A Tale of Two Ports: The Economic Geography of Inter-City Rivalry

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Abstract

This paper examines how two geographically separated ports compete for a market consisting of manufacturing firms located between the two ports. There is a firm in each port, and these two firms, taking the infrastructure provided by their governments as given, compete in a Bertrand sense. The governments, however, can also compete in terms of investment in infrastructure. This paper shows that there are cases in which both the firm and the government in the port that has a longer history in the market may have the first mover advantage. In particular, the government can provide a credible threat by overinvesting in infrastructure.

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1 Introduction

This paper presents a model of rivalry between two cities that are distribution centres for goods produced in a long narrow region connecting the two cities.

Rivalry between cities is a common feature of economic life. An example that comes immediately to mind is the potential rivalry between Hong Kong and Shanghai. There are many other examples in the world that fit the present description. For example, in the United States, San Francisco and Los Angeles, and Seattle and Tacoma have long histories of competition. In Australia, Sydney and Melbourne have been known to be rivals for a long time, both in the commercial and in the cultural spheres. In Canada, Montreal and Toronto used to be (almost) equal competitors. In Germany, Dresden and Leipzig have a long history of rivalry. Hong Kong and Singapore are possible contenders for being the principal financial centre for East and Southeast Asia (not including Japan.)

When cities compete, the respective city governments may have an incentive to interfere, possibly to help locally based firms. This may reflect the desire of each city to maximize a conventional social (or, rather, city-based) welfare function. Alternatively, one may adopt the political-economy view that city officials want to be reelected, and their campaign funds can be increased if they help local businesses (possibly at the expenses of local tax payers.)

Our paper is an attempt to model the commercial rivalry between two cities. We abstract from considerations such as population size, agglomeration effects, labor market externalities, or diversification of business activities[1]. Our main focus is on the investment in city infrastructure, such as the road system, the law enforcement and regulatory system (e.g., how effective is the body that regulates the activities of the city stock exchange in Hong Kong, as compared with Singapore?). We model the rivalry between the two cities as follows. We assume that in each city there is a single distributor (or a cartel of distributors). The two distributors compete in service prices that they offer to manufacturing firms that are located in a long, narrow region joining the two cities. Each distributor's cost of supplying distribution services depends on the quality of the city's infrastructure, the provision of which is the responsibility of the city's government. The two governments compete against each other, because each wants its city to be the most prosperous centre of commerce.

In the model considered in the present paper, there are two levels of

[1] For models that deal with these issues, see, for example, North (1955), Thisse (1987), Glaeser et al. (1992), Fujita and Thisse (1996), Long and Soubeyran (1998), and Fujita, Krugman and Venebles (1999).

competition, which take place consecutively. First, the governments of the cities compete in terms of investment in infrastructure. Then, taking the government investments as given, the two distributors compete. On the firm level, we examine both a Bertrand-Nash equilibrium and a Stackelberg leader-follower equilibrium between the two distributors. The latter equilibrium is the one the distributor that develops earlier wants to achieve, but its successful achievement of this equilibrium depends on certain restrictive assumptions. On the government level, a Nash equilibrium is possible if the cities are symmetric. However, if a city is developed earlier, it has the first mover advantage so that when a rival city tries to emerge with possible jumps in infrastructure investments, it has an incentive to increase its investment with the purpose of discouraging the investment by its rival. Such a preemptive investment by the first mover could be credible because investment that was made earlier is not reversible.

In section 2 we present the basic model and determine the equilibrium when one of the ports is the only supplier of the distribution services to the production firms. In section 3, we characterize the Nash-Bertrand equilibrium between two distributors, one located in each city. Section 4 analyzes the case in which one distributor has the first mover advantage, that is, this is a Stackelberg game between the two distributors. Section 5 studies the optimal unilateral intervention from the point of view of one city, under the assumption that the two distributors play a Nash-Bertrand game. In section 6, we deal with a simultaneous-move game between the two cities, while section 7 discusses the Stackelberg game between the two cities. In both of these sections, we maintain the assumption that the two distributors play a Nash-Bertrand game. The last section offers some concluding remarks.

2 The Model

The two cities are denoted by N and S (North and South), respectively.[2] The distance between them is normalized at unity. S is located at point 0 and N is located at point 1. There are two types of firms: production firms, and distribution firms. Production firms form a continuum, and are uniformly distributed between S and N. A production firm is indexed by x, where x is its distance from S and $1-x$ is its distance from N. These firms produce the

[2] Our convention of naming the two cities comes from the fact that in many of the examples cited earlier, the two cities are roughly in a North-South positions; for example, Hong Kong-Shanghai, Seattle-Tacoma, and San Francisco-Los Angeles.

same good.[3]

For simplicity, assume that each production firm produces either one unit of output, or nothing. The cost of producing the output is a given constant, which is normalized as zero. The cost to firm x of shipping its output from location x to S is $bx^2/2$, and the cost of shipping the output from location x to N is $(b/2)(1-x)^2$.[4]

In city S, there is a single distribution firm labeled distributor S (or, equivalently, a cartel of distributors) that provides the service of sending (and marketing) the goods to a foreign market. The cost of providing this service is C_S per unit of good sold, which is taken as given by the distributor. The cost C_S depends on the quality of the city's infrastructure, I_S, (such as the quality of the road network, and of the law enforcement system), as described by the following function,

$$C_S = C_S(I_S),$$

where $C_S'(I_S) < 0$ and $C_S''(I_S) > 0$.[5] It should be noted that I_S is labeled as investment for simplicity, but it is in fact the accumulated investment in the past less depreciation. It is therefore a stock, not a flow.

Distributor charges a price $\theta_S \geq C_S$, the same for all production firms, for its distribution service.[6] The world price of the good is P, assumed to be given exogenously.

In city N, there is a rival distributor labeled distributor N, with unit cost of distribution C_N given by

$$C_N = C_N(I_N),$$

where $C_N'(I_N) < 0$, $C_N''(I_N) > 0$.

3 Monopoly Distributor

We begin by focusing on one distributor. Suppose that initially the cost of the rival distributor in N is so high that it cannot compete with distributor S, e.g., the quality level I_N is so low that $C_N(I_N) > P$. Then the southern distributor is a monopolist.

[3]Many features of this model are borrowed from the spatial competition model of Hotelling (1927).

[4]Our assumption that the transport cost is quadratic in distance is in line with d'Aspremont et al. (1979).

[5]This means that an increase in the quality of the city's infrastructure will lower the cost of the distribution service, but the rate of change in C_S with respect to I_S increases with the latter.

[6]Charging the same price to all production firms implies no price discrimination by each distribution firm.

3.1 Optimal Production of Distributor S

The objective of distributor S is to maximize its profit by choosing a service price θ_S and a cut off point $x^m \leq 1$ such that all firms with distance $x > x^m$ will be excluded. For firms that are not excluded, the participation constraint is that they should earn non-negative profit. Formally, the distributor seeks to

$$\max_{\theta_S, x^m} \pi_S = \int_0^{x^m} (\theta_S - C_S) dx = (\theta_S - C_S) x^m, \tag{1}$$

subject to

$$P - \theta_S - \frac{b}{2} x^2 \geq 0, \qquad \forall x \in [0, x^m], \tag{2}$$

and

$$0 \leq x^m \leq 1. \tag{3}$$

It is convenient to define the mark-up on distribution cost as z,

$$z = \theta_S - C_S. \tag{4}$$

It is clear that problem (1) is equivalent to

$$\max_{z, x^m} \pi_S = z x^m, \tag{5}$$

subject to (3) and

$$P - C_S - z - \frac{b}{2} (x^m)^2 \geq 0. \tag{6}$$

To analyze problem (5), we make use of Figure 1. In the (x, z) space, we first construct various iso-profit contours, each of which corresponds to the same profit. Each iso-profit contour, such as XZ, has a slope of

$$\left.\frac{\partial z}{\partial x}\right|_{\mathrm{XZ}} = -\frac{z}{x} < 0, \tag{7}$$

and is convex to the origin. Condition (6) is the area bounded from above by curve PC, which has a slope of

$$\left.\frac{\partial z}{\partial x}\right|_{\mathrm{PC}} = -bx < 0, \tag{8}$$

and is concave to the origin. Constraint (3) refers to the region bounded by the vertical axis and the vertical line $x = 1$. The problem of the firm is to choose (x, z) that reaches the highest iso-profit contour subject to the constraints (6) and (3). Depending on the nature of the solution, two cases can be distinguished.

