On the Optimal Timing of Foreign Trade

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Abstract

In this paper, we consider a two-sector, dynamic model of an economy in which one of the sectors are growing through learning by doing while the other one is stagnant along an autarkic balanced growth path. The resulting declining relative price thus implies that accelerated industrialization through production subsidy could improve the lifetime welfare of this closed economy.

This model is used to study the effects of trade between two economies with similar structures. Trade and the pattern of production in the present economy depends not only on its characteristics such as technologies, preferences, knowledge accumulation rates, and labor force endowments, but also on the timing of trade. There are some cases in which the pattern of trade could switch and some cases in which the economy exports the "wrong" good in the sense that the actual pattern of trade is not the same as what it would be should no trade be allowed in the first place.

It is shown that whether free trade is beneficial to a small open economy in the long run depends on what good is being produced and exported, and also on when free trade is first allowed. Thus free trade that starts as soon as possible may not be good, but if the timing of free trade can be chosen, it is gainful. This paper also derives the optimal timing of trade under an optimal production subsidy to promote industrialization.

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1 Introduction

Should a country allow foreign trade? This is one of the oldest questions in the theory of international trade. From the mercantilists and the Classical economists like Adam Smith and David Ricardo to modern economists like Paul Samuelson and Murray Kemp, the gains from trade has been an important topic in the theory and research.[1] More recently, there has been growing interest in examining whether trade is gainful in a dynamic context. For example, Kemp and Long (1979), Binh (1985), Serra (1991), and Kemp and Wong (1995) analyze the welfare implications of trade on economies with overlapping generations.[2] Grossman and Helpman (1991), Baldwin (1992), and Taylor (1994), on the other hand, focus on how trade may affect the welfare of economies with endogenous growth.

While extending the traditional static models of international trade to a dynamic context does permit the analysis of the effects of trade on factor accumulation and/or technology, these papers share one common features. They all examine economies that are characterized by steady states with constant relative prices both before and after trade.[3] The implication is that if foreign trade is good, it does not matter whether foreign trade is allowed now or some finite time in the future.[4] This is ironic because while these papers are examining various dynamic models, the conclusion drawn is that timing of foreign trade does not matter.

The purpose of this paper is to consider explicitly a new dimension in the theory of international trade: time. It argues that there are many cases

[1] The literature on gains from trade is huge. A recent survey and extension is given in Wong (1995, Chapters 8 and 9).

[2] While examples have been constructed to show that with overlapping generations uncompensated trade is Pareto inferior to autarky, Kemp and Wong (1995) aruge that there are four compensation schemes that a government may use to ensure that trade is Pareto improving.

[3] It is common in the growth literture to postulate constant relative prices in the steady states. See Bond, Wang, and Yip (1997) for a general characterization of this class of growth models.

[4] In general, welfare is measured by the summation of discounted utility stream from now to infinity in a steady state. In other words, the welfare remains unchanged if the summation starts, say, two periods later.

in which the time dimension of foreign trade is crucial in determining the resource allocation, pattern of trade, and welfare of an economy. In these cases, it is no longer good enough to ask "Should a country allow foreign trade?". Instead, we should ask, "When should a country allow foreign trade?". This question not only dominates the former one, but also highlights the fact that there is one more dimension in the government trade policy that has not received enough attention in the literture.[5]

In introducing the time dimension of foreign trade, we depart from the usual practice in the literature of considering constant relative prices in a steady state of an economy. Instead, we construct a model in which at a balanced-growth-path (BGP) equilibrium, an economy is still experiencing changing relative prices. The feature, which is consistent with some stylized facts, allows us to examine the implications of permitting foreign trade at different points of time.[6] To achieve this objective, we consider a dynamic model with two sectors that follow different growth paths, even at a BGP equilibrium. One sector, named agriculture, employs only labor input from a constant labor work force. The other sector, denoted as manufacturing, uses both labor input and knowledge in production with knowledge being accumulated through an "external" learning-by-doing process.[7] As a result, the former sector stagnates while the latter expands over time, leading to a changing relative price along a balanced growth path.

Using the present model, we show that international trade depends not only on the characteristics of an economy and those of the rest of the world, but also on *when* free trade is allowed. The reason is that the relative prices in the economy and the rest of the world are changing over time, and in many cases, the comparative advantage of the economy and its patterns of production and trade also depend on time. It is thus no longer meaningful to ask what the comparative advantage of an economy is. Instead, one should ask what its comparative advantage at a particular point in time is and how it may change over time.

This paper examines how the lifetime welfare of an economy may be

[5]Allowing free trade at infinity is equivalent to permitting autarky forever.

[6]For example, some manufacturing goods such as computers, TV sets and radios are getting cheaper relatively to many other products over time.

[7]The learning-by-doing process is external to the firms in the sense that when firms make their production decisions, they take knowledge as given. See, for example, Boldrin and Scheinkman (1988), Young (1991), and Matsuyama (1992). A recent survey of economic growth and international trade is in Long and Wong (1997).

affected by trade. With no transition as the economy moves from autarky to free trade, the model allows us to measure the dynamic gains from trade directly along a balanced growth equilibrium path. Unlike some other papers, we ask not whether free trade is gainful, but whether free trade starting *from a particular time* is beneficial. We further derive the *optimal timing and subsidy* of foreign trade.

Since the lifetime welfare of an economy depends on the changing comparative advantage of an economy, it is natural to investigate whether it is worth altering the economy's comparative advantage through some policy such as production subsidy. This is an old question linked to the infant-industry argument.[8] In this model, we provide a new look to this argument and analyze when it makes sense to provide a production subsidy and delay the occurrence of trade.

This paper is organized as follows. In Section 2, we describe the closed economy. As explained, the balanced growth path of this economy is characterized by a declining relative price of a good labeled manufacturing. It is not surprising to find that this economy is distorted as there exists an external effect from the learning-by-doing process of knowledge accumulation. So policies like a production subsidy will improve its lifetime welfare and we derive the optimal production subsidy formula. Section 3 analyzes free

[8]Earlier dynamic analyses of the infant industry argument include Clemhout and Wan (1970) and Bardhan (1971). The former studied the optimal pricing policy in a small dynamic two-sector model of international trade with the learning-by-doing technological change. Depending upon the length of the planning horizon and the external world prices, the optimal policy took one of the three modes - protected the slower learner; protected first the slow learner then the fast learner; and protected always the fast learner. In the latter, a simple dynamic 2x2 Heckscher-Ohlin model with learning-by-doing in one of the sectors was built to analyze infant-industry protection. The stock of experience accumulated through learning-by-doing was external to the individual firm. In the steady-state equilibrium, the relative price of the two goods was constant and the sector with learning-by-doing was under-produced. Thus, an optimal subsidy was called for and it steadily decreased (increased) to the stationary level when the initial stock of experience was below (above) its steady-state level. See also Krugman (1984).

However, the empirical evidence on the infant industry argument is mixed. Krueger and Tuncer (1982) performed an empirical test on the infant industry argument using Turkish data and concluded that the infant industry argument was not supported. On the other hand, Harrison (1984) argued that if one applied formal statistical analysis on the Turkish data used in the Krueger-Tuncer study, a statistically significant positive relationship between increased protection and higher productivity could be found. This then casted doubt on the evidence against infant industry argument provided by Krueger and Tuncer.

trade and the pattern of production of the economy. Section 4 examines the dynamic gains from free trade. In section 5, the optimal timing of trade, with and without a suitably chosen production subsidy, is studied. The last section concludes.

2 The Closed Economy

Consider a two-sector, dynamic economy. Two homogeneous consumption goods, which for convenience are labeled agriculture and manufacturing, are produced by competitive firms.

2.1 Technology

The production of agricultural good (good A) requires only labor input, and its sectoral production function can be written as:

$$X_t^A = AL_t^A, \tag{1}$$

where X_t^A is the agricultural output, L_t^A is the labor input at any time $t \in [0, \infty]$, and $A > 0$ denotes the constant labor productivity. Since A is constant, it is equal to the marginal as well as average product of labor of the sector.

Production of the manufacturing good (good M) requires two inputs: labor (L^M) and an intangible capital (M),

$$X_t^M = F(M_t, L_t^M), \tag{2}$$

where X_t^M is the manufacturing output. The intangible capital mimics the concept of "experience" or "knowledge" in production, which is taken by the firms as constant at any time t, but it increases over time according to the following learning-by-doing process:

$$\dot{M}_t = \mu X_t^M = \mu F(M_t, L_t^M), \tag{3}$$

where $\mu > 0$ is a measure of the effectiveness of learning by doing and a dot above a variable means its time derivative.[9] By condition (3), M_t plays the role of the engine of growth in the model. We assume that the initial value of the intangible capital, M_0, is given. For perpetual growth in this economy,

[9] We do not consider depreciation of knowledge capital. See also Matsuyama (1992)

we assume that the production function, F, is subject to constant returns in M_t and take the following form:

$$F(M_t, L_t^M) = BM_t L_t^M, \tag{4}$$

where $B > 0$ is the technology index, which is constant over time. Firms take M_t at any time as given, perceiving that their output level is proportional to labor employment. As a result, we have a Ricardo-Viner model at hand as in Matsuyama (1992). Choosing the agricultural good as the numeraire, we denote the relative consumers price of the manufacturing good by p_t. In addition, we consider a production subsidy of constant ad valorem rate of $s > -1$ on the manufacturing sector so that the domestic manufacturing price faced by producers becomes $(1+s)p_t$.[10] Perfect and costless mobility of labor between the two sectors with positive outputs implies equalization of wage rates:

$$A = (1+s)p_t BM_t. \tag{5}$$

For simplicity, we assume that the economy is endowed with a constant labor force, L.

