

# Industrialization, Economic Growth, and International Trade

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## Abstract

This paper analyzes the relationship between economic growth, industrialization, and international trade in a two-sector endogenous growth model. With learning by doing, the manufacturing sector grows over time, but the agricultural sector experiences no learning by doing and does not grow. Along a balanced growth path, the economy grows perpetually, with the relative price of manufacturing declining continuously. The effects of trade with the rest of the world are analyzed. It is shown that the growth rate of the rest of the world could have major impacts on the pattern of production, the pattern of trade, and growth of the economy. In particular, if the economy remains diversified under trade, its growth can keep in pace with the rest of the world. However, if it produces agriculture only, no growth will be experienced. Moreover, there is an upper bound on the growth rate of the economy. If the growth rate of the rest of the world is higher than this limit, the economy cannot catch up and will eventually end up with complete specialization in agriculture.

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# 1 Introduction

Industrialization and international trade have long been regarded as two of the most important engines of growth for many countries. Ever since the industrial revolution that has brought rapid growth to various countries, industrialization has been regarded by many governments as the key to fast growth. Many government policies have been geared to promote the development of the manufacturing sector, very often at the expense of other sectors such as agriculture. International trade is commonly treated as an important factor of growth [e.g., Boldrin and Scheinkman (1988), Young (1991), and Wong (1995)]. All Asian countries that have experienced rapid growth in the previous decades are open economies, and this fact has great influence on the trade policies of many developing countries.

The objective of this paper is to examine the relationship between industrialization, economic growth, and international trade. In particular, it analyzes how industrialization and international trade may affect the growth performance of a country.

To analyze this relationship, we construct a simple two-sector endogenous growth model. The two sectors are conveniently called the manufacturing sector and the agricultural sector. The manufacturing sector grows over time due to both the accumulation of physical capital as a result of investment, and the accumulation of human capital through learning by doing. The growth of the manufacturing sector pulls the economy with it. The agricultural sector of the economy, however, does not have any learning by doing effect, implying no growth in technology. Along a balanced growth path (BGP), the two sectors are growing at different rates, implying a continuous decline in the relative price of manufacture.<sup>1</sup>

It is interesting to compare the present result of falling relative price of manufacturing with the well-known Prebisch-Singer hypothesis that developing countries are facing declining relative prices of primary products. While the hypothesis has been supported in some empirical studies,<sup>2</sup> it remains a

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<sup>1</sup>The feature of the present model that the relative price of manufacturing is declining along a BGP is quite different from many of the existing models; for example, Bond, Wang and Yip (1997) and Bond and Trask (1997).

<sup>2</sup>See, for example, Sarkar (1986, 1994, 1997).

controversial issue in the literature. Two remarks can be offered here. First, while the two goods in the present model are conveniently labeled manufacturing and agriculture, they can be relabeled for any pair of goods under consideration. Our results suggest only that whether the relative price of one good in a closed economy increases or decreases over time depends on the learning-by-doing effects in the sectors. Second, our model does not predict that a small economy will be facing deteriorating term of trade because the pattern of trade of the economy has to be determined endogenously. If, for example, the economy exports the agriculture, its term of trade is actually improved.<sup>3</sup>

In the present model, since manufacturing is getting cheaper relative to agriculture, many new questions arise. For example, how may the growth rate of the rest of the world affect this economy's pattern of trade, pattern of production, and growth? Can the economy ever catch up with the rest of the world? What are the conditions under which an economy can remain diversified, and can that be sustainable? What are the features of an economy if it is completely specialized in one good? How can we say about the growth rate of the economy if it is completely specialized under trade? Answers to these questions will be provided later in this paper.

In the next section, the features of the model are presented and its properties such as the balanced growth path of a closed economy will be derived. Section 3 analyzes the economy under free trade in the case in which both goods are produced (diversification) along a balanced growth path. Whether diversification can be sustained will be investigated. Section 4 turns to the case in which the economy is completely specialized in the production of the agriculture good. The alternative case in which it is completely specialized in producing the manufacturing good will be studied in Section 5. These three sections show that the patterns of production of the economy depend crucially on the growth rate of the world. The last section summarizes the main results and offers some concluding remarks.<sup>4</sup>

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<sup>3</sup>See Conway and Darity (1991), Darity (1990), and Dutt (1988) for related work.

<sup>4</sup>While our paper is limited to a positive theory of trade and industrialization, the analysis provides here certainly has strong welfare and policy implications. In another paper (Wong and Yip, 1998), we examine how the welfare of an economy may be affected by trade and industrialization in a dynamic model, and argue that an economy can still gain from trade even if its terms of trade deteriorate over time. We also identify some cases in which policies such as production subsidies can be used to promote the welfare of an open economy.

## 2 A Closed Economy

Consider an economy of constant size of population,  $L$ . Two types of homogeneous products, agricultural good (good A) and manufacturing good (good M), are produced by competitive firms using constant-returns technologies. Both goods are consumed, but the manufacturing good can also be invested to increase the physical capital stock.<sup>5</sup>

### 2.1 Production

The production of agricultural good is done by competitive firms using only labor input. The sectoral production function at any point of time  $t$  can be written as:

$$X_t^A = AL_t^A, \quad (1)$$

where  $X_t^A$  is the output,  $L_t^A$  is the homogeneous labor (number of workers) input, and  $A > 0$  denotes the constant labor productivity. Since  $A$  is constant, it is equal to the marginal as well as average product of labor of the sector.

Production of the manufacturing good requires two inputs: capital ( $K_t$ ) and labor ( $M_t L_t^M$ ),

$$X_t^M = F(K_t, M_t L_t^M), \quad (2)$$

where  $X_t^M$  is the output and  $L_t$  is the number of workers employed. The variable  $M_t$  is a labor productivity index, meaning that  $M_t L_t^M$  is regarded as the effective labor input. The production function in (2) satisfies the following assumption:

**Assumption A1.** *The production function of the manufacturing sector is twice differentiable and linearly homogeneous in factor inputs, and satisfied  $F_i > 0$  and  $F_{ii} < 0$ , where  $F_i$  denotes the partial derivative of  $F$  with respect to the  $i$ th argument ( $i = 1, 2$ ).*

**Two Cases:** *As  $L_t^M$  approaches zero, either (Case I)  $F_2$  approaches infinity (the Inada condition); or (Case II)  $F_2$  is bounded from above.*

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<sup>5</sup>Our model differs from the Ricardo-Viner-Jones model of Matsuyama (1992).

All markets are competitive. Define the capital-labor ratio in the manufacturing sector as  $k_t \equiv K_t/(M_t L_t^M)$ . The manufacturing production function can be rewritten as

$$X_t^M = M_t L_t^M f(k_t), \quad (3)$$

where  $f(k_t) \equiv F(k_t, 1)$ . With cost minimization, the wage rate,  $w_t$ , and rental rate of capital,  $r_t$ , in terms of the manufacturing good are equal to

$$\begin{aligned} w_t &= w(k_t) = f(k_t) - k_t f'(k_t) \\ r_t &= r(k_t) = f'(k_t). \end{aligned}$$

Firms are competitive, taking prices and the labor productivity as given.