The Whole Market Case (Corner Solution) — This case, in which distributor S serves all the production firms, is defined by the following condition:

$$P - C_S \geq \frac{3b}{2}. \tag{9}$$

To see why there is a corner solution, note that at $x^m = 1$, conditions (9) and (6) (the latter with an equality) imply that at $(1,\, P - C_S - b)$

$$\left.\frac{\partial z}{\partial x}\right|_{\text{PC}} \geq \left.\frac{\partial z}{\partial x}\right|_{\text{XZ}}, \tag{10}$$

or that contour XZ is at least as steep as curve PC. This is the case shown in Figure 1 (with a strict inequality in condition (10)). The optimal point with the highest possible profit is E.

At this point, condition (6) implies that the optimal price of the good is $\theta_S^* = P - b/2$, giving a profit to the firm of

$$\pi_{SM} = \pi_{SC}^*(C_S) = P - C_S - \frac{b}{2}. \tag{11}$$

The Partial Market Case (Interior Solution) — This case, in which distributor S serves some of the production firms closer to itself, is defined by the following condition:

$$P - C_S < \frac{3b}{2}. \tag{12}$$

Let us suppose that starting from point E there is a drop in the price P, or a rise in the cost C_S, or both, so that curve PC shifts down to, say, $\tilde{\text{P}}\tilde{\text{C}}$, satisfying condition (12). This produces a point of tangency, $\tilde{\text{E}}$, between $\tilde{\text{P}}\tilde{\text{C}}$ and iso-profit contour $\tilde{\text{X}}\tilde{\text{Z}}$, it is optimal for distributor S to serve all the production firms within the interval $(0,\, \tilde{x})$. Using (7) and (8), with (6) holding as an equality, we get the solution to the profit-maximizing problem: $z^* = 2(P - C_S)/3$, and

$$x^{m*} = \sqrt{2(P - C_S)/(3b)} \tag{13}$$

$$\theta_S^* = \frac{2P + C_S}{3}. \tag{14}$$

Firm S is able to get a profit of

$$\pi_{SM} = \pi^*_{SI}(C_S) = \frac{1}{\sqrt{b}}\left[\frac{2(P - C_S)}{3}\right]^{3/2}, \tag{15}$$

which is what contour $\tilde{\mathrm{X}}\tilde{\mathrm{Z}}$ represents. Note that in these two cases with all production firms as price takers distributor S will set the service price θ_S so that the production firm being served and farthest away from port S earns a zero profit.

3.2 Optimal Investment by Government S

The government of port S is to choose the optimal investment in infrastructure, I_S, which determines the cost of distribution firm S takes as given. The government chooses I_S to maximize the welfare function V_S of the port defined as

$$V_S = \pi_{SM} - (1 + \beta)I_S, \tag{16}$$

where β is the marginal cost of public finance[7], and π_{SM} is the profit of the distribution firm as a monopolist, defined by either (11) or (15) depending on whether an interior solution exists. Problem (16) yields the first order condition

$$\pi'_{SM}(C_S)C'_S(I_S) = (1 + \beta). \tag{17}$$

In condition (17), $\pi'_{SM}(C_S) = -1$ and $\pi''_{SM}(C_S) = 0$ in the Whole Market case, or $\pi'_{SM}(C_S) = -\sqrt{2(P - C_S)/(3b)}$ and $\pi''_{SM}(C_S) = [2/(3b)]^{1/2}[P - C_S]^{-1/2}$ in the Partial Market case.

When given the market price of the good, condition (17) is illustrated by schedule ABFDE in Figure 2. This schedule has two components. Segment ABFD is a vertical line, corresponding to the Whole Market case ($C_S < P - 3b/2$), in which (17) reduces to an equation with one unknown, I_S. Denote the unique solution by I^0_S. Curve DE corresponds to the Partial Market case ($C_S > P - 3b/2$). Condition (17) is differentiated to give the slope of curve DE:

$$\left.\frac{\mathrm{d}C_S}{\mathrm{d}I_S}\right|_{\mathrm{DE}} = -\frac{\pi'_{SM}C''_S}{C'_S\pi''_{SM}} < 0. \tag{18}$$

Also illustrated in Figure 2 is schedule CI, which represents $C_S = C_S(I_S)$, which, by assumption, is downward sloping and convex to the origin. The intersection between schedules ABFDE and CI gives the optimal infrastructure investment. At the initial market price, the optimal point occurs at F in

[7] It measures the deadweight loss caused by raising an extra dollar of tax.

the figure, with an infrastructure investment of I_S^0, leading to a service cost of C_S^*. Distributor S chooses to serve the whole market.[8]

Suppose now that there is a significant decrease in P to, say, $\tilde{P}$. As a result, the curve that represents condition (17) shifts to AB$\tilde{\text{F}}$G, with its portion B$\tilde{\text{F}}$G corresponds to the case in which distributor S serves only part of the market. With the slope in (18) sufficiently large in magnitude, B$\tilde{\text{F}}$G cuts schedule CI at point $\tilde{\text{F}}$, leading to an optimal infrastructure investment of $\tilde{I}_S^0$ and a corresponding service cost of $\tilde{C}_S^*$.[9] Making use of Figure 2 and the previous analysis, the following proposition can easily be established, the proof of which is given in the appendix:

Proposition 1: (a) If currently $C_S \leq P - 3b/2$, a small rise in P will not affect the government's infrastructure investment while distributor S continues to serve the whole market. The service price charged by distributor S increases by the same amount. (b) If currently $C_S > P - 3b/2$, a small rise in P will lead to an increase in the government's infrastructure investment and a fall in the service cost, while distributor S serves more firms. The change in the service price charged by distributor S is determined by condition (14).[10]

4 Duopoly in Distribution

Now consider the duopoly case, in which both distributors S and N may provide services to the production firms. These two distributors set service prices, θ_S and θ_N, respectively. To simplify the analysis, we assume that the

[8] As an example, one may take the special functional form

$$C_S(I_S) = \frac{\gamma_S}{I_S},$$

where γ_S is a positive parameter. Assumingt that the market price is high enough so that firm S serves the whole market, we obtain the optimal investment condition

$$I_S = \left[\frac{\gamma_S}{1+\beta}\right]^{1/2}.$$

[9] To satisfy the second-order condition, schedule GB is steeper than schedule CI, as shown in Figure 2. See the appendix for a proof.

[10] Note that for simplicity we have not explicitly imposed the condition that investment is irreversible, i.e., $I_S \geq 0$. Under irreversibility, there is an asymmetry between an increase in P and a decrease in P. While a government may be interested in increasing its investment in response to an increase in P, it does not destroy some of its previous investment in the case of a drop in P.

market price is high enough so that if one distributor serves the market, it serves the whole market (the Whole Market case).[11]

Production firms take the service prices θ_N and θ_S as given, and choose the distributor with the lower distribution cost. Denote x_c as the firm indifferent between dealing with N or S, satisfying

$$\theta_S + \frac{bx_c^2}{2} = \theta_N + \frac{b(1-x_c)^2}{2},$$

or

$$x_c = \frac{1}{2} + \frac{1}{b}\left(\theta_N - \theta_S\right). \tag{19}$$

Given that $x_c \in [0,1]$, three cases can be distinguished:[12]

- **Case S**: $\theta_N - \theta_S \geq b/2$. From (19), $x_c = 1$, with all production firms served by distributor S.
- **Case N:** $\theta_S - \theta_N \geq b/2$, implying that $x_c = 0$, with all production firms served by distributor N.
- **Case B:** $|\theta_N - \theta_S| < b/2$. In this case, $0 < x_c < 1$, with distributor S serving firms located at $(0,\ x_c)$ while distributor N serving those at $(x_c,\ 1)$.

4.1 Reaction Functions of the Distributors

Define $y \equiv \theta_N - \theta_S$, and $\phi(y) \equiv x_c$. Let us first consider distributor S, which maximizes its profit, taking θ_N as given:

$$\pi_S = \int_0^{\phi(y)} (\theta_S - C_S)dx = (\theta_S - C_S)\phi(y), \tag{20}$$

subject to

$$P - \theta_S - b[\phi(y)]^2/2 \geq 0. \tag{21}$$

Condition (21), which makes sure that all production firms served by the distributor do not have negative profits, is assumed to be not binding. The reaction curve of distributor S is derived as follows and is illustrated in Figure 3:

Case N (region A): In this region, $\phi(y) = 0$, and is bounded below by the line XKL, $\theta_S = \theta_N + (b/2)$, with distributor S serving no firm.

[11] This simplifies the analysis and guarantees interactions between the two distributors.

[12] In all these cases, the market price is assumed to be high enough so that no production firms concerned make a loss.