2.2 Preferences

Consumption of the two goods (C_t^A and C_t^M) are chosen optimally by a representative agent. Assume that the instantaneous utility function of the representative agent at time t is given by $\beta \ln C_t^A + \ln C_t^M$, where $\beta > 0$. The optimization problem of the representative agent is to choose the consumption stream to maximize lifetime welfare,

$$W = \max \int_0^\infty (\beta \ln C_t^A + \ln C_t^M) e^{-\rho t} \mathrm{d}t, \tag{6}$$

subject to a standard budget constraint

$$C_t^A + p_t C_t^M = AL_t^A + (1+s)p_t BM_t L_t^M - T_t, \tag{7}$$

as well as (3), where ρ is the rate of time preferences and T_t, treated as constant by the agent, denotes the lump-sum tax used to finance the production

[10] If $s < 0$, it is a production tax. If $s = 0$, consumers prices are equal to producers prices.

subsidy. Letting λ_t be the costate variable associated with (3), the first-order conditions for the optimization problem are

$$\begin{aligned} \beta p_t/C_t^A &= 1/C_t^M \qquad (8) \\ \dot{\lambda}_t &= \rho\lambda_t - \lambda_t\mu B L_t^M - \beta(1+s)p_t B L_t^M/C_t^A, \qquad (9) \end{aligned}$$

as well as (3), (7) and the transversality condition. Given a Cobb-Douglas type utility function, the representative agent chooses to consume both goods at all finite, positve prices.

In this section, we consider a closed economy, meaning that equilibrium of the goods market is described by

$$\begin{aligned} C_t^M &= BM_tL_t^M, \qquad (10) \\ C_t^A &= AL_t^A. \qquad (11) \end{aligned}$$

Equilibrium of the labor market is

$$L_t^A + L_t^M = L. \qquad (12)$$

By making use of the production functions (1) and (4), and the equilibrium condition (12), the production possibility frontier (PPF) of the economy at time t is described by the following equation:

$$X_t^A = AL - \frac{A}{BM_t}X_t^M. \qquad (13)$$

The marginal rate of transformation (MRT) of the economy, denoted by q_t, is equal to the magnitude of the slope of the PPF, or, by (13), equal to

$$q_t \equiv -\frac{\mathrm{d}X_t^A}{\mathrm{d}X_t^M} = \frac{A}{BM_t}. \qquad (14)$$

Because the intangible capital M_t is growing over time, the MRT is declining at the same rate. By condition (5), the producers' price ratio is equal to the MRT, $q_t = (1+s)p_t$.

The optimality and equilibrium conditions can be rewritten as follows:

$$\begin{aligned} C_t^A &= \beta p_t C_t^M \qquad (15) \\ \frac{\dot{\lambda}_t}{\lambda_t} &= \rho - \mu B L_t^M - \frac{1+s}{\lambda_t M_t} \qquad (16) \\ \dot{M}_t &= \mu B M_t L_t^M \qquad (17) \end{aligned}$$

as well as (5) and (11). Next, combining (5), (10)–(12), and (15), we get

$$L^M = \frac{(1+s)L}{1+s+\beta}. \tag{18}$$

Condition (18) has four implications. First, $L^M \in (0, L)$ for any finite $s > -1$, meaning that the economy is diversified. Second, the equilibrium value of L^M is independent of prices. Third, L^M is constant in equilibrium. This further implies that L^A and thus consumption and production of the agricultural good is constant in equilibrium. Fourth, a rise in s increases L^M:

$$\frac{\partial L^M}{\partial s} = \frac{\beta L}{(1+s+\beta)^2} > 0, \tag{19}$$

i.e., an increase in s induces more labor from the agricultural sector to the manufacturing sector, encouraging the production of the latter.

2.3 Balanced Growth Path

The balanced growth path (BGP) equilibrium of the economy is defined as a situation in which all endogenous variables are changing at constant rates (not necessarily the same). Based on this definition, the autarkic BGP equilibrium of the economy is described by the following proposition:

Proposition 1 *The autarkic BGP equilibrium of the economy is a situation where C_t^M, X_t^M, and M_t (λ_t and p_t) are growing (declining) at a common constant rate of g^a while C_t^A, X_t^A, $L_{t,}^A$ and $L_t^M$are stationary over time.*

Proof. By condition (18), L_t^M is constant over time. Let the BGP equilibrium growth rate of M_t be g^a. Then (5) yields $\dot{p}_t/p_t = -g^a$. Next, (11) and (1) give constancy of L_t^A, X_t^A and C_t^A, while (15) in turn implies that C_t^M (hence X_t^M) is growing at the same rate g^a. Finally, condition (16) implies that λ_t is declining at the same rate g^a. ■

Using Proposition 1, we can derive the BGP growth rate. Imposing the BGP equilibrium restrictions on (15) – (17), we get

$$C^A = \beta p_t^a C_t^M \tag{20}$$

$$-g^a = \rho - \mu B L^M - (1+s)/M_t^a \lambda_t^a \tag{21}$$

$$g^a = \mu B L^M, \tag{22}$$

where the superscript "a" is used to denote the autarkic BGP value of a variable. Note that in (20), by Proposition 1, the consumption of the agricultural good is constant. Let us denote this level by C^{Aa}. From (22), we note that the BGP growth rate of this closed economy is proportional to the employment, and thus output level, in the manufacturing sector. The maximum growth rate of this economy based on learning by doing, denoted by $\bar{g}$, is

$$\bar{g} = \mu BL. \tag{23}$$

>From (11), (12), and (18), we have

$$C^{Aa} = \frac{\beta AL}{1+s+\beta}. \tag{24}$$

Next, substituting (18) into (22), we get

$$g^a = \frac{(1+s)\mu BL}{1+s+\beta}. \tag{25}$$

Condition (25) suggests that the growth performance of the economy is contributed by the productivity of the manufacturing sector, B, knowledge accumulation through the learning by doing mechanism, μ, and the production subsidy, s. In this paper, we are particularly interested in the growth effect of a subsidy, and would like to express g^a as a sole function of s. For simplicity and unless confusion arises, we write g^a instead of $g^a(s)$. Two subsidy rates that receive the most attention in the present paper are $s = 0$ and the optimal subsidy, s^{a*}. The growth rate with zero subsidy is equal to $g^a(0) \equiv \mu BL/(1+\beta)$, while the optimal subsidy and the corresponding growth rate will be defined and derived later.

The effects of the subsidy on the growth rate are shown by the following derivatives:

$$\frac{\partial g^a}{\partial s} = \frac{\mu\beta BL}{(1+s+\beta)^2} > 0 \tag{26}$$

$$\frac{\partial^2 g^a}{\partial s^2} = -\frac{\mu\beta BL}{(1+s+\beta)^3} < 0. \tag{27}$$

Conditions (26) and (27) imply that g^a is strictly increasing and concave in s. Based on these two conditions, the dependence of the growth rate on s is illustrated by schdule GG in Figure 1. It is clear from condition (25) that g^a is bounded from above by $\bar{g} \equiv \mu BL$, but it is approaching $\bar{g}$ as s approaches infinity.

By condition (5), the term $p_t M_t$ is constant. Since M_0 is given, it is required that the initial autarkic price ratio, p_0^a, has to adjust instantaneously to satisfy the labor mobility condition, i.e.,

$$p_0^a = \frac{A}{(1+s)BM_0}. \tag{28}$$

Condition (28) suggests that when given M_0, an increase in s lowers the required initial price level.

To close this subsection, we briefly discuss the transitional dynamics of the closed economy. Defining $m_t = M_t \lambda_t$ and using (16) and (17), we can summarize the dynamics in the following linear autonomous ordinary differential equation:

$$\dot{m_t} = \rho m_t - (1+s). \tag{29}$$

We illustrate (29) in Figure 2, which highlights the fact that m_t is unstable. Thus, there cannot be any transition in terms of m_t in the closed economy and the BGP equilibrium must be achieved through the instantaneous adjustment of the shadow price of knowledge.

2.4 Welfare

With no transition in our closed economy, we can study the welfare consequence of the production subsidy by focusing exclusively on the BGP equilibrium. In this subsection, we derive the formula for the optimal production subsidy.

Making use of condition (24) and the fact that p_t is declining at the rate of g^a along a BGP, condition (6) can be simplified to give the autarkic lifetime welfare

$$W^a = \frac{1}{\rho}\left[(1+\beta)\ln C^{Aa} - \ln \beta - \ln p_0^a\right] + \frac{g^a}{\rho^2}. \tag{30}$$

By condition (30), and using (18), (24), and (28), an increase in s affects the autarkic welfare through three channels: a drop in C_a^A, a drop in p_0^a, and a rise in g^a. The first channel leads to a negative effect while the other two produce positive effects.

To get an understanding of these three effects, the welfare function in (30) is differentiated with respect to s to give

$$\frac{\mathrm{d}W}{\mathrm{d}s} = \frac{\mu\beta BL}{\rho^2(1+s+\beta)^2} - \frac{s\beta}{\rho(1+s)(1+s+\beta)}. \tag{31}$$

In general, the sign of $\mathrm{d}W/\mathrm{d}s$ in (31) is ambiguous. However, the condition shows that if s is zero or sufficiently small, the derivative is positive. This means that a small production subsidy is welfare improving. We also note that if s is sufficiently large, C^A is so small that $\ln C^A$ is very negative, leading to a welfare below the level with no intervention. We thus conclude that a positive, optimal subsidy exists.

To derive the optimal subsidy, s^{a*}, we set $\mathrm{d}W^a/\mathrm{d}s = 0$ and rearrange the terms to obtain

$$s^{a*} = \frac{g^a(s^{a*})}{\rho}, \tag{32}$$

where $g^a(s)$ is given by (25). Conditions (32) and (25) can be combined to give the optimal production subsidy. Graphically, this is the intersecting point, denoted by point S, between schedule GG and a ray with a slope of ρ in Figure 1. As shown earlier, schedule GG is positively sloped and concave, with a vertical intercept of $g^a(0)$. This means that the optimal subsidy exists and is unique. The economy remains diversified.

That the optimal production subsidy is positive reflects the fact that firms do not take into cosideration the dynamic learning by doing effect when choosing employment level. A subsidy as described by (32) is needed to achieve dynamic efficiency.