The growth of the sector, and thus that of the economy, comes from an increase in the manufacturing labor productivity  $M_t$  over time. In the present paper, we follow the tradition of Romer (1986) and model the endogenous growth as the result of an external learning-by-doing process.<sup>6</sup> In particular, the increase in labor productivity is assumed to be given by the following condition:

$$\dot{M}_t = \mu X_t^M = \mu F(K_t, M_t L_t^M), \quad (4)$$

where  $\mu > 0$  is a measure of the effectiveness of learning by doing. At any point of time,  $M_t$  is taken as given and no firm or individual will take condition (4) into consideration.

**Condition C1.** *The learning-by-doing (LBD) effects in the manufacturing sector are weak in the sense that  $\mu$  is sufficiently small.*

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<sup>6</sup>Our assumption of the existence of externality in the accumulation of human capital, which allows us to keep the assumption of perfect competition, is common in the endogenous growth literature. One limitation of this assumption is that firms and individuals do not take into account the learning-by-doing in their choice of optimal actions. An alternative and interesting way to endogenize growth is to internalize the learning-by-doing process so that firms may be facing decreasing costs. The resulting internal economies of scale usually lead to imperfect competition in a dynamic model. See Romer (1987 and 1990) for work along this line.

Condition C1 is required for some of the results derived below. Choosing the agricultural good as the numeraire, we denote the relative price of the manufacturing good by  $p_t$ . In addition, we consider a production subsidy of constant ad valorem rate of  $s \geq 0$  on the manufacturing sector so that the domestic producers' price of manufacturing becomes  $(1+s)p_t$ . Perfect mobility of labor between the two sectors with positive outputs implies equalization of wage rates:

$$A = (1+s)p_t M_t w(k_t). \quad (5)$$

## 2.2 Consumption and Investment

Consumption of the two goods ( $C_t^A$  and  $C_t^M$ ) and investment ( $\dot{K}_t$ ) are decided by a representative agent.<sup>7</sup> For simplicity, no depreciation of physical capital is considered. Assume that the instantaneous utility function of the representative agent at time  $t$  is given by  $\beta \ln C_t^A + \ln C_t^M$ , where  $\beta > 0$ . The optimization problem of the representative agent is to choose the consumption and investment streams to

$$\max \int_0^\infty (\beta \ln C_t^A + \ln C_t^M) e^{-\rho t} dt$$

subject to the standard budget constraint

$$C_t^A + p_t(C_t^M + \dot{K}_t) = AL_t^A + (1+s)p_t F(K_t, M_t L_t^M) - T_t, \quad (6)$$

as well as (4), where  $\rho$  is the rate of time preference and  $T_t$  denotes the lump-sum tax used to finance the production subsidy. Let  $\lambda_t^M$  and  $\lambda_t^C$  be the costate variables associated with (4) and (6) respectively. Then the first-order conditions for the optimization problem are

$$\beta/C_t^A = \lambda_t^C/p_t \quad (7)$$

$$1/C_t^M = \lambda_t^C \quad (8)$$

$$\dot{\lambda}_t^C = \rho \lambda_t^C - [(1+s)\lambda_t^C + \mu \lambda_t^M] F_1 \quad (9)$$

$$\dot{\lambda}_t^M = \rho \lambda_t^M - [(1+s)\lambda_t^C + \mu \lambda_t^M] F_2 L_t^M, \quad (10)$$

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<sup>7</sup>An alternative model, suggested by Eric Bond, is one in which investment is determined not by a representative agent but by competitive investors who take prevailing prices as given. This implies that intertemporal distortions in investment may be created. We avoid this approach in order not to model the behavior of the investors separately.

as well as (4), (6) and the transversality conditions.

For a closed economy, equilibrium of the goods market requires that

$$C_t^M + \dot{K}_t = F(K_t, M_t L_t^M) \quad (11)$$

$$C_t^A = A L_t^A. \quad (12)$$

Equilibrium of the labor market is described by

$$L_t^A + L_t^M = L. \quad (13)$$

The optimality and equilibrium conditions can be rewritten as follows:

$$C_t^A = \beta p_t C_t^M \quad (14)$$

$$\frac{\dot{\lambda}_t^C}{\lambda_t^C} = \rho - \left(1 + s + \frac{\mu}{q_t}\right) r(k_t) \quad (15)$$

$$\frac{\dot{\lambda}_t^M}{\lambda_t^M} = \rho - [\mu + (1 + s)q_t] L_t^M w(k_t) \quad (16)$$

$$\dot{K}_t = M_t L_t^M f(k_t) - C_t^M \quad (17)$$

$$\dot{M}_t = \mu M_t L_t^M f(k_t) \quad (18)$$

as well as (5) and (12), where  $q_t \equiv \lambda_t^C / \lambda_t^M$ . The utility function implies that both goods are consumed at all positive prices, and thus the present economy when closed is diversified.

### 2.3 Balanced Growth Path

The balanced growth path (BGP) equilibrium of the economy is defined as a situation in which all endogenous variables are changing at constant rates (not necessarily the same) while the capital-labor ratio of the manufacturing sector remains constant over time. Based this definition, the autarkic BGP equilibrium of the economy is described by the following proposition:

**Proposition 1** *The autarkic BGP equilibrium of the economy with a given  $s \geq 0$  is a situation where  $C_t^M$ ,  $X_t^M$ ,  $M_t$  and  $K_t$  ( $\lambda_t^C$ ,  $\lambda_t^M$ , and  $p_t$ ) are growing (declining) at a common constant rate of  $g^a$  while  $C_t^A$ ,  $X_t^A$ ,  $L_t^A$ ,  $L_t^M$  and  $k_t$  are stationary over time.*



The proofs of this and some other propositions are given in the appendix. Using Proposition 1, we can derive the BGP growth rate. Imposing the BGP equilibrium restrictions on (14) – (18), we get<sup>8</sup>

$$C^A = \beta p_t C_t^M \quad (19)$$

$$-g^a = \rho - \left(1 + s + \frac{\mu}{q}\right) r(k) \quad (20)$$

$$-g^a = \rho - [\mu + (1 + s)q] L^M w(k) \quad (21)$$

$$g^a = f(k)/k - c \quad (22)$$

$$g^a = \mu L^M f(k) \quad (23)$$

$$A = (1 + s)p_t M_t w(k) \quad (24)$$

where  $c \equiv C_t^M / K_t$  and condition (24) comes from equalization of the wage rates in the two sectors. From (20) – (21), we have

$$r(k) = q L^M w(k). \quad (25)$$

Combining (20), (23) and (25), we get

$$(1 + s - \mu L^M k) r(k) = \rho. \quad (26)$$

Next, using (12), (19) and (24), we have

$$\beta c = \left(\frac{L - L^M}{L^M}\right) \frac{(1 + s)w(k)}{k}. \quad (27)$$

Finally, manipulating (22), (23) and (27), we obtain

$$(1 - \mu L^M k) f(k) = \left(\frac{L - L^M}{L^M}\right) \frac{(1 + s)w(k)}{\beta}. \quad (28)$$

Conditions (26) and (28), which describe the autarkic equilibrium, form a system of two equations with two unknowns,  $L^M$  and  $k$ .

