Fundamental Trade Theorems under External Economies of Scale

Kar-yiu Wong[1]
University of Washington

July 31, 2000

[1]Department of Economics, Box 353330, University of Washington, Seattle, WA 98195-3330, U. S. A.; Tel. 1-206-685-1859; Fax: 1-206-685-7477; e-mail: karyiu@u.washington.edu; http://faculty.washington.edu/karyiu/

Abstract

This paper examines the validity of the five fundamental theorems in the positive theory of international trade in a basic model of external economies of scale. It shows that some of these theorems are valid in more cases than the literature suggests. In particular, if global changes under the specified adjustment mechanism are allowed, the Rybczynski and Stolper-Samuelson Theorems are always valid, whether or not a production equilibrium is stable. Modified forms of the Law of Comparative Advantage and the Heckscher-Ohlin Theorem in the presence of externality are derived. Conditions under which the Factor Price Equalization Theorem is valid will be derived.

Thanks are due to Koji Shimomura for helpful comments. Any remaining errors and shortcomings are mine.

1 Introduction

The positive theory of international trade based on the neoclassical framework is characterized by the following five fundamental theorems:

1. The Rybczynski Theorem;
2. The Stolper-Samuelson Theorem;
3. The Law of Comparative Advantage;
4. The Heckscher-Ohlin Theorem; and
5. The Factor Price Equalization Theorem.

All these theorems are proved in the neoclassical framework, which is characterized by some strong assumptions about the technologies and preferences of the countries. Among them are perfect competition and constant returns in all sectors.

Economists have long recognized the existence of externality and increasing returns in economies. Marshall (1879, 1890) is among the first that provides formal analysis of the implications of externality. One common approach used in the literature is to allow economies of scale that are external to firms. This approach is convenient and popular because it provides one way to include economies of scale without having to sacrifice the assumption of perfect competition.

Much work has been done on examining the features of economies and international trade in the presence of external economies of scale. While many results have been obtained, there has been limited success in evaluating the validity of all of the above theorems for economies with external economies of scale. Part of the reason is that model with externality gets complicated and most results are mixed, being sensitive to the specifications of the model. As a result, papers generally concentrate on some special cases such as the existence of only one factor, or the case in which there is one country uniformly bigger than the other one. However, the assumption of only one factor or both countries with the same capital-labor ratio in general is not suitable for analyzing these theorems. Furthermore, some of the results derived using these assumptions cannot be generalized to higher-dimensional frameworks.

The purpose of this paper is to examine the validity of these five theorems in a basic model of externality. The basic model is so constructed so that it is similar to a neoclassical framework with one exception: There are two countries, two factors and two sectors. Markets are competitive, and countries have identical technologies

and preferences although they may have different factor endowments. One of the sectors is subject to the traditional constant returns technologies while the other sector is subject to economies of scale. The existence of economies of scale is what separates the present basic model from the neoclassical framework. By making use of the present model, we can investigate how economies of scale may affect the validity of these five theorems of international trade.

Existing work on economies of scale in the trade literature usually considers comparative static properties with marginal changes. This paper takes a broader approach in which both marginal and finite changes are considered. This approach allows us to derive stronger results concerning the validity of these theorems.

This paper also examines some other related trade results. One interesting issue is about the effects of economies of scale on the existence of trade. Existing work usually suggests that economies of scale are a determinant of trade. This paper, however, argues that economies of scale are not sufficient for foreign trade. Three forces that affect foreign trade can be identified: comparative-advantage, scale-economies, and factor-price. Models with economies of scale, which contribute to trade, have the factor-price effect, which works against trade, as long as there are two or more factors. After deriving several important properties of an open economy subject to increasing returns, this paper shows that at least weaker versions of the Law of Comparative Advantage, the Heckscher-Ohlin and Factor Price Equalization Theorems can be stated.

In Section 2, we introduce a basic one-country model with two factors and two sectors. We examine several properties of the model, including the validity of the Rybczynski and Stolper-Samuelon Theorems. In Section 3, we derive the autarkic equilibrium of a closed economy, and investigate how the autarkic equilibrium may be affected by changes in factor endowments. Section 4 analyzes an open economy. In particular, it derives the offer curve of an open economy. Section 5 focuses on free trade between two economies. The stability conditions of a trade equilibrium are established. It is also shown that the Factor Price Equalization Theorem holds under certain conditions. In Section 6, we investigate whether increasing returns are sufficient for international trade. Section 7 establishes the Law of Comparative Advantage in the presence of external economies of scale. The relationship between factor endowments and patterns of trade is derived in Section 8, while Section 9 examines the relationship between commodity prices and factor prices under free trade. The last section concludes.

2 A Basic Model

This section introduces a basic model of external economies of scale and investigates its properties. For the time being, we focus on the home country. The economy is endowed with capital and labor of exogenously given amounts, K and L, respectively, which are used to produce two homogeneous goods labeled 1 and 2. The technologies of the sectors can be described by the following production functions:

$$Q_1 = h_1(Q_1)F_1(K_1, L_1) \quad (1)$$
$$Q_2 = h_2 F_2(K_2, L_2), \quad (2)$$

where Q_i is the output of good i, $i = 1, 2$, and K_i and L_i are respectively the capital and labor inputs in sector i. Function $F_i(K_i, L_i)$ is increasing, linearly homogeneous, concave and differentiable in factor inputs. Function $h_1(Q_1)$ and h_2 are regarded as constant by all firms, just like technology indices. In (2), h_2 is truly a constant, but in (1) $h_1(Q_1)$ depends on the sectoral output, while each firm takes as given, hence the source of externality.[1] Function $h_1(Q_1)$ satisfies the following conditions: $h_1 = h_1(Q_1) > 0$ for all $Q_1 > 0$, $h_1(0) = 0$, and $h_{11}(Q_1) \equiv \mathrm{d}h_1/\mathrm{d}Q_1 > 0$. Note that the sign of h_{11} means that this paper focuses on the case of external increasing returns.[2] However, for some of the results, we will also discuss the implications if sector 1 is subject to decreasing returns. For sector 2, however, h_2 is a constant, meaning that the sector exhibits constant returns. By the choice of unit, h_2 is set to unity. The rest of the economy is characterized by the neoclassical features, including perfect sectoral factor mobility and perfect price flexibility. We add the further assumption that sector 1 is capital intensive at all factor prices.[3]

The basic model described above is very useful in the present analysis. On the one hand, except for one feature, it is the same as the neoclassical framework: the feature being that sector 1 is subject to external economies of scale. The model can thus be most useful to determine how the presence of externality may affect the theory of international trade. For example, it is interesting to find out whether existence of

[1] For some fundamental concepts about externality and its use in the theory of international trade, see Wong (2000a).

[2] Kemp and Shimomura (1995) suggest another approach in which all firms are identical and internalize fully the impacts of an increase in its inputs on the aggregate output. In this case, the externality disappears. Under their approach, perfect competition will also disappear and the firms act jointly as a monopoly.

[3] See Wong (2000a) for an analysis of international trade using a more general model, and Wong (2000b) for an extension of the present model with international trade in goods and capital movement.

increasing returns is a factor of international trade. On the other hand, the present model has two factors, and thus can be used to examine how factor endowments may affect international trade. Moreover, it reduces to the one-factor models commonly used in the literature. We can use the model to find out why one-factor models are so special.

Define the rate of variable returns to scale (VRS) of sector 1, for all $Q_1 > 0$, by $\varepsilon_{11} \equiv Q_1 h_{11}(Q_1)/h_1(Q_1)$, which is of the same sign as $h_{11}(Q_1)$.[4] In other words, sector 1 is subject to external increasing returns if and only if ε_{11} is positive. To get positive social marginal products of factors in sector 1, it is assumed that $\varepsilon_{11} < 1$. Denote the supply price of good i by p_i^s.[5] Choosing good 2 as the numeraire, $p_2^s = 1$.

We employ the approach of virtual system introduced in Wong (1995). Define the virtual output of sector i by

$$\tilde{Q}_i = F_i(K_i, L_i). \tag{3}$$

A comparison of equation (3) with the production functions (1) and (2) reveals that $\tilde{Q}_i = Q_i/h_i$. Since function $F_i(K_i, L_i)$ has the properties of a neoclassical production function, the virtual system behaves like the neoclassical framework.[6] Define the virtual price as $\tilde{p}_i^s \equiv h_i p_i^s$, $i = 1, 2$. Since $h_2 = 1$, $\tilde{p}_2^s = 1$. Let us write $p_1^s = p^s$ and $\tilde{p}_1^s = \tilde{p}^s$. We can also define the virtual GDP (gross domestic function) function, $g(\tilde{p}^s, K, L)$, where K and L are the given capital and labor endowments in the economy. This function behaves like the neoclassical GDP function; in particular, its derivatives with respect to the virtual prices represent the virtual outputs, $\tilde{Q}_i(\tilde{p}^s, K, L)$. Using the definition of $\tilde{Q}_i$, we have

$$Q_1 = h_1(Q_1)\tilde{Q}_1(\tilde{p}^s, K, L) \tag{4}$$

$$Q_2 = \tilde{Q}_2(\tilde{p}^s, K, L). \tag{5}$$

Equations (4) and (5) give the link between the virtual and real systems. Differentiate

[4] Equation (1) can be inverted to give a reduced-form production function $Q_1 = H_1(K_1, L_1)$, which is homothetic. See Wong (2000a) for the proof and more discussion.

[5] The supply price of good 1 is defined as the minimum price of the good that will make the profits of the firms in the sector 1 non-negative.

[6] Recall that the present framework is the same as the neoclassical framework except that sector 1 is subject to external economies of scale. The definition of the virtual outputs is to eliminate the economies-of-scale effect.

these two equations, using the definitions of $\tilde{p}_i^s$, and rearrange the terms to yield:

$$\begin{bmatrix} \alpha_{11} & 0 \\ \alpha_{21} & 1 \end{bmatrix} \begin{bmatrix} \mathrm{d}Q_1 \\ \mathrm{d}Q_2 \end{bmatrix} = \begin{bmatrix} h_1^2 \tilde{Q}_{1p} \\ h_1 \tilde{Q}_{2p} \end{bmatrix} \mathrm{d}p^s + \begin{bmatrix} h_1 \tilde{Q}_{1K} \\ \tilde{Q}_{2K} \end{bmatrix} \mathrm{d}K + \begin{bmatrix} h_1 \tilde{Q}_{1L} \\ \tilde{Q}_{2L} \end{bmatrix} \mathrm{d}L, \tag{6}$$

where $\alpha_{11} = 1 - \varepsilon_{11} - \varepsilon_{11}\eta_{1p}$, $\alpha_{21} = -\eta_{2p}\varepsilon_{11} > 0$, $\eta_{1p} \equiv \tilde{p}^s \tilde{Q}_{1p}/\tilde{Q}_1$, and $\tilde{Q}_{ip} \equiv \partial \tilde{Q}_i/\partial \tilde{p}^s$. Note that η_{1p} is the price elasticity of the virtual supply of good 1; similarly, $\eta_{2p} = \tilde{p}^s \tilde{Q}_{2p}/\tilde{Q}_2$. Assuming a strictly convex virtual production possibility frontier, we have $\tilde{Q}_{1p}, \eta_{1p} > 0$ and $\tilde{Q}_{2p}, \eta_{2p} < 0$. Consider the following condition:

Condition E. $\varepsilon_{11} < 1/(1 + \eta_{1p})$.

Lemma 1. If sector 1 is subject to mild increasing returns so that condition E holds, or if sector 1 is subject to decreasing return, then $\Phi > 0$.

Define η_{ij}, $i = 1, 2$ and $j = K, L$, as the elasticity of the virtual output of good i with respect to the endowment of factor j, while prices are kept constant; for example, $\eta_{1K} = K\tilde{Q}_{1K}/\tilde{Q}_1$. For the virtual system, the Rybczynski Theorem implies that $\eta_{1K}, \eta_{2L} > 1$ and $\eta_{1L}, \eta_{2K} < 0$. Condition (6) is solved for output changes:

$$\hat{Q}_1 = \frac{\eta_{1p}}{\Phi}\hat{p}^s + \frac{\eta_{1K}}{\Phi}\hat{K} + \frac{\eta_{1L}}{\Phi}\hat{L} \tag{7}$$

$$\hat{Q}_2 = -\frac{(1-\varepsilon_{11})\eta_{2p}}{\Phi}\hat{p}^s + \left[\eta_{2K} - \frac{\varepsilon_{11}\eta_{2p}\eta_{1K}}{\Phi}\right]\hat{K} + \left[\eta_{2L} - \frac{\varepsilon_{11}\eta_{2p}\eta_{1L}}{\Phi}\right]\hat{L}, \tag{8}$$

where $\Phi = \alpha_{11}$. In many cases, the present model can be analyzed more conveniently in terms of output ratio, which is defined as $z \equiv Q_1/Q_2$. Let us use a "hat" to denote the proportional change of a variable; for example, $\hat{z} \equiv \mathrm{d}z/z$. Conditions (7) and (8) can be combined to give

$$\hat{z} = \frac{\mu}{\Phi}\hat{p}^s + \frac{\sigma}{\Phi}\hat{K} + \frac{\zeta}{\Phi}\hat{L}, \tag{9}$$

where

$$\begin{aligned}
\mu &= \eta_{1p} - (1-\varepsilon_{11})\eta_{2p} > 0 \\
\sigma &= \eta_{1K}(1-\varepsilon_{11}\eta_{2p}) - \eta_{2K}\Phi \\
\zeta &= \eta_{1L}(1-\varepsilon_{11}\eta_{2p}) - \eta_{2L}\Phi \\
\sigma+\zeta &= (\eta_{1K}+\eta_{1L})(1-\varepsilon_{11}\eta_{2p}) - (\eta_{2K}+\eta_{2L})\Phi \\
&= \varepsilon_{11}\left(1+\frac{g\eta_{1p}}{Q_2}\right) > 0.
\end{aligned}$$

Lemma 2. (a) We have $\sigma > 0$. (b) If condition E is satisfied, then $\zeta < 0$.

Proof. (a) Expand the definition of σ to give

$$\begin{aligned}
\sigma &= \eta_{1K} - \eta_{2K}(1-\varepsilon_{11}) + \varepsilon_{11}(\eta_{2K}\eta_{1p} - \eta_{1K}\eta_{2p}) \\
&= \eta_{1K} - \eta_{2K}(1-\varepsilon_{11}) + \frac{\varepsilon_{11}pK}{Q_1Q_2}\left(\frac{\partial Q_2}{\partial K}\frac{\partial Q_1}{\partial \tilde{p}^s} - \frac{\partial Q_1}{\partial K}\frac{\partial Q_2}{\partial \tilde{p}^s}\right). \qquad (10)
\end{aligned}$$

Use subscripts to denote partial derivatives for the virtual GDP function; for example, $g_{1L} = \partial^2 g/\partial\tilde{p}^s\partial L$. Then we have

$$\frac{\partial Q_2}{\partial K}\frac{\partial Q_1}{\partial \tilde{p}^s} - \frac{\partial Q_1}{\partial K}\frac{\partial Q_2}{\partial \tilde{p}^s} = g_{2K}g_{1p} - g_{1K}g_{2p}. \qquad (11)$$

Since the output function Q_1 is homogeneous of degree zero in commodity prices, we have $0 = pg_{1p} + g_{2p}$, where by Young's theorem, g_{2p} is equal to the differentiation of Q_1 with respect to the virtual price of good 2. Similarly, $g_K = r$, which is linearly homogeneous in commodity prices, implying that $r = pg_{K1} + g_{K2} = pg_{1K} + g_{2K}$. Substitute these two conditions into (11) and then the result into (10). We have

$$\sigma = \eta_{1K} - \eta_{2K}(1-\varepsilon_{11}) + \frac{\varepsilon_{11}\eta_{1p}rK}{Q_2} > 0. \qquad (12)$$

(b) This part of the proposition follows immediately condition E and Lemma 1. ■

Note that the sign of σ does not depend on condition E. An alternative, but sometimes useful, formulation of (9) is

$$\hat{z} = \frac{\mu}{\Phi}\hat{p}^s + \frac{\sigma+\zeta}{\Phi}\hat{K} - \frac{\zeta}{\Phi}(\hat{K}-\hat{L}). \qquad (13)$$

For an economy with fixed endowments, the supply price elasticity of z is equal to μ/Φ.

2.1 Price-Output Response

Condition (9) or (13) can be used to derive several important properties of the present model. We first consider the response of outputs to changes in prices. We say that the output-price response is normal (or perverse) if an increase in the price supply p^s induces an increase (or decrease) in the output ratio z. Note that we need to distinguish between small changes (local responses) and finite changes (global responses) of the variables.

Condition H. Function $h_1(Q_1)$ is (a) bounded from above when Q_1 approaches zero; and (b) bounded from below when Q_2 approaches zero (or when Q_1 approaches its maximum value).

There are cases in which function $h_1(Q_1)$ satisfies condition H; for example, $h_1(Q_1) = Q_1^a$, where $a \in (0,1)$, which may or may not be a constant, or $h_1(Q_1)$ is a polynomial function in Q_1. We assume condition H in the present paper.

Lemma 3. Given condition H, z approaches zero as p^s does, and is sufficiently large when p^s is.