Finally, it is straightforward to see from (25) and (32) that the optimal subsidy is decreasing in both β and ρ, but increasing in μ, B and L.

3 Free Trade and Production Patterns

We now allow free trade between the economy introduced above, which is now called the home economy, and the rest of the world (ROW). To simplify our analysis, the following assumptions are made: (a) The home economy is small as compared with the ROW in the sense that the economic conditions in the ROW are not affected by its trade with the economy. (b) The structure of the ROW is the same as the home economy under consideration. (c) At the time when trade is allowed, both the economy and the ROW are at their own BGP equilibrium. (d) For the time being, no production subsidy by any country is considered both before and after trade. More specifically, free trade is considered. (e) There is no international spillover of knowledge, meaning that the home economy learns from its own manufacturing production only.

Denote the exogenously given BGP growth rate of the ROW by $g^w > 0$, and the relative price of manufacturing in the ROW at time t by $p_t^w > 0$, which is decreasing at a rate of g^w.

It is well known in the literature that for a Ricardo-Viner economy under free trade and facing given world prices, its patterns of trade and production can be determined by comparing (i) the MRT of the economy with (ii) the prevailing world prices. In a static model, both MRT of the economy and the world price ratio are stationary. In the present model, however, both the economy's MRT and the world price ratio change over time. For a small open economy, its MRT depends not only on its technology, but also on how trade may affect its patterns of production and trade. In other words, the comparative advantage of the economy may change over time and may depend on whether trade exists in the first place.

If trade exists beginning from $t = 0$, we can distinguish between two different concepts of comparative advantage of the economy at any time $t \geq 0$. The first one is its *potential* comparative advantage, which is what it would have should trade not be allowed between the economy and the rest of the world from $t = 0$. Its potential comparative advantage is determined by comparing the MRT of the economy under autarky at time t, q_t^a, with the prevailing world price ratio, p_t^w at the same time. The second concept is its *actual* comparative advantage, which is determined by comparing its actual MRT under trade, q_t^f, with the prevailing world price ratio, p_t^w. When trade first exists at $t = 0$, $q_0^f = q_0^a$. After that, the MRT of the economy may follow a different path as compared with its path under autarky, implying that q_t^f may be greater or less than q_t^a, depending on the pattern of production, as will be explained later.

For the time being, we will focus on the acutal comparative advantage of the economy. At any time t the economy is (a) completely specialized in agriculture if

$$p_t^w < q_t^f; \tag{33}$$

or (b) completely specialized in manufacturing if

$$p_t^w > q_t^f. \tag{34}$$

Using conditions (33) and (34), the patterns of production are analyzed as follows.

3.1 Specialization in Agriculture: $\mathbf{SA}_0$

Suppose that free trade is allowed at $t = 0$, and that $p_0^w < q_0^a$. This means that the economy immediately is specialized in agriculture. Let us call this case SA_0, i.e., specialization in agriculture if free trade starts at $t = 0$.

The question is, can this pattern of production be sustained? To answer this question, note that with no production of the manufacturing good, there is no learning by doing. With the given intangible capital, M_0, and no depreciation, the production possibility frontier remains stationary. On the other hand, the world price is declining over time. This means that condition (33) is always satisfied, or that specialization in agriculture is sustained.

Another point to note is that with costless and frictionless labor movement, the economy reaches the new production point immediately after free trade. The economy then stays there forever. So there is no transition under free trade.

We now derive the BGP of the economy. With production of agriculture only, the national income in terms of agriculture is constant and equal to AL. With the Cobb-Douglas type utility function, the optimal consumption allocations are:

$$C_t^A = \left(\frac{\beta}{1+\beta}\right) AL \tag{35}$$

$$p_t^w C_t^M = \left(\frac{1}{1+\beta}\right) AL. \tag{36}$$

Thus, the home economy exports $AL/(1+\beta)$ units of the agricultural good to the rest of the world in exchange for the equivalent value of the manufacturing good at the world price p_t^w. Since the world price is falling, the quantity of the manufacturing good imported is growing over time. We summarize the characterization of the BGP equilibrium in the following proposition:

Proposition 2 *Suppose that $p_0^w < q_0^a$ and that free trade is allowed at $t \geq 0$. (a) The economy is completely specialized in agriculture. This production pattern is sustainable. (b) The free-trade BGP with specialization in agriculture* *is a situation where C_t^M is growing at the given rate of g^w while C_t^A and X_t^A are stationary over time. The home economy exports agriculture of the amount of $AL/(1+\beta)$ and imports an equal value of manufacturing.*

Although there is no knowledge accumulation in this case, the domestic consumption possibility frontier is moving out at the rate of g^w. This is due to the continuing improvement of the terms of trade.

3.2 Specialization in Manufacturing: SM_0

Consider now the case in which $p_0^w > q_0^a$. If free trade is allowed at $t \geq 0$, the home economy will be completely specialized in manufacturing. Using the notation introduced above, let us call this case SM_0.

The question again is, can this pattern of production be sustained? To answer this question, let us examine how this economy may change over time. With all of its labor allocated to the manufacturing sector, the economy achieves its maximum growth rate, $\overline{g} = \mu BL$, which is simply the growth rate of the intangible capital. By (14), the MRT of the economy, q_t^f, is declining at the rate of $\overline{g}$.

Since the pattern of production depends on the difference between the world price ratio and the MRT of the economy, two subcases can be identified: (a) $g^w \leq \bar{g}$; (b) $g^w > \bar{g}$.

3.2.1 Case SM_0(a): $g^w \leq \bar{g}$

In this subcase, condition (34) is always satisfied and the pattern of production, with specialization in manufacturing, is sustained. As a result, the national income at any time is given by $p_t^w BM_tL$ (in terms of the agriculture good), which is increasing at the rate of $\overline{g} - g^w$. The optimal consumption allocations are:[11]

$$C_t^A = \left(\frac{\beta}{1+\beta}\right) p_t^w B M_t L \quad (37)$$

$$p_t^w C_t^M = \left(\frac{1}{1+\beta}\right) p_t^w B M_t L. \quad (38)$$

Thus, in each period t, the home economy exports $\beta p_t^w B M_t L/(1+\beta)$ units (in terms of the agricultural good) of the manufacturing good to the rest of the world in exchange for the same value of the agricultural good at the world price p_t^w. We characterize the BGP equilibrium of this regime in the following proposition:

Proposition 3 *Suppose that $p_0^w > q_0^a$, $g^w \leq \bar{g}$, and that free trade is allowed at $t \geq 0$. (a) The economy is completely specialized in manufacturing, and this production pattern is sustainable. (b) The free-trade BGP with complete specialization is a situation where C_t^M, X_t^M, and M_t are growing at a constant rate of $\bar{g} = \mu B L$, while C_t^A is growing at a rate of $\bar{g} - g^w$. The economy exports $B M_t L/(1+\beta)$ units of the manufacturing goods and imports an equal value of agriculture.*

[11] Formally speaking, the optimization problem of the represntative agent is given by

$$W^{FT} = \max \int_0^\infty (\beta \ln C_t^A + \ln C_t^M) e^{-\rho t} dt$$

subject to the budget constraint

$$C_t^A + p_t^w C_t^M = p_t^w B M_t L,$$

and the knowledge accumulation equation

$$\dot{M}_t = \mu B M_t L.$$

Defining $\widetilde{m}_t = M_t \widetilde{\lambda}_t$ where $\widetilde{\lambda}_t$ denotes the costate variable associated with the above knowledge accumulation equation. The dynamics of the model exhibited this case is given by the following linear autonomous ordinary differential equation:

$$\dot{\widetilde{m}}_t = \rho \widetilde{m}_t - (1+\beta)/B,$$

which implies that $\widetilde{m}_t$ is unstable. Following the same argument adopted in the autarkic section, there cannot be any transition in this case.

Proof. Since M_t is growing at the rate of $\overline{g}$, from $X_t^M = BM_tL$ and (38), both X_t^M and C_t^M will also be growing at the same rate. In addition, p_t^w is declining at the given rate of g^w, condition (37) implies that C_t^A is growing at the rate of $(\overline{g} - g^w)$. Finally, the pattern of trade follows directly from the optimal consumption allocations (37) and (38). ■

3.2.2 Case SM$_0$(b): $g^w > \bar{g}$

In this subcase, because the world's price ratio is declining at a rate faster than that of the economy's MRT, there exists a time $t = t^M$ so that $p_t^w = q_t^f$. See Figure 3. At this point, the world's price line coincides with the economy's PPF. For $t > t^M$, $p_t^w < q_t^f$, meaning that the economy has a comparative advantage in and exports agriculture. In other words, the economy switches its comparative advantage at $t = t^M$, and the initial specialization in manufacturing is not substainable.

This result is similar to a result in Wong and Yip (1999), and indicates the fact that if the rest of the world grows too fast, faster than what the economy can potentially follow, the economy eventually will turn to specialization in agriculture.

When the economy is completely specialized in agriculture, the analysis is similar to that in the previous subsection and is omitted here.

Proposition 4 *Suppose that $p_0^w > q_0^a$, $g^w > \bar{g}$, and that free trade is allowed at $t \geq 0$. The economy is initially completely specialized in and exports manufacturing. There exists a time $t^M > 0$, beyond which the economy exports agriculture.*

The above two proposition can be combined together to give an alternative result:[12]

Corollary *Suppose that $p_0^w = q_0^a$ and that free trade is allowed at $t \geq 0$. The economy is completely specialized in the production of agriculture (manufacturing) if $g^w > (<)\ g^a$. The patterns of production and trade are sustainable.*

[12] In proving this proposition, it is noted that if $g^w < g^a$, then $g^w < \overline{g}$, meaning the pattern of production with an export of manufacturing is sustainable.