**Proposition 2** *Given the Inada conditions, an autarkic equilibrium that satisfies conditions (26) and (28) exists. If the learning by doing effect is not significant (condition C1), and if at the autarkic equilibrium the schedule that represents condition (28) is positively sloped, the autarkic equilibrium is unique.*

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<sup>8</sup>Note that endogeneous variables that stay stationary along a BGP have no time subindex.

For meaningful comparative static experiments, the autarkic equilibrium is assumed to be unique. Note that the Inada conditions assumed in the above proposition are stronger than what are needed for the existence of a BGP.

Once  $L^M$  and  $k$  are determined, the autarkic BGP growth rate ( $g^a$ ), agriculture employment, output, and consumption, and the values of  $p_t K_t$  and  $p_t M_t$  can be obtained from (23), (13), (1), (19), and (24). Note that  $C_t^M$ ,  $M_t$  and  $K_t$  increase at a rate of  $g^a$  over time. By (3), the output of manufacturing along a BGP rises at the same rate, implying that  $p_t X_t^M$  is a constant. The national income of this economy in terms of agriculture is equal to  $Y^a = X^A + p_t X_t^M$ , which, by the above analysis, is constant over time.

The BGP equilibrium can be illustrated in Figure 1. The value of national income, which is the maximum possible output of agriculture,  $AL$ , are marked on the vertical axis. The budget line of the economy, MN, which has a slope of  $-p_t$ , can be drawn, where  $p_t$  is the prevailing price ratio. The corresponding values of  $K_t$  and  $M_t$  are used to construct the production possibility frontier (PPF), shown as ST in the diagram. This frontier touches line MN at point Q, the autarkic production point. The consumption point is at C, where QC represents the level of investment. Suppose at a later time  $t'$  the relative price  $p'_t$  is known (noting that its rate of decline is equal to  $g^a$ ). The above argument can be used again to construct the new budget line (MN') and the new PPF (ST') in Figure 1. The new production point (Q') is shown, where points Q and Q' are on the same horizontal line because the output of agriculture is constant. The new consumption point is at C', where Q'C' of manufacturing has been invested.

## 2.4 Comparative Statics

To determine the effects of a change in some exogenous variables ( $\mu, L, \rho, s$ ), totally differentiate (26) and (28) and rearrange terms to give

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} dL^M \\ dk \end{bmatrix} = \begin{bmatrix} rkL^M & 0 & 1 & -r \\ fkL^M & (1+s)w/\beta L^M & 0 & b \end{bmatrix} \begin{bmatrix} d\mu \\ dL \\ d\rho \\ ds \end{bmatrix}$$

where  $a_{11} \equiv -\mu kr < 0$ ,  $a_{12} \equiv (1+s - \mu L^M k)r' - \mu L^M r < 0$ ,  $a_{21} \equiv (1+s)wL/[\beta(L^M)^2] - \mu kf$ ,  $a_{22} \equiv (1 - \mu L^M k)r - \mu L^M f - [(L - L^M)(1 +$

$s)w'/(βL^M)]$ ,  $b ≡ (1 - μL^Mk)f/(1 + s) > 0$ . Solving the above matrix equation, we obtain

$$\frac{dL^M}{dρ} = \frac{a_{22}}{D} \quad (29)$$

$$\frac{dL^M}{dμ} = \frac{kL^M(ra_{22} - fa_{12})}{D} \quad (30)$$

$$\frac{dL^M}{dL} = -\frac{a_{12}w(1 + s)}{βL^MD} \quad (31)$$

$$\frac{dL^M}{ds} = -\frac{ra_{22} + ba_{12}}{D} \quad (32)$$

$$\frac{dk}{dρ} = -\frac{a_{21}}{D} \quad (33)$$

$$\frac{dk}{dμ} = \frac{kL^M(fa_{11} - ra_{21})}{D} \quad (34)$$

$$\frac{dk}{dL} = \frac{a_{11}w(1 + s)}{βL^MD} \quad (35)$$

$$\frac{dk}{ds} = \frac{ra_{21} + ba_{11}}{D}, \quad (36)$$

where  $D ≡ a_{11}a_{22} - a_{12}a_{21}$  is the determinant of the matrix on the LHS. In general, the sign of  $D$  is ambiguous. For the time being let us focus mainly on the case in which the learning-by-doing effect is not significant, that is,  $μ$  is sufficiently small (Condition C1). In this case, we have  $a_{21} > 0$  and  $D ≈ -wLr'/[β(L^M)^2] > 0$ .

Invoking condition C1, we have the following unambiguous comparative statics results:

$$\frac{dL^M}{dL} > 0, \quad \frac{dk}{dρ} < 0, \quad \frac{dk}{dμ} < 0, \quad \frac{dk}{dL} < 0. \quad (37)$$

The effects of labor accumulation on economic growth can be derived from (23):

$$\frac{dg^a}{dL} = \frac{μw}{βL^MD} (-fa_{12} + rL^Ma_{11}) > 0, \quad (38)$$

where  $-fa_{12} + rL^M a_{11} = \mu L^M r w - (1 - \mu L^M k) f r' > 0$ . Also,

$$\frac{dg^a}{d\mu} = \frac{\mu k L^M}{D} \left[ f \left( -fa_{12} + rL^M a_{11} \right) + r \left( -rL^M a_{21} + fa_{22} \right) \right] + L^M f \quad (39)$$

$$\frac{dg^a}{d\rho} = \frac{\mu}{D} \left( -rL^M a_{21} + fa_{22} \right) < 0 \quad (40)$$

$$\frac{dg^a}{ds} = \frac{\mu}{D} \left[ b \left( rL^M a_{11} - fa_{12} \right) + r \left( rL^M a_{21} - fa_{22} \right) \right] > 0. \quad (41)$$

Finally, note that the two equilibrium conditions, (26) and (28), do not contain the technology index  $A$ . We thus have the interesting result that both  $L^M$  and  $k$ , and thus the growth rate ( $g^a$ ), are independent of the value of  $A$ . This result, which is not that straightforward, is due to the fact that  $M_t$  is a stock variable and cannot adjust instantaneously, implying that a change in  $A$  affects the relative price  $p_t$  proportionately. Thus all quantity variables such as production, consumption, and growth remain unchanged.

### 3 A Diversified, Small Open Economy

Suppose now that the economy introduced above, which from now on is called the home economy, is allowed to trade freely with the rest of the world (ROW). To simplify our analysis, we make the following assumptions:

- (a) The home economy is small as compared with the ROW in the sense that the economic conditions in the ROW are not affected by its trade with the economy.
- (b) The structure of the ROW is the same as the home economy.
- (c) At the time when trade is allowed, both the economy and the ROW are at their own BGP equilibrium.
- (d) There is no production subsidy in the home economy.
- (e) There is no international spillover, meaning that the home economy learns from its own manufacturing production only.