Case S (region B): In this region, $\phi(y) = 1$ and is bounded above by the line, YJGH, $\theta_S = \theta_N - (b/2)$, with distributor S capturing the whole market. Note that $\theta_S = P - b/2$, the monopolistic service price, when $\theta_N \geq P$.

Case B (region D): In this region, $\phi(y) \in (0, 1)$, and is bounded by lines XKL and YJGH. The reaction function in this region is obtained from the solution to problem (20) and is given by

$$\theta_S = R_S(\theta_N) = \frac{b}{4} + \frac{1}{2}\left[C_S + \theta_N\right]. \tag{22}$$

In the diagram, the line represented by (22) cuts YJGH at point G, where $\theta_N = C_S + (3/2)b$. Combining the above results, we conclude that the reaction curve of distributor is given by schedule EFGHI in Figure 3. We now have the following interesting result:

Proposition 2 (Limit Pricing): Assume that condition (9) holds. If distributor N charges θ_N so that $P > \theta_N > C_S + (3/2)b$, then distributor S will charge a service price that captures the whole market for itself. This service price is below the monopoly price.

Note that in Proposition 2, condition (9) implies that distributor S as a monopolist will capture the whole market. If $P > \theta_N > C_S + (3/2)b$, the distributor's reaction curve is given by GH in Figure 3. The corresponding service price charged by S is less than the monopolist price, $P - b/2$, but S captures the whole market with distributor N serving no production firm. This case is interesting as the presence of distributor N poses as a threat to distributor S, who chooses to set a price lower than the monopolist price. On the other hand, distributor S sets a price low enough to deter N from entering the market.

The reaction function for distributor N can be constructed in a similar way. If it has a segment in the interior of region D, then that segment satisfies the following equation

$$\theta_N = \frac{b}{4} + \frac{1}{2}\left[C_N + \theta_S\right] \tag{23}$$

We now derive the equilibrium. We assume that both distributors take the infrastructure in each city (and thus its service cost and that of its competitor) as given, and compete in a Bertrand fashion so that they simultaneously choose a service price. The Nash equilibrium is described by the following proposition:

Proposition 3 (Nash equilibrium): Assume that condition (9) holds. (a) If $C_N \geq C_S + (3/2)b$, distributor S captures the whole market. In the subcase in which $C_N < P$, distributor S charges a service price of

$$\theta_S = C_N - \frac{b}{2}. \tag{24}$$

If $C_N \geq P$, distributor S will charge the monopoly price $\theta_S = P - (b/2)$.

(b) If $C_N < C_S + (3/2)b$, and $C_S < C_N + (3/2)b$, or more concisely, $|C_N - C_S| < (3/2)b$, then we have an interior Nash equilibrium, satisfying both conditions (22) and (23), and the equilibrium charges are

$$\theta_S^* = \frac{b}{2} + \frac{1}{3}\left(2C_S + C_N\right), \tag{25}$$

and

$$\theta_N^* = \frac{b}{2} + \frac{1}{3}\left(2C_N + C_S\right), \tag{26}$$

provided that P is sufficiently great so that the indifferent production firm x_c earns non-negative profit.[13]

(c) If $C_S \geq C_N + (3/2)b$, distributor N captures the whole market . In the subcase in which $C_S < P$, distributor N charges a service price of

$$\theta_N = C_S - \frac{b}{2}. \tag{27}$$

If $C_S \geq P$, distributor N will charge the monopoly price $\theta_N = P - (b/2)$.

Remarks: The above proposition indicates that (i) distributor S will use the limit pricing strategy if C_N is below P but is sufficiently greater than C_S (precisely, $C_N \geq C_S + (3/2)b$). If C_N falls, then the limit price θ_S, given by (24) will also be adjusted downwards, and (ii) in the case where both distributors have positive market shares, a fall in C_N will cause both equilibrium charges to fall, but θ_S falls by less than θ_N.

4.2 Services Costs and Nash Equilibrium

Construct a diagram (Figure 4) in the (C_N, C_S) space, for a given P, which we assume to be greater than $(3/2)b$. We first examine how **Case S** is affected by the distributors' service costs. In region W_S, which is defined by $C_N > P > C_S + (3/2)b$, firm S is the monopoly that serves the whole market, and its profit is

$$\pi_{SM} = P - C_S - (b/2) = \pi_{SM}(C_S), \tag{28}$$

[13] This condition requires that $P \geq \theta_S^* + (b/2)x_c^2$ where $x_c = (1/2) + (1/3)b(C_N - C_S)$ and where θ_S^* is as given above.

where the subscript M indicates that the distributor is in effect a full monopolist. In triangle $\mathrm{T_N}$, which is defined by $(3/2)b < C_N < P$, $0 < C_S < P - (3/2)b$, and $C_S < C_N - 3b/2$, both distributors offer a service price less than P, but distributor S captures the whole market by setting the limit price $\theta_{SL} = C_N - (b/2)$, thus earning the profit

$$\pi_{SL} = \theta_{SL} - C_S = C_N - C_S - (b/2), \tag{29}$$

where the subscript L indicates that the distributor is charging a limit price. We now turn to **case N**. In region $\mathrm{W_N}$ defined by $C_S > P > C_N + (3/2)b$, firm N is the monopoly that serves the whole market, and its profit is

$$\pi_{NM} = P - C_S - (b/2) = \pi_{NM}(C_S). \tag{30}$$

Similarly, in triangle $\mathrm{T_S}$ defined by $(3/2)b < C_S < P$, $0 < C_N < P - (3/2)b$, and $C_N < C_S - 3b/2$, both distributors offer a service price less than P, but distributor N captures the whole market by setting the limit price $\theta_{NL} = C_S - (b/2)$, thus earning the profit

$$\pi_{NL} = \theta_{NL} - C_N = C_S - C_N - (b/2) > b. \tag{31}$$

Finally, for **case B**, if the cost configuration (C_N, C_S) is in the region X defined by the square $\{(C_N, C_S) : 0 \le C_N \le P,\ 0 \le C_S \le P\}$ excluding the two triangles $\mathrm{T_N}$ and $\mathrm{T_S}$ and a small region Q in the north-east corner of this square,[14] then we have a unique and interior Nash equilibrium, i.e., both distributors have positive market shares. The market share of distributor S at this equilibrium is

$$x_S = \frac{1}{2} + \frac{1}{3b}\left[C_N - C_S\right],$$

and its Nash equilibrium profit is

$$\pi_{SI}^{nash} = \left[\theta_S - C_S\right] x_S = \frac{1}{b}\left[\frac{b}{2} + \frac{1}{3}\left(C_N - C_S\right)\right]^2, \tag{32}$$

where the subscript I indicates that the equilibrium is an interior one.

[14] In the small region Q, the costs of both distributors are so close to P that some subset of production firms are not served by either distributor. The lower boundary of Q is given by the condition that the marginal production firm earns zero profit and is indifferent between the two distributors.

5 Stackelberg Leadership by A Distributor

In the preceding sections, it was assumed that the two distributors play a Nash-Bertrand game, choosing their service prices simultaneously. There are, however, some cases in which one of them can play as a Stackelberg leader. One possible case is now described.

To describe such a possibility, let us first investigate more properties of the distributors' reaction curves. Consider Figure 5, which is obtained from Figure 3, with some added details. Distributor S's reaction function in Figure 3 is reproduced in Figure 5, and is labelled JNKH, where K is the kink. The co-ordinates of points J, K and H are respectively $(0,\, C_S/2+b/4)$, $(C_S+3b/2,\, C_S+b)$ and $(P,\, P-b/2)$. In the region in which both distributors serve some production firms (region D in Figure 3), the profit of distributor S is given by

$$\pi_S = [\theta_S - C_S]\, x_c = [\theta_S - C_S] \left[\frac{1}{2} + \frac{1}{b}(\theta_N - \theta_S)\right]. \tag{33}$$

The loci of $(\theta_N,\, \theta_S)$ that give the same profit of distributor S, which can be called an iso-profit contour, has the slope

$$\frac{\partial \theta_S}{\partial \theta_N} = -\frac{\partial \pi_S/\partial \theta_N}{\partial \pi_S/\partial \theta_S} = \frac{1}{1-k}, \tag{34}$$

where

$$k = \frac{b + 2(\theta_N - \theta_S)}{2[\theta_S - C_S]}.$$

Note that the slope of an iso-profit contour at a point on line JK, part of the distributor's reaction curve, is infinity, implying that $k = 1$. This further implies that the slope of the contour is positive (negative) above (below) JK. In Figure 5, a Stackelberg equilibrium is depicted at point S, where distributor N's reaction curve LNM is tangent to distributor S's iso-profit curve π_S^s, which is higher that what distributor S can get at a Nash equilibrium. Since the slope of distributor N's reaction curve NM is equal to 2, $k = 1/2$ at point S.

