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The Law of Comparative Advantage without Social Utility Functions*

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Abstract:

This paper derives sufficient conditions under which the Law of Comparative Advantage and the General Law of Comparative Advantage are true when the preferences of the trading countries may not be represented by "well-behaved" social utility functions. It shows that in the neoclassical framework with convex technologies, profit maximization and Walras's Law, the laws of comparative advantage under a natural trade are still valid if either the General Law of Demand or the Weak Axiom of Revealed Preference holds, or if losers are compensated using lump-sum transfers or consumption taxes.

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1. Introduction

Ever since the work of David Ricardo (Ricardo, 1817), the Law of Comparative Advantage has become one of the most important theorems in the theory of international trade. This law, which states that each country imports the good which is relatively more expensive under autarky and that the world price ratio under natural trade is bounded by the autarkic price ratios in the countries, is established in a very simple framework with two goods and two countries.[1] Efforts have been made to extend it to a higher dimensional framework (Dixit and Norman, 1980; Deardorff, 1980; Ethier, 1984; Wong, 1995), and the major result is summarized by the General Law of Comparative Advantage.[2] The general law, which states that a country *on average* imports the goods which are relatively more expensive under autarky, is weaker than the Law of Comparative Advantage.

Previous work that proves the Law or the General Law of Comparative Advantage is based on the assumption that a "well-behaved" social utility function exists in each country.[3] The most important feature of a social utility function is that for the economy the Weak Axiom of Revealed Preference (WARP) is true.

To the best of our knowledge, no work in the literature has examined whether the laws are still valid when social utility functions do not exist. It is well known that without a social utility function, for the economy the WARP may not hold, and it is not clear what conditions will be needed to guarantee the validity of the laws.

The purpose of this note is to provide some answers to this open question, and derive other, possibly weaker, conditions that will generate the laws of comparative advantage. We do keep the neoclassical assumptions of convex technology, perfect competition, and Walras's Law.

2. The Main Results

Consider a two-country (labeled home and foreign) framework with $n \geq 2$ tradable goods.[4] For the home country, define $\mathbf{p} \in R_+^n$ as the vector of commodity prices, $\mathbf{D} \in R_+^n$ as the vector of consumption, $\mathbf{S} \in R_+^n$ as the vector of production outputs, and $\mathbf{M} \equiv \mathbf{D} - \mathbf{S}$ as the vector of imports. A superscript "a" is used to denote the value of a variable under autarky while the value of the variable under trade has no such superscript. The corresponding variables of the foreign country are distinguished by an asterisk.

The General Law of Comparative Advantage: It states that

$$(\mathbf{p}^a - \mathbf{p}^{a*}) \cdot \mathbf{M} \geq 0. \tag{1}$$

This law can be stated in its stronger version: If $\mathbf{p}^a \neq \mathbf{p}^{a*}$ then $(\mathbf{p}^a - \mathbf{p}^{a*}) \cdot \mathbf{M} > 0$.

The Law of Comparative Advantage: In the special case in which $n = 2$, if $(p_2^a/p_1^a) > (p_2^{a*}/p_1^{a*})$, then

(a) $-M_1 \equiv E_1 \geq 0$ and $M_2 \geq 0$;

(b) $(p_2^a/p_1^a) \geq (p_2^w/p_1^w) \geq (p_2^{a*}/p_1^{a*})$ with at least one strict inequality, where p_i^w is the world price of good i under natural trade.

For the time being, we focus on a general model with $n \geq 2$. Let us consider the following three conditions:

(L1) $\mathbf{p}^a \cdot \mathbf{D}^a \leq \mathbf{p}^a \cdot \mathbf{D}$;

(L2) $\mathbf{p}^a \cdot \mathbf{S}^a \geq \mathbf{p}^a \cdot \mathbf{S}$;

(L3) $\mathbf{p}^a \cdot \mathbf{D}^a = \mathbf{p}^a \cdot \mathbf{S}^a$.

Lemma 1. Given conditions (L1) to (L3), the General Law of Comparative Advantage is valid.

Proof. These three conditions are combined to give

$$\mathbf{p}^a \cdot \mathbf{M} = \mathbf{p}^a \cdot (\mathbf{D} - \mathbf{S}) \geq \mathbf{p}^a \cdot (\mathbf{D}^a - \mathbf{S}^a) = 0. \tag{2}$$

For the foreign country, condition (2) gives

$$\mathbf{p}^{a*} \cdot \mathbf{M}^* \geq 0. \tag{3}$$

In a trade equilibrium, $\mathbf{M} + \mathbf{M}^* = 0$. Thus conditions (2) and (3) can be combined together to give equation (1). □

Note that the stronger version of the law (with a strict inequality in (1) when the countries have different autarkic prices) holds if at least (L1) or (L2) is satisfied with a strict inequality.