4 Dynamic Gains From Free Trade

In this section, we examine the welfare effects of free trade and analyze whether free trade benefits dynamically the economy. It is assumed in this section that free trade, if it is allowed, exists when $t = 0$, while the analysis of the timing of free trade will be provided later.

As will be seen later, the gains from trade depends crucially on the pattern of trade (and production). As a result, our analysis will be divided into two parts, one for each type of production pattern.

4.1 Case SA$_0$: $p_0^w < q_0^a$

In this case, as analyzed earlier, the economy will be completely specialized in agriculture, whether or not the world or the home economy grows faster. However, since we have to compare the lifetime welfare under free trade with that under autarky, we have to take into account the switching of the potential comparative advantage of the economy. In other words, we divide this case into two subcases, depending on whether the world or the economy when closed grows faster.

4.1.1 Case SA$_0$(a): $g^a \leq g^w$

In this subcase, as analyzed earlier, the economy is completely specialized in agriculture and the pattern of production is sustainable. As a result, we can simply compare the welfare of the economy under free trade with the autarkic welfare.

Substitute the BGP values of consumption given by conditions (35) and (36) into the welfare function in (6), which is then simplified to give

$$W^A = \frac{1}{\rho}\left[(1+\beta)\ln\left(\frac{\beta AL}{1+\beta}\right) - \ln\beta - \ln p_0^w\right] + \frac{g^w}{\rho^2}. \tag{39}$$

We now compare this welfare function with that under autarky, which is given by (30). After simplification, we have

$$W^A - W^a = \frac{g^w - g^a}{\rho^2} + \frac{\ln p_0^a - \ln p_0^w}{\rho} > 0, \tag{40}$$

which shows that free trade is unambiguously better than autarky.

This result is not surprising and can be explained intuitively. When free trade is first allowed, the economy instantaneously receives the static gains from trade. As time goes on, the world price ratio drops, and should the economy allow no trade in the first place its PPF will shift out, pivoting around its vertical intercept. Since $g^a \leq g^w$, the free-trade consumption possibility frontier of the economy is always above (except at the vertical intercept) the potential PPF, meaning that no matter when free trade is allowed, the comparative advantage of the economy is always in the agricultural good. As a result, the economy can get static gains from trade at all time, and over time the intertemporal welfare is higher under free trade.

The expression in (40) gives the gains from trade, which can be decomposed into two positive terms.[13] The first term can be termed the *growth effect*, which highlights the improvement in the growth rate of the consumption possibility frontier brought by the world, while the second terms can be called the *dynamic terms of trade effect*, which comes from the difference between the world price ratio and the autarkic price ratio.

4.1.2 Case SA$_0$(b): $g^a > g^w$

In this subcase, as analyzed before, if free trade starts at $t = 0$, the economy exports agriculture forever. However, because $g^a > g^w$, if in a regime in which free trade is not allowed until at least $t = t^A > 0$, as shown in Figure 4, there is a reversal of potential comparative advantage, with manufacturing as the exportable when free trade is allowed.[14] This means that with free trade starting from $t = 0$, then after $t = t^A$ the economy actually is exporting the "wrong" good. Because of this, the net dynamic gains from free trade starting from the beginning becomes ambiguous.

To see this point more clearly, refer again to condition (40). The second term on the RHS remains to be positive while the first term is now negative, making the sign of the welfare change ambiguous. Using condition (40), we can say that the economy gains from trade if and only if

$$\rho \left(\ln p_0^a - \ln p_0^w \right) > g^a - g^w. \tag{41}$$

Note that condition (41) is satisfied if $g^w \geq g^a$.

[13] When $g^w = g^a$, the first term is zero.

[14] The time $t = t^A$ is when $q_t^a = p_t^w$.

Proposition 5 *Suppose that* $p_0^w < q_0^a$ *and that free trade exists starting from* $t = 0$. *A necessary and sufficient condition for a positive dynamic gain from trade is given by (41). A sufficient condition for a positive dynamic gain from trade is* $g^w \geq g^a$.

4.2 Case SM$_0$: $p_0^w > q_0^a$

In this case, when free trade is first allowed, the economy has a comparative advantage in manufacturing and exports the good. Whether this pattern of trade can be sustained depends on g^w and $\bar{g}$. The following welfare analysis can thus be divided into two parts.

4.2.1 Case SM$_0$(a): $g^w \leq \bar{g}$

In this subcase, the pattern of trade is sustained, with the economy exporting manufacturing. The consumption of the two goods at time t is given by conditions (37) and (38). Substitute these values into the welfare function (6) and simplify the expression to give

$$W^M = \frac{1+\beta}{\rho}\left[\ln\left(\frac{\beta BL}{1+\beta}\right) + \ln p_0^w + \ln M_0\right] - \frac{1}{\rho}(\ln\beta + \ln p_0^w) + \frac{g^w}{\rho^2}. \quad (42)$$

To determine the gains from trade, we subtract the autarkic lifetime welfare in (30) from (42) to yield, after simplifying the terms,

$$W^M - W^a = \frac{\beta(\ln p_0^w - \ln p_0^a)}{\rho} + \frac{\beta(\bar{g} - g^w) + (\bar{g} - g^a)}{\rho^2} > 0, \quad (43)$$

where the sign is based on the given conditions and the fact that $\bar{g} > g^a$. For convenience, we follow the notation introduced above to call the first term on the RHS of (43) the dynamic terms of trade effect and the second term the growth effect. Note that the dynamic terms of trade effect is due to the economy's comparative advantage, while the growth effect comes from the growth differentials. It can further be noted that the growth effect is still postive even if $\bar{g} = g^w$.

The positive gain from trade in the present case as shown in (43) is not surprising. As free trade is first allowed, the economy gets static gains from trade by exporting manufacturing, the good in which it has a comparative advantage. As the economy grows, its comparative advantage remains unchanged because its growth rate, $\bar{g}$, is not less than that of the world. Thus the economy continues to gain from trade.

4.2.2 Case SM_0(b): $g^w > \bar{g}$

In this case, the world grows so fast that the economy cannot catch up and that it eventually becomes specialized in agriculture. As a result, its intertemporal welfare is no longer given by (42), and (43) cannot be used to show the welfare change.

To determine the gains from trade, let us examine more closely the pattern of trade. Consider Figure 5, which shows a possible adjustment of the world's price ratio, p_t^w, the economy's MRT under autarky, q_t^a, and the economy's MRT if free trade is allowed from $t = t_0$, q_t^f. Note that Figure 5 is just Figure 3 with the schedule for q_t^a added. The relative slopes of the schedules are based on $g^w > \bar{g} > g^a$.

Figure 5 highlights two important points. First, at $t = t^M$, there is a switch in the actual compartive advantage of the economy. Second, at $t = t^a$, there is a switch in the potential comparative advantage of the economy. Using these two points, we can divide the whole time period into three regions: (i) $t \in [0, t^a)$ (ii) $t \in [t^a, t^M]$ and (iii) $t > t^M$. In region (i), $q_t^a < p_t^w$, and the economy is exporting manufacturing when free trade is allowed. This means that the economy is exporting the good in which it has a comparative advantage. In region (iii), $q_t^a > p_t^w$, and with free trade starting from $t = 0$, the economy is exporting agriculture, the "right" good in the sense that this good is what the econmy will export should it prohibit free trade until $t > t^M$. Therefore we expect that the economy gains in these two regions. In region (ii), however, we get a different result. We have $q_t^a > p_t^w$ but the economy exports the "wrong" good in the sense that should free trade not be allowed in the first place, the comparative advantage of the economy in this region is in agriculture, not manufacturing that is being exported when free trade is allowed at $t = 0$.

The above analysis then suggests that free trade may not be good. To analyze this result more rigorously, we derive the welfare of the economy with free trade starting from $t = 0$, using the fact that it exports manufacturing when $t < t^M$ and exports agriculture when $t > t^M$. Using again (6) and the corresponding consumption derived earlier, we have the lifetime welfare of the economy equal to

$$
\begin{aligned}
W^{MA} &= \int_0^{t^a} \left\{ (1+\beta) \ln \left(\frac{\beta BL}{1+\beta} p_t^w M_t^f \right) - \ln p_t^w \right\} e^{-\rho t} \mathrm{d}t \\
&+ \int_{t^a}^{t^M} \left\{ (1+\beta) \ln \left(\frac{\beta BL}{1+\beta} p_t^w M_t^f \right) - \ln p_t^w \right\} e^{-\rho t} \mathrm{d}t \\
&+ \int_{t^M}^{\infty} \left\{ (1+\beta) \ln \left(\frac{\beta AL}{1+\beta} \right) - \ln p_t^w \right\} e^{-\rho t} \mathrm{d}t - \frac{\ln \beta}{\rho}. \qquad (44)
\end{aligned}
$$

To compare the welfare function in (44) with the autarkic welfare function in (30), we disaggregate the latter into three corresponding components, using the fact that $A = Bp_t^a M_t$, to give

$$
\begin{aligned}
W^{a} &= \int_0^{t^a} \left\{ (1+\beta) \ln \left(\frac{\beta BL}{1+\beta} p_t^a M_t \right) - \ln p_t^a \right\} e^{-\rho t} \mathrm{d}t \\
&+ \int_{t^a}^{t^M} \left\{ (1+\beta) \ln \left(\frac{\beta BL}{1+\beta} p_t^a M_t \right) - \ln p_t^a \right\} e^{-\rho t} \mathrm{d}t \\
&+ \int_{t^M}^{\infty} \left\{ (1+\beta) \ln \left(\frac{\beta AL}{1+\beta} \right) - \ln p_t^a \right\} e^{-\rho t} \mathrm{d}t - \frac{\ln \beta}{\rho}. \qquad (45)
\end{aligned}
$$

Substracting W^a in (45) from W^{MA} in (44) and rearranging the terms, we get

$$
\begin{aligned}
W^{MA} - W^{a} &= \int_0^{t^a} \{ (1+\beta)(\bar{g} - g^a)t + \beta(\ln p_t^w - \ln p_t^a) \} e^{-\rho t} \mathrm{d}t \\
&+ \int_{t^a}^{t^M} \{ (1+\beta)(\bar{g} - g^a)t + \beta(\ln p_t^w - \ln p_t^a) \} e^{-\rho t} \mathrm{d}t \\
&+ \int_{t^M}^{\infty} \{ (\ln p_t^a - \ln p_t^w \} e^{-\rho t} \mathrm{d}t. \qquad (46)
\end{aligned}
$$

The three terms on the RHS of (46) correspond to the welfare differentials in the three regions explained above. Based on the given conditions, it is easy to determine that the first and third terms are positive while the second term is in general ambiguous. As a result, the sign of the overall welfare change is ambiguous. One sufficient condition for a positive gain is that $\bar{g}$ is sufficiently close to g^a.