Proof. Because the virtual system behaves like a neoclassical system, $\tilde{Q}_1$ approaches zero when $\tilde{p}^s$ is sufficiently small. Given condition H(a), $\tilde{p}^s$ approaches zero when p^s does, and Q_1 and z approach zero when $\tilde{Q}_1$ does. Combining these results, we conclude that Q_1 and z approach zero when p^s does. On the other hand, given condition H(b), $\tilde{p}^s$ is sufficiently large when p^s is. The corresponding virtual and real outputs of good 2 are sufficiently small, i.e., z is sufficiently large. ■

Figure 1 shows a possible supply price schedule KLMN. Given condition H, p^s is small when z is small, but is large when z is large, which means that at least part of the schedule is positively sloped. The figure shows the case in which the supply price schedule is partly positively sloped (segments KL and MN) and partly negatively sloped (segment LM). The slope of the supply price schedule is equal to

$$\frac{\mathrm{d}p^s}{\mathrm{d}z} = \frac{p^s}{z}\frac{\Phi}{\mu},$$

which is positive if and only if $\Phi > 0$. By condition (13), the *local* price-output response in a small neighborhood around a production equilibrium is normal if and only if $\Phi > 0$. Therefore the slope of the supply price schedule is positively sloped if and only if the local price-output response is normal.

To see the above point more clearly, suppose that the initial relative price is p^1, as shown in Figure 1. The three points, A, B, and C, at which a horizontal line through p^1 cuts the supply price schedule, show three possible output ratios, with points A and C on a positively sloped segment and point B on a negatively sloped segment. Let there be a small rise in the relative price of good 1 to p', shifting the price line up. The three points of intersection move to A$'$, B$'$, and C$'$, as shown in the diagram. If we compare a point of intersection between the initial price ration and schedule p^s with the new one in the same neighborhood, for example, point A compared with A$'$, B with B$'$, or C with C$'$, then we can conclude that the local price-output response is normal if the initial point is either A or C, but it is perverse if the initial point is B. If, however, the initial price is p^2, which cuts the supply price schedule once, then the schedule must be positively sloped at the point of intersection and so the local price-output response must be normal.

By the Correspondence Principle (Samuelson, 1947), there is a correspondence between comparative statics and stability of an equilibrium. We now make use of this principle to provide an alternative point of view. To introduce stability, we follow Ide and Takayama (1991, 1993) and assume that when the firms are facing a given price ratio $\bar{p}$ (as in the case of a small open economy facing a given world price ratio), the output ratio adjusts according to the following condition:

$$\dot{z} = \beta(\bar{p} - p^s) = \phi(z), \tag{14}$$

where $\beta > 0$ is a constant, and the dependence of p^s on the output ratio is given by (13). Condition (14) implies that firms in sector 1 will have incentives to increase their outputs if the prevailing relative price of good 1 is higher than the supply price of good 1. Differentiate both sides of (14) to give:

$$\mathrm{d}\dot{z} = -\frac{\beta p \Phi}{z\mu}\mathrm{d}z = \phi'(z)\mathrm{d}z. \tag{15}$$

Local stability requires that in the small neighborhood around a production equilibrium $\phi'(z) < 0$, or that $\Phi > 0$, or that the supply price schedule be positively sloped. Note that such type of stability is sometimes called Marshallian stability because it is based on output adjustment. In Figure 1, it is then said that production equilibria A and C are locally stable while B is locally unstable. This corresponds to the result that points A and C give normal price-output responses (shifting to A$'$ and C$'$, respectively, after an increase in p^s), while point B gives a perverse response (shifting to B$'$). The above results are summarized as follows:

Lemma 4. The following statements are equivalent: (a) $\Phi > 0$, evaluated at a production equilibrium. (b) The local price-output response is normal. (c) The

supply price schedule is locally positively sloped. (d) A production point is locally Marshallian stable.

Note that the statements in Lemma 4 describe the properties of the model in a small neighborhood around a production equilibrium. By the lemma, it is often argued that if a production equilibrium is not stable, then the price-output response is perverse and normal comparative static results will not be obtained. However, such pessimism is very often misplaced because unstable equilibria are nearly always not observed. Samuelson (1971) further argues that if finite changes are considered, an equilibrium that is unstable locally can become stable for a finite change. He calls this the Global Correspondence Principle. This principle can be applied here. Suppose that point B is initial equilibrium point. After an exogenous rise, the price ratio is higher than the supply price ratio, and according to the adjustment rule in (14), z will increase. This process will continue until z reaches point C$'$, instead of decreasing to B$'$. As a result, the output response is normal, not perverse, despite the fact that the supply price schedule is negatively sloped at B. We summarize the above results in the following proposition:

Proposition 1 *The price-output response is locally normal if and only if* $\Phi > 0$. *If finite changes are considered with the adjustment rule (14), the price-output response is always normal, irrespective to the sign of* Φ.

2.2 Effects of Changes in Factor Endowments

Suppose that there is an increase in the endowment of one of the factors while the commodity prices are fixed (as in the case of a small open economy). The effects on the outputs of the sectors are given by conditions (7) and (8). In particular, if $\Phi > 0$, then an increase in a factor endowment will increase the output of the sector that uses the factor intensively and decrease the other output. Since sector 1 is subject to increasing returns, if $\Phi > 0$, then Φ is less than unity. Noting that η_{1K}, $\eta_{2L} > 1$, we can conclude that an increase in a factor endowment will in fact increase, by a greater proportion, the output of the sector that uses the factor intensively. This is the Rybczynski Theorem. Furthermore, if the economy increases in size in an uniform way with no change in prices, i.e., $\hat{K} = \hat{L} > 0$, then by condition (9), there is an increase in the output ratio. This effect, which is called the scale effect, is absent in the neoclassical framework because an increase in the size of the economy under constant prices will increase both outputs by the same proportion.

Conditions (7) and (8) show that the Rybczynsk Theorem holds under certain conditions. What happens if these conditions are not satisfied? First, suppose that

sector 1 is subject to decreasing returns, then $\varepsilon_{11} < 0$. This implies that $\Phi > 1$, meaning that the output of a sector will increase, but not necessarily by a greater proportion, if the economy is endowed with more of the factor used intensively in the sector although it will decrease if the other factor endowment increases. This means that part of the Rybczynski Theorem holds.

The validity of the Rybcyznski Theorem requires that $\Phi > 0$. Of course, we have to ask, what happens if $\Phi < 0$? Similarly questions have been asked in the literature: Since the Rbycyznski Theorem in the presence of external economies of scale depends on certain conditions, what happens if these conditions are not satisfied.[7] The usual reaction is a pessimistic one. Mayer (1974), Chang (1981), and Ide and Takayama (1991) argue that if the production equilibrium is (locally) Marshallian stable and if Jones's (1968) assumptions concerning the demands for factors are made, then outputs respond normally (and locally) to prices and the Rybczynski theorem holds (in terms of changes in outputs).[8] However, all of these papers are concentrating on marginal changes. As the previous subsection argues, a distinction should be made between marginal and finite changes. In the real world, especially if we are trying to compare two countries, finite changes are far more relevant and important. A perverse comparative static result for small changes may become normal if finitie changes are considered. For this reason, let us broaden the analysis as follows.

In Figure 2 we show a possible supply price schedule KLMN and the given supply price $\bar{p}$, where the price line cuts the schedule at points A, B, and C. As analyzed, $\Phi > 0$ at points A and C, but $\Phi < 0$ at point B. To examine finite output changes, suppose that there is a small increase in the capital endowment, $\Delta K > 0$. Condition (9) implies that the supply price schedule shifts to K′LB′MN′ (the dotted curve), which cuts the given price line at points A′, B′, and C′.[9] It is clear from the diagram that if local output changes are considered, i.e., shifts from A to A′, B to B′, and C to C′, then the local capital-output response is normal if and only if the supply price schedule is positively sloped. In other words, the theorem, for local changes, holds for points A and C but not for B.

Suppose that the economy is indeed initially at point B. The new supply price schedule K′LB′MN′, which represents a higher capital endowment in the economy, shows that at the initial output ratio z^B, the supply price shifts down to $p^{s\prime}$, which is less than the prevailing price ratio $\bar{p}$. According to the adjustment rule (14), there

[7] See, for example, Jones (1968), Inoue (1981), and Tawada (1989).

[8] See also Kemp (1969), Panagariya (1980), Tawada (1989), Wong (1995, Chapter 5).

[9] A turning point of the supply price schedule remains unchanged. To see why, note from (9) that at a turning point, either μ approaches zero or Φ approaches infinity. The condition further implies that at a turning point and if z is held constant, a change in K will not change the price.

will be an increase in z. In fact, z will increase until it reaches $z^{C\prime}$. Furthermore, we already know that if the initial point is C, the percentage increase in the output of good 1 is greater than that of capital, meaning that for the movement from point B to C$'$ the percentage increase in good 1 is even greater.

The same analysis can be applied to analyze a small increase in the labor endowment under constant commodity prices. Thus we have the following proposition:

Proposition 2 *(**Rybczynski Theorem in the Presence of External Economies of Scale**) Given constant commodity prices, a small increase in a factor endowment will increase, by a greater proportion, the output of the sector that uses the factor intensively and decrease the other output if (a) $\Phi > 0$, and only local changes are considered; or (b) finite changes and the adjustment rule (14) are considered. If $\varepsilon_{11} < 0$, then $\Phi > 0$ and the theorem remains valid in a local sense except that the percentage increase in the output of a good may not be greater than the percentage increase in the factor which is used intensively in the sector.*

The above proposition is interesting for several reasons. First, external decreasing returns in one of the sectors does not violate the Rybczynski Theorem in a local sense, at least in terms of the sign of output changes. Second, for external increasing returns in one sector, if $\Phi > 0$, then the theorem is valid in a local sense even in terms of percentage changes. Third, if finite changes are considered, then the theorem is true whether or not the sector is subject to increasing or decreasing returns. The last point is especially interesting because the theorem is valid even if $\Phi < 0$.

2.3 Effects of Changes in Commodity Prices

We now turn to the relationship between commodity and factor prices. The first thing we need to find out is whether there is any difference between the factor prices in the real and virtual systems. Since the sectors are competitive, the factor prices can be determined by

$$w = p^s h_1(Q_1) F_{1L}(K_1, L_1) = F_{2L}(K_2, L_2) \tag{16}$$

$$r = p^s h_1(Q_1) F_{1K}(K_1, L_1) = F_{2K}(K_2, L_2), \tag{17}$$

where $h_1(Q_1)F_{1j}(K_1, L_1)$ is the private marginal product of factor j in sector 1, $j = K$, L, with $h_1(Q_1)$ taken as given by the firms. Using the definition of the virtual supply price, (16) and (17) can be written as

$$w = \tilde{p}^s F_{1L}(K_1, L_1) = F_{2L}(K_2, L_2) \tag{18}$$

$$r = \tilde{p}^s F_{1K}(K_1, L_1) = F_{2K}(K_2, L_2). \tag{19}$$

Conditions (18) and (19) are the same as the factor prices defined in the virtual system, with $F_i(K_i, L_i)$ treated as the production function of sector i. As the real and virtual systems have the same factor prices.

Let us for the time being focus on the virtual system. Under diversification the equilibrium virtual unit costs are related to the virtual and real prices in the following ways:

$$\tilde{c}_1(w,r) = \tilde{p}^s = h_1(Q_1)p^s \tag{20}$$

$$\tilde{c}_2(w,r) = 1, \tag{21}$$

where good 2 is the numeraire. To evaluate the effects of an increase in the supply price, differentiate both sides of (20) and (21) and rearrange terms to give

$$\begin{bmatrix} \varphi_{w1} & \varphi_{r1} \\ \varphi_{w2} & \varphi_{r2} \end{bmatrix} \begin{bmatrix} \hat{w} \\ \hat{r} \end{bmatrix} = \begin{bmatrix} (1-\varepsilon_{11})/\Phi \\ 0 \end{bmatrix} \hat{p}^s, \tag{22}$$

where $\varphi_{ji} > 0$ is the elasticity of virtual unit cost of sector i with respect to a change in factor price j, $i = 1, 2$, $j = w, r$; for example, $\varphi_{r1} \equiv (r/\tilde{c}_1)(\partial \tilde{c}_1/\partial r)$. Solving the above conditions, we get

$$\hat{w} = \frac{(1-\varepsilon_{11})\varphi_{r2}}{\Phi D}\hat{p}^s \tag{23}$$

$$\hat{r} = -\frac{(1-\varepsilon_{11})\varphi_{w2}}{\Phi D}\hat{p}^s, \tag{24}$$

where $D \equiv \varphi_{w1}\varphi_{r2} - \varphi_{w2}\varphi_{r1}$ is the determinant of the matrix in (22). Because sector 1 is capital intensive, $D < 0$. Thus a small increase in the supply price of good 1 will marginally raise the rental rate and lower the wage rate if $\Phi > 0$, i.e., if the price-output response is normal. Note further that in (24), $\varphi_{w2} > |D|$, due to the Stolper-Samuelson effect in the virtual system, and $(1-\varepsilon_{11}) > \Phi$ if $\varepsilon_{11} > 0$ and $\Phi > 0$. Thus $\hat{r} > \hat{p}^s$, i.e., the percentage increase in the rental rate is greater than that in the supply price. If, however, $\varepsilon_{11} < 0$, then $\Phi > 1 - \varepsilon_{11} > 0$. This implies that the percentage increase in the rental rate may not be greater than the percentage increase in p^s.

The above result, which is about small changes in prices, relies on normal price-output response. However, we mentioned earlier that if finite changes are considered, the price-output response must be normal. This means that the above result will hold for finite changes. To see this point more clearly, let us consider the case in which sector 1 is subject to strong increasing returns so that $\varepsilon_{11} > 0$ and $\Phi < 0$.

Suppose that there is a small increase in p^s. We show earlier that Q_1 will go up if the adjustment rule (14) is assumed. Because of increasing returns, $h_1(Q_1)$ will go up, meaning that $\tilde{p}^s$ will go up, by a percentage greater than that of p^s. Thus r will rise by a greater proportion, and w will drop.

The above result can be illustrated in Figure 3, which shows the unit cost schedules of the two sectors corresponding to the supply prices of the sectors, $\hat{p}^s$ and 1. Because sector 1 is capital intensive, unit cost schedule $\tilde{c}_1 = \tilde{p}^s = h_1(Q_1)p^s$ is less steep than schedule $\tilde{c}_2 = 1$ at the initial point E.[10] Suppose that there is a small increase in p^s. If $\Phi > 0$, then there is an increase in $\tilde{p}^s$. This will shift schedule up to $\tilde{c}_1 = h_1(Q_1')p^s$, where Q_1' is the new output level, which is higher than the initial one.[11] So r will go up by a greater percentage, and w will fall. If sector 1 is subject to decreasing returns, then $\tilde{p}^s$ will go up with p^s, though possibly by a smaller percentage. So r will go up, maybe by a smaller percentage, and w will fall. The above results are summarized by the following proposition:

Proposition 3 ***(Stolper-Samuelson Theorem in the Presence of External Economies of Scale)****. A small increase in the price of one good will increase the real reward of the factor used intensively in the production of the good but will lower that of the other factor if (a) sector 1 is subject to weak increasing returns so that* $\Phi > 0$*; or (b) finite changes in outputs are allowed under the adjustment rule (14), whether or not sector 1 is subject to increasing returns. If sector 1 is subject to decreasing returns and if only local changes are considered, an increase in the price of one good will increase the reward, but not necessarily the real reward, of the factor used intensively in the production of the good but will lower that of the other factor.*

3 Autarkic Equilibrium

To analyze autarkic equilibrium of the economy, let us introduce its preferences. We assume that there exists a social utility function of the economy, which is increasing, homothetic, differentiable, and quasi-concave in the two goods. Denote the demand price ratio by p^d for a given consumption ratio z.[12] Due to homotheticity and strict

[10] The slope of a unit-cost schedule is equal to the (negative) ratio of labor to capital employed in the sector.

[11] It can be shown that the percentage change in $\tilde{p}^s$ is equal to $[(1-\varepsilon_{11})/\Phi]\hat{p}^s$. Thus if $\varepsilon_{11} > 0$ and $\Phi > 0$, the percentage increase in $\tilde{p}^s$ is greater than that of p^s. If, however, $\varepsilon_{11} < 0$, which implies that $\Phi > (1-\varepsilon_{11}) > 0$, and $\tilde{p}^s$ will increase by a smaller percentage than p^s.

[12] The demand price p^d is defined as the maximum price that the consumers are willing to pay for a given basket consisting of a given ratio of good 1 to good 2, z.

quasi-concavity, p^d can be expressed as a decreasing function of the quantity ratio, z: $p^d = \gamma(z)$, where $\gamma'(z) < 0$. Define $\psi \equiv -\hat{z}/\hat{p} > 0$ as the price elasticity of demand. The demand price can be illustrated in Figure 4 by a downward sloping schedule.

An autarkic equilibrium of the economy is represented by the output ratio that gives:

$$p^s = p^d = p^a, \tag{25}$$

where superscript "a" denotes the autarkic equilibrium value of a variable. The autarkic equilibria in three possible cases are shown in panels (a) to (c) in Figure 4. (Ignore the dotted schedule for the time being.) By (25), an autarkic equilibrium occurs when the supply price schedule cuts the demand price schedule. Panels (a) and (b) show a unique autarkic equilibrium while panel (c) shows an economy with three autarkic equilibria. Furthermore, in panel (a), point A occurs at a point on a positively-sloping part of the supply price schedule, while in panel (b), it occurs at a point on a negatively-sloping part of the schedule.