Denote the exogenously given BGP growth rate of the ROW by  $g^w > 0$ . Furthermore, let us denote the relative price of manufacturing in the ROW at time  $t$  by  $p_t^w > 0$ , which is decreasing at a rate of  $g^w$ .

### 3.1 Pattern of Production

To determine the home country's pattern of production, let us substitute (1) into (2) and use the labor market equilibrium condition  $L_t^M + L_t^A = L$  to get an implicit expression for the equation of the home economy's production possibility frontier:

$$X_t^M = F\left(K_t, M_t\left(L - \frac{X_t^A}{A}\right)\right). \quad (42)$$

Differentiating both sides of condition (42) and rearranging terms, we get the marginal rate of transformation,  $MRT$  (the time subscripts being suppressed for simplicity):

$$MRT \equiv -\frac{dX^A}{dX^M} = \frac{A}{MF_2}. \quad (43)$$

Define the extreme values of  $MRT$  at time  $t$  as (see Figure 2)

$$\underline{\chi}_t \equiv -\left.\frac{dX^A}{dX^M}\right|_{L^M \rightarrow 0} = -\frac{A}{MF_2(K, 0)} \quad (44)$$

$$\bar{\chi}_t \equiv -\left.\frac{dX^A}{dX^M}\right|_{L^M \rightarrow L} = -\frac{A}{MF_2(K, ML)}. \quad (45)$$

If  $F(\cdot, \cdot)$  satisfies the Inada condition (i.e.,  $\lim_{L^M \rightarrow 0} F_2 = \infty$ ), then  $\underline{\chi}_t = 0$  at all times. If the Inada condition is not satisfied,  $\underline{\chi}_t$  and  $\bar{\chi}_t$  generally change over time.

Depending on the technologies of the home economy and the value of  $p_t^w$  at any time  $t$ , three patterns of production in the economy under the following conditions can be identified:

- (a) complete specialization in agriculture:  $p_t^w \leq \underline{\chi}_t$ ;
- (b) diversification with positive production of both goods:  $\bar{\chi}_t > p_t^w > \underline{\chi}_t$ ;
- (c) complete specialization in manufacturing:  $p_t^w \geq \bar{\chi}_t$ .<sup>9</sup>

In the present section, we focus on the case in which the economy diversifies under free trade along a BGP.

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<sup>9</sup>The three conditions can be written in an equivalent way: (a)  $A \geq p_t^w M_t w(K, 0)$ ; (b)  $A = p_t^w M_t w(K_t, M_t L_t^M)$  for  $0 \leq L^M \leq L$ ; and (c)  $A \leq p_t^w w(K_t, M_t L)$ .

## 3.2 Free-Trade Balanced Growth Path

Since the derivation of the optimal investment and equilibrium conditions are similar to the one described in the previous section for a closed economy, the details are skipped. The equilibrium conditions along a BGP are:

$$C_t^A = \beta p_t^w C_t^M \quad (46)$$

$$\frac{\dot{\lambda}_t^C}{\lambda_t^C} = \rho - \left(1 + \frac{\mu}{q_t}\right) r(k_t) \quad (47)$$

$$\frac{\dot{\lambda}_t^M}{\lambda_t^M} = \rho - (\mu + q_t) L_t^M w(k_t) \quad (48)$$

$$A = p_t^w M_t w(k_t) \quad (49)$$

$$p_t^w \dot{K}_t = AL_t^A + p_t^w M_t L_t^M f(k_t) - p_t^w C_t^M - C_t^A \quad (50)$$

$$\dot{M}_t = \mu M_t L_t^M f(k_t), \quad (51)$$

where  $q_t \equiv \lambda_t^C / \lambda_t^M$ .

**Proposition 3** *Under free trade with the ROW in which the relative price of manufacturing,  $p_t^w$ , is declining at the constant rate of  $g^w$ , the BGP under diversification of the home economy is a situation in which  $C_t^M$ ,  $X_t^M$ ,  $M_t$  and  $K_t$  ( $\lambda_t^C$ ,  $\lambda_t^M$ ) are growing (declining) at a common constant rate of  $g^w$  while  $C_t^A$ ,  $k_t$ ,  $X_t^A$ ,  $L_t^A$ , and  $L_t^M$  are all positive and stationary over time.*

The BGP under diversification of the home economy can be derived as follows. From (47) and (48) and the fact that both  $\lambda_t^C$  and  $\lambda_t^M$  are decreasing at the same rate of  $g^w$ , we get

$$\mu L^M r w = (\rho + g^w - \mu L^M w)(\rho + g^w - r). \quad (52)$$

Condition (51) and the fact that  $M$  is growing at the rate  $g^w$  are combined together to give

$$g^w = \mu L^M f(k). \quad (53)$$

Substituting (53) into (52) and rearranging the terms, we get

$$\rho = (1 - \mu L^M k) r. \quad (54)$$

Based on conditions (54) and (53), let us define the following two functions:

$$\Phi(L^M, k; g^w) \equiv \rho - (1 - \mu L^M k)r(k), \quad (55)$$

$$\Theta(L^M, k; g^w) \equiv g^w - \mu L^M f(k). \quad (56)$$

The above analysis suggests that the BGP of the economy is described by  $\Phi(L^M, k; g^w) = 0$  and  $\Theta(L^M, k; g^w) = 0$ .

The derivatives of  $\Phi(L^M, k; g^w)$  are (denoted by subindices):

$$\begin{aligned} \Phi_L &= \mu r k > 0 \\ \Phi_k &= \mu L^M r - (1 - \mu L^M k)r' = \mu L^M r - \rho r'/r > 0 \\ \Phi_g &= 0. \end{aligned}$$

When given  $g^w$ , condition  $\Phi(L^M, k; g^w) = 0$  is depicted by schedule  $\Phi$  in Figure 3. The slope of the schedule is equal to

$$\left. \frac{dk}{dL^M} \right|_{\Phi} = -\frac{\Phi_L}{\Phi_k} = -\frac{rk}{L^M r - \rho r'/(r\mu)} < 0. \quad (57)$$

The partial derivatives of function  $\Theta(k, L^M; g^w)$  can be obtained in a similar way:

$$\begin{aligned} \Theta_L &= -\mu f < 0 \\ \Theta_k &= -\mu L^M r < 0 \\ \Theta_g &= 1 > 0. \end{aligned}$$

In Figure 3, the condition  $\Theta(k, L^M; g^w) = 0$  when given  $g^w$  is illustrated by schedule  $\Theta$ . The slope of the schedule is given by

$$\left. \frac{dk}{dL^M} \right|_{\Theta} = -\frac{\Theta_L}{\Theta_k} = -\frac{rk + w}{L^M r} < 0. \quad (58)$$

By comparing the expressions in (57) and (58) and noting that  $r' < 0$ , it is easy to see that schedule  $\Theta$  is steeper than schedule  $\Phi$  at a point of intersection (if exists).

