Curvature of the Production Possibility Frontier under External Economies of Scale: Was J. Tinbergen Wrong?

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Abstract

This paper challenges the current view in the economics literature that J. Tinbergen was wrong in drawing the production possibility frontier (PPF) of an economy strictly convex (concave) to the origin when the output of a decreasing-returns (increasing-returns) sector is sufficiently small. This paper provides a systematic analysis of the curvature of the PPF of an economy under external economies, and shows that a PPF with the Tinbergen curvature can exist for some classes of economies.

1. Introduction

In the economics literature, it is a common practice to introduce the existence of external economies of scale (or variable returns to scale) as a way to relax the assumption of constant-returns technologies while keeping the condition of perfect competition. Such type of externality, which is frequently called Marshallian externality, can be found in economic fields such as international trade and public finance. See, for example, three recent surveys by Helpman (1984), Krugman (1987), and Wong (1995, Chapter 5).

Despite numerous efforts and developments, there still remain controversies and open questions related to economies that are subject to external economies. One issue that has kept the interest of economists for a long time is the curvature of the production possibility frontier (PPF) of such an economy.

One of the earliest economists who analyzed how externality may affect the curvature of an economy's PPF is J. Tinbergen. Tinbergen (1945, 1954) considers an economy with two goods which are produced by one single factor, labor, with sector 1 being subject to increasing returns to scale (IRS) and sector 2 subject to decreasing returns to scale (DRS). He illustrates the PPF of such an economy in a simple diagram (Tinbergen, 1945, p. 192; 1954, p. 181), which is drawn here as Figure 1.[1,2] The main feature of his diagram is that the PPF is strictly convex to the origin near the good-1 (IRS) axis and strictly concave to the origin near the good-2 (DRS) axis. Let us call such a shape of the PPF near the axes the *Tinbergen curvature*.

For some time since Tinbergen's work, it has been a common practice to draw PPF of an economy under external economies with the Tinbergen curvature.[3]

In 1969, Herberg and Kemp (1969) derive conditions for economies in the presence of external economies that have PPF strictly convex (resp. concave) to the origin when the output of an IRS (resp. DRS) sector approaches zero, irrespective of the returns to scale in the other sector. Such a curvature of a PPF, which is here called the *Herberg-Kemp curvature*, is opposite to the Tinbergen curvature. This result is a strong one in the sense that in the cases analyzed the curvature of the PPF in the relevant region depends only on the returns to scale of the sector whose output approaches zero but not on those of other sectors.

In 1981, Panagariya (1981) develops a very simple external-economies framework with two sectors and one factor. Assuming homogeneous production functions, he shows that the PPF of the economy is strictly convex to the origin when the output of the IRS sector is small, and strictly concave to the origin when the output of the DRS sector is small. His diagram, which is redrawn in Figure 2, is completely opposite to that of Tinbergen. (We reverse the axes of the Panagariya diagram so that it is easier to compare it with the Tinbergen diagram.)

Applying, incorrectly, the Herberg-Kemp proposition,[4] Panagariya criticizes Tinbergen's diagram and argues that the Tinbergen diagram is wrong (Panagariya, 1981, p. 226).[5] This criticism is echoed in Helpman's authoritative survey (Helpman, 1984): "Starting from Tinbergen (1945), it became *a common mistake* to draw this curve with reversed concavity-convexity properties." (Helpman, 1984, p. 343, footnote 21, present emphasis.)

It is no doubt that the Panagariya-Helpman criticism is now widely accepted and that the Tinbergen diagram is regarded as wrong. Clearly the pendulum of the profession has swung from totally accepting the Tinbergen diagram to totally rejecting his diagram.

But, has the pendulum swung too far?

Yes, it has, argues the present paper: There do exist some economies subject to external economies that have a PPF with the Tinbergen curvature. This means that Figure 1 is compatible with some classes of economies with an IRS sector 1 and a DRS (or CRS) sector 2. The main conclusion of this paper is that Tinbergen was not wrong.

To draw this conclusion, this paper begins with an in-depth analysis of the curvature of the PPF of an economy subject to external economies. It then derives conditions for economies characterized by a PPF with the Herberg-Kemp curvature, and conditions for economies characterized by a PPF with the Tinbergen curvature. Some numerical examples are presented to show that a PPF with either type of curvature is possible. This paper concludes with a discussion of why we should be interested in whether an economy has a PPF with the Herberg-Kemp or Tinbergen curvature.

Section 2 of this paper presents the model and an approach to derive the curvature of the PPF of an economy. Sections 3 and 4 examine conditions for the Herberg-Kemp curvature and the Tinbergen curvature, respectively. Conditions for and examples of PPFs with these curvatures are presented. Section 5 provides some concluding remarks and explains why we would be interested in knowing whether the curvature of the PPF of an economy under external economies has the Herberg-Kemp or Tinbergen curvature.