Proposition 6 *Suppose that* $p_0^w > q_0^a$ *and that free trade exists starting from* $t = 0$. *(a) If* $g^w \leq \bar{g}$, *then the economy gains from trade. (b) If* $g^w > \bar{g}$, *the economy gains from trade in a dynamic context if and only if the expression in condition (46) is positive. If it is further given that* g^a *is sufficiently close to* $\bar{g}$, *then the dynamic gain from trade is positive.*

The results in cases SA_0(a) and SM_0(a) can be combined to give the following proposition, the proof of which is straightforward and omitted here.

Proposition 7 *Suppose that initially* $q_0^a = p_0^w$. *Then free trade starting from* $t = 0$ *is beneficial (or not harmful in the singular case in which* $g^a = g^w$ *because no trade exists).*

The patterns of trade and production in different cases, and also the welfare effects of trade to be derived later, are summarized by the table in the appendix.

5 Optimal Timing of Trade

So far, we have been assuming that free trade starts from $t = 0$, and showed that free trade may not be gainful. However, since the comparative advantage and the pattern of trade may change over time, it is natural to ask two questions. First, if free trade is to be allowed, can the economy do better by delaying the timing for free trade, i.e., allowing autarky up to a certain point $t_0 > 0$, and then free trade thereafter? Second, can the economy do even better by choosing the optimal timing of trade and at the same time providing a subsidy to alter the pattern of trade? We try to answer these two questions below.

5.1 Free Trade

In this subsection, we focus on the first of the two questions, and try to determine the optimal time t_0 from which free trade starts in each of the above four cases. Let us consider first case SM_0(a), and assume that the economy prohibits trade from $t = 0$ to $t = t_0 > 0$ and then allows free trade, with specialization in manufacturing. Its lifetime welfare, $\widetilde{W}$, as a function of t_0, is equal to

$$\begin{aligned}\widetilde{W}(t_0) &= \int_0^{t_0}\left[(1+\beta)\ln\left(\frac{\beta AL}{1+\beta}\right)-\ln\beta\right]e^{-\rho t}dt-\int_0^{t_0}\ln p_t^a e^{-\rho t}dt\\ &\quad+\int_{t_0}^{\infty}\left[(1+\beta)\ln\left(\frac{\beta BLp_t^w M_t}{1+\beta}\right)-\ln\beta\right]e^{-\rho t}dt-\int_{t_0}^{\infty}\ln p_t^w e^{-\rho t}dt\\ &= K_0-(\ln p_0^a-\ln p_0^w)\int_0^{t_0}e^{-\rho t}dt-(g^a-g^w)\int_0^{t_0}te^{-\rho t}dt\\ &\quad+(1+\beta)(\overline{g}-g^w)\int_{t_0}^{\infty}te^{-\rho t}dt, \end{aligned} \tag{47}$$

where $K_0 \equiv \{(1+\beta)\ln[\beta AL/(1+\beta)] - \ln\beta - \ln p_0^w\}/\rho - g^w/\rho^2$. Note that $p_0^a > p_0^w$ and $\overline{g} > g^a > g^w$; so by (47) $\widetilde{W}$ is decreasing in t_0. We thus conclude that if free trade is allowed, it should be allowed as soon as possible. This result is not surprising because as shown earlier free trade is beneficial.

The same analysis and result also apply to case SA_0(a), in which free trade starting from $t_0 = 0$ is gainful: So allowing free trade sooner is better.

We now turn to the other two cases, SA_0(b) and SM_0(b), in which free trade starting from $t = 0$ may be harmful because sometimes the "wrong" good is exported. It is therefore reasonable to ask whether free trade should be delayed.

Let us first consider case SA_0(b). Figure 4 shows two regions: (i) $t \in [0, t^A]$ and (ii) $t > t^A$, where $t^A < \infty$ is the time at which the MRT of the economy under autarky is equal to the world's relative price. Let us examine separately the possibility of having the commencement of free trade in each of these two regions. If free trade is to start in region (ii), with $t_0 > t^A$, the economy is specialized in manufacturing, and this pattern of production is sustainable. Using the above analysis and the expression of $\widetilde{W}(t_0)$ given by (47), we can conclude that if free trade is to start in region (ii), the optimal value of t_0 is t^A, with the resulting welfare equal to $\widetilde{W}(t^A)$. The same analysis can be applied to show that if free trade is allow in region (i), the optimal value of t_0 is 0, with the corresponding welfare being equalt to W^A. The following rule can thus be established for the optimal time, $\hat{t}_0$, at which free trade starts:

$$\hat{t}_0 = \begin{cases} 0 & \text{if} \quad W^A \geq \widetilde{W}(t^A) \\ t^A & \text{if} \quad W^A < \widetilde{W}(t^A). \end{cases} \tag{48}$$

Note that allowing autarky of the economy at all times is the same as setting t_0 to be infinity. In other words, the autarkic welfare is equal to $W^a \equiv \widetilde{W}(\infty)$.

As shown earlier, $\widetilde{W}(t_0)$ is strictly decreasing in t_0, implying that $\widetilde{W}(t^A) > W^a$. The rule in (48) guarantees that the economy will get a welfare not less than $\widetilde{W}(t^A)$. As a result, free trade under the rule is better than no trade.

Case SM_0(b) can be analyzed in a similar way. At time 0, q_t^a is less than p_t^w, but since in this case $g^a < \overline{g} < g^w$, there exists a time t^a so that $q_t^a > p_t^w$ for $t > t^a$. So two regions can be identified: (i) $t \in [0, t^a]$ and (ii) $t > t^a$, and if free trade is to start in each of these two regions, it should start as soon as possible. Denote the lifetime welfare of the economy under autarky in the period $[0, t^a]$ and free trade for $t > t^a$ by $\overline{W}(t^a)$, which is given by

$$\begin{aligned}
\overline{W}(t^a) = & \int_0^{t^a} [(1+\beta)\ln(\frac{\beta AL}{1+\beta}) - \ln\beta] e^{-\rho t} dt - \int_0^{t^a} \ln p_t^a e^{-\rho t} dt \\
& + \int_{t^a}^{\infty} [(1+\beta)\ln(\frac{\beta AL}{1+\beta}) - \ln\beta] e^{-\rho t} dt - \int_{t^a}^{\infty} \ln p_t^w e^{-\rho t} dt \\
= & [(1+\beta)\ln(\frac{\beta AL}{1+\beta}) - \ln\beta]/\rho - \int_0^{t^a} \ln p_t^a e^{-\rho t} dt - \int_{t^a}^{\infty} \ln p_t^w e^{-\rho t} dt.
\end{aligned}$$

Subtracting $\overline{W}(t^a)$ from the lifetime welfare W^M under free trade starting from $t = 0$, we have

$$W^M - \overline{W}(t^a) = (1+\beta)(\ln p_0^w - \ln q_0^a)/\rho + \int_0^{t^a} (\ln p_t^a - \ln p_t^w) e^{-\rho t} dt. \quad (49)$$

Noting that $q_0^a < p_0^w$ and $g^w > \overline{g} > g^a$, the welfare differential given by (49) has ambiguous sign. Therefore the rule for the optimal time for free trade is

$$\hat{t}_0 = \begin{cases} 0 & \text{if} \quad W^M \geq \overline{W}(t^a) \\ t^a & \text{if} \quad W^M < \overline{W}(t^a). \end{cases} \quad (50)$$

Using an argument similar to the one given earlier, it can be shown that free trade under rule (50) is better than no trade. The results obtained are summarized in the following proposition:

Proposition 8 *Suppose that the government is considering allowing free trade. In cases SA_0(a) and SM_0(a), free trade should be allowed from the beginning. In case SA_0(b), the optimal timing of free trade is given by condition (48), while in case SM_0(b), the rule for optimal timing of free trade is given by condition (50). In each of these cases, free trade with the stated rule is better than no trade.*

5.2 Trade with A Production Subsidy

In Section 2, we showed that the government of a closed economy can improve the lifetime welfare of the representative agent by imposing a suitable production subsidy. In this section, we follow the same approach and try to see whether it makes sense to impose a production subsidy and choose the optimal time when trade is allowed.

We begin with the cases in which the economy exports agriculture under free trade, i.e., cases SA_0(a) and SA_0(b). In these cases, the point for imposing a production subsidy is to protect domestic manufacturing producers so that the economy can be specialized in manufacturing. The advantage of this pattern of production is that the economy keeps on growing with a rate even higher than that under autarky, $\overline{g}$. The question we face is whether the present sacrifice due to the costs associated with the subsidy can be outweighed by long-run benefit that comes from growth.