Now suppose that currently the infrastructure investments by government N is very low, making the service cost of distributor N very high, so that distributor N chooses not to serve any production firms. So distributor S is a monopolist and charges a service price of $\theta_S^m = P - b/2$. Suppose now that government invests significantly in infrastructures, lowering distributor N's service cost so that its reaction curve is now represented by LNSM in Figure 5. If both distributors play Bertrand, the equilibrium is at point N. Distributor S will observe a considerable drop in its service price (to θ_S^n) and profit.

Suppose now that, shortly before the entry of distributor N, distributor S lowers its service price to θ_S^s, which is lower than θ_S^m but higher than θ_S^n. If distributor N is convinced that this is what distributor S is committed to, it will choose the Stackelberg point S.

There is, however, another possible case which is less intuitive. Suppose that the service cost faced by distributor N is in fact higher so that its reaction curve is $\tilde{\text{L}}\tilde{\text{M}}$ in Figure 5. With this reaction curve, the Stackelberg equilibrium shifts to a point like $\tilde{\text{S}}$. In the case shown in the diagram, point $\tilde{\text{S}}$ is above the horizontal line at $P - b/2$, meaning that anticipating the entrance of a rival distributor S *raises* its service price.

Such a case might seem counter-intuitive, but can be explained intuitively. In the present model, while distributor S captures only part of the market after the entrance of distributor N, when it raises its service price it anticipates that distributor N will react with a price at a level high enough so that distributor S will not lose a big market share. This case can also be explained in an alternative way. When a distributor is in a monopoly situation, there is a trade-off between profit per unit of service sold, and the number of units of service sold. When it has a rival as a follower, the trade-off is still there, but it is somewhat altered: under monopoly, when a firm changes its price, it is moving along a *given* demand curve described by $x = [(2/b)(P - \theta_S)]^{1/2}$, but with the existence of a follower, when distributor S changes its price, it is moving along a quite different demand curve: $x_c = (1/2) + (1/b)[\theta_N - \theta_S]$, where $\theta_N = \theta_N(\theta_S)$.

Of course, the Stackelberg equilibrium depends on the assumption that distributor N is convinced that θ_S^s is irreversible. This would be the case if distributor S chooses a service price and spends resources on fixing and announcing it. The more resources distributor S spends on fixing the price, the more costly it is to change it later, and the more convincing the pricing policy is.[15] Usually the existing firm is in a better position in choosing the service price first, meaning that distributor S, being the first in business, is more likely to have the first mover advantage. If, however, distributor N believes that distributor S's chosen service price is reversible at low costs, the more likely equilibrium is the Nash equilibrium.

A Numerical Example: This numerical example shows the possibility that distributor S charges more in the duopoly case than in the monopoly case. Let $P = 2$, $C_S = 0.5$, $C_N = 1.2$, and $b = 1$. The monopoly price for distributor S is 1.5. At that price, it serves the whole market, and its monopoly profit is 1. Now, consider the entry of distributor N. If S maintains

[15] Of course, the cost of the resources on fixing the price will come from the distributor's profit.

the previous monopoly service price $\theta_S = 1.5$, then N will charge $\theta_N = 1.6$, and S's market share will fall to 0.6, and its profit will be $\pi_S = 0.6$. Clearly, S can do better by maximizing (33) where $\theta_N = \theta_N(\theta_S)$ as given by (23). This yields $\theta_S = 1.60$, and $\theta_N = 1.65$, resulting in a smaller market share for S, $x_c = 0.55$ and a higher profit, $\pi_S = 0.605$.

6 Optimal Unilateral Intervention

Assume that initially the costs are C_S^0 and C_N^0, and the equilibrium is an interior one, i.e., both distributors serve some production firms. Suppose that the government of city S wants to maximize the welfare function V_S by choosing investment level I_S, taking as given the amount I_N. We assume that government S first chooses a new (possibly higher) investment level, with government N remaining passive. Both distributors take the investment levels as given and compete in a Bertrand fashion.

Let us assume that by spending $I_S > 0$, the cost C_S will fall to a level below C_S^0. This fall is represented by a function $F_S(I_S)$, where $F_S(0) = 0$, $F_S'(.) < 0$, and $F_S''(.) > 0$, and $C_S = C_S^0 - F_S(I_S)$.[16] The government of city S maximizes

$$V_S = \frac{1}{b}\left[\frac{b}{2} + \frac{1}{3}(C_N - C_S)\right]^2 - (1+\beta)I_S. \tag{35}$$

The first-order condition is

$$\frac{\mathrm{d}V_S}{\mathrm{d}I_S} = -\omega(I_N, I_S^*)F_S'(I_S^*) - (1+\beta) \leq 0, \ = 0 \text{ if } I_S^* > 0, \tag{36}$$

where

$$\omega(I_N, I_S^*) = \frac{1}{3} + \frac{1}{9b}\left(C_N^0 - F_S(I_N) - C_S^0 + F_S(I_S^*)\right) > 0, \tag{37}$$

and at an interior Nash equilibrium $|C_N - C_S| < (3/2)b$. It follows that condition (36) is satisfied at some $I_S^* > 0$ if $|F'(0)| > (1+\beta)/\omega(I_N', 0)$, that is, if the first dollar spent on investment has a very substantial marginal effect on cost reduction. In what follows we assume that this is the case, so that, given I_N, at the optimal investment level I_S^*, we have

$$F'(I_S^*) = -(1+\beta)/\omega(I_N, I_S^*). \tag{38}$$

The second-order condition is

$$\frac{\mathrm{d}^2V_S}{\mathrm{d}I_S^2} = -\omega(I_N, I_S^*)F''(I_S^*) - \frac{1}{9b}\left[F'(I_S^*)\right]^2 \equiv \Delta_S < 0, \tag{39}$$

[16] Notice that it is assumed that the fall in cost is independent of the existing level of cost. A more general formulation would have C_S^0 appear as a parameter in the function F_S.

which is satisfied.

Proposition 4: Assume that $|F'_S(0)| > (1+\beta)/\omega(I_N, 0)$. Then the optimal policy for the government of S is to raise taxes to invest in the city's infrastructure, until condition (38) is satisfied.

Remark: Because of upward sloping reaction functions, if we are considering optimal output tax, we would get a result similar to that of Eaton and Grossman (1986), who found that if a foreign firm and a home firm compete in prices (and their reaction functions have positive slope) then the optimal policy for the home government is to increase the cost of the home firm by taxing its exports. In our model, we deal with modifying real costs, and the optimal policy is to reduce the cost of the home firm, if the initial cost is high. There are two features worth noting. Firstly, since the production firms that are located on the long narrow region linking N and S are the purchasers of the services, they are the analog of the consumers in the third market in the Eaton-Grossman model. However, in our model, the "demand" for service of S, given by (19), remains unchanged if both distributors slightly raise their service prices by the same amount. This is not the case in the Eaton-Grossman model. The second feature is that while in the Eaton-Grossman model a dollar of subsidy reduces the cost of the home firm by a dollar, in our model a dollar of spending in infrastructure reduces the cost of the distributor by F'_S which is not a constant (recall that $F''_S > 0$).

7 Simultaneous Government Policies

Now suppose that both cities are engaged in the infrastructure-investment game. The governments first choose an investment level simultaneously. Then the distributors, taking these investment levels as given, compete in a Bertrand fashion.

For a given initial pair (C_S^0, C_N^0), let us find the reaction function of each city. From (36), we obtain city S's reaction function $I_S = r_S(I_N)$, with a negative slope given by

$$\left.\frac{\mathrm{d}I_S}{\mathrm{d}I_N}\right|_S \equiv r'_S(I_N) = \frac{1}{\Delta_S}\left[\frac{F'_N F'_S}{9b}\right] < 0, \tag{40}$$

where Δ_S is defined in (39). By condition (40), the magnitude of the slope is less than unity if $|F'_S(I_S)| \geq |F'_N(I_N)|$. The reaction function $I_S = r_S(I_N)$ is illustrated in Figure 6. Note that there is a maximum value of city S's investment, as represented by the flat part AB of the reaction schedule.