2. A Model of External Economies

Consider an m-factor, n-sector framework, $m \geq 1$ and $n \geq 2$. Because the PPF is the focus of this paper, it is convenient to limit the present analysis to a two-sector model, that is, $n = 2$. Some of the factors may be sector specific, but at least one of them is mobile between sectors.[6] Following the tradition of the literature, we assume output-generated external effects and homothetic production functions. Let the production function of sector i, $i = 1, 2$, be

$$Q_i = h_i(Q_i)\widetilde{F}_i(\mathbf{v}_i), \tag{1}$$

where Q_i is the output, and $\mathbf{v}_i$ is the vector of m factor inputs. Function $\widetilde{F}_i(\mathbf{v}_i)$ behaves like a neoclassical production function: twice differentiable, linearly homogeneous and concave in factor inputs. Function $h_i(Q_i)$, where $h_i(Q_i) > 0$ for all $Q_i > 0$, is perceived by each firm as constant. From the society's point of view, function $h_i(Q_i)$ represents the external economies of scale, and if its derivative, $h_i'(Q_i)$, is positive (negative, or zero), then the sector is subject to increasing (decreasing, or constant) returns to scale, i.e. IRS (DRS, or CRS). Taking prices as given, firms are competitive.[7]

The rate of variable returns to scale (VRS), ε_i, for sector i is defined to be the elasticity of function $h_i(Q_i)$, i.e.,

$$\varepsilon_i \equiv \frac{Q_i}{h_i(Q_i)} \frac{\mathrm{d}h_i(Q_i)}{\mathrm{d}Q_i} = h_i'(Q_i)\widetilde{F}_i.$$

The sign of ε_i is the same as that of $h_i'(Q_i)$, meaning that it is positive if and only if the production function exhibits IRS. To have positive social marginal products of factors in sector i, it is assumed that $\varepsilon_i < 1$.

Denoting the price of good i by p_i, define $\widetilde{Q}_i \equiv Q_i/h_i(Q_i)$ and $\widetilde{p}_i \equiv p_i h_i(Q_i)$, which are called the virtual output and virtual price, respectively. A GDP (gross domestic product) function of the economy can be defined as follows:

$$g(\widetilde{p}_1, \widetilde{p}_2, \mathbf{v}) = \max_{\widetilde{Q}_1, \widetilde{Q}_2, \{\mathbf{v}_i\}} \widetilde{p}_1\widetilde{Q}_1 + \widetilde{p}_2\widetilde{Q}_2 \quad \text{s.t.} \quad \widetilde{F}_i(\mathbf{v}_i) \geq \widetilde{Q}_i, \quad i = 1, 2,$$
$$\mathbf{v}_1 + \mathbf{v}_2 \leq \mathbf{v}, \tag{2}$$

where $\mathbf{v}$ is the vector of factor endowments in the economy.

Function $g(\widetilde{p}_1, \widetilde{p}_2, \mathbf{v})$ as defined by condition (1) behaves, in terms of $\widetilde{p}_1$, $\widetilde{p}_2$ and $\mathbf{v}$, like the traditional GDP function. The derivative of the function with respect to $\widetilde{p}_i$ is equal to

$$\frac{\partial g}{\partial \widetilde{p}_i} = \widetilde{Q}_i(\widetilde{p}_1, \widetilde{p}_2, \mathbf{v}). \tag{3}$$

Using the definition of $\widetilde{Q}_i$, we have

$$Q_i = h_i(Q_i)\widetilde{Q}_i(\widetilde{p}_1, \widetilde{p}_2, \mathbf{v}) \qquad i = 1, 2. \tag{3$'$}$$

(3′) represents a system of 2 equations. When commodity prices and factor endowments are given, this system of equations can be solved for the outputs, although the solution may not be unique.

In our analysis, it is convenient to distinguish between the *real system* which is represented by $(\{Q_i\}, \{p_i\})$, and the *virtual system* which is represented $(\{\widetilde{Q}_i\}, \{\widetilde{p}_i\})$. Since function $\widetilde{F}_i(\mathbf{v}_i)$, $i = 1, 2$, behaves like a traditional production function, the virtual system is similar to the neoclassical framework.