To answer this question, let us first focus on case SA_0(a). The subsidy policy under consideration consists of a production subsidy imposed on manufacturing according to the following rule:

$$\widehat{s}_t = \begin{cases} q_t^a - p_t^w & \text{if} \quad q_t^a > p_t^w \\ 0 & \text{if} \quad q_t^a \leq p_t^w. \end{cases} \tag{51}$$

According to the above rule, $\widehat{s}_t$ declines over time until q_t^a is less than p_t^w. As a result of the subsidy, trade would lead to complete specialization in and an export of manufacturing, with the economy growing at a rate of $\overline{g}$, which is higher than the autarkic growth rate. The resulting lifetime welfare is equal to

$$\begin{aligned} W^{Ms} &= \frac{1}{\rho}\left[(1+\beta)\ln\left(\frac{\beta BLp_0^w M_0}{1+\beta}\right) - \ln\beta - \ln p_0^w\right] \\ &+ \frac{(1+\beta)(\overline{g}-g^w)+g^w}{\rho^2}. \end{aligned} \tag{52}$$

To compare this subsidy policy with the free-trade policy, subtract W^A from W^{Ms} as given in (52) to give

$$\begin{aligned} W^{Ms} - W^A &= \frac{1+\beta}{\rho}\left[\ln\left(\frac{\beta BLp_0^w M_0}{1+\beta}\right) - \ln\left(\frac{\beta AL}{1+\beta}\right)\right] \\ &+ \frac{(1+\beta)(\overline{g}-g^w)}{\rho^2} \end{aligned}$$

$$= \frac{1+\beta}{\rho}\left[\ln p_0^w - \ln q_0^a\right] + \frac{(1+\beta)(\overline{g} - g^w)}{\rho^2} \tag{53}$$

Let us introduce the following condition:

Condition C: $W^{Ms} - W^A$ as given by (53) is positive.

The subsidy policy is a good one if and only if condition C is satisfied. On the RHS of (53), the first term represents the drop in welfare due to a shift-in of the consumption possibility frontier as the economy is completely specialized in the "wrong" good, and the second term represents the welfare change due to a change in the growth rate. The first term is negative, but the second term is positive if and only if $\overline{g} > g^w$. Thus a necessary condition for a dynamic gain from trade and industrialization is that the economy's maximum growth rate is greater than the world's growth rate.

We now turn to case SA_0(b), in which there is a switch in the potential comparative advantage of the economy. Consider now the following policy: The economy stays closed from $t = 0$ to $t = t^s$, and then a constant production subsidy, s, is imposed so that the economy is completely specialized in and exports manufacturing. We need to determine the optimal values of t^s and s.

Under this policy, $C^A = \beta AL/(1 + s + \beta)$. The lifetime welfare of the economy then becomes

$$\begin{aligned} W^s &= \int_0^{t^s} \left[(1+\beta)\ln\left(\frac{\beta AL}{1+s+\beta}\right) - \ln\beta\right] e^{-\rho t}dt - \int_0^{t^s} \ln p_t^a e^{-\rho t}dt \\ &+ \int_{t^s}^{\infty} \left[(1+\beta)\ln\left(\frac{\beta BLp_t^w M_t}{1+\beta}\right) - \ln\beta\right] e^{-\rho t}dt - \int_{t^s}^{\infty} \ln p_t^w e^{-\rho t}dt \\ &= \left[(1+\beta)\ln\left(\frac{1+\beta}{1+s+\beta}\right)\right] \int_0^{t^s} e^{-\rho t}dt + \widetilde{W}(t^s). \end{aligned} \tag{54}$$

Since $\ln[(1 + \beta)/(1 + s + \beta)] < 0$, the first term on the RHS of (54) is decreasing in t^s. We need to find how W^s is dependent on the time when trade is allowed. Note that because the production subsidy needed to alter the pattern of trade creates a distortion in resource allocation, its magnitude should be just slightly big enough to close the gap between p_t^w and q_t^a. Thus if trade is allowed earlier, a bigger subsidy is needed to reverse the pattern of trade. In other words, to minimize the cost, s should be negatively related to

t^s, and we denote the rate of change of s with respect to t^s by s'. See Figure 4. Differentiate W^s with respect to t^s and simplify the terms to get,

$$\begin{aligned}\frac{\mathrm{d}W^s}{\mathrm{d}t^s} &= \frac{(1+\beta)s'}{1+s+\beta}\frac{1-e^{-\rho t^s}}{\rho} + (1+\beta)e^{-\rho t^s}\ln\left(\frac{1+\beta}{1+s+\beta}\right) \\ &- (\ln p_0^a - \ln p_0^w)e^{-\rho t^s} \\ &- [(g^a - g^w) + (1+\beta)(\overline{g} - g^w)]t^s e^{-\rho t^s}. \qquad (55)\end{aligned}$$

The derivative in (55) is negative, meaning that trade under this subsidy policy should be allowed as soon as possible. In other words, if trade with an export of manufacturing is to be allowed, it should be allowed when $t^s = 0$.

Thus the optimal way to impose a subsidy, if it is to be imposed, is to follow the rule given by (51), with the lifetime welfare of the economy given by (52). The alternative policy is free trade, with complete specialization in agriculture and the corresponding welfare given by (39). The difference in the two welfare levels is given by (53), in which the first term is negative while the second term is positive. Therefore the subsidy policy dominates the free-trade policy if and only if condition C is satisfied.

We now turn to case SM_0(b). As again shown in Figure 5, our analysis can be limited to the following two policies: trade with possibly a subsidy beginning in region (i) $t_0 \in [0, t^a]$ and trade with possibly a subsidy beginning in region (ii) $t_0 > t^a$, with autarky before t_0. If trade is to begin in region (i), a subsidy has no production and growth effect because the economy is already completely specialized in manufacturing under free trade. Our earlier analysis can also be applied to show that in this case trade (with or without subsidy) should be allowed as soon as possible, with the resulting welfare equal to W^M. If trade is to begin in region (ii) and if trade is free, the economy will export agriculture. So starting from $t = t^a$, this subcase is similar to case SA_0(a), and a production subsidy may improve welfare in some cases, some would argue. However, a careful thought would tell us that the optimal subsidy is zero: We showed earlier that the optimal subsidy is zero if $\overline{g} < g^w$, but $\overline{g} < g^w$ is what we have in this case. Therefore in the present case the question is what the optimal value of t_0 is. To answer this question, we use again condition (50).

Proposition 9 *Suppose that the government can choose the optimal manufacturing subsidy and the optimal timing of foreign trade. In cases SA_0(a)* *and SA_0(b), free trade should start as soon as possible, and the optimal subsidy is zero if and only if condition C is violated. If condition C is satisfied, the subsidy is given by rule (51). In cases SM_0(a) and SM_0(b), no subsidy should be provided. In case SM_0(a), free trade should be allowed as soon as possible. In case SM_0(b), the rule for the optimal starting time for free trade is given by (50). In all cases, trade under the specified rule is better than no trade.*

Three remarks about the arguments for and against the use of production subsidy in various cases as suggested by above proposition are in order. First, the proposition can be used to shed some light on the so-called infant-industry argument for protection. In cases SA_0(a) and SA_0(b), the economy has a comparative advantage in agriculture and will export the good under free trade. Free trade is gainful, so long as the economy can choose the optimal timing of trade. However, by specializing in agriculture, the economy sees a drop in its growth rate (down to zero) under free trade, as its manufacturing sector shrinks and disappears. This is a disasater to a government that tries to maximize the growth rate of its economy. Protecting its manufacturing sector with a production subsidy seems to be a logical way to improve its growth rate.[15] However, the use of such a subsidy to promote industrialization may or may not be good for the economy's lifetime welfare, as condition (53) shows. If a subsidy is justified, as rule (51) shows, it should decrease over time until it is zero. Another remark about the results in the above proposition is that if the economy would export manufacturing under free trade, no subsidy should be needed because it is ineffective in increasing the economy's growth rate. The third remark is that the above proposition can be applied in the following special case to give

Proposition 10 *If $q_0^a = p_0^w$, then the optimal production subsidy for the economy is zero, and free trade should start from $t = 0$, irrespective to the growth rates of the economy and the rest of the world.*

The proof of this proposition is straightforward and omitted. Note that by Proposition 7, free trade starting from $t = 0$ is gainful (so long as $g^a \neq g^w$). Proposition 10 states that this is the optimal trade policy, with no production subsidy.

[15] ¿From the theory of distortions, it is clear that a production subsidy dominates a tariff in protection the manufacturing sector. See Bhagwati (1971).

6 Concluding Remarks

In this paper, we have introduced a model with an economy that has its relative price declining at a constant rate over time along a balanced growth path. Using this model, we brought out a new dimension of trade and welfare that has generally been neglected in the literature: time. Not only are the patterns of production and trade dependent on when trade is allowed, the effects of government policies could also depend on when the policies are imposed and when the economy starts trading with other economies.

In this paper, we found cases in which arguments for encouraging the production of manufacturing can be provided, i.e., industrialization. This is not surprising since learning by doing that occurs in the manufacturing production is the engine of growth for the closed economy. However, we showed that under free trade, it is better in some cases to allow the economy to be specialized in agriculture even though no learning by doing is experienced. This result is less intuitive, but not difficult to understand: The gains from trade come from the expansion of its consumption possibility frontier as its terms of trade are improving continuously. This result thus shows that an economy can gain not only from its learning by doing, but also from the learning by doing in the rest of the world through continuous improvement in the terms of trade.

Because of the time dimension of trade, in the present model it is not good enough to ask whether free trade is gainful. Rather, the correct questions to ask are whether free trade that starts now is gainful, and whether free trade that starts at a later time is gainful. For the small, open economy under consideration, the answer to the former question is ambiguous but that to the latter question is in the affirmative.

In the present model, the ignorance of the learning by doing effect by the firms represents a distortion in the economy. Such an externality means that the firms will in general underproduce manufacturing. This explains why in a closed economy the optimal subsidy to maximize the lifetime welfare of the economy is positive. Under free trade, the distortion may or may not disappear. For example, if the economy would export manufacturing under free trade, a production subsidy is not necessary. If the economy's initial comparative advantage is in agriculture, a producton subsidy on manufacturing is beneficial under certain conditions. This justifies the use of a subsidy to industrialize the economy in these cases.