This is the optimal investment of the government when distributor S is a monopolist.[17]

Similarly, for city N, the reaction function of government N is $I_N = r_N(I_S)$, which satisfies

$$\left.\frac{\mathrm{d}I_N}{\mathrm{d}I_S}\right|_N \equiv r'_N(I_S) = \frac{1}{\Delta_N}\left[\frac{F'_N F'_S}{9b}\right] < 0, \tag{41}$$

where Δ_N is defined in a similar way to Δ_S above. The reaction function is illustrated in Figure 6, with CD representing the maximum value of government N's investment.

In general, there is the possibility of multiple equilibria, as illustrated in Figure 6. If the initial costs C_S^0 and C_N^0 are identical, and the functions $F_N(.)$ and $F_S(.)$ are the same, then there exists a symmetric Nash equilibrium that is stable, see point E_3 in Figure 6. However, there may exist non-symmetric stable equilibria, such as points E_1 and E_5.

In the case of multiple equilibria, it can be shown, by construction of iso-welfare curves, $V_i =$ constant $(i = S, N)$, that city N prefers E_5 to E_3 to E_1, and similarly, city S prefers E_1 to E_3 to E_5.

In what follows, we will restrict attention to the case where there is only one Nash equilibrium. We assume that such equilibrium is stable. See Figure 7, where the unique Nash equilibrium is E.

8 Stackelberg Game between Cities

As explained earlier, it is possible that one of the cities, say S, develops first. Suppose that initially city N has a very low investment in infrastructure, such as I_N^0 so that distributor N cannot compete with distributor S.[18] In the absence of active investment by city N, the government of S and distributor S reach the monopoly equilibrium as described in Section 3. This equilibrium can be represented by a point in between A and B in Figure 7. Note that the investment by city N that corresponds to point B, denoted by I_N^B in Figure 7, is the one that will yield a service cost to distributor N equal to the market price of the good, $C_N = P$. With this service cost and market price, distributor N will not want to provide any distribution service.

Suppose now that city N invests in infrastructure. What is the new equilibrium? The reaction curve of city S is ABEF. Since city S develops earlier, it has the first mover advantage. Note that AB is a horizontal line, showing

[17] Point B gives the value of I_N that gives a service cost $C_N = P$.

[18] In other words, we assume that $C_N(I_N^0) > P$.

the maximum strategic investment of city S if it takes city N's investment as given.

If city N decides to increase its infrastructure investment, we have to consider its reaction curve. Suppose that at a lower service cost, its reaction curve is represented by KLMECD as shown in the diagram. The intersecting point between the two reaction curves, point E, gives the Nash equilibrium. To reach this point, city S has to invest $I_S^E < I_S^0$, yielding a welfare of $\bar{V}_S$ to city S. Since infrastructure investment considered here is an accumulated investment, it makes no sense to destroy some of the investment made previously. So city S will more likely keep the existing investment level. Taking this as given, city N will react with an investment of I_N^M, with the equilibrium given by point M. City S will get a welfare level higher than $\bar{V}_S$.

City S, however, can in general do better than point M. In Figure 7, one of its iso-welfare contour, denoted by $\hat{V}_S$, touches city N's reaction curve at L. This is the Stackelberg point, with city S acting as the leader and city N as the follower. Since city S has the first mover advantage, it can increase its investment level to I_S^L at the time when city N is about to enter the market. Reacting to this level, city N will choose the investment level I_N^L. Thus city S successfully achieves the welfare level $\hat{V}_S$, which is higher than $\bar{V}_S$.

It is often said that a Stackelberg equilibrium is not a likely outcome in an oligopoly market because it is difficult for agents to make credible threats. In this case, the threat made by city S is credible because the increase in investment is not reversible.

In the above case with the Stackelberg equilibrium at L, city N enters the market, makes some investment, and captures part of the market. Other cases can be considered. Consider the case in which an iso-welfare contour of city S touches N's reaction curve at point K or higher (instead of point L). Starting from the monopoly case and anticipating the entrance of city N, city S makes a preemptive move, increasing its investment to I_S^K. In this case, city N will have no incentive to invest because it does not make sense to make distributor N competitive, recalling that with its investment corresponding to point B or K, distributor N's service cost is at least as high as the market price, P. This result is summarized in the following proposition:

Proposition 5 (Limit Investing): If city S has the first mover advantage, it can increase its investment to a level at which city N responds with zero investment. There are cases in which city S will choose to increase its investment to a level to discourage city N from helping distributor N to enter the market.

9 Concluding Remarks

We have modelled the rivalry between two cities, focussing on infrastructure investment. We have shown that at the firm level, equilibrium may involve limit pricing by a distributor. At the city level, preemptive investment by the city that exists first is possible. Such preemptive investment could discourage the latecomer from entering the market.

There are a few issues that could be taken up in future research. Firstly, what is the optimal cooperative outcome, where the cooperation may be between the two cities, at the expense of the production firms in the long narrow strip that join them? One can also involve the welfare of the owners of these firms as well. Secondly, cities may compete in more than one dimension. When the model is extended to allow for this, is it possible that the non-cooperative equilibrium may maximize, rather than minimize, the difference between the two cities? Could it be the case that one city becomes a financial centre while the other becomes a goods distribution centre?

Appendix

Proof of Proposition 1: The proof of part (a) is simple and is omitted. To prove (b), write the first-order condition (17) as

$$G(P, I_S) \equiv \pi'_{SM}(C_S(I_S))C'_S(I_S) - (1+\beta) = 0. \tag{42}$$

The second-order condition is

$$\frac{\partial G}{\partial I_S} = \left(C'_S\right)^2 \pi''_{SM} + \pi'_{SM} C''_S,$$

is assumed to be negative, i.e.,

$$-\frac{\pi'_{SM} C''_S}{C'_S \pi''_{SM}} < C'_S,$$

or in Figure 2, curve CI is less steep than the GB curve. The response of I_S to an increase in P is obtained from (42):

$$\frac{\mathrm{d}I_S}{\mathrm{d}P} = -\frac{\partial G/\partial P}{\partial G/\partial I_S} = \frac{\pi''_{SM} C'_S}{\left(C'_S\right)^2 \pi''_{SM} + \pi'_{SM} C''_S} > 0. \tag{43}$$

From (13), (14) and (43),

$$\begin{aligned} \frac{\mathrm{d}x^{m*}}{\mathrm{d}P} &= \sqrt{\frac{3b}{8(P - C_S)}} \left(1 - C'_S \frac{\mathrm{d}I_S}{\mathrm{d}P}\right) > 0 \\ \frac{\mathrm{d}\theta^*_S}{\mathrm{d}P} &= \frac{2}{3} + \frac{1}{3} C'_S \frac{\mathrm{d}I_S}{\mathrm{d}P}. \end{aligned}$$