Totally differentiate both sides of $Q_i = h_i\widetilde{Q}_i$ to give

$$\mathrm{d}Q_i = \widetilde{Q}_i h_i' \mathrm{d}Q_i + h_i \mathrm{d}\widetilde{Q}_i = \frac{h_i}{(1-\varepsilon_i)} \mathrm{d}\widetilde{Q}_i. \tag{4}$$

Let us define $\psi_i = h_i/(1-\varepsilon_i) > 0$. Condition (4) can be written alternatively as

$$\mathrm{d}Q_i = \psi_i \mathrm{d}\widetilde{Q}_i. \tag{4$'$}$$

Condition (4′) can be used to determine the marginal rate of transformation, MRT, which is equal to

$$\frac{\mathrm{d}Q_2}{\mathrm{d}Q_1} = \frac{\psi_2}{\psi_1}\frac{\mathrm{d}\widetilde{Q}_2}{\mathrm{d}\widetilde{Q}_1} = \frac{h_2(1-\varepsilon_1)}{h_1(1-\varepsilon_2)}\frac{\mathrm{d}\widetilde{Q}_2}{\mathrm{d}\widetilde{Q}_1}. \tag{5}$$

Because the virtual system behaves like a neoclassical framework, the set of feasible $(\widetilde{Q}_1, \widetilde{Q}_2)$ is convex and the frontier of this set is negatively (or non-positively) sloped. Let us denote the virtual production possibility frontier (PPF) by $\widetilde{Q}_2 = T(\widetilde{Q}_1)$, $T' \le 0$ and $T'' \le 0$. The MRT given by (5) reduces to

$$\frac{\mathrm{d}Q_2}{\mathrm{d}Q_1} = \frac{\psi_2}{\psi_1}T'. \tag{5$'$}$$

We make the following assumption:

Condition A: $-T'$ has positive, finite lower and upper bounds, and $-T''$ has a finite upper bound.

By (5′) and condition A, the real PPF (or simply PPF) of the present economy under external economies of scale is negatively sloped.

To determine the curvature of the PPF, totally differentiate both sides of (5′) to give[8]

$$\begin{aligned} \frac{\mathrm{d}^2 Q_2}{\mathrm{d}{Q_1}^2} &= T'' \frac{\psi_2}{\psi_1} \frac{\mathrm{d}\widetilde{Q}_1}{\mathrm{d}Q_1} + T' \frac{\psi_1 \psi_2' (\mathrm{d}Q_2/\mathrm{d}Q_1) - \psi_2 \psi_1'}{{\psi_1}^2} \\ &= \frac{\psi_2}{{\psi_1}^2} \Big[T'' + \psi_2' (T')^2 - \psi_1' T' \Big]. \end{aligned} \tag{6}$$

To determine the sign of $\mathrm{d}^2 Q_2/\mathrm{d}{Q_1}^2$, we have to evaluate ψ_i'. First note that

$$\varepsilon_i' = \frac{h_i(h_i' + Q_i h_i'') - Q_i (h_i')^2}{{h_i}^2} = \frac{h_i'(1-\varepsilon_i) + Q_i h_i''}{h_i}.$$

Using this condition, we have

$$\psi_i' = \frac{h_i'(1-\varepsilon_i) + h_i \varepsilon_i'}{(1-\varepsilon_i)^2} = \frac{2h_i'(1-\varepsilon_i) + Q_i h_i''}{(1-\varepsilon_i)^2}. \tag{7}$$

The value of ψ_i' as given by (7) is substituted into (6) to determine the curvature of the PPF. In general, the sign of ψ_i' is ambiguous, implying that the curvature of the PPF is ambiguous, too.

3. The Herberg-Kemp Curvature

Let us consider the following condition:

Condition HK1: As Q_i approaches zero, ψ_i' approaches ∞ (resp. $-\infty$) while ψ_j' remains finite, $j \neq i$, if $\varepsilon_i >$ (resp. $<$) 0.

Condition HK2: As Q_i approaches zero, ψ_j' approaches ∞ (resp. $-\infty$) while ψ_i' remains finite, $j \neq i$, if $\varepsilon_i >$ (resp. $<$) 0.

These conditions are used to give the following results:

Proposition 1. (a) Given conditions A and HK1, the PPF has the Herberg-Kemp curvature. (Herberg and Kemp, 1969.)
(b) Given conditions A and HK2, the PPF has the Herberg-Kemp curvature.

Proposition 1 can be proved easily by making use of condition (6). Given condition HK1, when $Q_i \to 0$, ψ_i' is so significant in magnitude that it dominates ψ_j', $j \neq i$, and by condition A, $\mathrm{d}^2 Q_2 / \mathrm{d}{Q_1}^2$ has the same sign as ψ_i' and ε_i. This implies that the PPF has the Herberg-Kemp curvature. A similar argument can be used to show that such a curvature exists if condition HK2 instead of condition HK1 is assumed.[9]

We now have to determine what types of economies would satisfy condition HK1 or HK2. We first consider condition HK1. Suppose that the production function of sector i as described by (1) is defined in such a way that $h_i(Q_i) = {Q_i}^{\varepsilon_i}$, where ε_i is a constant. The production function is then said to be homogeneous in factor inputs. Since ε_i is the elasticity of $h_i(Q_i)$, it is equal to the rate of VRS and is constant for the assumed technology. Differentiation of $h_i(Q_i) = {Q_i}^{\varepsilon_i}$ gives

$$h_i' = \varepsilon_i Q_i^{\varepsilon_i - 1}$$
$$h_i'' = -\varepsilon_i (1 - \varepsilon_i) Q_i^{\varepsilon_i - 2}.$$

Using these two conditions, condition (7) reduces to

$$\psi_i' = \frac{\varepsilon_i}{(1 - \varepsilon_i) Q_i^{1 - \varepsilon_i}}. \tag{8}$$

Condition (8) shows that ψ_i' has the same sign as ε_i, i.e., ψ_i' is positive if and only if sector i exhibits IRS. We now have the following result:

Lemma 1. If sector i's rate of VRS is constant, then condition HK1 is satisfied for that sector.