Proposition 4 *Given condition H, an autarkic equilibrium exists. If the supply price schedule is positively sloped, then the autarkic equilibrium is unique.*

Proof. Given condition H and the properties of the demand price schedule, $p^d > p^s$ when z is small, and $p^d < p^s$ when z is large. By continuity, there exists one (or more) z that satisfies condition (25). If the supply price schedule is monotonely positively sloped, then there can only be one intersection, i.e., the equilibrium is unique. ■

Like what we did earlier, we now analyze the stability of an autarkic equilibrium. Because the prices faced by firms may change as the outputs vary, the Ide-Takayama adjustment rule can be modified as follows:

$$\dot{z} = \beta(p^d - p^s) = \phi(z), \tag{26}$$

where β is a positive constant. The rationale behind (26) is that if $p^d > p^s$, then firms in sector 1 will have incentives to increase their production, causing z to rise. Differentiate both sides of (26), evaluating them in a region close to the autarkic equilibrium, to give

$$\phi'(z) = -\frac{\beta p^a}{z\gamma\mu}\frac{1}{\theta}, \tag{27}$$

where $\theta \equiv 1/(\gamma\Phi + \mu)$, which is positive if and only if

$$\Phi > -\frac{\mu}{\gamma}. \tag{28}$$

For local stablity, we require that $\phi'(z) < 0$, which is satisfied if $\theta > 0$. In other words, condition (28) is a necessary and sufficient condition for local stability of an autarkic equilibrium under the adjustment rule (26). Note that condition (28) can be satisfied even if Φ is slightly negative. Thus we have

Lemma 5. An autarkic equilibrium is locally stable if and only if condition (28) is satisfied. A sufficient condition for a locally stable autarkic equilibrium is that $\Phi > 0$.

Lemma 5 can be illustrated in Figure 4. In panel (a), the autarkic equilibrium is on a segment of the supply price schedule which is positively sloped, i.e., $\Phi > 0$, while in panel (b), $\Phi < 0$ at the autarkic equilibrium. In both cases, the autarkic equilibrium is stable.[13] In panel (c), the demand price schedule cuts the supply price schedule at points A, B, and C. Based on the above analysis, points A and C are locally stable while point B is locally unstable.[14]

Proposition 5 *Given condition H, the autarkic equilibrium with the lowest output ratio and the one with the highest output ratio are locally stable. If the autarkic equilibrium is unique, it is locally stable.*

To develop the analysis further, let us first examine the effects of a change in factor endowments on an autarkic equilibrium. Substitute the demand elasticity into (13). Using the equilibrium condition (25) and rearranging terms, we have

$$\hat{p}^a = -\theta \left[\sigma \hat{K} + \zeta \hat{L}\right] = -\theta \left[(\sigma + \zeta)\hat{K} - \zeta(\hat{K} - \hat{L})\right]. \tag{29}$$

Condition (29) suggests that how the autarkic equilibrium price is affected by a change in a factor endowment depends on, among other things, the sign of θ, which is related to the local stability of the equilibrium. Thus if $\theta > 0$, then p^a drops when there is a small increase in K or the size of the economy, but p^a increases when there is a small increase in L. Note that by (29), $-\theta(\sigma + \zeta)$ is a measure of the *scale* effect, i.e., the percentage change of p^a when both factors are increased by the same proportion. The scale effect is absent in a neoclassical framework.

With finite changes allowed, the effects of a change in the factor endowment will be normal, independent of the sign of θ. This point is illustrated in Figure 4. For

[13] In both cases, $p^d > p^s$ when $z < z^a$ but $p^d < p^s$ when $z > z^a$.

[14] To see why point B in panel (c) of Figure 4 is unstable, we can note that if z is slightly greater than the output ratio at point B, z^B, then $p^d > p^s$, and so z increases, or if z is slightly less than z^B, then $p^d < p^s$, and so z decreases.

example, if there is an increase in capital endowment ($\hat{K} > 0$, $\hat{L} = 0$) or an increase in the size of the economy ($\hat{K} = \hat{L} > 0$), then the supply price schedule in all these panels will shift to, say, the position represented by the dotted schedule. In panels (a) and (b), or for points A and C in panel (c), the autarkic point is locally stable, as $\theta > 0$. Point B in panel (c) is not locally stable. However, instability of point B is true in a local sense only. After an increase in K or the size of the economy so that the supply price schedule shifts to the dotted curve, the demand price is higher than the supply price at the initial output ratio, z^B. According to the adjustment rule (26), z increases. In fact, z will continue to increase until point C$'$ is reached. This represents a drop in p^s. A similar result can be obtained for the case in which there is an increase in L.

Proposition 6 *An increase in the capital (or labor) endowment or the size of the economy will lower (or raise) the autarkic price ratio p^a if (a) the autarkic equilibrium is locally stable; or (b) finite changes in z under the adjustment rule (26) are allowed.*

The effects of an increase in the size of an economy on the autarkic price ratio is well known from the work of Markusen and Melvin (1981), Tawada (1989), and Ide and Takayama (1993). The above proposition goes beyond their work by examining not only marginal changes but also finite changes. Our analysis suggests that the effects of changes in factor endowments are normal even if the autarkic equilibrium is locally unstable. This result is significant as two trading countries generally have factor endowments quite different from each other.

4 An Open Economy

We now analyze foreign trade. Consider the economy described above with fixed factor endowments. Free trade is allowed with other countries. For simplicity, no cross-country externality is assumed, meaning that trade does not affect the technologies in the economy, except through a change in the output of good 1 and function $h_1(Q_1)$. With homothetic preferences, denote the Marshallian demand for good i, $i = 1, 2$, by $C_i(p, g(p, K, L))$, where $g(p, K, L)$ is the GDP function of the economy. The export supply of good i is equal to

$$E_i(p, K, L) = Q_i(p, K, L) - C_i(p, g(p, K, L)). \tag{30}$$

Subscripts are again used to denote partial derivatives of the export supply functions; for example, $E_{1p} \equiv \partial E_1/\partial p$. Variable E_{1p} measures the change in the export supply

due to a small increase in the price ratio, and is called the price-export response. We say that the price-export response is normal (or perverse) if $E_{1p} > (<)\ 0$. Furthermore, let us define E^a_{1p} as the value of E_{1p} evaluated at the autarkic point. If $E^a_{1p} > (<)\ 0$, it means that if the prevailing world price of good 1 is slightly higher than the autarkic supply price of good 1, the economy tends to export (import) good 1.

The offer curve of the economy can be derived from the export supply functions. Alternatively, it can be derived graphically using Figure 5. Panels (a) and (b) correspond to panels (a) and (b) of Figure 4, respectively. The autarkic equilibrium is unique, with the autarkic price ratio equal to p^a. For a reason given below, we will focus on a unique autarkic equilibrium, and will not considered a case similar to panel (c) of Figure 4.

Imagine that the economy is facing given world prices. Consider an arbitrary price ratio p^1 and assume that it is slightly higher than the autarkic price ratio p^a. The corresponding price line cuts the demand price schedule once at point D, but cuts the supply price schedule at one or more points. The diagram shows the case in which there are three points of intersection on schedule p^s, points E, F, and G. As a result, three possible values of excess demand/supply of good 1 are created, represented by line segments DE, DF, and DG, where the signs of these three values depend on the relative position of point D: in panel (a), all of them represent excess supply of good 1 while in panel (b), DE and DF are excess demand while DG means excess supply.[15]

These excess demands/supplies are used to give the three points of the economy's offer curve corresponding to the price ratio p^1. Panels (a) and (b) of Figure 6 are obtained from panels (a) and (b) of Figure 5, respectively. For example, the three line segments in panel (a) of Figure 5 can be used to determine the export supply of good 1 corresponding to OH, OJ, and OK in panel (a) of Figure 6. Similarly, line segments DE, DF, and DG in panel (b) can be used to determine the import demand for/supply of good 1 corresponding to OH, OJ, and OK in panel (b) of Figure 6.

Figures 5 and 6 reveal one interesting feature of the present model under external economies of scale: Whether a production equilibrium (when facing a given price ratio) is locally stable depends on whether the supply price schedule is strictly increasing. If the schedule is partly positively sloped and partly negatively sloped, the production equilibrium within a certain range of price ratio will not be unique. However, even with multiple production equilibria, the autarkic equilibrium may be

[15] It should be noted that DE, DF, and DG are the gaps between the output ratios corresponding to the demand price and supply price. They are not equal to, but can be used to determine, the excess demand/supply of the two goods.

unique.

Similar steps can be taken to derive the excess demand/supply under other possible prices. The excess demands and supplies can then be used to trace out the offer curve of the economy, as panels (a) and (b) of Figure 6 show. The offer curve for an economy with external economies of scale has some properties similar to those of an offer curve in a neoclassical framework: (i) It passes through the origin and is tangent at the origin to the price line representing the autarkic price ratio.[16] (ii) Sooner or later it bends toward the import axes, when the income effect of a change in the price ratio outweighs the substitution effect. (iii) In the absence of international transfer or factor movement, the offer curve appears in the first and third quadrants only.

One important feature of the offer curve, which is crucial for some of the results derived below, is its curvature at the origin. In panel (a) of Figure 6, it is convex toward the quadrant with positive export of both goods, while in panel (b) it is concave to the positive export quadrant. A careful examination of the panels will reveal that in panel (a), a small rise in the price ratio from the autarkic level will create an excess supply of good 1 and excess demand for good 2, i.e., $E^a_{1p} > 0$. In panel (b), if p^1 is slightly greater than the autarkic level, an excess demand for good 1 and excess supply of good 2 are created, i.e., $E^a_{1p} < 0$.

Lemma 6. The price-export response at the origin is normal if the price-output response is normal at the autarkic point.

Proof. Suppose that the economy is initially under autarky and there is a small increase in the relative price of good 1. Since the demand price schedule is negatively sloped, on the demand side there will be a drop in the output ratio. If the price-output response is normal at the autarkic point, then the supply price schedule is positively sloped, meaning that on the supply side there is a rise in the output ratio. Thus an excess supply of good 1 is created, and the price-export response is normal. ■

Lemma 6 can be illustrated in Figure 6. Note that in panel (a), which shows a normal price-output response, the price-export response is normal. However, a normal price-output response is only a sufficient, but not necessary, condition for a normal price-export response.

Lemma 7. If an economy has only one factor and a stable autarkic equilibrium, then its export-price response is perverse at the autarkic equlibrium point.

[16] This result can be proved by differentiating the trade balance equation, $pE_1 + E_2 = 0$, and evaluating it at the autarkic point.

Proof. If there is only factor, the virtual system reduces to a Richardian system with a linear production possibility frontier. The elasticity of supply of good 1 η_{11} approaches infinity, implying that Φ is negative. This means that the supply-price schedule is negatively sloped. If the autarkic equilibrium is stable, the economy is the one described by panel (b) of Figure 5 or 6. As a result, its offer curve has the perverse curvature at the origin. ■

One-factor models of externality have been used extensively in the literature: Either (1979, 1982), Panagariya (1981), Helpman (1984), Krugman (1987), Tawada (1989), and Kemp and Schweinberger (1991) are examples. Lemma 7 shows that one-factor models have a very special prpoerty: the export-price response of the economy at a stable autarkic point is perverse.

Our analysis suggests that when there are two factors or more, the curvature of the economy's offer curve may be normal at the origin. However, most papers with two-factor externality models focus on the cases in which the price-export response at the origin is perverse; for example, Kemp (1969), Melvin (1969), and Chacholiades (1978).

5 International Trade

We are now ready to extend the above model to analyze international trade. Call the above economy home, which is allowed to trade freely with another country labeled foreign. Both economies have the same structure, with identical technologies and preferences, although their factor endowments may be different. As mentioned, cross-country externality is assumed to be absent. Let us use asterisks to denote the variables of the foreign country; for example, $E_1^*(p^*, K^*, L^*)$ represents the foreign export supply of good 1.

A free-trade equilibrium between the countries can be described by the following equations:

$$E_1(p, K, L) + E_1^*(p^*, K^*, L^*) = 0 \tag{31}$$

$$p = p^*. \tag{32}$$

Condition (31) describes the equilibrium condition of the good-1 market while (32) is the result of free trade and zero transport cost. By Walras' Law, these two conditions imply equilibrium of the good-2 market.

Graphically, a free-trade equilibrium is represented by an intersection (except at the origin) or a tangency between the offer curves of the two countries. Panels (a) and

(b) of Figure 7 show two different possible cases in which OC and OC* are the offer curves of the home and foreign countries, respectively: In panel (a), five equilibria, A, B, C, D, and O, can be identified while in panel (b), there are three equilibria, A, B, and O. Note that an unique autarkic equilibrium in each country does not exclude the existence of multiple trade equilibria.

Since trade between two countries can give rise to multiple equilibria, we can compare different equilibria by analyzing their stability. Following an approach suggested by Marshall (1979, 1980), suppose that there exists a competitive intermediary which is able to ship good 1 between the countries.[17] Consider an equilibrium volume of home export of good 1 E_1^0 while at a particular point of time, the intermediary transport a volume E_1', which is equal to or close to E_1^0. Note that both E_1^0 and E_1' may be negative, representing import of good 1. If $E_1' = E_1^0$, trade is in equilibrium and the intermediary should keep this amount of export of good 1. If $E_1' \neq E_1^0$, what would the intermediary do?

To answer the above question, invert the home export function $E_1' = E_1(p, K, L)$ and the foreign import function $-E_1' = -E_1^*(p^*, K^*, L^*)$, with factor endowments given, to give $p' = \rho(E_1')$ and $p^{*\prime} = \rho^*(E_1')$.[18] The functions can be interpreted as the price ratios in the two countries so that they are willing to trade E_1' of good 1, with the appropriate values of good 2 needed to balance trade under p' (for home) or $p^{*\prime}$ (for foreign). If $p' = p^{*\prime}$, E_1' is an equilibrium volume. If $p' \neq p^{*\prime}$, we assume that the intermediary varies the trade volume according to the following equation:

$$\dot{E}_1 = \gamma(p^{*\prime} - p') = \gamma(\rho^*(E_1') - \rho(E_1')) = \psi(E_1'), \tag{33}$$

where $\gamma > 0$ is a constant. Equation (33) is analogous to the adjustment equation (26), with a similar interpretation. The rationale is that for the firms in the home sector 1, they compare p' with $p^{*\prime}$ to determine whether they should increase or decrease their export of good 1: p' is the break-even price and is what they can get by selling their output at home while $p^{*\prime}$ is what they can get by exporting their output to foreign. Thus if $p^{*\prime} > p'$, export of good 1 is encouraged, i.e., $\dot{E}_1 > 0$.

Differentiate equation (33) and rearrange the terms to give

$$\psi'(E_1') = \gamma\left(-\frac{1}{E_{1p^*}^*} - \frac{1}{E_{1p}}\right) = -\gamma\left(\frac{1}{E_{1p^*}^*} + \frac{1}{E_{1p}}\right). \tag{34}$$

For a (locally) stable equilibrium, a necessary and sufficient condition is that $\psi'(E_1) < 0$ in the small neighborhood around the equilibrium point. Define $\delta_e \equiv pE_{1p}/E_1$ and

[17] The intermediary can be a large group of companies that do not have any monopoly power.

[18] With E_1' sufficiently close to E_1^0, we do not have to worry about multiple solution.

$\delta_m \equiv -pM_{2p}/M_2$, where $M_i \equiv -E_i$ is home's import of good i, with two similar variables defined for foreign, which are denoted with asterisks. Condition (34) can be written in alternative forms:

$$\psi'(E_1') = -\frac{\gamma p^0}{E_1^0}\left(\frac{1}{\delta_m^*} + \frac{1}{\delta_e}\right) = -\frac{\gamma p^0}{E_1^0}\left(\frac{\delta_m + \delta_m^* - 1}{\delta_m^*(\delta_m - 1)}\right), \tag{35}$$

where we have used the result $\delta_e = \delta_m - 1$.[19] Conditions (34) and (35) can be used to establish the following proposition:

Proposition 7 *A necessary and sufficient condition for a (locally) Marshallian stable trade equilibrium is that* $(\delta_m + \delta_m^* - 1)/[\delta_m^*(\delta_m - 1)] > 0$. *A sufficient condition for a (locally) Marshallian stable trade equilibrium is either (i)* $\delta_m > 1$ *and* $\delta_m^* > 0$; *or (ii)* E_{1p}, $E_{1p^*}^* > 0$ *at the equilibrium point.*

Note that the first sufficient condition, $\delta_m > 1$ and $\delta_m^* > 0$ means that the offer curves of both countries are positively sloped. Given this condition, the well-known Marshallian-Lerner condition is also satisfied. If this sufficient condition is not guaranteed, then the Marshallian-Lerner condition is neither necessary nor sufficient for local Marshallian stability.[20]

Marshallian stability of a trade equilibrium can be analyzed graphically. Consider the no-trade equilibrium represented by point O (the origin) in panel (a) of Figure 7. To determine its stability, assume that an arbitary amount of good 1 E_1' is moved from home to foreign. Let the corresponding points on the home offer curve (thick one) and foreign offer curve (thin one) be G and H, respectively. The equilibrium price ratio p' of home (or $p^{*\prime}$ of foreign) is given by the slope of a ray from origin to point G (or H). The diagram shows that $p' > p^{*\prime}$. According to (33), $\dot{E}_1 < 0$. Graphically, the movement of E_1 is indicated by an arrow in Figure 7. Similar analysis can be used to show that trade equilibria A, O and D are stable while B and C are unstable. In panel (b), trade equilibria A and B are stable while point O is unstable.

[19] This result can be obtained by differentiating the trade balance equation $pE_1 = M_2$. See, for example, Ethier (1995, p. 101).

[20] The Marshallian-Lerner condition is based on price adjustment, or sometimes called the Walrasian adjustment.

6 Are Increasing Returns Sufficient for International Trade?

This is one of the oldest questions in the theory of international trade under externality. Modern analysis of this question can be traced back to the work of Kemp (1969) and Melvin (1969) for models with two factors of production. Both of them focus on the cases in which the no-trade equilibrium between two countries are unstable, and suggest that foreign trade is a natural consequence in the presence of external increasing returns. Melvin even conclude that increasing returns are a determinant of trade. Later work such as Chacholiades (1978) follows the same argument and makes the link between increasing returns and foreign trade. This conclusion is further reinforced by the work with one factor of production by Ethier (1979, 1982), Helpman (1984), Krugman (1987, 1994), and Kemp and Schweinberger (1991). Their work provides more rigorous analysis of the production decisions of firms and demonstrates how trade may exist.