Figure 3 shows the case in which schedules  $\Phi$  and  $\Theta$  intersect at point E, which represents the BGP equilibrium  $(\tilde{L}^M, \tilde{k})$ . Diversification under free

trade means that  $\tilde{L}^M \in (0, L)$ . Using conditions (57) and (58), such an equilibrium, if exists, is unique.

Once  $\tilde{L}^M$  and  $\tilde{k}$  are known, the rest of the endogenous variables can be determined easily by making use of the optimality and equilibrium conditions derived earlier. Note that both  $p_t^w K_t$ ,  $p_t^w M_t$  and  $p_t^w C_t^M$  are stationary, while  $K_t$ ,  $M_t$  and  $C_t^M$  are increasing at a rate of  $g^w$  along a BGP.

The change in production and consumption over time can also be illustrated in a diagram similar to Figure 1, with a horizontal line representing the locus of production point (because the production of good A is constant) and another horizontal line representing the locus of consumption point (because the consumption of good A is constant). In the presence of trade, these two horizontal lines may not coincide: The economy exports (imports) good A if and only if the production locus is higher than the consumption locus.

We now determine whether diversification under free trade is sustainable, i.e., whether the condition  $\bar{\chi}_t > p_t^w > \underline{\chi}_t$  can hold over time. To answer this question we need to determine how  $\bar{\chi}_t$ ,  $p_t^w$  and  $\underline{\chi}_t$  change over time. First note that  $p_t^w$  decreases at a rate of  $g^w$ . Next, if  $F_2(K, L^M)$  approaches infinity as  $L^M$  approaches zero, we have  $\underline{\chi}_t = 0$  always. If, when  $L^M \rightarrow 0$ ,  $F_2(K, L^M)$  is bounded from above and is independent of  $K$ ,<sup>10</sup> meaning that  $\underline{\chi}_t$  decreases at a rate of  $g^w$  along a BGP. Finally, we turn to the change of  $\bar{\chi}_t$ . Differentiate both sides of (45) to give

$$\widehat{\bar{\chi}}_t = -(1 + \sigma_{ML})\widehat{M} - \sigma_K\widehat{K}, \quad (59)$$

where “hats” denote growth rates of variables and  $\sigma_i$  is the elasticity of function  $F_2(K, ML)$  with respect to variable  $i$ ,  $i = K, ML$ . Along a BGP, because both  $M$  and  $K$  grow at a rate of  $g^w$ , (59) reduces to

$$\widehat{\bar{\chi}}_t = -(1 + \sigma_{ML} + \sigma_K)g^w = -g^w, \quad (60)$$

where because  $F_2(K, ML)$  is homogeneous of degree zero in  $K$  and  $ML$ ,  $\sigma_{ML} + \sigma_K = 0$ . Condition (60) implies that  $\bar{\chi}_t$  decreases at a rate of  $g^w$  along the BGP. In other words,  $\bar{\chi}_t > p_t^w > \underline{\chi}_t$  can hold over time.

**Proposition 4** *Given Assumption A1, the free-trade BGP equilibrium under diversification of the home economy is unique and sustainable.*

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<sup>10</sup> $F_2(K, L^M)$  is homogenous of degree zero, implying that  $KF_{21} + ML^M F_{22} = 0$ . Given part (b) of assumption A1 and when  $L^M \rightarrow 0$ ,  $F_{21} \rightarrow 0$  for any positive amount of  $K$ .



### 3.3 Trading Regimes indexed by $g^w$

The role of the ROW's growth on the home economy's BGP with diversification can be analyzed further. Let us imagine that there is a continuum of hypothetical trading regimes, in each of which there is one different growth rate of the ROW.<sup>11</sup> We want to examine the balanced growth path of the home economy if it is in different trading regimes.

Let us begin with the regime in which  $g^w$  is the same as the home economy's autarkic growth rate,  $\bar{g}^a$ . In this case, the values of  $L^M$  and  $k$  that satisfy (54) and (53) are the same as those that satisfy (26) and (28). If at the time of allowing free trade the autarkic relative price is the same as that of the ROW, then no trade exists and the no-trade situation continues indefinitely.

Suppose now that there is a small change in  $g^w$  so that the home economy remains diversified under trade. Differentiating both  $\Phi(L^M, k; g^w) = 0$  and  $\Theta(L^M, k; g^w) = 0$  as defined in (55) and (56), treating  $g^w$  as a parameter and rearranging terms, we get

$$\begin{bmatrix} \Phi_L & \Phi_k \\ \Theta_L & \Theta_k \end{bmatrix} \begin{bmatrix} d\tilde{L}^M \\ d\tilde{k} \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} dg^w.$$

Solving the two equations to give the effects of a change in  $g^w$ :

$$\frac{d\tilde{L}^M}{dg^w} = \frac{\Phi_k}{\tilde{D}} > 0, \quad (61)$$

$$\frac{d\tilde{k}}{dg^w} = \frac{-\Phi_L}{\tilde{D}} < 0, \quad (62)$$

where  $\tilde{D} = \mu^2 L^M r w - (1 - \mu L^M k) \mu f r' > 0$ . Condition (61) implies that  $d\tilde{L}^A/dg^w < 0$  and  $dX^A/dg^w < 0$ .

The effects of a change in  $g^w$  can be illustrated in Figure 4. When there is an increase in  $g^w$ , schedule  $\Theta$  shifts up to, say,  $\Theta'$  while schedule  $\Phi$  does not move, with the new equilibrium point depicted by point E', showing an increase in  $\tilde{L}^M$  but a drop in  $\tilde{k}$ . These results are summarized in the following proposition.

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<sup>11</sup>Note that we are comparing the balanced growth paths of the home economy in different regimes with different growth rates of the ROW. For simplicity, we just say an increase or decrease in  $g^w$  as we shift from one regime to another.

**Proposition 5** *As  $g^w$  increases slightly, there is an increase in the BGP values of  $L^M$ ,  $M$ , and  $X^M$ , but a drop in the BPG values of  $k$  and  $X^A$ .*

The pattern of trade of the home economy along a BGP in different regimes can be derived as follows. Define  $Z^M \equiv X^M - C^M - \dot{K}$  (time subscript dropped for simplicity) as the export supply of good M,  $Z^A \equiv X^A - C^A$  as the export supply of good A, and  $E = C^A + pC^M$  as the national expenditure. Whether trade exists, the budget constraint (or the Walras Law, i.e.,  $Z^A + pZ^M = 0$ ) of the economy implies that

$$E = X^A + p(X^M - I), \quad (63)$$

where we let  $I \equiv \dot{K}$ . We are now ready to state and prove the following proposition:

**Proposition 6** *Suppose that the home country trades freely with the ROW and remains diversified. (a) If the ROW grows slightly faster than the home economy and if the home country's investment does not rise as a result of trade, then the home country exports manufacturing and imports agriculture. (b) If the ROW grows slightly slower than the home economy and if the home country's investment does not fall, then the home economy imports manufacturing and exports agriculture.*