Lemma 1 can be proved by using equation (8). With constant rates of VRS, when $Q_i \to 0$, $\psi_i' \to \infty$ $(-\infty)$ if sector i is subject to IRS (DRS), while ψ_j' is finite, $j \neq i$. This gives condition HK1.[10] This lemma can be combined with Proposition 1 (a) to give:

Proposition 2. Given condition A, if both sectors' production functions are homogeneous, then the PPF has the Herberg-Kemp curvature.

Models with externality and homogeneous production functions appear in many papers: for example, Ethier (1979, 1980), Panagariya (1981), Helpman (1984), and Markusen and Melvin (1981, 1984). In the models of Panagariya, Ethier and Helpman, there is only one factor. Thus the virtual system is a Ricardian one and condition A is necessarily satisfied; $T' < 0$ and $T'' = 0$. This implies that the PPF must have the Herberg-Kemp curvature. When there are more than one (mobile) factors in the economy, as in the model of Markusen and Melvin, condition A is not necessarily true and the PPF may not have the Herberg-Kemp curvature.

While it is recognized that condition HK1 (plus condition A) is sufficient for a PPF with the Herberg-Kemp curvature, condition HK2 has received much less attention, if any. We now examine the kind of economies that would satisfy this condition. Suppose that $h_i = b_i^{-1} \exp[b_i Q_i - 1]$, $i = 1, 2$, where $b_i > 0$. Choose the units of factors/goods so that $\widetilde{F}_i(\mathbf{v}) = 1$, where $\mathbf{v}$ is the vector of given factor endowments. The production function as given by (1) implies that the maximum output of good i (when only this good is produced) is equal to $1/b_i$. The derivatives of h_i are $h_i' = \exp[b_i Q_i - 1]$ and $h_i'' = b_i \exp[b_i Q_i - 1]$. The rate of VRS is equal to $\varepsilon_i = b_i Q_i > 0$, meaning that both sectors are subject to IRS. By condition (7), ψ_i' is equal to

$$\psi_i' = \frac{2h_i'(1 - \varepsilon_i) + Q_i h_i''}{(1 - \varepsilon_i)^2} = \frac{(2 - b_i Q_i) \exp[b_i Q_i - 1]}{(1 - b_i Q_i)^2}. \tag{9}$$

Lemma 2. If $h_i = b_i^{-1} \exp[b_i Q_i - 1]$, $i = 1, 2$, where $b_i > 0$, then condition HK2 is satisfied for both sectors.

Lemma 2 can be proved by noting that condition (9) implies that as $Q_i \to 0$ (and as $Q_j \to 1/b_j$, $i \neq j$), (a) $\psi_j' \to \infty$; and (b) $\psi_i' \to 2 \exp[-1]$. The lemma and Proposition 1 are used to give the following proposition:

Proposition 3. Given condition A, if $h_i = b_i^{-1} \exp[b_i Q_i - 1]$, $i = 1, 2$, where $b_i > 0$, then the PPF of the economy has the Herberg-Kemp curvature.

4. The Tinbergen Curvature

We now relax the assumption that the rates of VRS are constant, and make no assumption about condition HK1 or HK2. Thus the Herberg-Kemp curvature may not exist. We need to determine whether a PPF may have the Tinbergen curvature. Consider the following two conditions:

Condition J1: As Q_i approaches zero, ψ_i' approaches $-\infty$ (resp. ∞) while ψ_j' remains finite, $j \neq i$, if $\varepsilon_i >$ (resp. $<$) 0.

Condition J2: As Q_i approaches zero, ψ_j' approaches $-\infty$ (resp. ∞) while ψ_i' remains finite, $j \neq i$, if $\varepsilon_i >$ (resp. $<$) 0.

These two conditions look similar to conditions HK1 and HK2 but produce different results.

Proposition 4. Given conditions A and J1 or J2, the PPF of an economy under external economies has the Tinbergen curvature.

Proposition 4 can be proved along the same line as that of Proposition 1. All of these propositions depend on the fact that if the relevant ψ_i' is infinite with a particular sign while ψ_j' is finite, $i \neq j$, then it dominates ψ_j' and determines the sign of the second-order-derivative term in condition (6) and thus the curvature of the PPF.

The more difficult part of the present analysis is to find out the types of technologies of an economy that imply a PPF with the Tinbergen curvature. This is what we now turn to.