Appendix: Summary of Results

	Cases			
	SA_0(a)	SA_0(b)	SM_0(a)	SM_0(b)
features	$q_0^a > p_0^w$, $g^a \leq g^w$	$q_0^a > p_0^w$, $g^a > g^w$	$q_0^a < p_0^w$, $\overline{g} \geq g^w$	$q_0^a < p_0^w$, $\overline{g} < g^w$
trade pattern	exports A, sustainable	exports A, sustainable	exports M, sustainable	exports M, then A
gains from free trade	$W^A > W^a$	$W^A \ ? \ W^a$	$W^M > W^a$	$W^M \ ? \ W^a$
optimal timing, free trade	$\widehat{t}_0 = 0$	$\widehat{t}_0 = 0$ or t^A	$\widehat{t}_0 = 0$	$\widehat{t}_0 = 0$ or t^a
optimal timing, possible subsidy	$\widehat{t}_0 = 0$ $\widehat{s} = 0$ if condition C is violated* or if $\overline{g} < g^w$	$\widehat{t}_0 = 0$ $\widehat{s} = 0$ if condition C is violated*	$\widehat{t}_0 = 0$ $\widehat{s} = 0$	$\widehat{t}_0 = 0$ or t^a $\widehat{s} = 0$

Note: *If condition C is satisfied, the required production subsidy on sector M is $\widehat{s}_t = q_t^f - p_t^w$ when $q_t^f \geq p_t^w$, and $\widehat{s}_t = 0$ when $q_t^f < p_t^w$.