The pattern of trade of the home economy suggested in Proposition 6 is not intuitive. One may think that because the manufacturing sector is the engine of growth, if the ROW grows faster then the small economy would have a comparative disadvantage in the good. Proposition 6 shows that this intuition is not correct. The rationale behind the proposition is that if the ROW grows faster, then along a BGP with diversification, the small economy has to catch up by producing more of the manufacturing good. This promotes the export of the good so long as the investment does not increase significantly. The case in which the ROW grows slower can be interpreted in the same way. This analysis shows that in the present model, the more appropriate way to predict the pattern of trade along a BGP is not to compare the autarkic relative prices of the economies at any point of time, since they keep falling, but to compare the BGP growth rates of the economies.<sup>12</sup>

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<sup>12</sup>In the static, neoclassical framework the comparative advantages of economies are

### 3.4 More Conditions for Diversification

In this subsection, we try to derive more conditions for diversification in the economy along a BGP under free trade. Recall that diversification requires  $\tilde{L}^M \in (0, L)$ . Now treating  $L^M$  as a parameter but  $k$  and  $g$  as variables, the BGP equilibrium conditions can be written as

$$\Phi(k, g; L^M) = 0 \quad (64)$$

$$\Theta(k, g; L^M) = 0. \quad (65)$$

When  $L^M \rightarrow 0$ , let  $(\underline{k}, \underline{g})$  solves the conditions, and when  $L^M \rightarrow L$ , let  $(\bar{k}, \bar{g})$  solves the conditions. By condition (55),  $\underline{k} = r^{-1}(\rho)$ , and by condition (56),  $\underline{g} = 0$ . The solution  $(\bar{k}, \bar{g})$  can be obtained by replacing  $L^M$  with  $L$  in conditions (55) and (56). Both  $\bar{k}$  and  $\bar{g}$  are finite.

It was derived earlier that as  $g^w$  rises, the locus of the equilibrium point shifts along schedule  $\Phi$  in Figure 4. Denote the vertical intercept of the schedule by point G, and the point on the schedule that corresponds to  $\tilde{L}^M = \bar{L}$  by point G'. As schedule  $\Phi$  is negatively sloped,  $\bar{k} < r^{-1}(\rho)$ . Therefore condition  $\tilde{L}^M \in (0, L)$  is equivalent to  $g^w \in (0, \bar{g})$ , or  $k \in (\bar{k}, r^{-1}(\rho))$ .<sup>13</sup> We now have the following proposition:

**Proposition 7** *The necessary condition for diversification in a BGP equilibrium under free trade is that  $g^w \in (0, \bar{g})$ .*

## 4 Complete Specialization in Agriculture

We now turn to another type of pattern of production under free trade: complete specialization in the agricultural good. This happens when  $0 < p_t^w \leq \underline{\chi}_t$ . However, recall that because  $p_t^w > 0$ , if the Inada conditions are satisfied, then  $\underline{\chi}_t = 0$ ,<sup>14</sup> meaning that the economy will never specialize in

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defined in terms of the autarkic relative prices. This is the Law of Comparative Advantage (Wong, 1995). This law has been extended to growing economies in their steady states in which autarkic relative prices are stationary. See, for example, Bond and Trash (1997). This law is not applicable in the present model, however.

<sup>13</sup>Notice that  $d\tilde{L}^M/dg^w > 0$  as derived earlier implies the monotonic relation between the two variables.

<sup>14</sup>To get  $\underline{\chi}_t = 0$ , all we need is that  $F_2(K, 0) \rightarrow 0$  as  $L^M \rightarrow 0$ .

the production of agriculture. So the analysis in this section is applicable in the cases in which the Inada conditions are not satisfied so that  $\underline{\chi}_t > 0$ .

When the economy produces only agriculture, we have  $\tilde{L}^A = L$  and  $\tilde{L}^M = 0$ . This means that the wage-equalization condition (49) is no longer valid, since manufacturing is not produced.<sup>15</sup> For the same reason, the LBD equation (51) is not applicable.

The absence of LBD effects under free trade has several implications. First, the stocks of physical capital and human capital become idle.<sup>16</sup> Second, the production of agriculture, which depends on  $L$  only, has a constant output over time. Third, the lack of learning by doing means that the transition of the economy to a new BGP under free trade could be simple. In particular, if labor can move between sectors instantaneously and costlessly, then shifting from diversification under autarky to specialization in agriculture under free trade would require only a quick jump to the new BGP. Fourth, since both  $K$  and  $M$  are constant over time,  $\underline{\chi}_t$  remains stationary over time. On the other hand, the world price  $p_t^w$  decreases at a rate of  $g^w$ . This means that specialization in agriculture is sustained over time.

The last two remarks have a further implication. Suppose that the economy diversifies right after having free trade with the ROW. The home economy then adjusts along a new path. Suppose at any time during the transition of the economy to a new BGP the world price  $p_t^w$  drops below  $\underline{\chi}_t$ , which in general is not a constant when the economy is adjusting. If labor movement between the sectors is instantaneous and costless, then the output of good M drops immediately to zero. This eliminates all LBD effects in the future, and the economy will remain specialized in agriculture over time.

**Proposition 8** *If at any time in the presence of free trade  $p_t^w < \underline{\chi}_t$ , the economy is completely specialized in agriculture. No learning by doing exists, and the pattern of production remains unchanged over time.*

When the home economy produces only agriculture, it trades with the rest of the world and chooses consumption so that the marginal rate of substitution is equal to the world relative price, as given by (46). However, the external terms of trade of the economy improve over time since the world

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<sup>15</sup>In fact, (49) is replaced by  $A > p_t^w M_t w(k_t)$ .

<sup>16</sup>Of course, the economy can sell physical and/or human capital, if tradable, to the ROW. The economy will receive a one-time payment.

relative price decreases continuously. The terms of trade improvement thus improves the welfare of the economy over time, allowing the economy to benefit from the LBD effect in the ROW.

## 5 Complete Specialization in Manufacturing

If  $p_t^w \geq \bar{\chi}_t$ , the home economy will specialize in the production of manufacturing. With learning by doing, labor productivity in the sector grows according to equation (4). Since the economy consumes both goods, it exports manufacture and imports agriculture at the prevailing world prices. Trade is still balanced due to the budget constraint:

$$p_t^w \dot{K}_t = p_t^w M_t L_t^M f(k_t) - p_t^w C_t^M - C_t^A. \quad (66)$$

As before, the representative agent consumes and accumulates physical capital to maximize her intertemporal welfare. Her maximization problem is similar to that analyzed before, except that production of agriculture is zero, implying that  $L^M = L$ . The first-order conditions are then given by (46) – (48), (66) and (51).

Can a BGP with complete specialization in manufacturing be sustained? Suppose that it can be. Recall the equilibrium conditions (64) and (65). We showed that with  $L^M = L$ , the solution to these two conditions is  $(\bar{k}, \bar{g})$ . This equilibrium is depicted by point G' in Figure 4. If  $g^w = \bar{g}$ , the BGP with manufacturing of the home economy with the same growth rate can be sustained. If the  $g^w > \bar{g}$ , then the home economy will not be able to catch up with the ROW. To see this point more clearly, note that with  $g^w > \bar{g}$ , the economy has to accumulate physical capital at a rate as required by condition (53). However, by doing so, condition (54) will be violated, meaning that the intertemporal welfare of the representative agent is not being maximized.