However, are increasing returns sufficient for international trade? All the work mentioned earlier seems to suggest that the existence of external economies of scale is sufficient for the existence of international trade. We now examine this issue more carefully. The best way to do that is to start with two economies that are exactly identical and see whether trade will exist. In particular, we consider again the two countries described above, each with a unique autarkic equilibrium, but we make the further assumption that the countries have identical technologies, preferences, and factor endowments.[21] This assumption is made in order to "neutralize" the comparative-advantage force; in other words, the countries do not have any comparative advantage as their autarkic equilibria are identical.

Figure 7 shows the offer curves of these identical countries, which are tangent to each other at the origin. This means that the origin represents a no-trade equilibrium. The discussion in the previous section shows that the case in panel (a) has a Marshallian stable no-trade equilibrium while in the case in panel (b) the no-trade equlibrium is Marshallian stable.

Will these economies trade? To answer this question, we have to determine the stability of the no-trade equilibrium. The following propositions are provided.

Proposition 8 *Given two identical countries with a unique autarkic equilibrium, the no-trade equilibrium is Marshallian stable if and only if the price-export response of each country is normal at the autarkic point.*

[21] The assumption of identical factor endowments will be relaxed later.

Proof. Because the countries are identical with an autarkic equilibrium, $E_{1p} = E^*_{1p}$ at the autarkic point. Therefore from condition (34), the no-trade equilibrium is (locally) Marshallian stable if and only if $E_{1p} > 0$, or if and only if the price-export response is normal. ■

Using the analysis in the previous section, we can argue that if the no-trade equilibrium is Marshallian stable [panel (a) of Figure 7], then no trade is expected even if free trade is allowed. If, however, the no-trade equilibrium is not Marshallian stable [panel (b) of Figure 7], then when free trade is allowed, a small disturbance will move the trade equilibrium away from the initial point. This means that trade will exist. However, what will each country export? That depends on the disturbance. If the disturbance brings the trade point slightly to the right in the diagram, then the trade equilibrium will move further away to the right, until point B is reached. In this case, the home country exports good 1. However, if the disturbance brings the trade point to the left, then eventually point A will be reached and the home country will export good 2. Thus we have:

Proposition 9 *If the no-trade equilibrium between two identical countries is Marshallian stable, then no trade is expected. If the no-trade equilibrium between two identical countries is Marshallian unstable, then trade exists after a small disturbance, the patterns of trade are indeterminate.*

Why, then, do most papers in the literature argue explicitly or implicitly that trade is evitable in the presence of increasing returns? Two reasons can be offered here. First, using a model with two factors, Melvin (1969), Kemp (1969), and Chacholiades (1978) consider only the cases in which the no-trade equilibrium is unstable. No mention and analysis have been provided to the case in which the no-trade equilibrium is stable. Second, Ethier (1979, 1982), Helpman (1984), Krugman (1987, 1994), and Kemp and Schweinberger (1991) focus on models with one factor. The presence of only one factor has a crucial implication, as the following proposition shows:

Proposition 10 *If each economy has only one factor and a unique, Marshallian stable autarkic equilibrium, then the no-trade equilibrium between two identical countries is Marshallian unstable, and the two countries trade after a small disturbance. The patterns of trade, however, are indeterminate.*

Proof. By Lemma 7, the export-price response of a one-factor economy is perverse at the autarkic point. Proposition 9 then gives the proposition. ■

Before we have a discussion of the current literature, let us provide some intuition of the results concerning the existence of trade. In the present model, three forces that affect trade can be identified: *comparative-advantage* (CA) force, *scale-economies* (SE) force, and *factor-price* (FP) force. The CA force exists when the two countries have different autarkic relative prices so that when free trade is allowed, by the Law of Comparative Advantage, each country exports the good which is relatively cheaper under autarky. In the example introduced earlier, the countries are identical and thus have the same autarkic relative price. So the CA force does not exist.

The FP force comes from changes in the unit costs of firms due to changes in relative factor prices. To see what this is, consider again a neoclassical framework with two identical countries (so that the CA force is zero) and no external economies of scale (so that the SE force is zero). We know that the no-trade equilibrium is stable and no trade will be expected. Suppose that there is a small disturbance in country A so that there is, say, a small increase in the production of the capital-intensive good and a small drop in the production of the labor-intensive good. Because of the factor intensity ranking, at the new production point under the initial autarkic prices, there is an excess demand for capital but an excess supply of labor, leading to a drop in the wage-rental ratio and causing a rise in the unit cost of good 1 relative to that good 2, i.e., there is a rise in the relative supply price of good 1. As a result, firms in sector 1 will experience a drop in profitability and would decrease their outputs, while the opposite will occur in sector 2. In other words, the initial changes in outputs due to the disturbance are not sustainable and the countries will return to the original no-trade equilibrium and trade does not exist. So the FP force works against trade.

To understand the SE force, which is due to the existence of external economies of scale, let us consider a one-factor framework with external economies of scale that has been analyzed a lot in the literature. Again assume that the countries are identical (so that the CA force is zero) with increasing returns in sector 1 but constant returns in sector 2. Since there is only one factor, the FP force does not exist. We are now left with the SE force. Suppose that free trade is allowed and that a small disturbance will occur so that there is, say, a small increase in the output of good 1 but a small drop in the output of good 2 in country A, but no disturbance in country B. Because of the existence of increasing returns, the increase in output will lower the average cost of firms in sector 1 of country A, making them more competitive than the counterpart in country B. As a result, they start exporting good 1 to country B. By capturing a bigger share of the market, country A's sector 1 firms will be able to lower the average cost even further, capturing a even bigger share of the market. This process continues until country B is completely specialized in producing good 2. As a result, trade exists, but we note that the pattern of trade is indeterminate because it

could be the sector-1 firms in country B that experience an initial increase in output, eventually leading to an export of the good. So the SE force works for trade.

Are increasing returns sufficient for international trade? If one considers a one-factor framework with two identical countries, then both the CA and FP forces are absent so that the SE force will lead to trade after a small disturbance. However, if we consider a two-factor framework with identical countries, then both the FP and SE forces exists, with the FP force working against trade but the SE force encouraging trade. Whether trade exists depends on which of these two forces is stronger. The above propositions give necessary and sufficient conditions for trade to exist although in the literature papers usually assume implicitly or explicitly that the SE force dominates.[22]

7 The Modified Law of Comparative Advantage

The Law of Comparative Advantage has two components:[23]

1. Under natural trade, each country exports the good in which it has a comparative advantage, i.e., the good which is relatively cheaper under autarky.

2. Under natural trade, the world price ratio is bounded by the autarkic price ratios of the countries.

Implicit in the law is that if the two countries do not have any comparative advantage, i.e., when they have the same autarkic price ratio, no trade will exist.

Is this law still true in the presence of external economies of scale? It is easy to see that point (2) of the law is not true. We have already seen that trade can exist between two identical countries with the same autarkic price ratio. When free trade exists, the world price ratio can be of any value, depending on where the offer curves of the countries meet. As a result, the world price ratio may not be bounded by the countries' autarkic price ratios, which are equal. [See, for example, Figure 7(b).]

[22]The answer to the question of whether increasing returns sufficient for trade could depend on what "increasing returns" is referring to. If it means only the scale-economies force defined in the paper, it is sufficient for trade. The purpose of this section is to argue that models with external economies of scale may not have trade between two identical countries unless there is only one factor.

[23]See Wong (1995, Chapter 3). The law is usually proved with the existence of a social utility function of each country, but Shimomura and Wong (1998) extend the law to cases in which social utility functions do not exist. The law refers to a two-good framework. See Wong (1995) for extensions of the law to higher dimensional frameworks.

Let us turn to point (1) of the law. Is it valid? Assuming the Marshallian adjustment rule, the answer to this question is in the affirmative.

Proposition 11 ***(Law of Comparative Advantage in the Presence of External Economies of Scale)****. Assuming the Marshallian adjustment rule, under free trade, each country exports the good in which it has a comparative advantage.*

Proof. Without loss of generality, assume that the home country has a comparative advantage in good 1, i.e., good 1 is relatively cheaper in home. This is the case shown in Figure 8. By the Marshallian adjustment rule described by (33), home exports good 1, until point C is reached. The diagram shows five trading equilibria, points A, B, C, D, and E. The present adjustment rule implies that points A, C, and E are stable, but if the countries start from the origin, they will reach point C, with both countries exporting the good in which they have a comparative advantage. ■

8 Factor Endowments and Patterns of Trade

One of the main results in the neoclassical framework is its prediction of countries' patterns of trade based on factor endowments. Attempts to extend this result to model with external economies of scale have not made much success. One reason may be that the models are complicated. Therefore papers usually consider special cases, such as the existence of one factor, one country being uniformly bigger than the other, or only small changes. However, we have seen that these approaches are very limited and could give misleading results. For example, in the presence of one factor, the price-export response at the no-trade point must be perverse, and no factor endowment ratios can be considered. The case with one country uniformly bigger than the other is limited because it entirely ignores the cases in which one country versus the other one is capital abundant. The consideration of small changes is not satisfactory because the factor endowments of countries usually are quite different from each other.

We now make use of the present approach and techniques described above to develop a much more general theory of international trade based on factor endowments. To illustrate the roles of factor endowments, we assume that the total capital and labor endowments in the world are fixed, and we consider the effects of different ways of allocating the factors between home and foreign. This case can conveniently be illustrated by the box diagram in Figure 9, the dimensions of which represent the world's factor endowments. Any point in the box represents different ways of factor

allocation between the countries. Point M, the mid-point of the diagonal, represents the case in which the two countries have the same factor endowments.

The previous sections analyze how the autarkic prices of a country may be affected by changes in factor endowments. The analysis is used to establish the following proposition:

Proposition 12 *Suppose that home is bigger than foreign in either of the following senses: (i) it has more capital but the same amount of labor; or (ii) it has uniformly more capital and more labor. Then home exports the good which is capital intensive and subject to increasing returns. If home has more labor but the same amount of capital, then home exports the good which is labor intensive and subject to constant returns.*

Proof. By Proposition 6, when a country gets bigger with more capital or more capital and labor of the same percentage, the autarkic price drops, so long as an autarkic equilibrium is Marshallian stable or3 finite changes in outputs due to factor endowment changes are allowed. Therefore if home is bigger than foreign in either of these ways, and since autarkic equilibrium is unique in both countries, home has a lower autarkic relative price of good 1. By the modified Law of Comparative Advantage, home exports good 1. Similarly, if home has more labor but the same amount of capital, it will have a higher autarkic relative price of good 1 and thus have a comparative advantage in good 2. ■

The existing literature has very little to say about the roles of factor endowments in predicting the patterns of trade. One exception is Markusen and Melvin (1981), who consider two countries of different sizes but with the same capital-labor ratio. They argue that there exists at least one stable equilibrium in the first quadrant with the bigger country (with the same capital-labor ratio) exporting good 1. Their result, strictly speaking, is not a theory of pattern of trade, as they do not argue that the bigger country will definitely export good 1. In the present paper, by making use of the adjustment mechanism described in (33), we show that the bigger country will in fact export good 1.

The above proposition argues further that the country with more capital (with the same amount of labor) will export good 1 and the one with more labor (with the same amount of capital) will import good 1.

More predictions about patterns of trade are given as follows. In Figure 9, we construct a pattern-of-trade schedule, which is denoted as PMT in the diagram. This is a schedule that represents various factor endowments with which the countries have

the same autarkic price ratio. Obviously it passes through point M and is continuous because of continuous technology and preferences. With these factor endowments, whether the countries will trade under free trade depends on whether the no-trade point is Marshallian stable. For factor endowments above (or below) schedule PMT, home has a lower (or higher) autarkic relative price of good 1, and by the Law of Comparative Advantage, will export good 1 (or 2).

Proposition 13 can be used to derive more properties of the pattern-of-trade schedule. Consider point A in the diagram, at which home has more capital but the same amount of labor as foreign. By the proposition, home exports good 1. So A must be above schedule PMT. Point B, on the other hand, represents an allocation of factors with which home is uniformly bigger than foreign. Again by the proposition, home exports good 1, meaning that B is above schedule PMT. Consider now point D, at which home has more labor but the same amount of capital as foreign. So home exports good 2, meaning that D must be below schedule PMT. Furthermore, since the countries are symmetric, segment PM is symmetric to segment MT. Let us draw a horizontal line through point M. The above results imply that schedule PMT must lie in the minor region bounded by the diagonal and the horizontal line through M, as the diagram shows.

The slope of schedule PMT can be derived as follows. Using condition (29) for home and foreign, with asterisks to denote foreign variables, we have

$$\hat{p}^a - \hat{p}^{*a} = -\theta \left[(\sigma + \zeta)\hat{K} - \zeta(\hat{K} - \hat{L}) \right] + \theta^* \left[(\sigma^* + \zeta^*)\hat{K}^* - \zeta^*(\hat{K}^* - \hat{L}^*) \right]. \quad (36)$$

Suppose that we start from a point such as M at which both countries have the same autarkic price ratio, condition (36) can be interpreted as a measure of the percentage change in p^a/p^{*a}. Along schedule PMT, the countries have the same autarkic price ratio, meaning that the percentage change in p^a/p^{*a} is zero. Setting the RHS of (36) to zero and using the conditions that $\mathrm{d}K = -\mathrm{d}K^*$ and $\mathrm{d}L = -\mathrm{d}L^*$, we have the slope of PMT as

$$\left.\frac{\mathrm{d}K}{\mathrm{d}L}\right|_{\mathrm{PMT}} = -\frac{\theta\zeta/L + \theta^*\zeta^*/L^*}{\theta\sigma/K + \theta^*\sigma^*/K^*}. \quad (37)$$

If the price-output response is always positive, all the parameters except ζ and ζ^* in (37) are positive while ζ and ζ^* are negative, implying that PMT is positively sloped. At point M, with $K = K^*$ and $L = L^*$, and the equality of parameters of the countries, the expression in condition (37) reduces to

$$\left.\frac{\mathrm{d}K}{\mathrm{d}L}\right|_{\text{PMT at M}} = -\frac{\zeta K}{\sigma L} > 0. \quad (38)$$

Note that because $\sigma > |\zeta|$, condition (38) implies that at point M in Figure 8 schedule PMT is positively sloped but less steep than the diagonal.

Figure 9 can now be used to derive a more general theory of pattern of trade. Let us define the following terms:

1. Home versus foreign is *relatively abundant* in capital if $K/L > K^*/L^*$.

2. Home versus foreign is *absolutely abundant* in capital if $K > K^*$.

Labor abundance in a relative or absolute sense and factor scarcity can be defined in a similar way. Obviously, relative abundance of a factor is neither sufficient nor necessary for absolute abundance in that factor, and a country is abundant relatively in a factor if and only if it is scarce relatively in another factor.

In Figure 9, schedule PMT and the diagonal divide the box into six regions, which are labeled (I) to (VI). Factor abundance/scarcity and patterns of trade are different in different regions. First, we note that home is absolutely capital abundant in regions (I), (V), and (VI), but relatively capital abundant in regions (I), (II), and (III). Labor abundance in other regions can also be found. In other words, in region (I) home is abundant in capital both in an absolute and in a relative senses, while in region (IV), it is abundant in labor in a relative sense while not abundant in capital. In terms of the patterns of trade, for an endowment point in regions (I), (II), or (VI), home exports good 1 while in other regions, home exports good 2. Thus we have,

Proposition 13 ***(Heckscher-Ohlin Theorem in the Presence of External Economies of Scale)****. A country exports the good subject to increasing returns if it is relatively and absolutely abundant in the factor used intensively in the good, or if it is uniformly bigger. A country exports the good subject to constant returns if it is relatively abundant in the factor used intensively in the good but not absolutely abundant in the other factor, or if it is uniformly smaller.*

The intuition behind this theorem is obvious. As explained, the response of the autarkic price ratio to factor endowments can be disaggregated into the scale effect and the factor-proportion effect. The scale effect implies that a country that is bigger uniformly will export the increasing-returns good, while the factor-proportion effect represents the possibility that a country has a comparative advantage in the good which uses its relatively abundant factor. In regions (I) and (IV), both effects work in the same direction so that the patterns of trade are predictable. In other regions, these two effects are opposite to each other, and the comparative advantages of the

countries depend on the strengths of the effects. In regions (II) and (III), home is relatively abundant in capital but absolutely scarce in capital. In region (II), the factor-proportion is stronger in region (II) so that home exports good 1, but in region (III), the scale effect is stronger in region (III) so that home's comparative advantage is good 2. Similarly, in regions (V) and (VI) the scale and factor-proportion effects are opposite to each other, with the factor-proportion effect stronger in region (V) and the scale effect stronger in region (VI).

9 International Trade and Factor Prices

Keeping the assumption that the two countries have identical technologies, we now examine the fifth trade theorem: free trade leads to equalization of factor prices. In some cases, we add the additional assumption that the countries have identical and homothetical preferences. This will help us provide some graphical illustration.

We note that under free trade, commodity prices in both countries are equalized, but the virtual commodity prices may not be. If the countries have different virtual commodity prices, then from the properties of the neoclassical framework we know that the countries must have different factor prices. Therefore to determine whether factor prices are equalized we have to determine whether the countries have the same virtual commodity prices. Because the virtual relative supply price is equal to $\tilde{p}^s = h_1(Q_1)p^s$, two countries have the same virtual supply price if and only if they have the same output of good 1 under free trade. Recall that condition (7) shows how the output of good 1 is affected by the supply price and factor endowments. We want to determine how the factor endowments should be redistributed between the countries so that under free trade they produce the same output level of good 1.