Suppose that $h_1 = b_1^{-1} \exp[b_1 Q_1 - 1]$, where $b_1 > 0$, while sector 2 is subject to DRS. Again, choose the units of factors/goods so that $\widetilde{F}_1(\mathbf{v}) = 1$.

Lemma 3. If $h_1 = b_1^{-1} \exp[b_1 Q_1 - 1]$, where $b_1 > 0$, while sector 2 is subject to DRS and if ψ_2' is finite in the region in which the output of good 2 is small, then condition J2 is satisfied for sector 2.

Lemma 3 can be proved by noting that when $Q_2 \to 0$ (and when $Q_1 \to 1/b_1$), $\psi_1' \to \infty$. If ψ_2' remains finite in this region, then condition J2 is satisfied. This lemma and Proposition 4 give the following proposition:

Proposition 5. Given condition A, if $h_1 = b_1^{-1} \exp[b_1 Q_1 - 1]$, where $b_1 > 0$, and if ψ_2' is finite in the region in which the output of good 2 is small, then the PPF of the economy has the Tinbergen curvature when Q_2 approaches zero.

In the case considered in the above proposition, the PPF of the economy has the Tinbergen curvature in the region close to the good-1 axis. One may wonder whether one can find economies with a PPF which has the Tinbergen curvature in the regions close to both axes. The following two examples show that such economies can be found.

Example 1: Suppose that $h_1 = b_1^{-1} \exp[b_1 Q_1 - 1]$ and $b_2^{-1} h_2 = \exp[1 - b_2 Q_2]$, where $b_1, b_2 > 0$. It can be shown that $\varepsilon_1 = b_1 Q_1 > 0$ and $\varepsilon_2 = -b_2 Q_2 < 0$ for $Q_1, Q_2 > 0$. This means that sector 1 is subject to IRS and sector 2 is subject to DRS. There is only one factor, labor, with its given endowment in the economy equal to unity. The maximum outputs of goods 1 and 2 (when only one good is produced) are $1/b_1$ and $1/b_2$, respectively.[11] With only one factor, the virtual economy is a Ricardian one, and the virtual PPF in terms of $\widetilde{Q}_1$ and $\widetilde{Q}_2$ is a straight line with a slope of -1, i.e., $T' = -1$ and $T'' = 0$. Thus condition A is satisfied. Since $\psi_2 > 0$, by condition (6), the sign of $\mathrm{d}^2 Q_2/\mathrm{d}Q_1{}^2$ is the same as that of $(\psi_1' + \psi_2')$. By condition (7), we have

$$\psi_1' = \frac{(2 - b_1 Q_1)\exp[b_1 Q_1 - 1]}{(1 - b_1 Q_1)^2} \tag{10a}$$

$$\psi_2' = \frac{-(2 + b_2 Q_2)\exp[1 - b_2 Q_2]}{(1 + b_2 Q_2)^2}. \tag{10b}$$

When $Q_1 \to 0$ (so that $Q_2 \to 1/b_2$), conditions (10) give

$$\psi_1' + \psi_2' \to 2\exp[-1] - 3/4 = -0.014 < 0, \tag{11a}$$

implying that the PPF of the economy is concave to the origin. When $Q_2 \to 0$ (so that $Q_1 \to 1/b_1$), conditions (10) give

$$\psi_1' + \psi_2' \to \infty, \tag{11b}$$

meaning that the PPF is strictly convex to the origin. Conditions (11) imply that the PPF of the economy has the Tinbergen curvature.

In the above example, condition J2 is satisfied for sector 2. However, conditions J1 and J2 are only sufficient conditions for the Tinbergen curvature of the PPF of an economy under external economies. In the following example, both conditions J1 and J2 are not satisfied, but the PPF of the economy has the Tinbergen curvature.

Example 2: Suppose that $h_i = a_i + b_i Q_i + c_i Q_i^2$, where all parameters are finite and $a_i > 0$. If b_i and/or c_i is negative, then a_i is sufficiently large so that h_i is positive for all relevant values of Q_i. It can be shown that

$$\varepsilon_i = \frac{Q_i(b_i + 2c_i Q_i)}{h_i} \tag{12a}$$

$$\psi_i' = \frac{2h_i(a_i b_i + 3a_i c_i Q_i - c_i^2 Q_i^3)}{(a_i - c_i Q_i^2)^2} \tag{12b}$$

There is only one factor, labor, with its endowed amount in the economy equal to unity. As we showed, the virtual economy is a Ricardian one, and condition A is satisfied. Let the parameters have the following values:

$$a_1 = 0.8 \qquad a_2 = 1.2$$

$$b_1 = 0.2 \qquad b_2 = -0.1$$

$$c_1 = 0 \qquad c_2 = -0.1$$

Using these values, we can determine that the maximum values of goods 1 and 2 (when the output of the other good is zero) are both equal to unity. Substituting these values into (12a), we can see that sector 1 is subject to IRS and sector 2 subject to DRS. As in the previous example, the sign of $\mathrm{d}^2Q_2/\mathrm{d}{Q_1}^2$ is the same as that of $(\psi_1' + \psi_2')$. When $Q_1 \to 0$ $(Q_2 \to 1)$, $\psi_1' + \psi_2' \to -0.18 < 0$, meaning that the PPF is strictly concave to the origin. When $Q_2 \to 0$ $(Q_1 \to 1)$, $\psi_1' + \psi_2' \to 0.3 > 0$, implying that the PPF is strictly convex to the origin. These results show that the PPF has the Tinbergen curvature.