We now derive more explicitly the equilibrium when  $g^w \geq \bar{g}$ . If the economy is growing at its maximum rate,  $\bar{g}$ , with complete specialization in manufacturing,  $\bar{\chi}_t$  is decreasing at the same rate. The minimum value of  $MRT$ ,  $\underline{\chi}_t$ , will either decrease at a rate of  $\bar{g}$  (if the Inada conditions are not satisfied) or will remain at a fixed level of zero (if the Inada conditions are satisfied). As a result, in the singular case in which  $g^w = \bar{g}$ , both  $p_t^w$  and  $\bar{\chi}_t$  are decreasing at a rate of  $\bar{g}$ , and the BGP under specialization in manufacturing can be sustained. If, however, the growth rate of the ROW is  $g^w > \bar{g}$ , then  $p_t^w$  decreases faster than  $\bar{\chi}_t$ . This means that sooner or later

$p_t^w$  is less than  $\bar{\chi}_t$ , and the economy starts producing both goods. However, because  $g^w > \bar{g}$ , a BGP with diversification does not exist. In this case, the BGP equilibrium of the economy is one with complete specialization in agriculture, which has been described in the previous section. However, if  $\underline{\chi}_t = 0$ , then the economy can be completely specialized in agriculture only asymptotically.

**Proposition 9** (a) *If  $g^w = \bar{g}$ , then a BGP equilibrium under free trade with complete specialization in manufacturing is sustainable. (b) If  $g^w > \bar{g}$ , then a BGP equilibrium with complete specialization in manufacturing under free trade is not sustainable. The BGP of the economy is one in which the economy is completely specialized in agriculture [asymptotically if the Inada conditions are satisfied].*

## 6 Concluding Remarks

We have constructed a model to analyze the relationship between industrialization, economic growth and international trade. This model has some interesting features that distinguish it well from many of the existing growth models in the literature. Probably the most notable one is the fact that the learning-by-doing effect exists in one of the sectors only, making labor in that sector more and more productive. In a closed economy, the widening gap in labor productivity in the two sectors does not imply the decline or disappearance of the stagnant (agriculture) sector, but it does lead to the constant drop in the relative price of the growing (manufacturing) sector. The feature of a declining relative price is consistent with what we usually observe for many manufacturing goods.

The determinants of the growth of the economy is the focus of this paper. When an economy is closed, its growth rate depends on the learning-by-doing effect, which is directly related to the relative output level of the manufacturing sector. This suggests the possible use of production subsidies to promote growth.

Under free trade, the effectiveness of production subsidies in promoting growth is generally limited. The growth of the economy depends on, among other things, its pattern of trade and the growth of the rest of the world. For example, if the economy is diversified along a balanced growth path, then its growth rate is pegged to that of the rest of the world. If the economy is

completely specialized in agriculture, then no learning-by-doing effects exist, and the home economy will have no incentive to invest in physical capital because physical capital is not used in the agricultural sector. The growth rate of the economy drops down to zero, showing the importance of the manufacturing sector in its growth performance. For the case of complete specialization in manufacturing, except in the singular case in which  $g^w = \bar{g}$ , a BGP with complete specialization in manufacturing does not exist. A BGP of the economy, if it exists, is complete specialization in agriculture [asymptotically if the Inada conditions are satisfied].

The last remark that may be pointed out is that this paper considers only domestic learning by doing as an engine of growth. There could be many other factors that improve the growth performance of an economy resulting from trading with other countries: for example, technology spillover effects, technology transfers through foreign direct investment and international labor migration, and imitation.<sup>17</sup> All these effects, which have been ignored here, could be the topics for future research.<sup>18</sup>

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<sup>17</sup>Wong (1997) shows that international labor migration from a more advanced country to a less advanced country can improve the human capital level in the latter country.

<sup>18</sup>For a recent survey of the effects of trade on growth, see Long and Wong (1997).

## Appendix

**Proof of Proposition 1:** Along a BGP equilibrium, by definition,  $k_t$  is constant. Condition (18) implies that  $L_t^M$  is constant over time. Let the BGP equilibrium growth rate of  $M_t$  be  $g^a$ . Then (5) yields  $\dot{p}_t/p_t = -g^a$ . Next, (12) and (1) give constancy of  $L_t^A$ ,  $X_t^A$  and  $C_t^A$ , while (14) and (17) in turn imply that both  $C_t^M$  and  $K_t$  (hence  $X_t^M$ ) are growing at the same rate  $g^a$ . Finally, condition (15) implies that  $q_t$  is a constant along a BGP, meaning that both  $\lambda_t^C$  and  $\lambda_t^M$  are declining at the same rate  $g^a$ . ■

**Proof of Proposition 2:** Let us first examine the properties of condition (26), and use superscript “ $I$ ” to denote the corresponding variables. As  $L^M \rightarrow 0$ ,  $k \rightarrow \underline{k}^I \equiv r^{-1}(\rho/(1+s)) > 0$ , where  $r^{-1}(\cdot)$  is the inverse function of the rental rate function. As  $L^M \rightarrow L$ ,  $k \rightarrow \bar{k}^I$ , where  $\bar{k}^I$  solves the following equation<sup>19</sup>

$$\mu L \bar{k}^I = 1 + s - \frac{\rho}{r(\bar{k}^I)}. \quad (67)$$

Furthermore, the rate of change of  $k$  with respect to  $L^M$  subject to condition I is equal to

$$\left. \frac{dk}{dL^M} \right|_I = \frac{\mu k r}{\rho r' / r - r \mu L^M} < 0, \quad (68)$$

where  $r' \equiv dr/dk < 0$ .

We now turn to condition (28) and use superscript “ $II$ ” to denote the corresponding variables. As  $L^M \rightarrow 0$ ,  $k \rightarrow \underline{k}^{II} \equiv 0$ , when given the Inada condition.<sup>20</sup> As  $L^M \rightarrow L$ ,  $k \rightarrow \bar{k}^{II} \equiv 1/(\mu L)$ . Note that when  $s$  is sufficiently

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<sup>19</sup>To see whether equation (67) has a solution, note that when  $k \rightarrow 0$ ,  $r(k) \rightarrow \infty$  by the Inada conditions, so that the LHS is less than the RHS; when  $k \rightarrow \infty$ ,  $r(k) \rightarrow 0$ , implying that the LHS is greater than the RHS. Continuity of the functions imply that at least one solution with  $k^I > 0$  exists. Note further that the LHS is increasing in  $k$  while the RHS is decreasing in  $k$ . Thus the solution is unique.