The sign of $\mathrm{d}\theta^*_S/\mathrm{d}P$ is ambiguous. ■

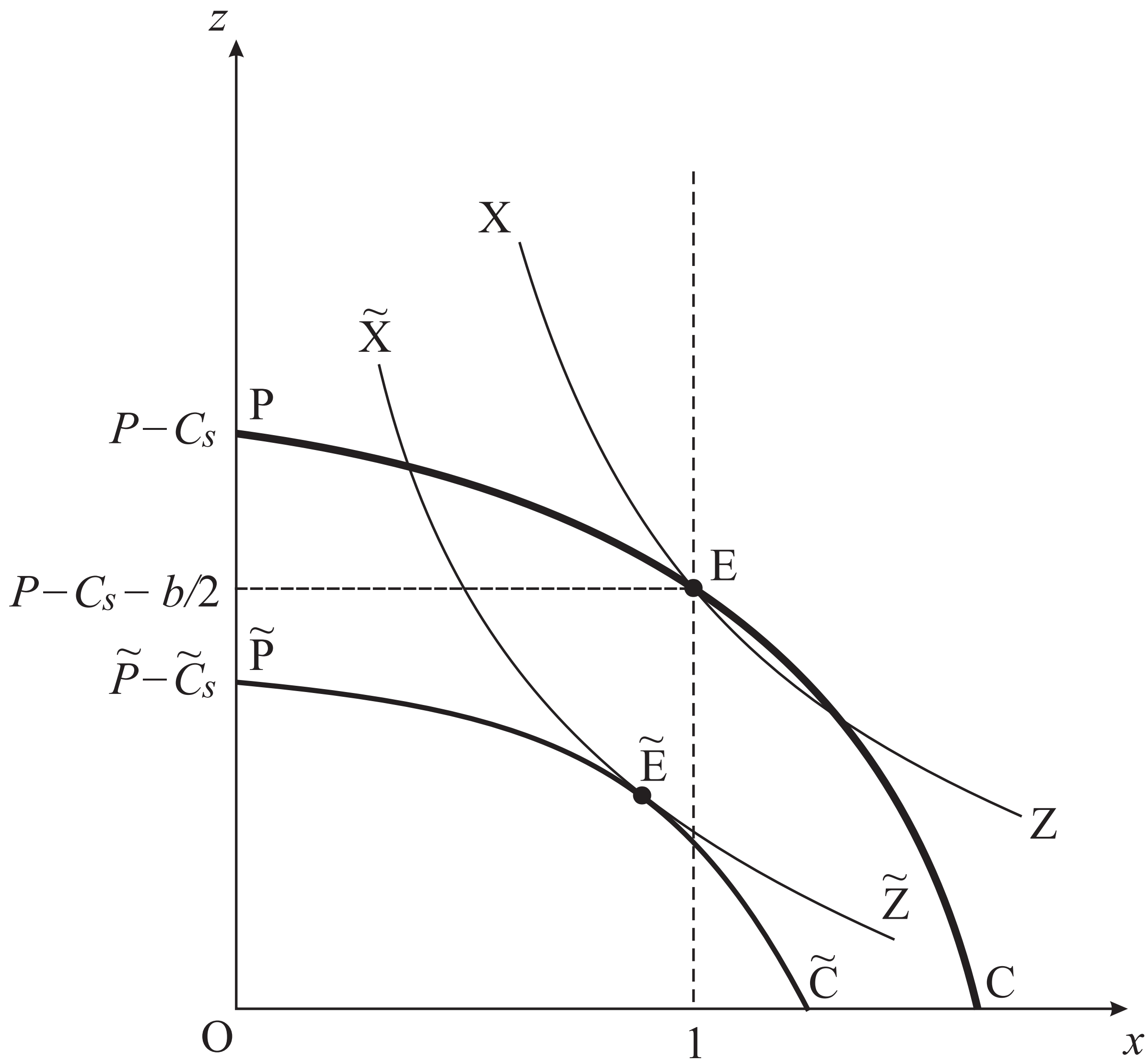

Figure 1

Monopoly: Optimal Market Size

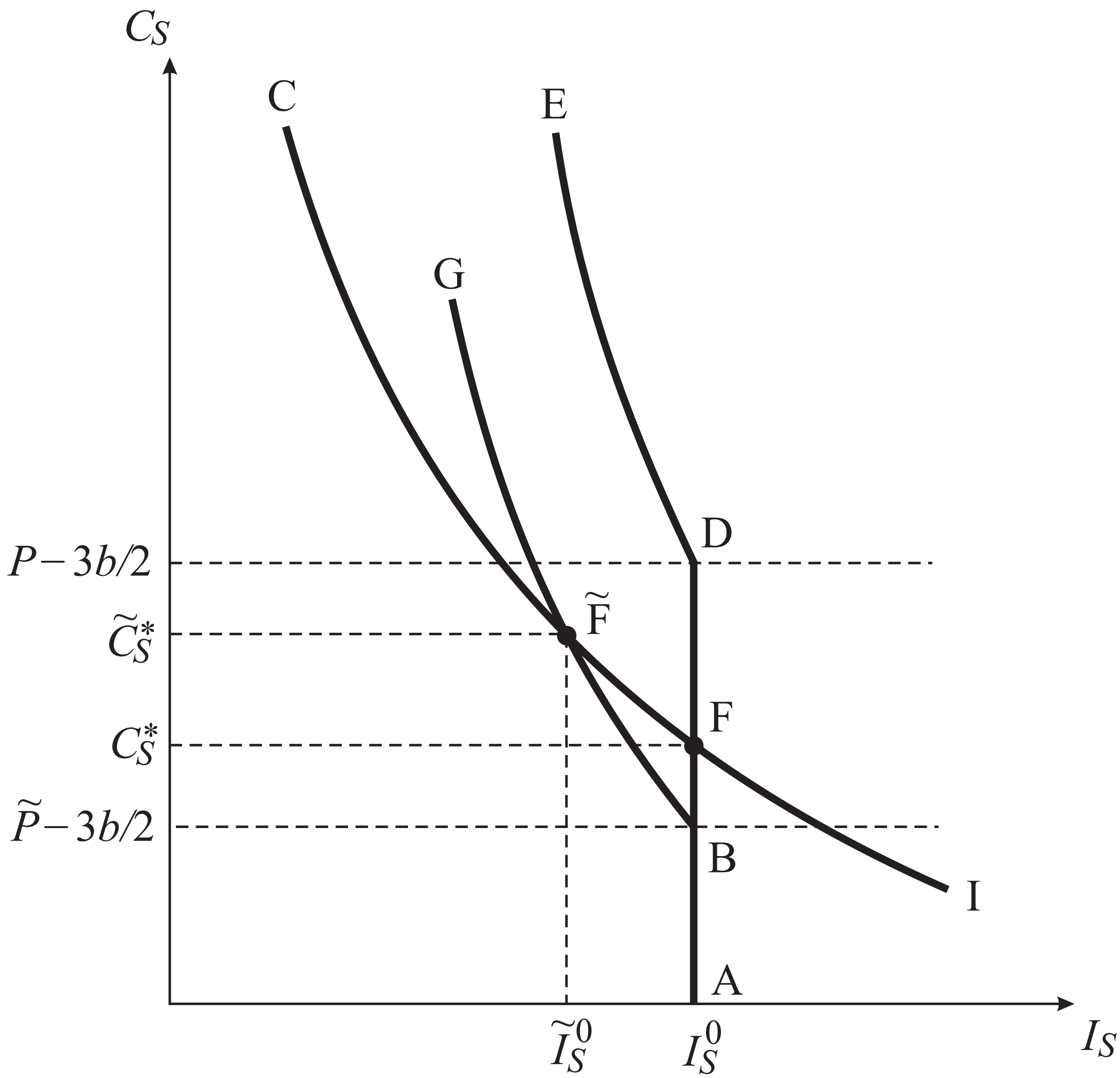

Figure 2

Monopoly: Optimal Investment