5. Concluding Remarks

The motive of writing this paper comes from some controversial statements in the literature concerning the curvature of the PPF of an economy subject to Marshallian externality. These statements can be summarized by the question, was Tinbergen wrong in drawing his diagram of an economy's PPF?

The conclusion of this paper is that Tinbergen was not wrong because there exist some classes of economies subject to external economies that have a PPF of the Tinbergen curvature. Drawing this conclusion is not the only contribution of this paper: It provides a systematic analysis of the curvature of an economy's PPF, and derives conditions for a PPF with the Tinbergen curvature and conditions for a PPF with the Herberg-Kemp curvature. As a matter of fact, Herberg and Kemp (1969) already showed that knowing that one sector is subject to IRS and one sector subject to DRS or CRS is not sufficient to tell the curvature of its PPF, even in the regions close to the axes and even if the production functions are homothetic. The last message is recognized in some other papers such as Clemhout and Wan (1970) and Mayer (1974),[12] but apparently it is missing in some later papers.

Perhaps one may ask why we should be interested in knowing whether the PPF of an economy has the Herberg-Kemp or Tinbergen curvature? Two answers can be offered. The simple answer is that because the curvature of the PPF of an economy under external economies of scale is in general ambiguous, it is nice to have some basic guidelines for drawing its PPF, at least in the regions close to the axes. The Herberg-Kemp and Tinbergen curvatures in some cases are the guidelines we need.

Another answer to the above question has to do with the relationship between the curvature of a PPF and the responses of outputs to price changes. It is well known that the price-output responses of an economy may be perverse and that generally whether these responses are normal cannot be inferred from the local curvature of the PPF (Jones, 1968; Herberg and Kemp, 1969; Kemp, 1969). One special case suggested by Markusen and Melvin (1981) is the one in which the production functions are homogeneous, i.e., with constant rates of VRS. They show that in this case the price-output responses are normal if and only if the PPF is strictly concave toward the origin. However, we showed that in this case condition HK1 is satisfied. Thus if condition A is also satisfied, the PPF has the Herberg-Kemp curvature. Therefore in this special case, whether the price-output responses are normal can be inferred from the curvature of the PPF.

The Markusen-Melvin result is consistent with a result in Herberg and Kemp (1969, Theorem 3) that in the region where the output of a good is close to zero, the price-output responses are perverse if and only if that sector is subject to IRS, irrespective of the returns in the other sector. This theorem is applicable even if the PPF of an economy does not have the Herberg-Kemp curvature. Thus if it is given that the PPF of an economy has the Tinbergen curvature (without the Markusen-Melvin assumption that the production functions are homogeneous), then by the Herberg-Kemp theorem, the price-output responses can again be inferred from the curvature of the PPF in the regions close to the axes: The price-output responses are perverse if and only if the PPF is strictly concave toward the origin. This result is opposite to the Markusen-Melvin one.

FOOTNOTES

1. In the section that describes and applies the diagram (Tinbergen, 1945, pp. 190–193; 1954, pp. 181–183), it has not been stated clearly which sector is subject to IRS. However, in another section, Tinbergen (1945, pp. 196–199) uses the example constructed by Frank Graham and makes it clear that sector 1 is subject to IRS and sector 2 subject to DRS. The diagram in Tinbergen (1954, p. 181) shows an equilibrium with an increase (resp. decrease) in the production of good 2 (resp. 1). Since this case refers to Graham's argument of loss from trade, it is therefore implicit in Tinbergen's argument that sector 1 is subject to IRS and sector 2 subject to DRS.
2. There are some errors in the diagram in Tinbergen (1945, p. 192). Some of them were corrected in Tinbergen (1954, p. 181).
3. Panagariya (1980, p. 221) mentions that similar diagrams showing the curvature of the PPF of such an economy also appear in books by Richard Caves, Charles Kindleberger, C. E. Staley, Richard Caves and Ronald Jones, and Ingo Walter.
4. Panagariya (1981, p. 221) writes, "It has been shown by Horst Herberg and Murray Kemp that, *given homothetic production functions with IRS in one industry and DRS in the other*, the production-possibilities frontier (PPF) is strictly concave to the origin near the IRS axis and strictly convex to the origin near the DRS axis (present emphasis)." This statement, which is not correct, is more than what Herberg and Kemp have claimed, because homotheticity of production functions is not sufficient for a PPF to have the Herberg-Kemp curvature.
5. On a separate point, Panagariya (1981) is right when he argues that Tinbergen does not recognize the divergence between the social and private marginal products at a competitive equilibrium.
6. Herberg and Kemp (1991) state that the Herberg-Kemp proposition (Herberg and Kemp, 1968) is valid if some factors are sector specific except when there is only one mobile factor. Wong (1996) argues that the proposition is also valid even if there is only one mobile factor.
7. It is assumed that firms, in choosing their optimal outputs, are taking the outputs of other firms as given. Kemp and Shimomura (1995) challenge this assumption and argue that identical firms will behave like a single monopolist, thus making external economies to scale not compatible