Let us first provide some qualitative analysis to bring out the intuition behind the results, and leave a rigorous analysis later. Suppose that the two countries are identical. Therefore the countries have the same autarkic equilibrium. If free trade is allowed, the no-trade point is an equilibrium, which is Marshallian stable if and only if the price-export response of each country at the origin is normal. If it is normal, then trade is zero when free trade is allowed. At this point, the two countries have the same supply price and the same output of good 1, implying that the countries have the same virtual supply price. Under diversification in production, the countries have the same factor prices.[24] If the export-price response is perverse at the no-trade point, then trade will exist after a small disturbance. However, the trade pattern is ambiguous. We do know that the country that exports good 1 must produce a larger

[24] Of course at this point the two countries do not trade even if free trade is allowed.

output of good 1 than the other country does, meaning that the former country will have a higher virtual supply price, and thus, by the Stolper-Samuelson Theorem, have a higher rental rate but a lower wage rate. In other words, if trade exists between two identical countries, then factor prices are not equalized: the rental rate is higher in the country that exports good 1.

From point M, assume that a certain amount of capital is transferred from foreign to home, i.e., home has more capital but the two countries have the same amount of labor endowment. The previous section shows that home will have a comparative advantage in good 1 and will export that good. Condition (7) shows that a small increase in the capital endowment, while the supply price and labor endowment are kept constant, will encourage the production of good 1. Since this result is valid for any commodity price ratio, we can say that if these two countries are trading freely (with the same commodity price ratio), then the country with more capital must be producing a higher output level of good 1. Furthermore, home has a higher rental rate and a lower wage rate. The same analysis and result exist if both countries have the same capital-labor ratio, with home being bigger.

Suppose now that home has slightly more labor but the same capital as compared with foreign. The previous section argues that home will export good 2. Furthermore, condition (7) indicates that home has a lower output level of good 1 under free trade. As a result, home has a lower virtual supply price, a lower rental rate and a higher wage rate under free trade.

The above analysis implies that by continuity of the export functions, there exist some factor endowment distributions with which the countries have the same output level of good under free trade. Assuming diversification in production, the countries will have the same virtual supply price and factor prices under trade. These factor endowments that imply factor price equalization are illustrated by schedule FME in Figure 10. Based on the above analysis, this schedule is in the minor region bounded by the diagonal of the box and a horizontal line passing through the mid-point, M.

This schedule can be analyzed more rigorously as follows. For the time being, we assume that the countries have identical technologies but not necessarily identical preferences. Differentiate (31), making use of (32), $\mathrm{d}K^* = -\mathrm{d}K$, and $\mathrm{d}L^* = -\mathrm{d}L$ to give

$$\left(E_{1p} + E^*_{1p^*}\right)\mathrm{d}p + \left(E_{1K} - E^*_{1K^*}\right)\mathrm{d}K + \left(E_{1L} - E^*_{1L^*}\right)\mathrm{d}L = 0. \tag{39}$$

Rearranging the terms in (39), we have

$$\left(E_{1p} + E^*_{1p^*}\right)\mathrm{d}p = \left(E^*_{1K^*} - E_{1K}\right)\mathrm{d}K + \left(E^*_{1L^*} - E_{1L}\right)\mathrm{d}L. \tag{40}$$

Equation (40) gives the effects of a reallocation of factor endowments on the free-trade price ratio. Schedule FME represents the locus of factor endowments that have

the same output of good 1 under free trade. Let us start from point M, assuming a normal price-export response. Let there be a small reallocation of factor endowments between the countries. By (7) and (8), the difference between the changes in the output levels of good 1 in the two countries is equal to

$$\hat{Q}_1 - \hat{Q}_1^* = \left(\frac{\eta_{1p}}{\Phi} - \frac{\eta_{1p^*}^*}{\Phi^*}\right)\hat{p} + \left(\frac{\eta_{1K}}{\Phi K} + \frac{\eta_{1K^*}^*}{\Phi^* K^*}\right)\mathrm{d}K + \left(\frac{\eta_{1L}}{\Phi L} + \frac{\eta_{1L^*}^*}{\Phi^* L^*}\right)\mathrm{d}L. \tag{41}$$

Now if we keep $Q_1 = Q_1^*$ as factor endowments are reallocated and free trade is allowed, (41) becomes

$$h_1\left(\frac{\tilde{Q}_{1p}}{\Phi} - \frac{\tilde{Q}_{1p^*}^*}{\Phi^*}\right)\mathrm{d}p + \left(\frac{\tilde{Q}_{1K}}{\Phi} + \frac{\tilde{Q}_{1K^*}^*}{\Phi^*}\right)\mathrm{d}K + \left(\frac{\tilde{Q}_{1L}}{\Phi} + \frac{\tilde{Q}_{1L^*}^*}{\Phi^*}\right)\mathrm{d}L = 0. \tag{42}$$

Note that $h_1(Q_1)$ depends on Q_1 only, and is thus the same in both countries. Substitute the value of $\mathrm{d}p$ in (40) into (42) to get

$$\begin{aligned}&\left[\Omega\left(E_{1K^*}^* - E_{1K}\right) + \left(\frac{\tilde{Q}_{1K}}{\Phi} + \frac{\tilde{Q}_{1K^*}^*}{\Phi^*}\right)\right]\mathrm{d}K + \\ &\left[\Omega\left(E_{1L^*}^* - E_{1L}\right) + \left(\frac{\tilde{Q}_{1L}}{\Phi} + \frac{\tilde{Q}_{1L^*}^*}{\Phi^*}\right)\right]\mathrm{d}L = 0,\end{aligned} \tag{43}$$

where

$$\Omega = \frac{h_1}{E_{1p} + E_{1p^*}^*}\left(\frac{\tilde{Q}_{1p}}{\Phi} - \frac{\tilde{Q}_{1p^*}^*}{\Phi^*}\right).$$

Equation (43) gives the slope of the schedule FME

$$\left.\frac{\mathrm{d}K}{\mathrm{d}L}\right|_{\mathrm{FME}} = -\frac{\Omega\left(E_{1L^*}^* - E_{1L}\right) + \left(\tilde{Q}_{1L}/\Phi + \tilde{Q}_{1L^*}^*/\Phi^*\right)}{\Omega\left(E_{1K^*}^* - E_{1K}\right) + \left(\tilde{Q}_{1K}/\Phi + \tilde{Q}_{1K^*}^*/\Phi^*\right)}. \tag{44}$$

In general, the slope of schedule FME is ambiguous.

To get more results, consider the special case in which the countries also have identical and homothetic preferences. Recall that in Figure 10, point M represents two countries that are identical. In the small region around point M, condition (44) reduces to

$$\left.\frac{\mathrm{d}K}{\mathrm{d}L}\right|_{\text{FME at M}} = -\frac{\tilde{Q}_{1L}}{\tilde{Q}_{1K}} > 0, \tag{45}$$

which shows that schedule FME is positively sloped at least in the neighborhood of point M. Furthermore, it is derived in the neoclassical framework that $\eta_{1L} = \lambda_{K2}/|\lambda|$ and $\eta_{1K} = -\lambda_{L2}/|\lambda|$, where λ_{j2} is the proportion of factor j employed in sector 2, and $|\lambda| = \lambda_{K2} - \lambda_{L2} < 0$, where the sign is due to the assumed factor intensity ranking.[25] Making use of these results, (45) reduces to

$$\left.\frac{\mathrm{d}K}{\mathrm{d}L}\right|_{\text{FME at M}} = -\frac{\eta_{1L}}{\eta_{1K}}\frac{K}{L} = \frac{\lambda_{K2}}{\lambda_{L2}}\frac{K}{L} = k_2 > 0, \tag{46}$$

where k_2 is the capital-labor ratio in sector 2. Comparing (38) and (46), we can see that schedule PMT is steeper than schedule FME at point M if and only if

$$\frac{|\zeta|}{\sigma} > \frac{\lambda_{K2}}{\lambda_{L2}}. \tag{47}$$

The following lemma gives a sufficient condition under which (47) is satisfied.

Lemma 8. If sector 1 is subject to mild external increasing returns so that $\Phi > 0$, then condition (47) holds.

Proof. Using the definition of ζ and σ, we have

$$-\frac{\zeta}{\sigma} = \frac{-\eta_{1L} + \eta_{2L}\Psi}{\eta_{1K} - \eta_{2K}\Psi}, \tag{48}$$

where $\Psi = \Phi/(1 - \varepsilon_{11}\eta_{2p})$. If sector 1 is subject to increasing returns and $\Phi > 0$, then $\Psi > 0$. Using the results that $\eta_{2L} = -\lambda_{K1}/|\lambda|$ and $\eta_{2K} = -\lambda_{L1}/|\lambda|$, (48) can be written as

$$-\frac{\zeta}{\sigma} = \frac{\lambda_{K2} + \lambda_{K1}\Psi}{\lambda_{L2} + \lambda_{L1}\Psi}. \tag{49}$$

If $\Psi > 0$ under the conditions stated earlier, the factor intensity ranking implies that $\lambda_{K1}\Psi > \lambda_{L1}\Psi$. Furthermore, $\lambda_{K2} < \lambda_{L2}$. Thus

$$\frac{\lambda_{K2}}{\lambda_{L2}} < \frac{\lambda_{K2} + \lambda_{K1}\Psi}{\lambda_{L2} + \lambda_{L1}\Psi}. \tag{50}$$

Combining (38), (46), and (49), we have (47) and the lemma. ■

Consider point G in Figure 10. If condition (47) holds, then point G is below schedule PMT in Figure 9; i.e., it represents a factor endowment distribution so that

[25] See Wong (1995, Chapter 2) for the proof of these results.

the home country exports good 2 under free trade. How can factor prices be equalized under free trade if factor endowments of the countries are given by point G? Such a possibility is illustrated by Figure 11. Schedules TT and T^*T^* represent the production possibility frontiers of the home and foreign countries, respectively. Points A and A^* are their autarkic points, with the home autarkic price line higher than the foreign autarkic price line, reflecting home's comparative advantage in good 2. Under free trade, the production points of the countries shift to Q and Q^*, respectively, while points C and C^* are their consumption points. Because of identical and homothetic preferences, points C and C^* are on the same line from the origin, and trade equilibrium means that line segment CQ equals line segment C^*Q^*. The diagram shows that both countries are diversified and have the same output levels of good 1 under free trade, thus implying factor price equalization.

However, it is known that for factor price equalization, both countries have to be diversified in production. This implies that generally the factor endowments of the countries should not be too different from each other; in other words, schedule FME does not exist too far away from point M. Furthermore, in terms of reallocation of factors between the countries, the countries are symmetric. This means that portion ME of schedule FME is a mirror image of portion FM. The above results are then summarized as follows:

Proposition 14 ***(Theorem of Factor Price Equalization in the Presence of External Economies of Scale)*** *Given free trade, zero transport costs, diversification, identical technologies, and equal output of good 1, the countries have the same factor prices. If it is further given that two countries have identical preferences, schedule FME in Figure 10 shows various factor endowment allocations with which the countries experience equal factor prices. It is positively sloped at least in the neighborhood around point M, and, given mild increasing returns in sector 1, is less steep than schedule PMT at point M.*

10 Concluding Remarks

In this paper, we developed a basic model of external economies of scale, and examined the validity of the five fundamental theorem in the positive theory of international trade.

We found that both the Rybczynski and Stolper-Samuelson Theorems are valid in more cases than the existing literature suggests. In particular, we showed that if global changes under the specified adjustment mechanism are allowed, both theorems are valid, whether or not the production equilibrium is stable.

We also found that the Law of Comparative Advantage and the Heckscher-Ohlin Theorem are valid under certain conditions. The validity of Factor Price Equalization Theorem, however, is more limited.

This paper also brings out some "new" points:

1. Some of the comparative static results in the theory of international trade may still be normal even if the equilibrium is unstable. This result is just an application of the Global Corresponding Principle, but the use of this principle in the theory of international trade is not common.

2. Increasing returns are not sufficient for foreign trade. As a matter of fact, the existing literature on whether increasing returns are a determinant of foreign trade is confusing. It is partly because those papers that have two factors usually focus on the cases with strong increasing returns so that the no-trade equilibrium is Marshallian unstable, and partly because there are quite a number of papers considering exclusively a one-factor model. As shown in this paper, models with one factor must have unstable no-trade equilibrium between two identical countries. This means that trade must occur between two identical economies after a small disturbance under free trade. However, this result cannot be extended to two-factor models because with more than one factors, the no-trade equilibrium could be stable, implying that trade may not occur even after a small disturbance.

The model developed in this paper, which is here called the basic model, has the crucial feature that it is the same as a neoclassical framework except that one of the sectors is subject to production externality. Making just one difference between the present model and the neoclassical model allows us to single out the roles of external economies of scale. It is special as compared with some of models in the literature, some of which consider cross-sector externality and some consider externality in both sectors. Furthermore, no international externality has been considered. However, this is a price we are willing to pay: allowing cross-sector externality, externality in both sectors, and international externality could "contaminate" the effects of external economies of scale.[26]

Of course it is interesting to note that the present model is still more general than the one-factor models, which have been used in various forms in many papers. Our model reduces to the one-factor model, and has been used to point out some of the special implications of the one-factor models.

[26] See Wong (2000a) for some extensions of the present model.

