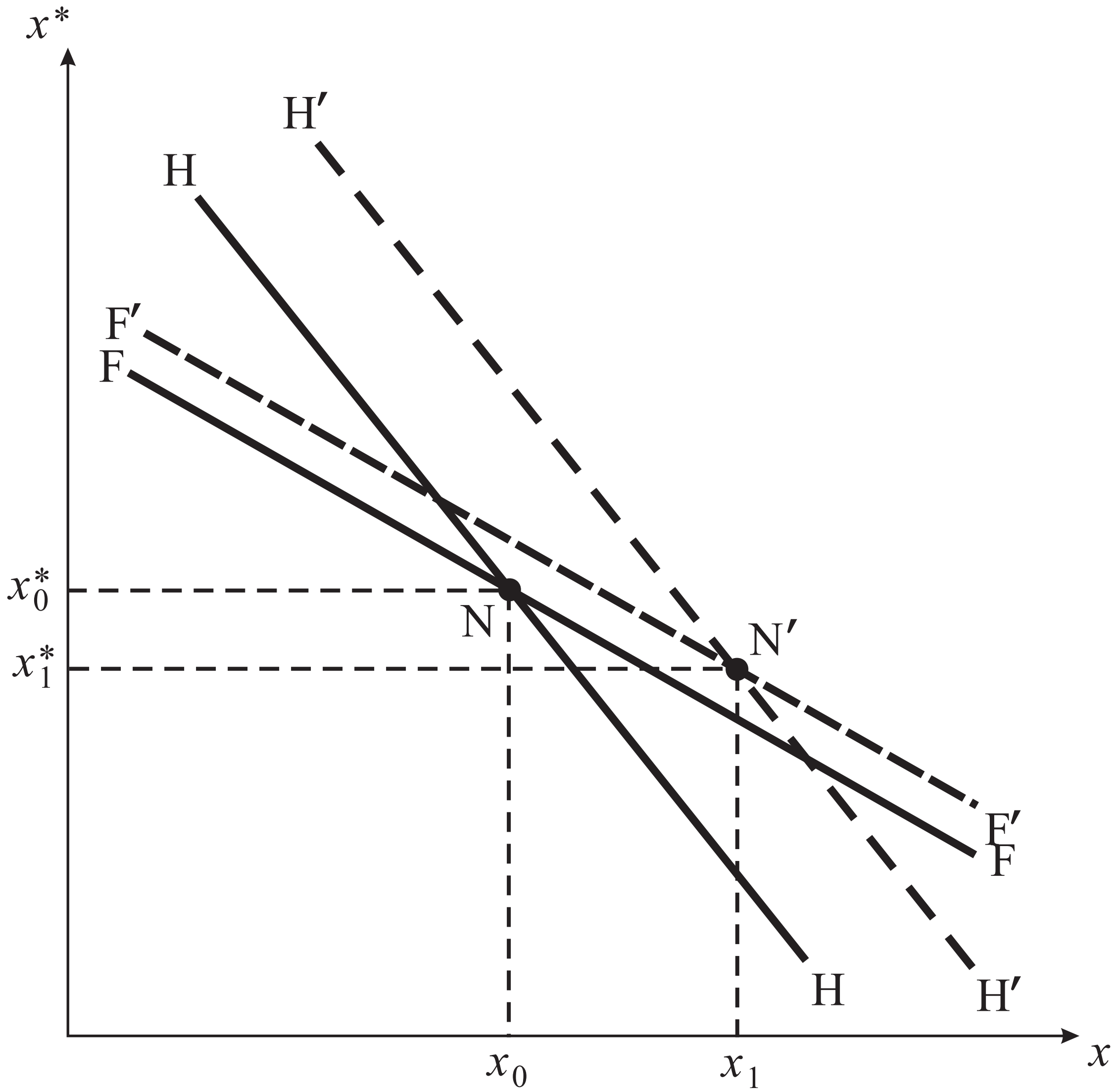

Figure 4:
The Intermediate-Input Market in Case 4

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