with perfect competition.

8. Equation (6), which appears in slightly different forms in many papers in the literature, can be traced back to Tinbergen (1945, p. 191).
9. Note that condition A is needed in the proof of Proposition 1. This condition, which is assumed only implicitly in Herberg and Kemp (1969), is first stated explicitly by Mayer (1974).
10. It is possible that as $Q_2 \to 0$, $\psi_2 \to 0$. In this case, we have

$$\lim_{Q_2 \to 0} \frac{\mathrm{d}^2 Q_2}{\mathrm{d}{Q_1}^2} = 0. \tag{13}$$

An example that gives condition (13) is provided by Mayer (1974). However, as Herberg and Kemp (1975) point out, condition (13) is not equivalent to the condition that $\mathrm{d}^2 Q_2/\mathrm{d}{Q_1}^2$ is positive for Q_2 positive but sufficiently small.

11. For $Q_2 > 0$, Q_1 is less than b_1 and ε_1 is less than unity.
12. In fact, Herberg and Kemp (1969) even provide an example in which the PPF of an economy does not have the Herberg-Kemp curvature. In this example, which they owe to Paul Samuelson, sector 1 is subject to constant returns and sector 2 is subject to IRS. The PPF is concave in the neighborhood of X_1-axis and convex in a neighborhood of the X_2-axis.