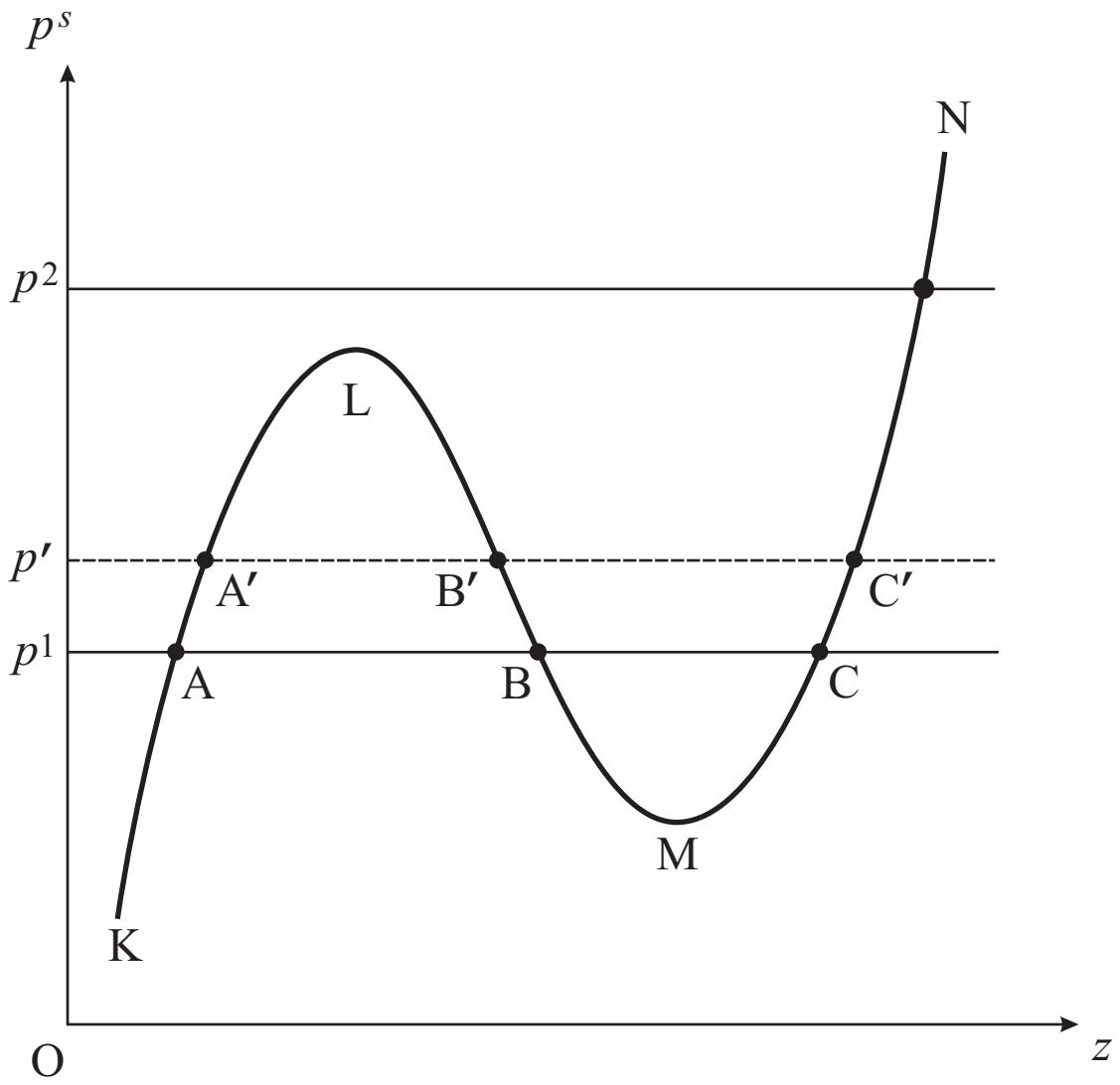

Figure 1

Price-Output Response

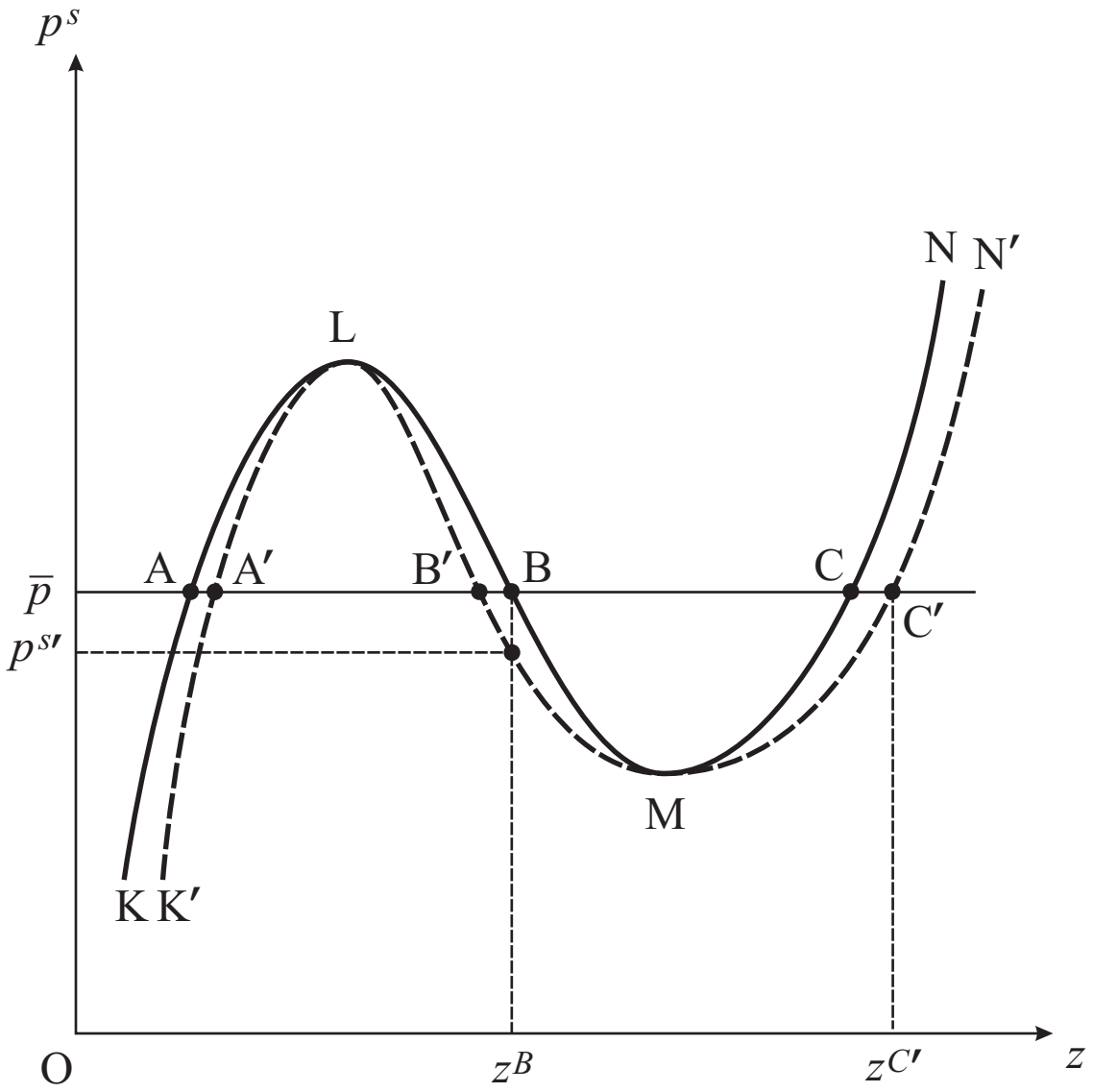

Figure 2

Capital-Output Response

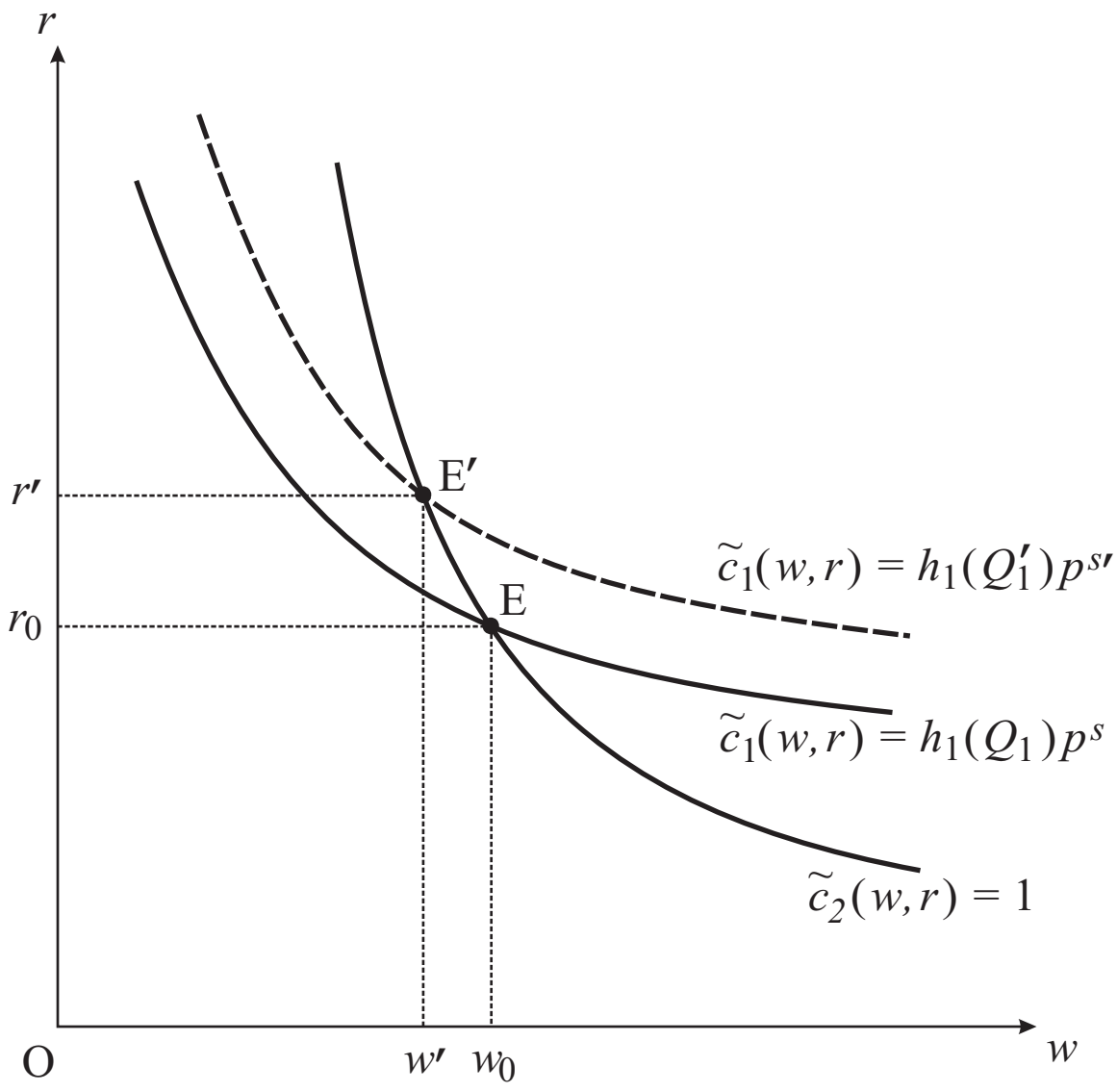

Figure 3

Proving the Stolper-Samuelson Theorem

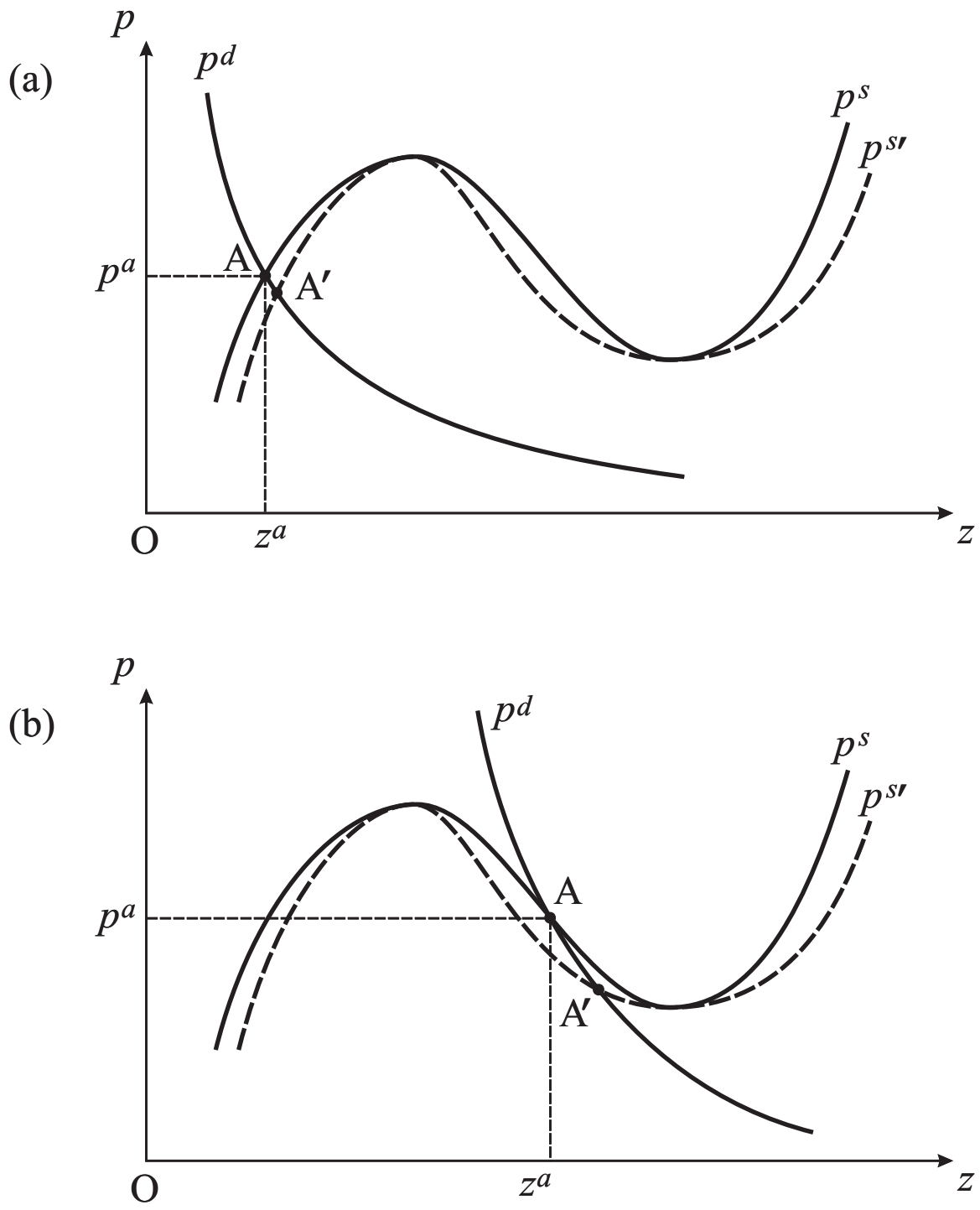

Figure 4

Stability of the Autarkic Equilibrium

(c)