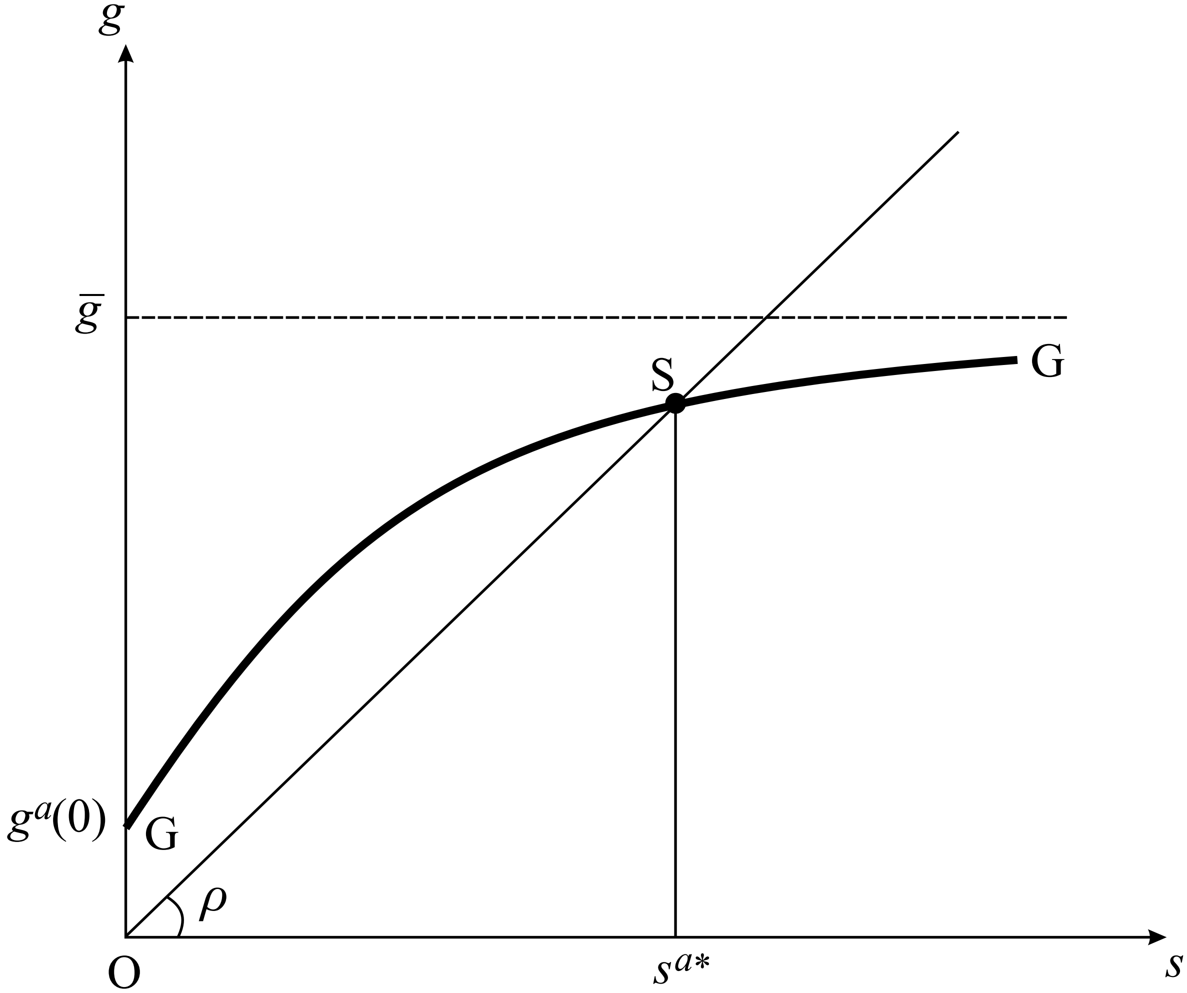

Figure 1

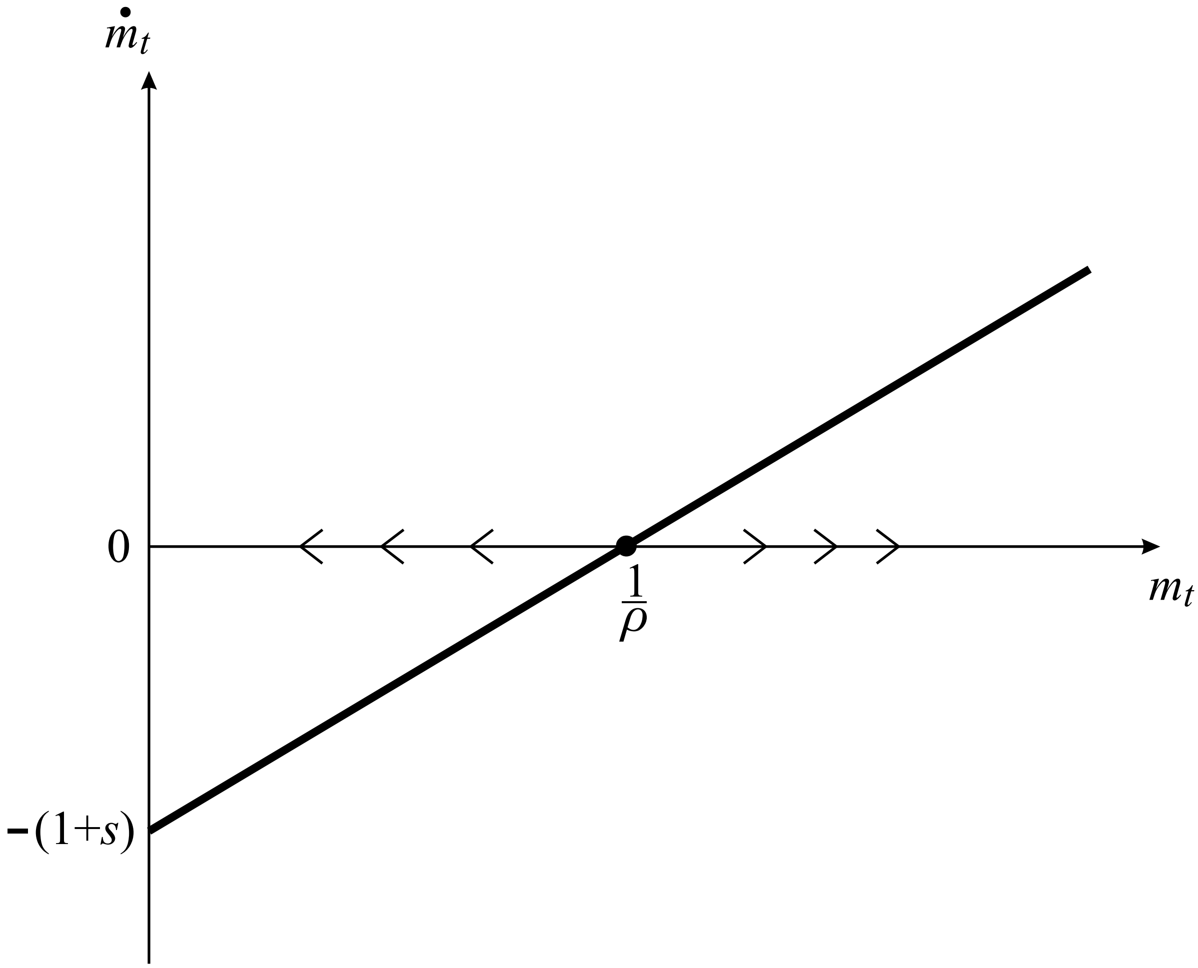

Figure 2

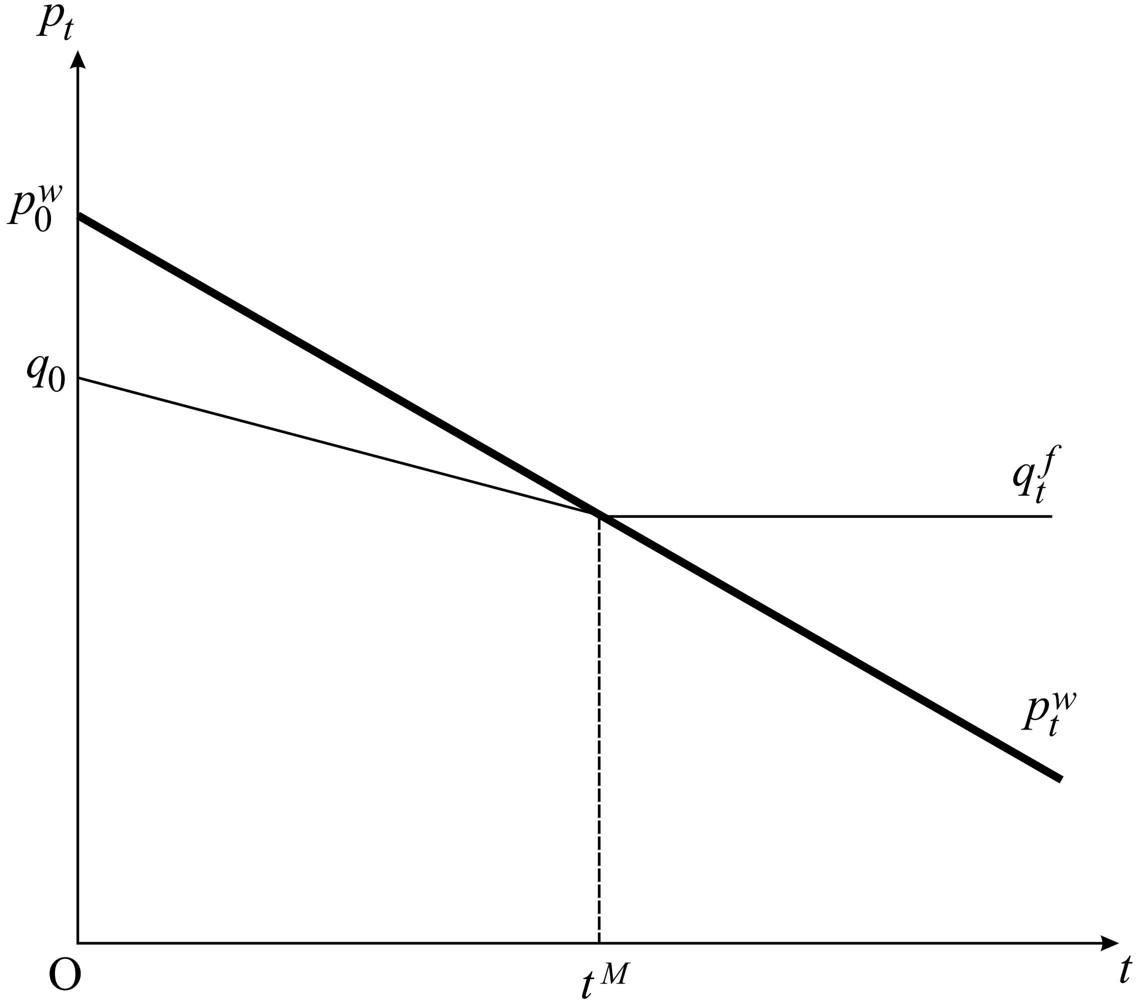

Figure 3

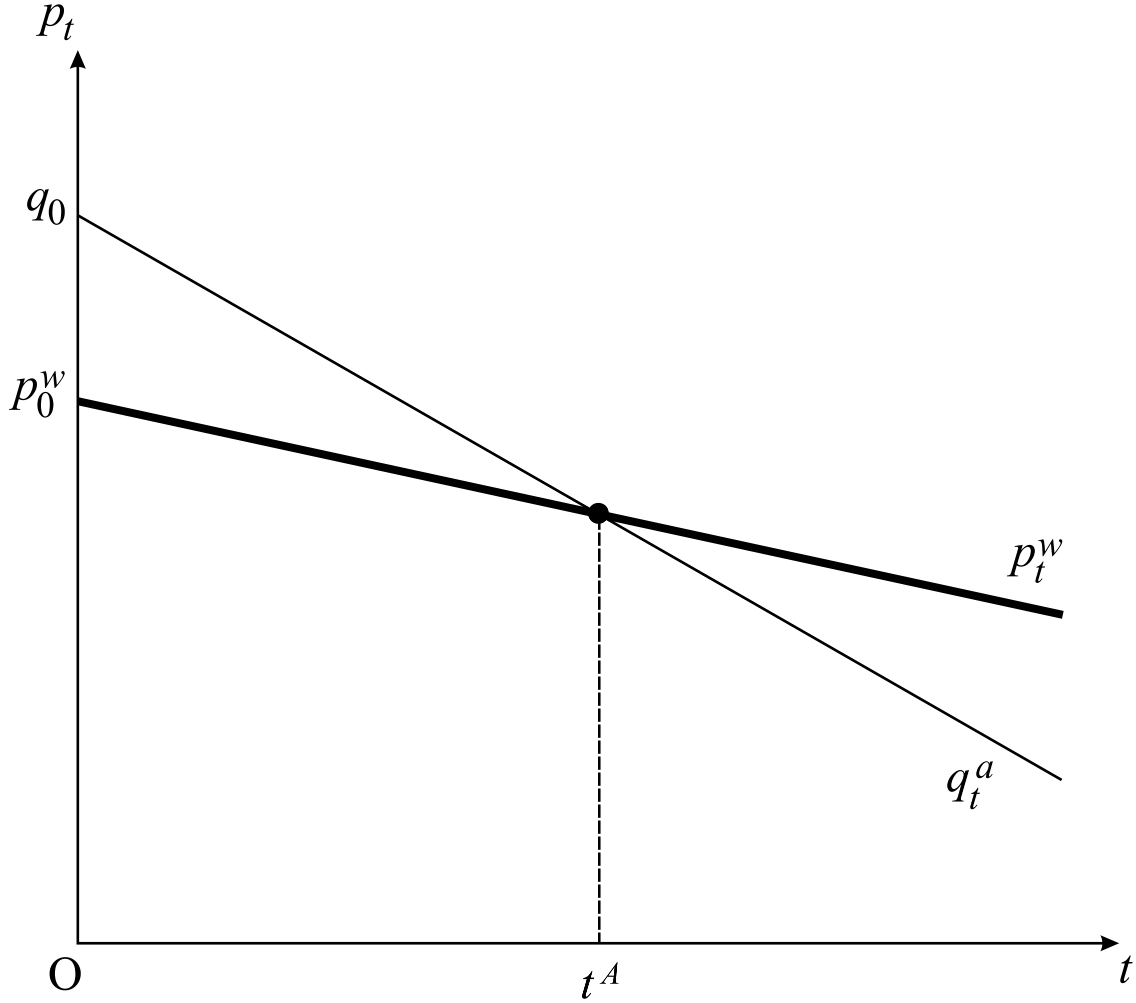

Figure 4

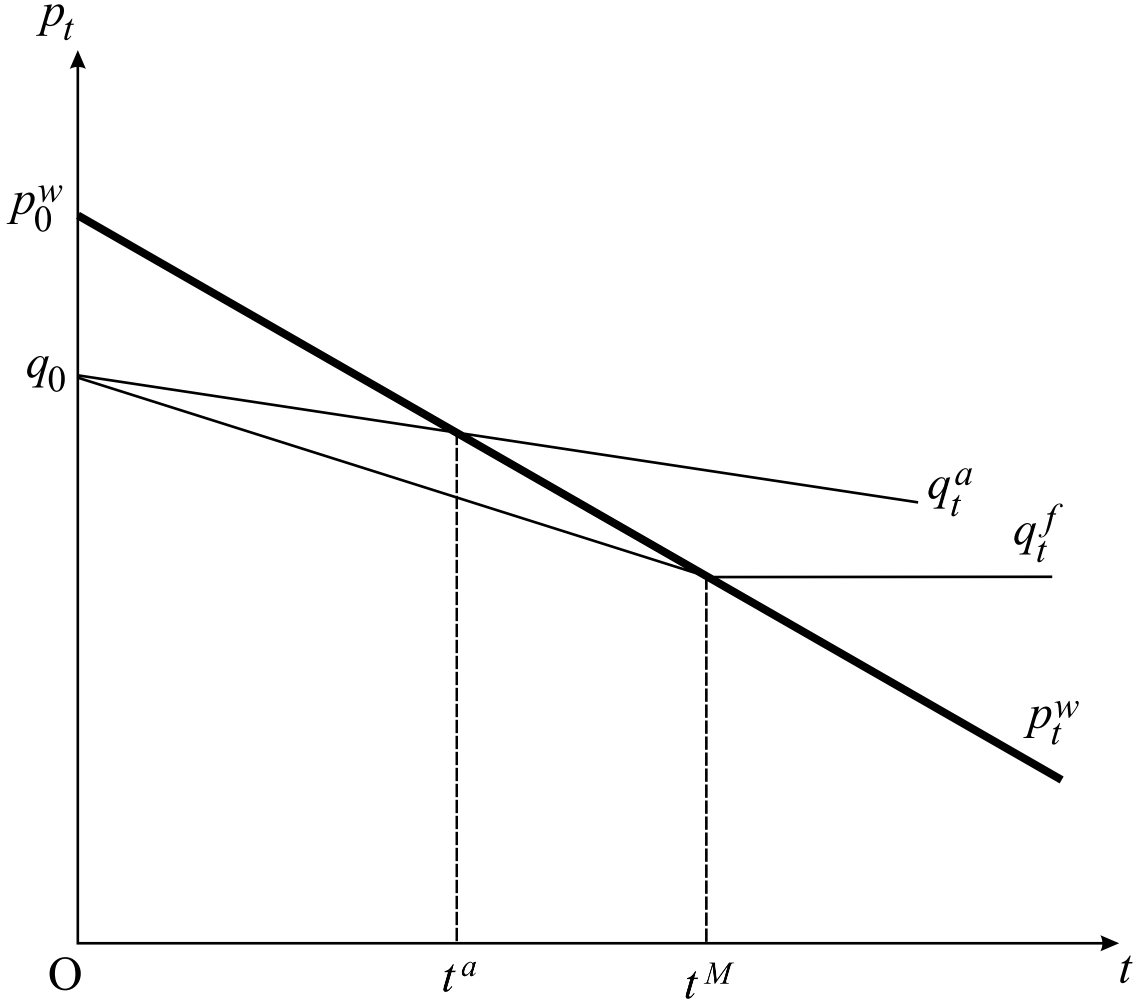

Figure 5

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