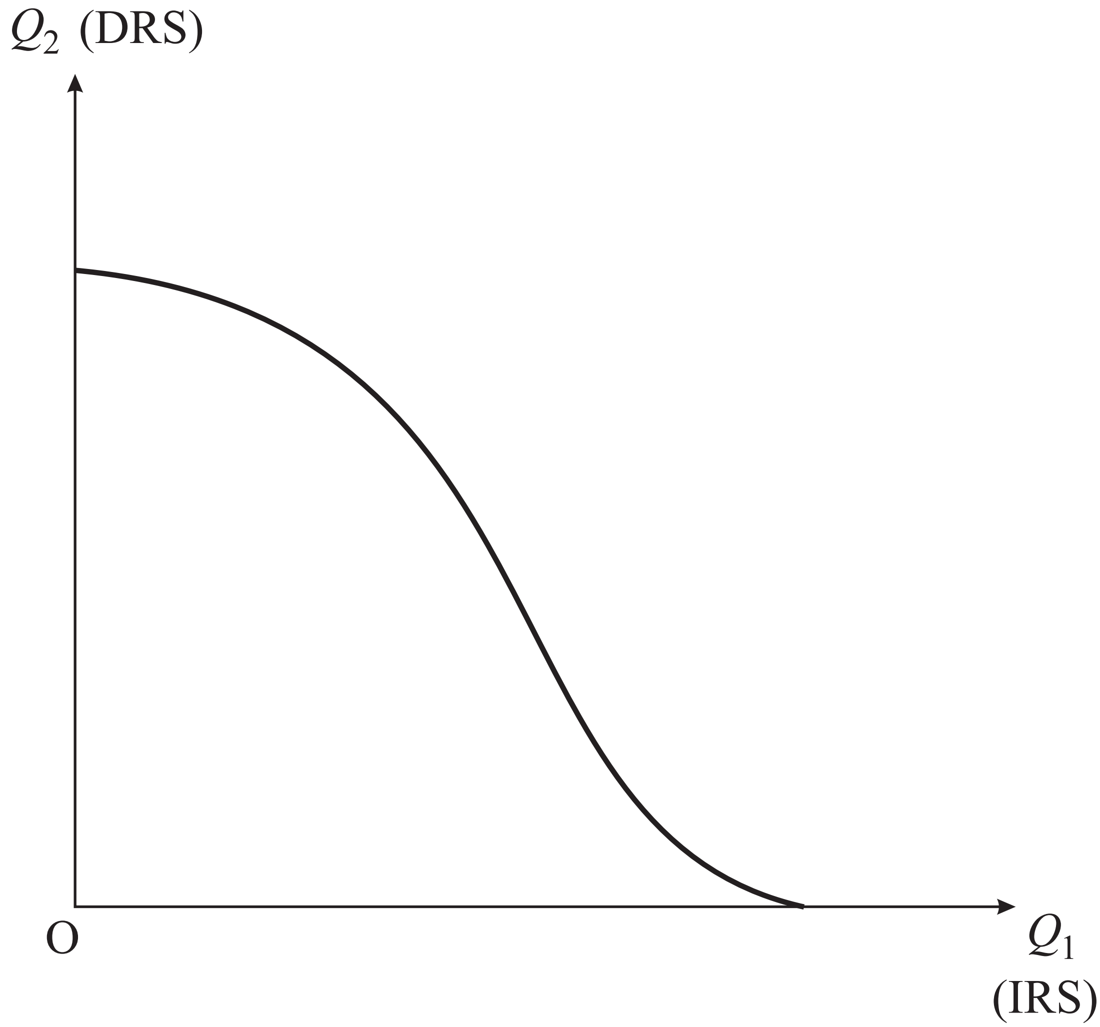

Figure 1

The Tinbergen Curvature

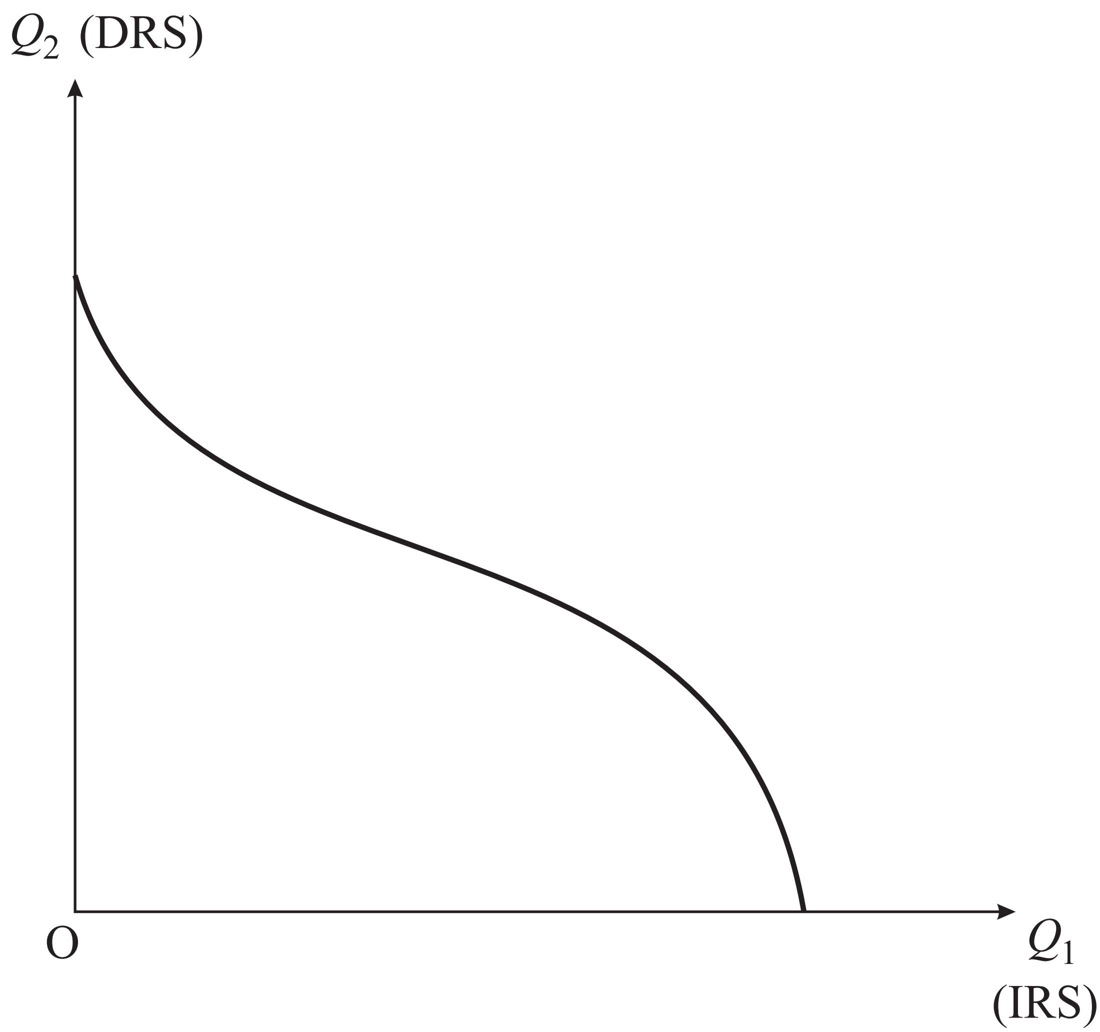

Figure 2

The Herberg-Kemp Curvature

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