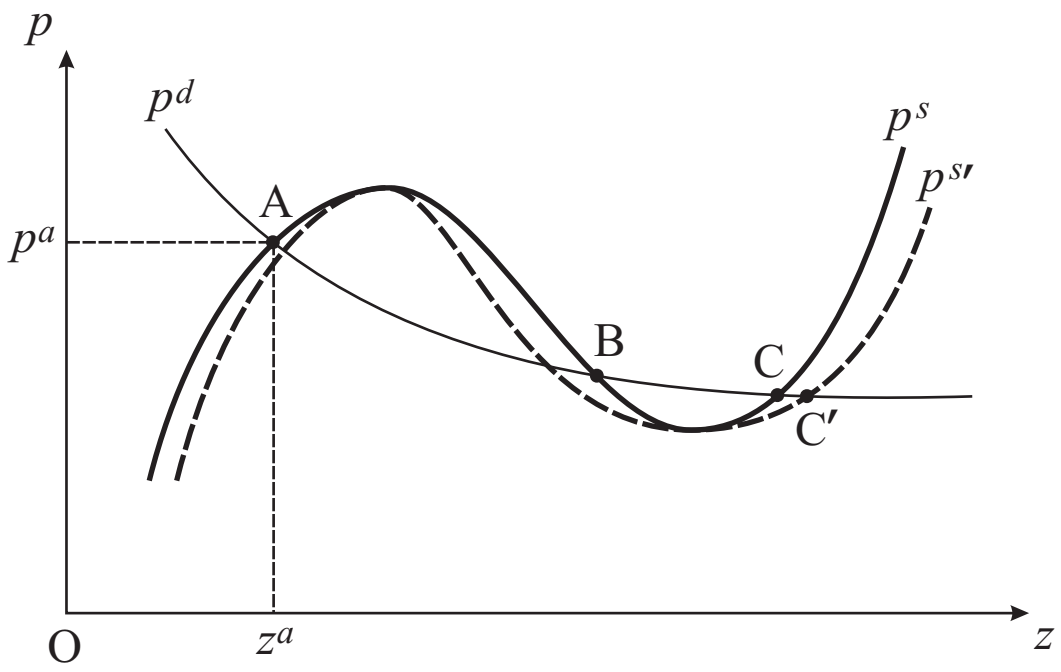

Figure 4

Stability of the Autarkic Equilibrium

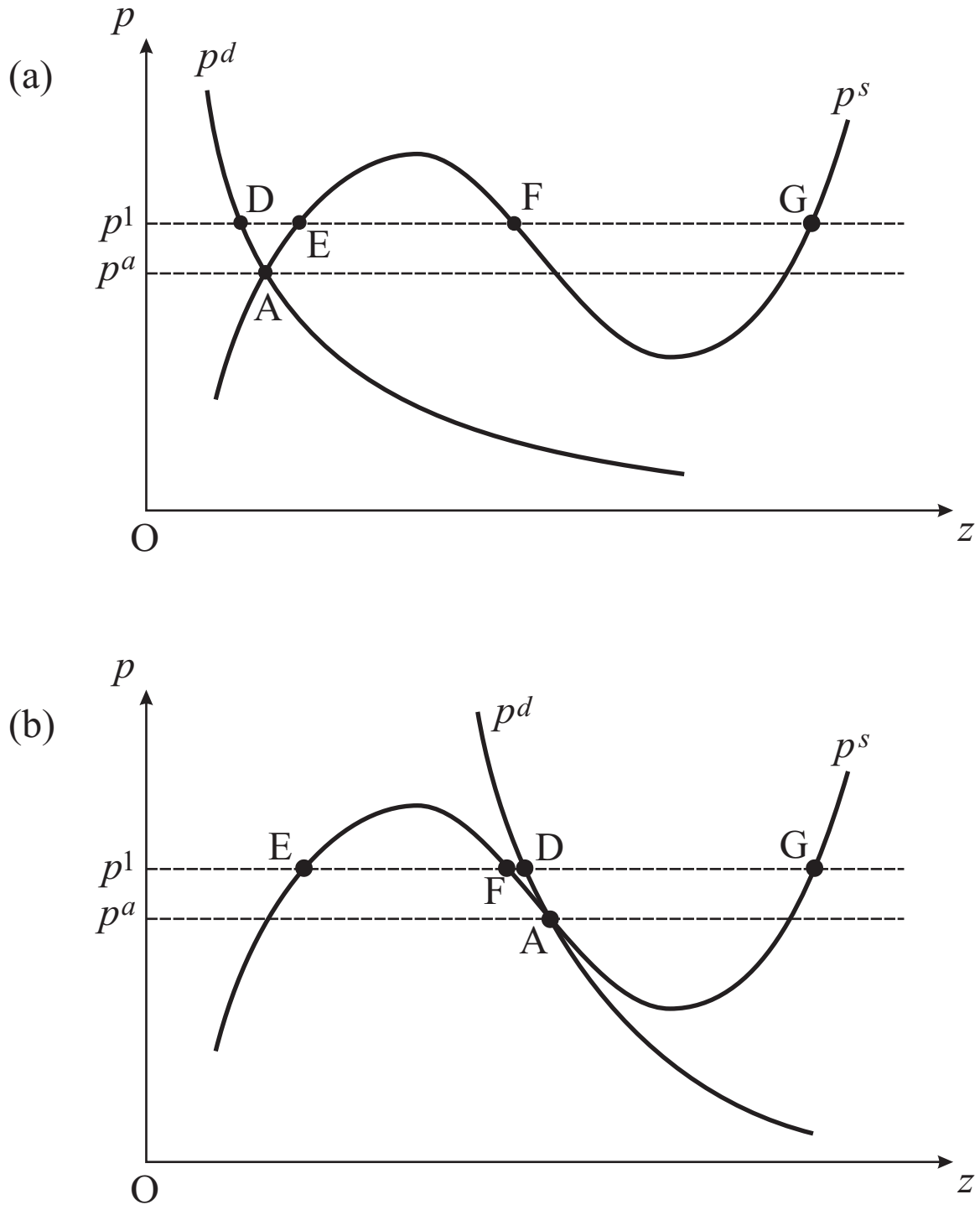

Figure 5

Derivation of the Offer Curve

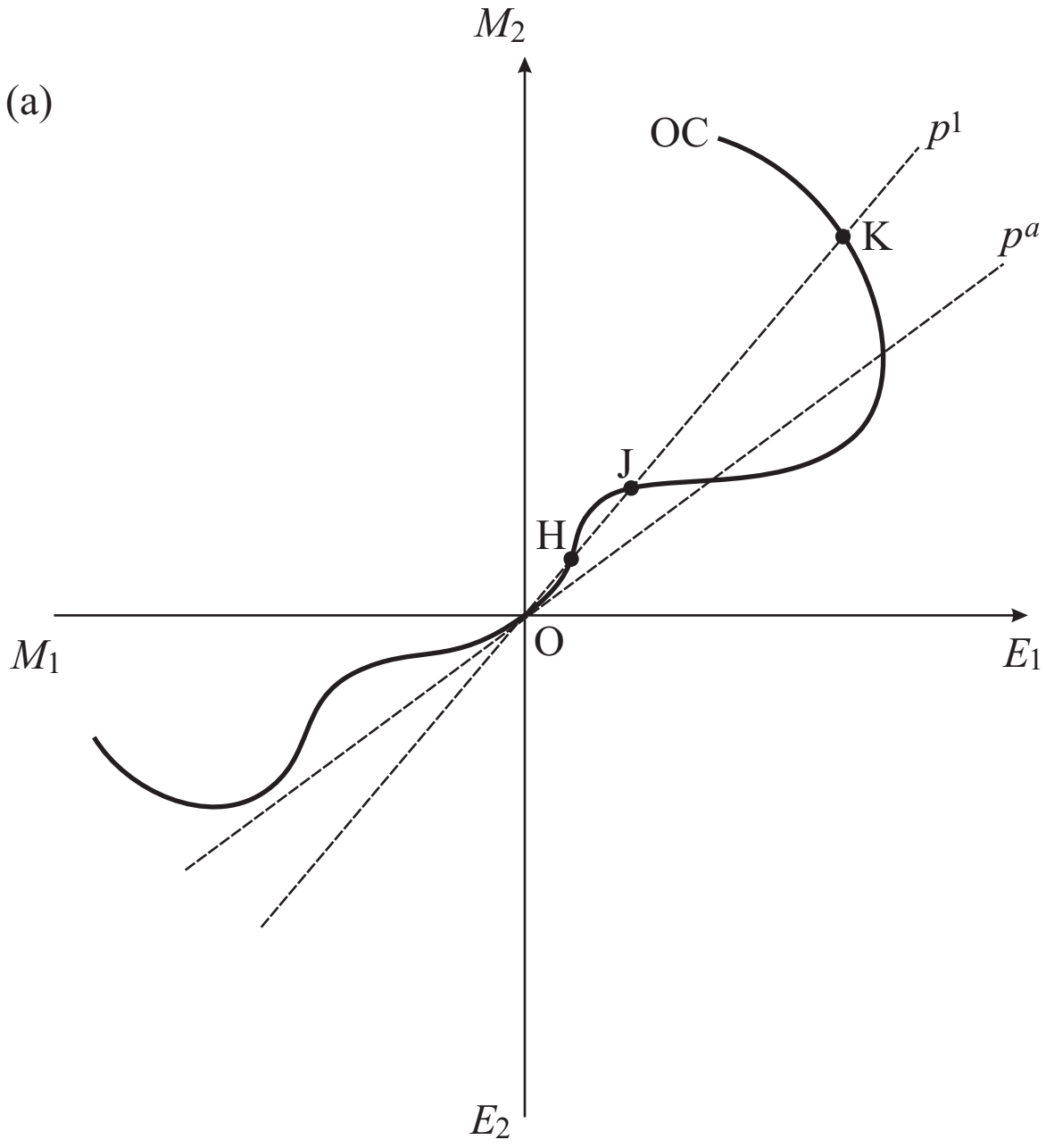

Figure 6

Offer Curve of the Economy

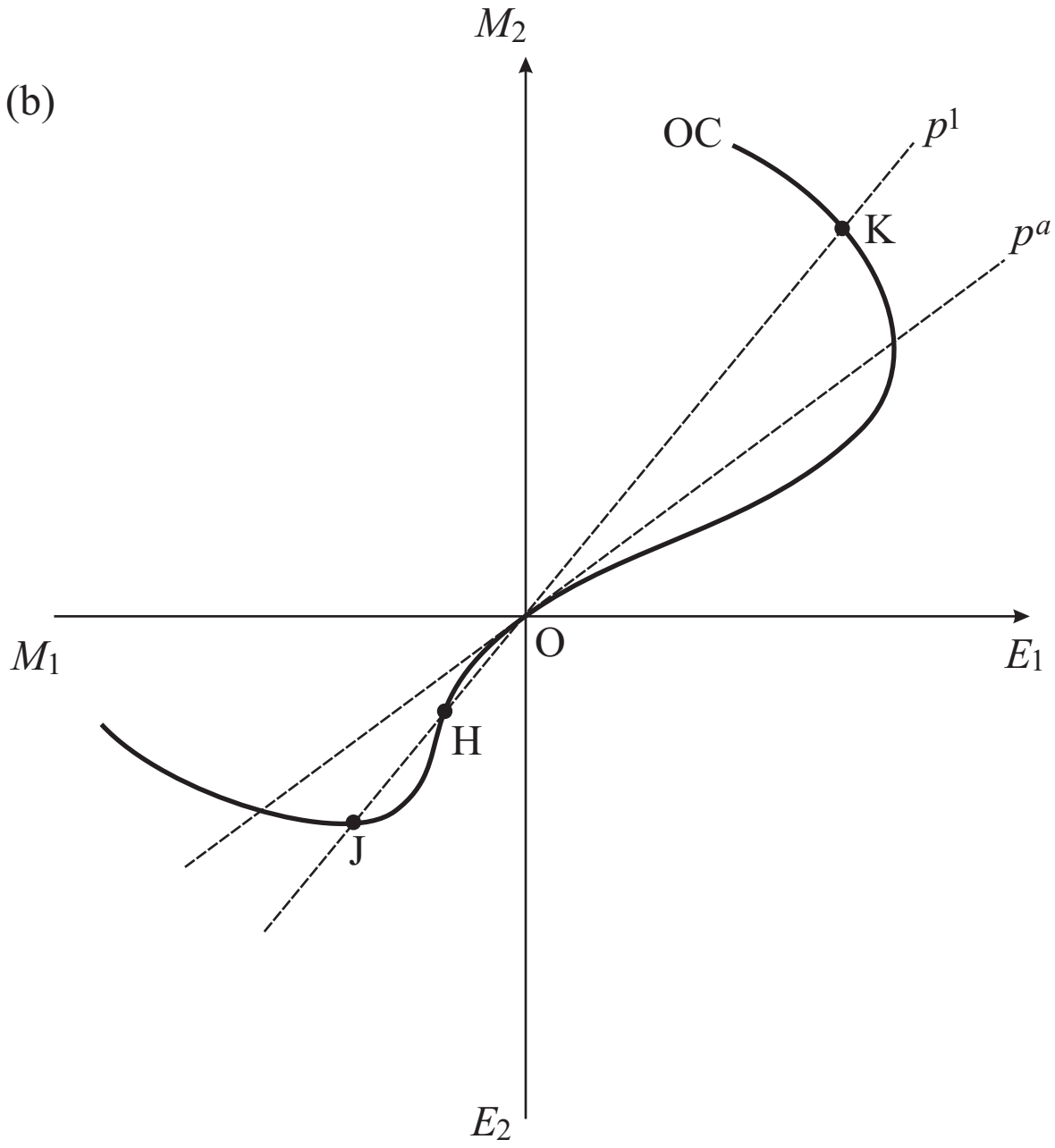

Figure 6

Offer Curve of the Economy

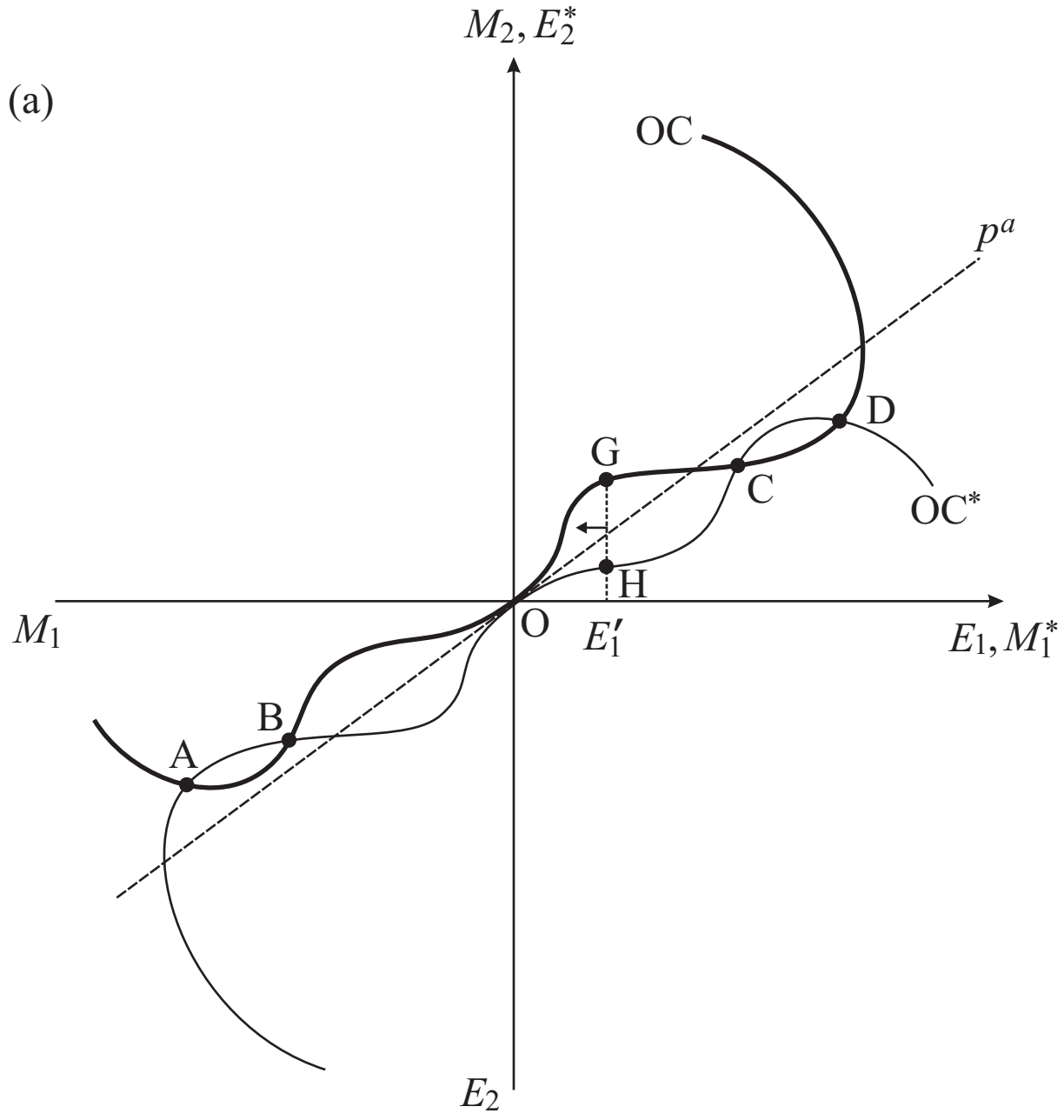

Figure 7

Stability of A Trading Point

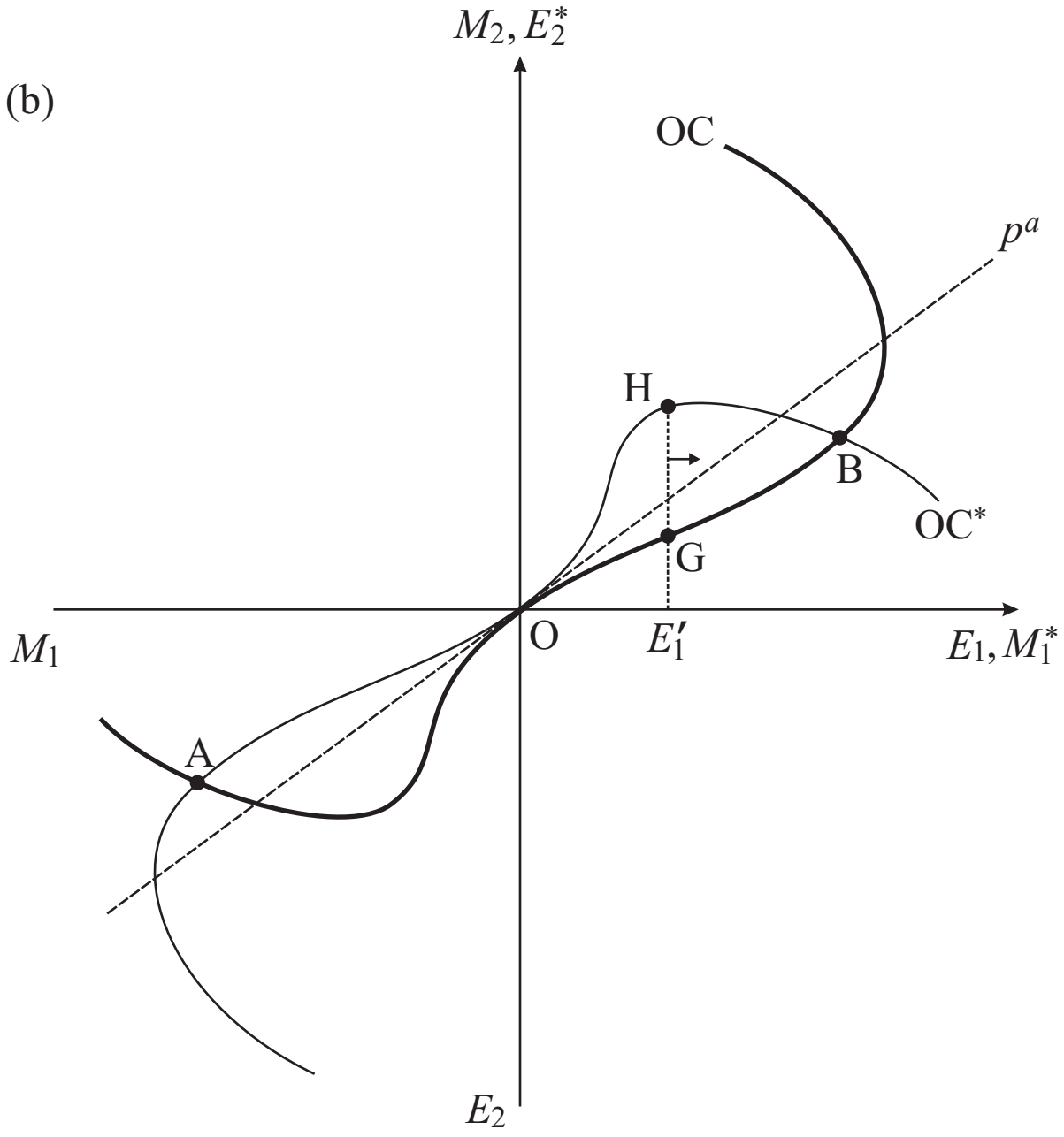

Figure 7

Stability of A Trading Point

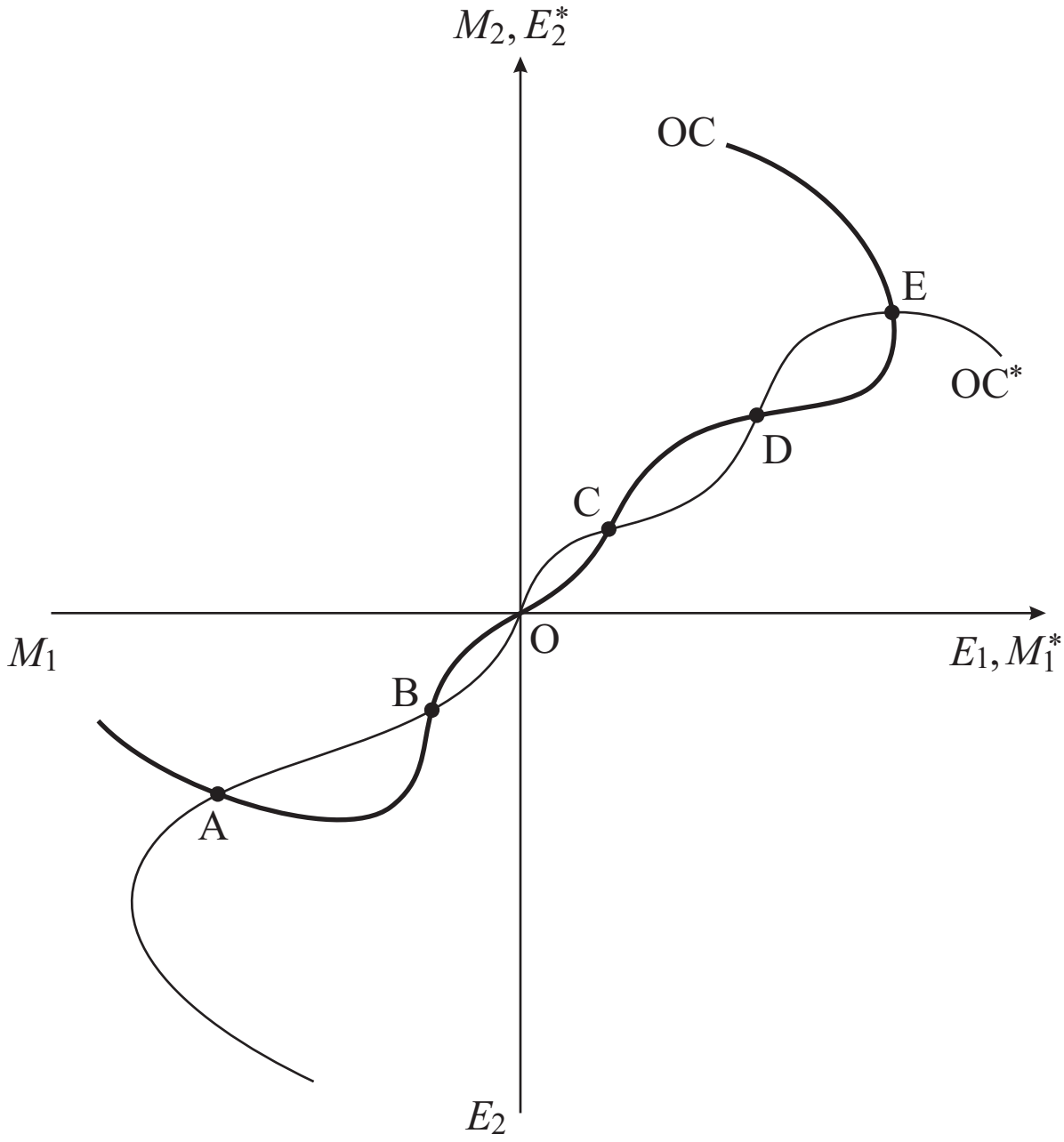

Figure 8

Law of Comparative Advantage

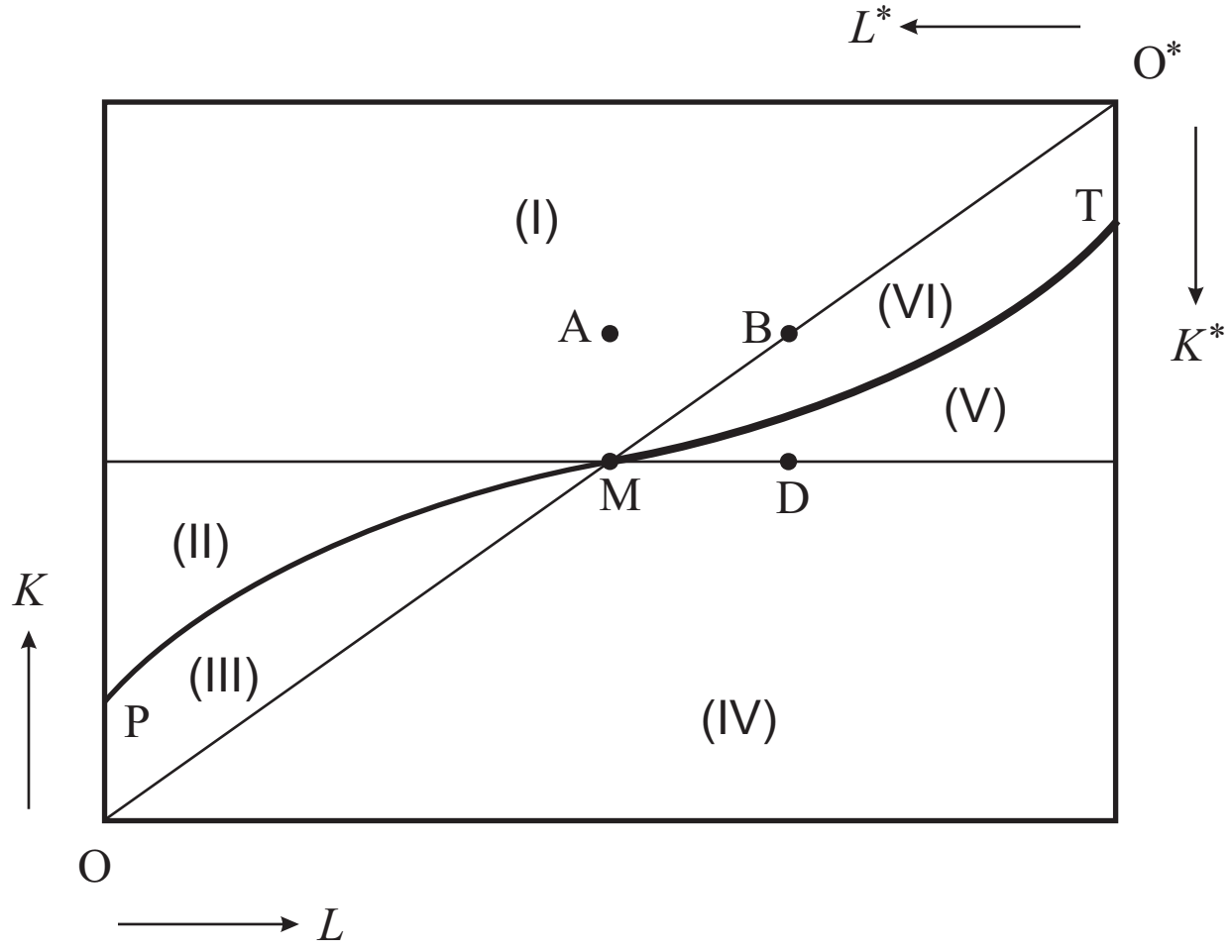

Figure 9

Box Diagram of Factor Endowments

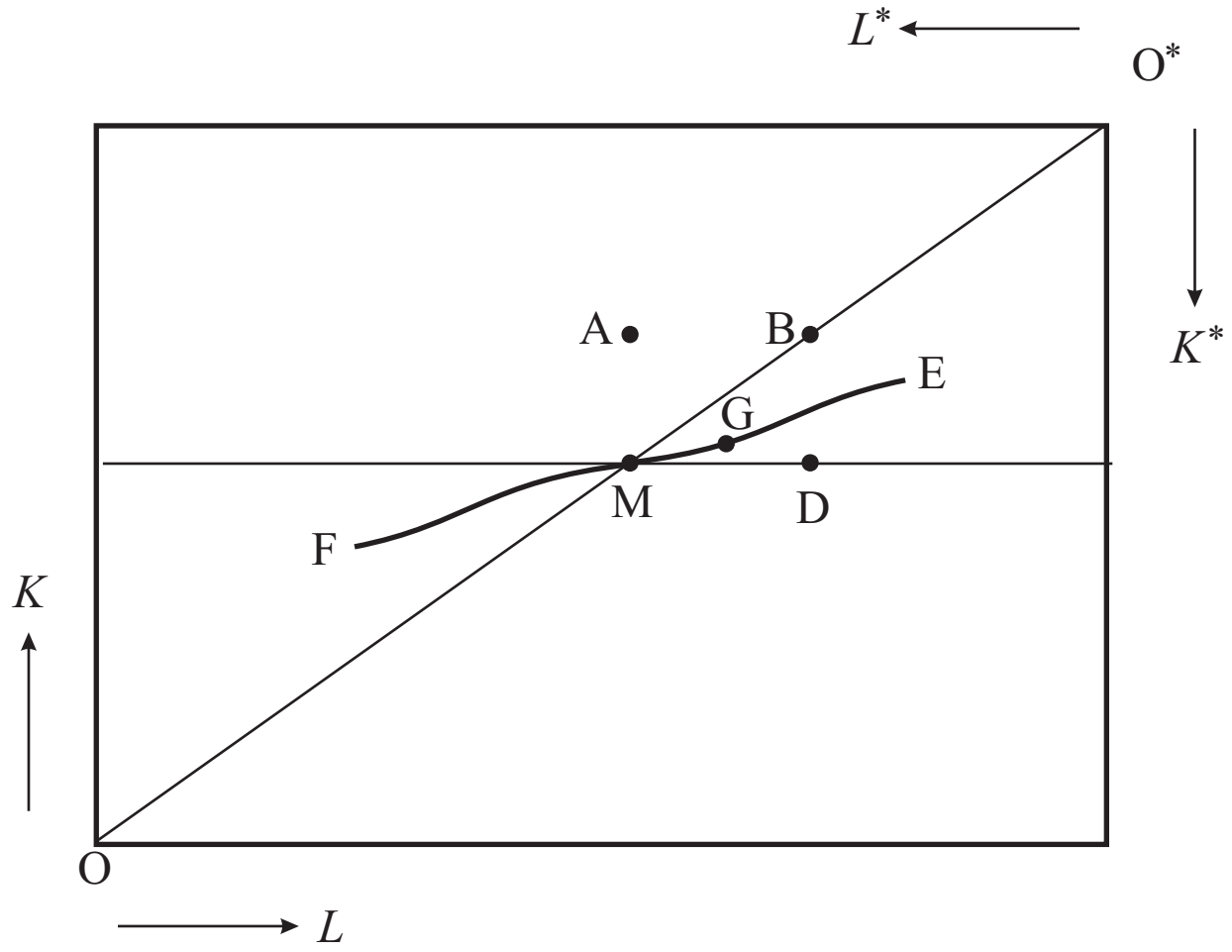

Figure 10

Factor Price Equalization Schedule

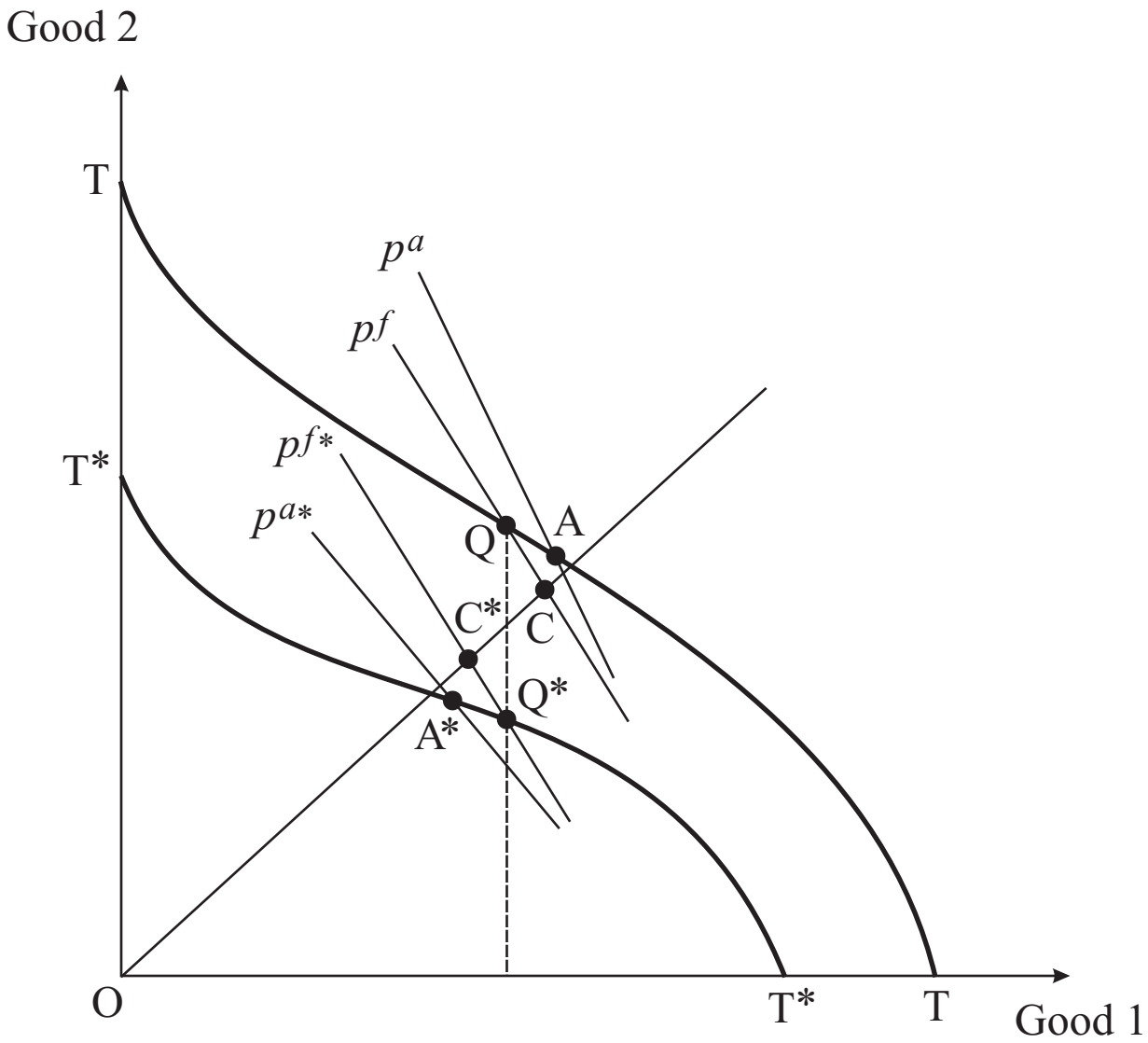

Figure 11

Factor Price Equalization under Free Trade

References

[1] Chacholiades, Miltiades (1978), *International Trade Theory and Policy*, New York: McGraw-Hill, Inc.

[2] Chang, Winston W. (1981), "Production Externalities, Variable Returns to Scale, and the Theory of Trade," *International Economic Review*, 22 (3): 511–525.

[3] Ethier, Wilfred (1979), "Internationally Decreasing Costs and World Trade," *Journal of International Economics*, 9: 1–24.

[4] Ethier, Wilfred (1982), "Decreasing Costs in International Trade and Frank Graham's Argument for Protection," *Econometrica*, 50 (5): 1243–1268.

[5] Helpman, Elhanan (1984), "Increasing Returns, Imperfect Markets, and Trade Theory," in Ronald W. Jones and Peter B. Kenen (eds.), *Handbook of International Economics*, Vol. I, Amsterdam: Elsevier Science Publishers, 325–365.

[6] Helpman, Elhanan and Paul R. Krugman (1985), *Market Structure and Foreign Trade*, Cambridge, Mass.: MIT Press.

[7] Ide, Toyonari and Akira Takayama (1991), "Variable Returns to Scale, Paradoxes, and Global Correspondence in the Theory of International Trade," in Akira Takayama, Michihiro Ohyama, and Hiroshi Ohta (eds.), *Trade, Policy, and International Adjustments*, San Diego: Academic Press, Inc.: 108–154.

[8] Ide, Toyonari and Akira Takayama (1993), "Variable Returns to Scale, Comparative Statics Paradoxes, and the Theory of Comparative Advantage," in Horst Herberg and Ngo Van Long (eds.), *Trade, Welfare, and Economic Policies: Essays in Honor of Murray C. Kemp*, Ann Arbor: University of Michigan Press: 67–119.

[9] Inoue, Tadashi (1981), "A Generalization of the Samuelson Reciprocity Relations, the Stolper-Samuelson Theorem and the Rybczynski Theorem under Variable Returns to Scale," *Journal of International Economics*, 11: 79–98.

[10] Jones, Ronald W. (1968), "Variable Returns to Scale in General Equilibrium Theory," *International Economic Review*, 9: 261–272.

[11] Kemp, Murray C. (1969), *The Pure Theory of International Trade and Investment*, Englewood Cliffs, N.J.: Prentice-Hall, Inc.