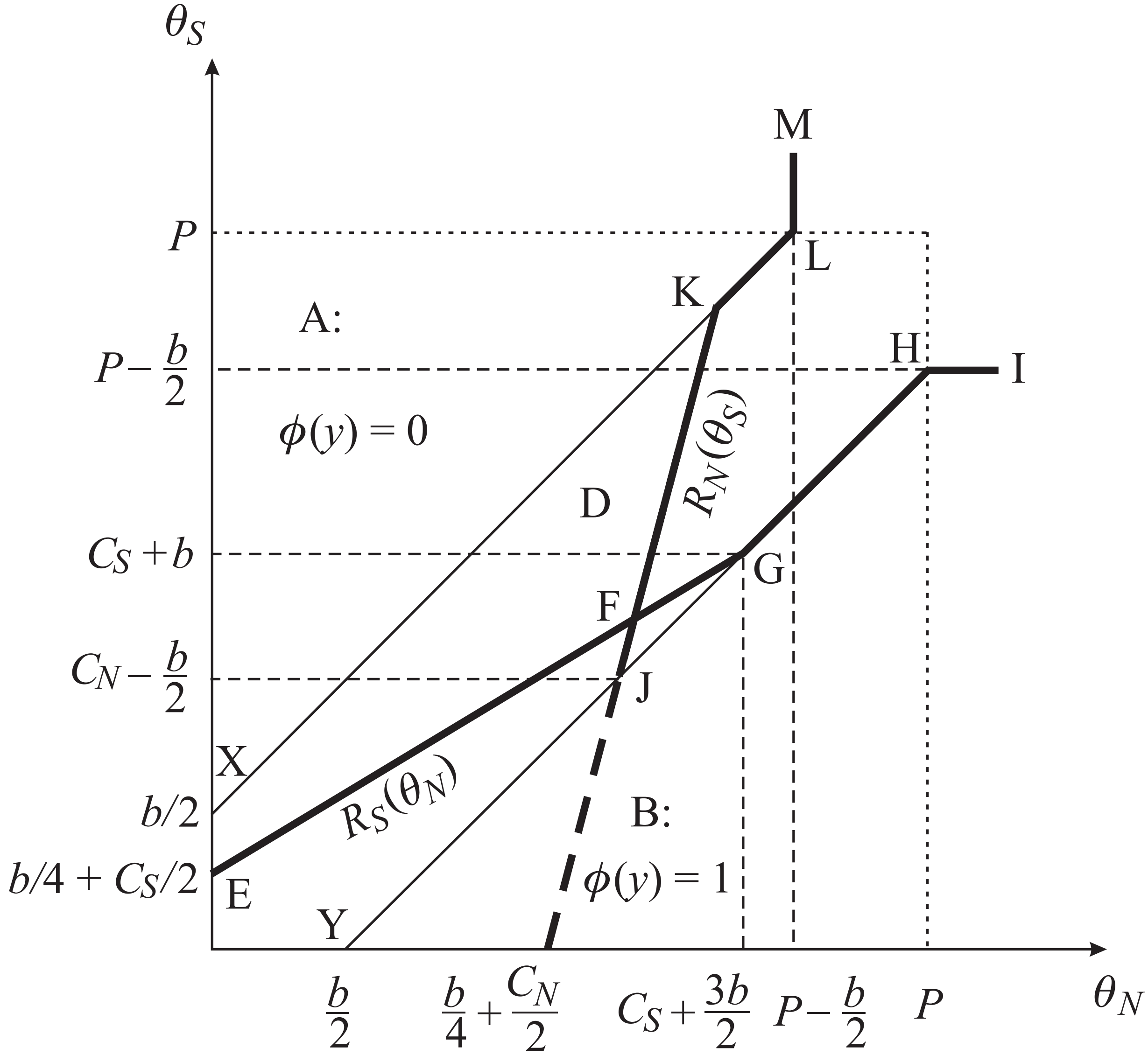

Figure 3

Reaction Functions and Nash Equilibrium

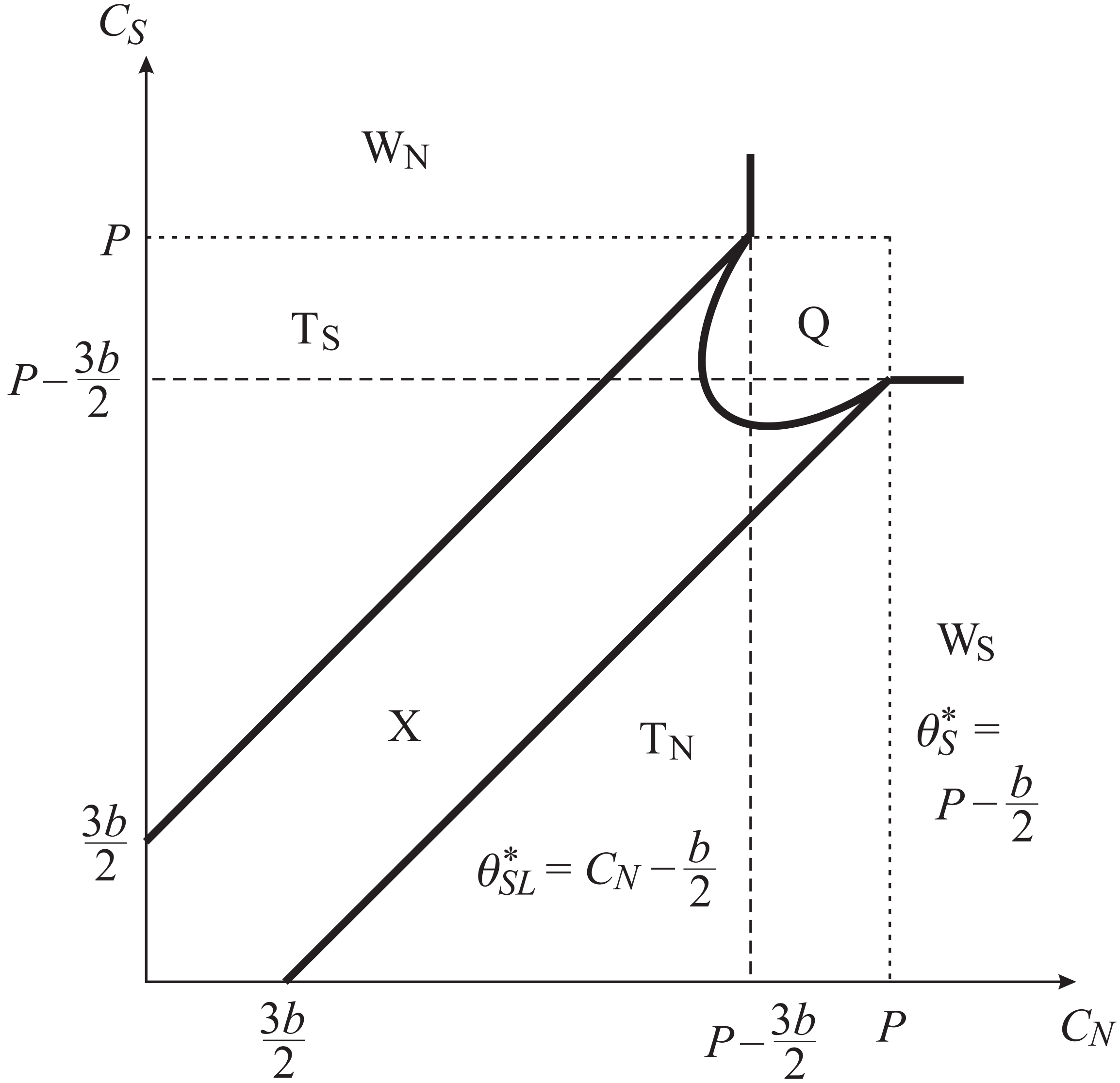

Figure 4

Nash Equilibrium and Cost Structures

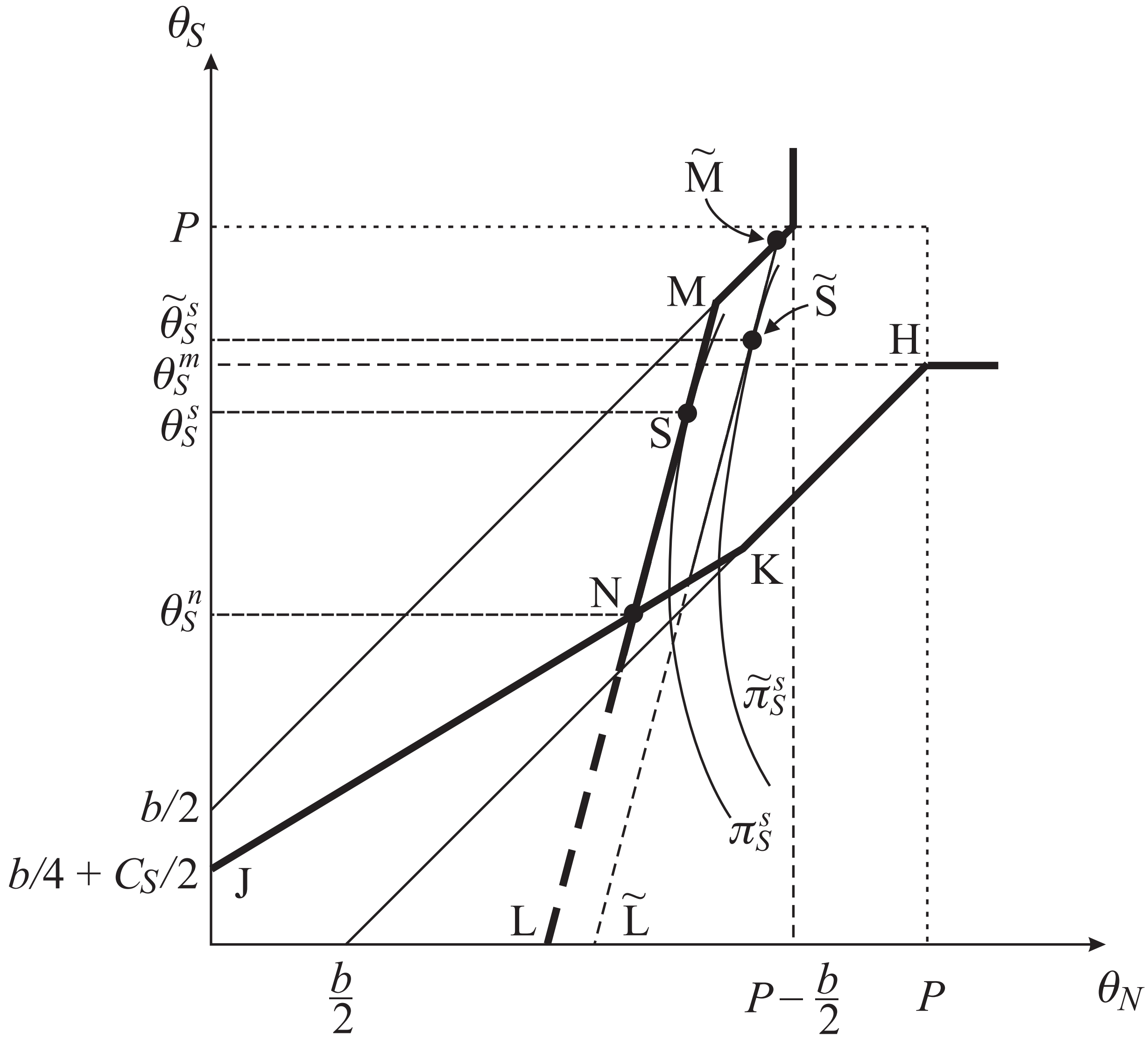

Figure 5

Distributor S as a Stackelberg Leader