The proof of Propositiion 1 is present in many places; e.g., Dixit and Norman (1980), Deardorff (1980), Ethier (1984), and Wong (1995). Condition (L3) is due to Walras's Law, which holds for all non-negative prices, and condition (L2) is due to convex technologies and perfect competition. When condition (L2) is true for any two non-negative price sets and their corresponding output levels, it is called the *Revenue Maximization condition*. Two sufficient conditions for condition (L1) have been derived in the literature: (i) A "well-behaved" social utility function exists; and (ii) Natural trade yields a higher social utility level.[5]

We now derive several other sufficient conditions for condition (L1) without relying on the existence of a social utility function. First of all, let us define the General Law of Demand. As the economy shifts from an autarkic equilibrium to a natural trade equilibrium, the General Law of Demand is said to be satisfied if:

$$(\mathbf{p} - \mathbf{p}^a) \cdot (\mathbf{D} - \mathbf{D}^a) \leq 0. \tag{4}$$

Condition (4) states that relative to the autarkic point, the changes in commodities prices are *on average* not positively related to the changes in demands. In the special case with only one commodity, this condition is equivalent to a non-positively sloped demand curve. Note that condition (4) is a local result, which is what we need in the present context, but it can be generalized to all possible prices.

Proposition 1. *Given the General Law of Demand, the Revenue Maximization condition and Walras's Law, condition (L1) holds.*

Proof. Expand condition (4) to give

$$\mathbf{p} \cdot \mathbf{D} + \mathbf{p}^a \cdot \mathbf{D}^a - \mathbf{p}^a \cdot \mathbf{D} - \mathbf{p} \cdot \mathbf{D}^a \leq 0. \tag{4$'$}$$

The Revenue Maximization condition (using natural-trade prices) implies that

$$\mathbf{p} \cdot \mathbf{S} \geq \mathbf{p} \cdot \mathbf{S}^a. \tag{5}$$

By Walras's Law, we have

$$\mathbf{p} \cdot \mathbf{S} = \mathbf{p} \cdot \mathbf{D}. \tag{6}$$

The autarkic equilibrium condition is

$$\mathbf{D}^a \leq \mathbf{S}^a. \tag{7}$$

Conditions (5) to (7) are combined together:

$$\mathbf{p} \cdot \mathbf{D} \geq \mathbf{p} \cdot \mathbf{D}^a. \tag{8}$$

Subtracting condition (8) from (4′) gives condition (L1). □

Note that in Proposition 1, to get condition (L1) we need not just the General Law of Demand, but also the Revenue Maximization condition and Walras's Law.

We now derive another condition for condition (L1).

Proposition 2. *Given the Weak Axiom of Revealed Preference (WARP) for the economy, the Revenue Maximization condition and Walras's Law, condition (L1) holds.*

Proof. Recall that condition (8) is implied by the Revenue Maximization condition and Walras's Law. If WARP is given, condition (8) implies

$$\mathbf{p}^a \cdot \mathbf{D} > \mathbf{p}^a \cdot \mathbf{D}^a,$$

which is condition (L1). □

Another sufficient condition for condition (L1) is derived as follows. First, we refer to a result in Wong (1995):

Proposition 3. *Assuming costless transfers or taxation and when given perfect information, natural trade is gainful in the sense that a government can use either lump-sum transfers or consumption taxes to make all individuals not worse off than under autarky. (Wong, 1995).*

Note that in Proposition 3, the gainfulness of a natural trade does not require the existence of a social utility function. As usual, a stronger result can be obtained if producers' prices change under natural trade and if production substitution is possible (or, for the case of lump-sum transfers, if consumption substitution exists for at least one consumer).[6] Under these conditions, some consumers can be made better off. Using this result, we have:

Proposition 4. *If under natural trade the government uses either lump-sum transfers or consumption taxes to make every household not worse off than under autarky, then condition (L1) holds.*

Proof. Let there be H households which may have different preferences and/or endowments. For the hth household, $h = 1, \ldots, H$, if it is not worse off under natural trade, then under the autarkic prices the trade consumption bundle cannot cost less than the autarkic consumption bundle, or the household would choose the trade consumption bundle under autarky, i.e.,

$$\mathbf{p}^a \cdot \mathbf{D}^{ha} \leq \mathbf{p}^a \cdot \mathbf{D}^h, \qquad \text{for all} \;\; h = 1, \ldots, H, \tag{9}$$

where the superscript "h" is used to denote the variables of the hth household. Summing up conditions (9) for all the households gives condition (L1). □

Note that in the presence of compensation, condition (L1) holds after compensation using either lump-sum transfers or consumption taxes has been made.