[12] Kemp, Murray C. and Albert G. Schweinberger (1991), "Variable Returns to Scale, Non-Uniqueness of Equilibrium and the Gains from International Trade," *Review of Economic Studies*, 807–816.

[13] Kemp, Murray C. and Koji Shimomura (1995), "The Apparently Innocuous Representative Agent," *Japanese Economic Review*, 46 (3): 247-256.

[14] Krugman, Paul R. (1987), "Increasing Returns and the Theory of International Trade," in Truman F. Bewley (ed.) *Advanced in Economic Theory, Fifth World Congress*, Cambridge: Cambridge University Press, 301–328.

[15] Krugman, Paul R. (1994), "Empirical Evidence on the New Trade Theories: The Current State of Play," in *New Trade Theories: A Look at the Empirical Evidence*, a CEPR report.

[16] Markusen, James R. and James R. Melvin (1981), "Trade, Factor Prices, and the Gains from Trade with Increasing Returns to Scale," *Canadian Journal of Economics*, 14 (3): 450–469.

[17] Marshall, Alfred (1879), *Pure Theory of Foreign Trade*, reprinted by London School of Economics and Political Science, 1930.

[18] Marshall, Alfred (1890), *Principles of Economics*, 8th edition, London: Macmillan, 1920.

[19] Melvin, James R. (1969), "Increasing Returns to Scale As A Determinant of Trade," *Canadian Journal of Economics*, 2: 389–402.

[20] Panagariya, Arvind (1980), "Variable Returns to Scale in General Equilibrium Theory Once Again," *Journal of International Economics*, 10: 499–526.

[21] Panagariya, Arvind (1981), "Variable Returns to Scale in Production and Patterns of Specialization," *American Economic Review*, 71 (1): 221–230.

[22] Samuelson, Paul A. (1947), *Foundation of Economic Analysis*, Cambridge, Mass.: Harvard University Press.

[23] Samuelson, Paul A. (1971), "On the Trail of Conventional Beliefs about the Transfer Problem," in Jagdish N. Bhagwati *et al.* (eds.), *Trade, Balance of Payments and Growth: Papers in International Economics in Honor of Charles P. Kindleberger*, Amsterdam: North-Holland, 327–351.

[24] Shimomura, Koji and Kar-yiu Wong (1998). "The Law of Comparative Advantage without Social Utility Function," *Review of International Economics*, 6: 401-406.

[25] Tawada, Makoto (1989), *Production Structure and International Trade*, Berlin: Springer-Verlag.

[26] Wong, Kar-yiu (1995), *International Trade in Goods and Factor Mobility*, Cambridge, Mass.: MIT Press.

[27] Wong, Kar-yiu (2000a), "Externality in the Theory of International Trade: Some Basic Concepts," *mimeo.*, University of Washington.

[28] Wong, Kar-yiu (2000b), "External Economies of Scale and International Trade: Further Analysis," *mimeo.*, University of Washington.

[29] Wong, Kar-yiu (2000c), "International Trade and Factor Mobility under External Economies of Scale," *mimeo.*, University of Washington.