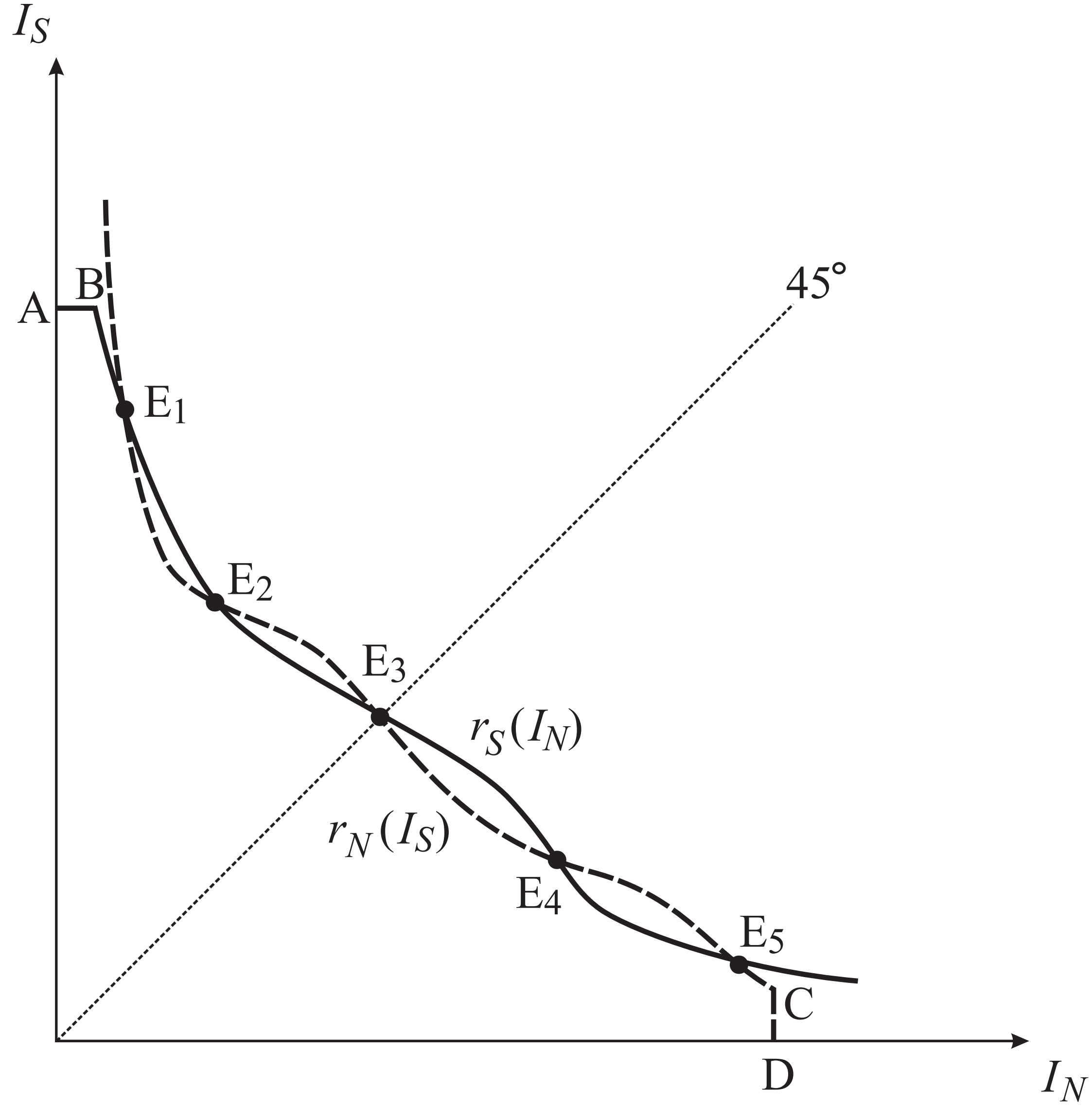

Figure 6

Nash Equilibria for Government Policies

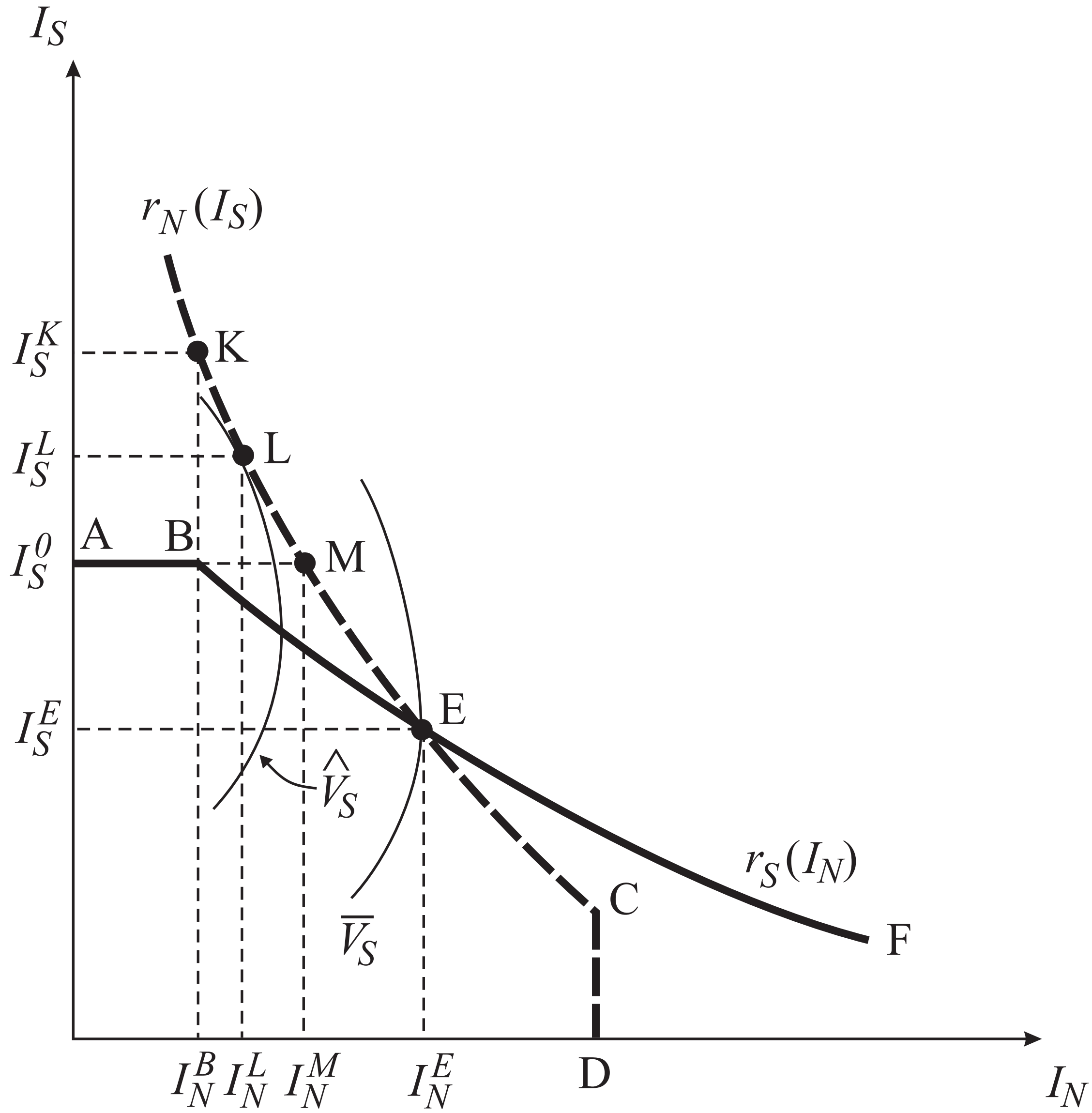

Figure 7

Port S as a Stackelberg Leader

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