We can now combine the above results together to give:

Proposition 5. *Suppose that natural trade exists between two countries and that the Revenue Maximization condition and Walras's Law hold for both countries. If, furthermore, either the General Law of Demand or the Weak Axiom of Revealed Preference holds for both countries, or if all households are made not worse off than under autarky by using either intra-country lump-sum transfers or consumption taxes, then the General Law of Comparative Advantage holds.*

Proof. By Propositions 1, 2, and 4, conditions (L1) to (L3) hold under the specified conditions. By Lemma 1, the General Law of Comparative Advantage is valid. □

Lemma 2. In the special case of two tradable goods, the Law of Comparative Advantage holds under the conditions specified in Proposition 5.

Proof. Suppose, without loss of generality, that $(p_2^a/p_1^a) > (p_2^{a*}/p_1^{a*})$. With a zero trade balance (Walras's Law), M_1 and M_2 must be of opposite sign.[7] Suppose that $M_1 > 0$ and $-M_2 \equiv E_2 > 0$. We want to show that this leads to a contradiction.

In the present two-good model with the assumed signs of M_1 and E_2, equation (2) reduces to

$$p_2^a E_2 \leq p_1^a M_1,$$

or, after rearranging terms,

$$\frac{p_2^a}{p_1^a} \leq \frac{M_1}{E_2}. \tag{10}$$

The same condition applies to the foreign country:

$$\frac{p_2^{a*}}{p_1^{a*}} \geq \frac{M_1}{E_2}, \tag{10$'$}$$

where $-M_1^* = M_1 > 0$ and $-E_2^* = E_2 > 0$. Combining these two conditions together gives

$$\frac{p_2^a}{p_1^a} \leq \frac{p_2^{a*}}{p_1^{a*}},$$

which contradicts the original condition: $(p_2^a/p_1^a) > (p_2^{a*}/p_1^{a*})$. Thus, we must have $E_1, M_2 > 0$. With this result, equation (2) reduces to

$$\frac{p_2^a}{p_1^a} \geq \frac{E_1}{M_2}. \tag{11}$$

Given Walras's Law, the trade balance is zero, i.e.,

$$p_1^w E_1 = p_2^w M_2. \tag{12}$$

Equations (11) and (12) are combined together to give

$$\frac{p_2^a}{p_1^a} \geq \frac{p_2^w}{p_1^w}. \tag{13}$$

By applying the same argument to the foreign country which imports good 1 and exports good 2, we have

$$\frac{p_2^{a*}}{p_1^{a*}} \leq \frac{p_2^w}{p_1^w}. \tag{14}$$

Equations (13) and (14) are then combined together to give

$$\frac{p_2^a}{p_1^a} \geq \frac{p_2^w}{p_1^w} \geq \frac{p_2^{a*}}{p_1^{a*}}, \tag{15}$$

with at least one strict inequality. □

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FOOTNOTES

1. Trade is said to be natural if the net tariff revenue is non-negative (Deardorff, 1980). It is also called "trade under self-financing tariffs" (Ohyama, 1972).
2. It has been shown that the Law of Comparative Advantage is not valid when there are more than two goods. See, for example, Dradbicki and Takayama (1979).
3. A social utility function is "well-behaved" if it is continuous, increasing and quasi-concave in consumption commodities.
4. The analysis can easily be extended to cover international factor mobility. See Wong (1995).
5. If a natural trade is gainful, then the consumption bundle $\mathbf{D}$ yields a higher (or not lower) utility than $\mathbf{D}^a$ does. That the former is not chosen under autarky must be because it requires an expenditure at least as high as that for $\mathbf{D}^a$. This gives condition (L1).
6. The use of consumption taxes to guarantee a gainful free trade was first suggested by Dixit and Norman (1980). This compensation policy was later extended to make sure that some individuals are made better off by requiring either the Weymark condition or the use of poll grants. See Wong (1995, Chapter 8) for more discussion.
7. Since the countries have different autarkic price ratios, they trade when it is allowed. So M_1 and M_2 are both non-zero.