<sup>20</sup>Rewrite equation (28) as  $\beta L^M (1 - \mu L^M k) f(k) = (L - L^M)(1 + s)w(k)$ . When  $L^M \rightarrow 0$ , the LHS of the equation approaches zero, requiring that the corresponding  $w(k) \rightarrow 0$ , or  $k \rightarrow 0$  by the Inada conditions.



small, equation (67) implies that  $\bar{k}^{II} > \bar{k}^I$ .<sup>21</sup> Furthermore, the rate of change of  $k$  with respect to  $L^M$  subject to conditions (28) and C1 is equal to

$$\left. \frac{dk}{dL^M} \right|_{II} = \frac{L(1+s)w}{L^M[(L-L^M)(1+s)w' - \beta r L^M]}. \quad (69)$$

From condition (69), the rate of change of  $k$  with respect to  $L^M$  approaches positive infinity as  $L^M \rightarrow 0$ . This rate of change is not continuous at  $(L-L^M)(1+s)w' = \beta r L^M$ , approaching positive infinity as  $[(L-L^M)(1+s)w' - \beta r L^M]$  approaches  $0^+$ , or negative infinity as  $[(L-L^M)(1+s)w' - \beta r L^M]$  approaches  $0^-$ . Because  $\underline{k}^I > \underline{k}^{II}$  and  $\bar{k}^I < \bar{k}^{II}$ , and due to continuity of the functions, there exists at least one autarkic equilibrium. Since the rate of change of  $k$  with respect to  $L^M$  for condition (26) is always negative, if the corresponding rate of change of  $k$  with respect to  $L^M$  for condition (28) at the autarkic equilibrium is always positive, the autarkic equilibrium is unique. ■

**Proof of Proposition 3:** By the definition of a BGP,  $k_t$  is constant and  $M_t$  is growing at a constant rate of  $g^w$ . Condition (51) implies that  $L_t^M$  (and hence  $L_t^A$ ) are stationary. This in turn yields a constant  $K_t/M_t$  ratio. Since  $\dot{p}_t^w/p_t^w = -g^w$ , (49) gives the BGP equilibrium growth rate of  $M_t$  (and hence  $K_t$ ) as  $g^w$ . Next, (46) and (50) give constancy of  $p_t^w C_t^M$  and  $C_t^A$  so that  $C_t^M$  is growing at the same rate  $g^w$ . Finally, from (8) and (47), both  $\lambda_t^C$  and  $\lambda_t^M$  are declining at the same rate  $g^w$ . ■

**Proof of Proposition 6:**(a) Condition (46) implies that  $E = (1 + 1/\beta)C^A$ , or that

$$\Delta E = (1 + 1/\beta)\Delta C^A. \quad (70)$$

If investment does not fall, then Proposition 5 implies that  $\Delta(X^M - I) > 0$  so that by condition (63)  $\Delta E > \Delta X^A$ . This result and condition (70) imply that

$$(1 + 1/\beta)\Delta C^A > \Delta X^A. \quad (71)$$

By Proposition 5,  $\Delta X^A < 0$ . If  $\Delta C^A > 0$ ,  $\Delta Z^A < 0$  and, by the Walras Law,  $\Delta Z^M > 0$ . If  $\Delta C^A < 0$ , then (71) implies that  $\Delta X^A < \Delta C^A < 0$ . So again,  $\Delta Z^A < 0$  and  $\Delta Z^M > 0$ . Part (b) can be proved in a similar way. ■

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<sup>21</sup>Actually what we need for this result is that  $s < \rho/r(k^I)$ .

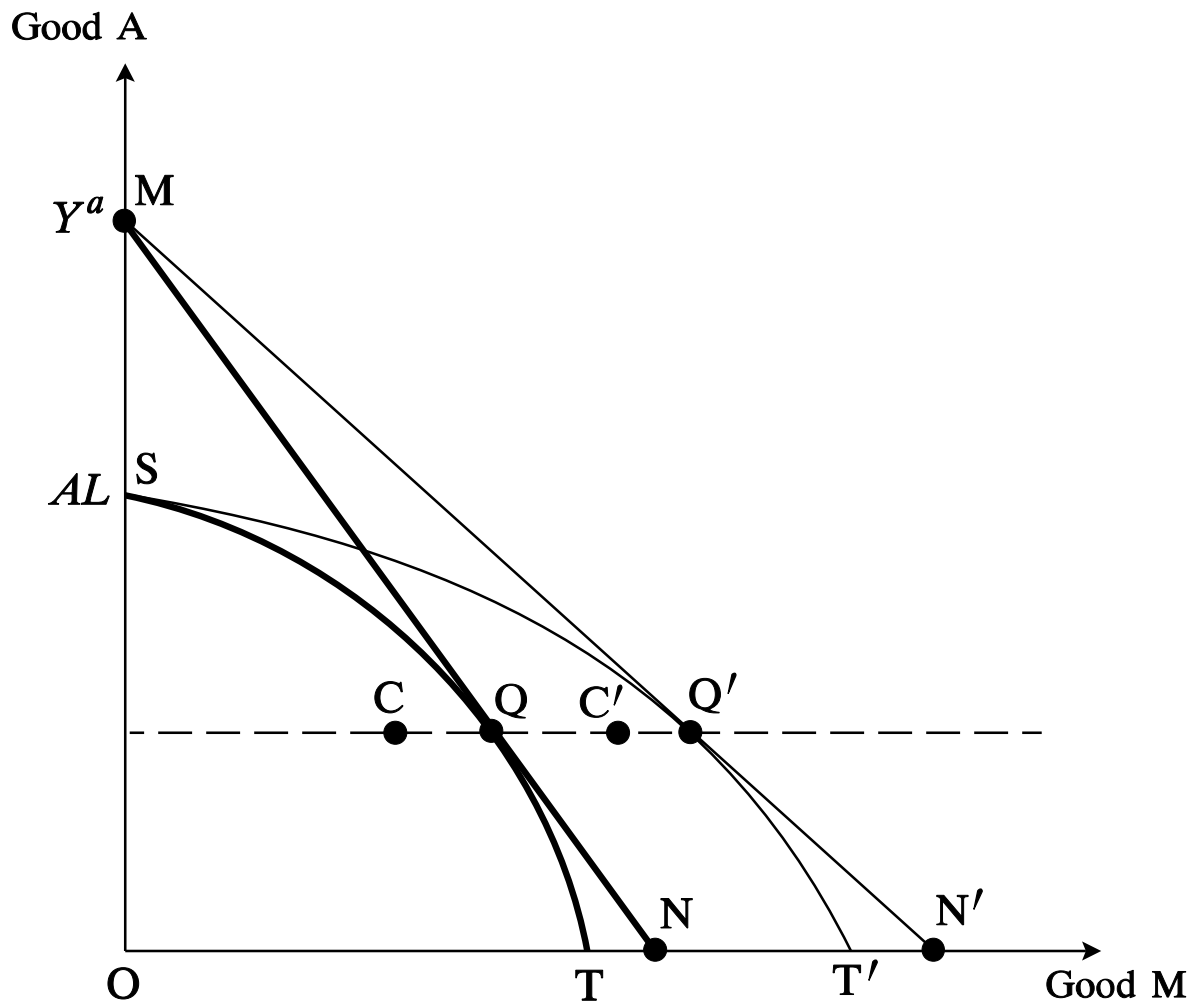


Figure 1

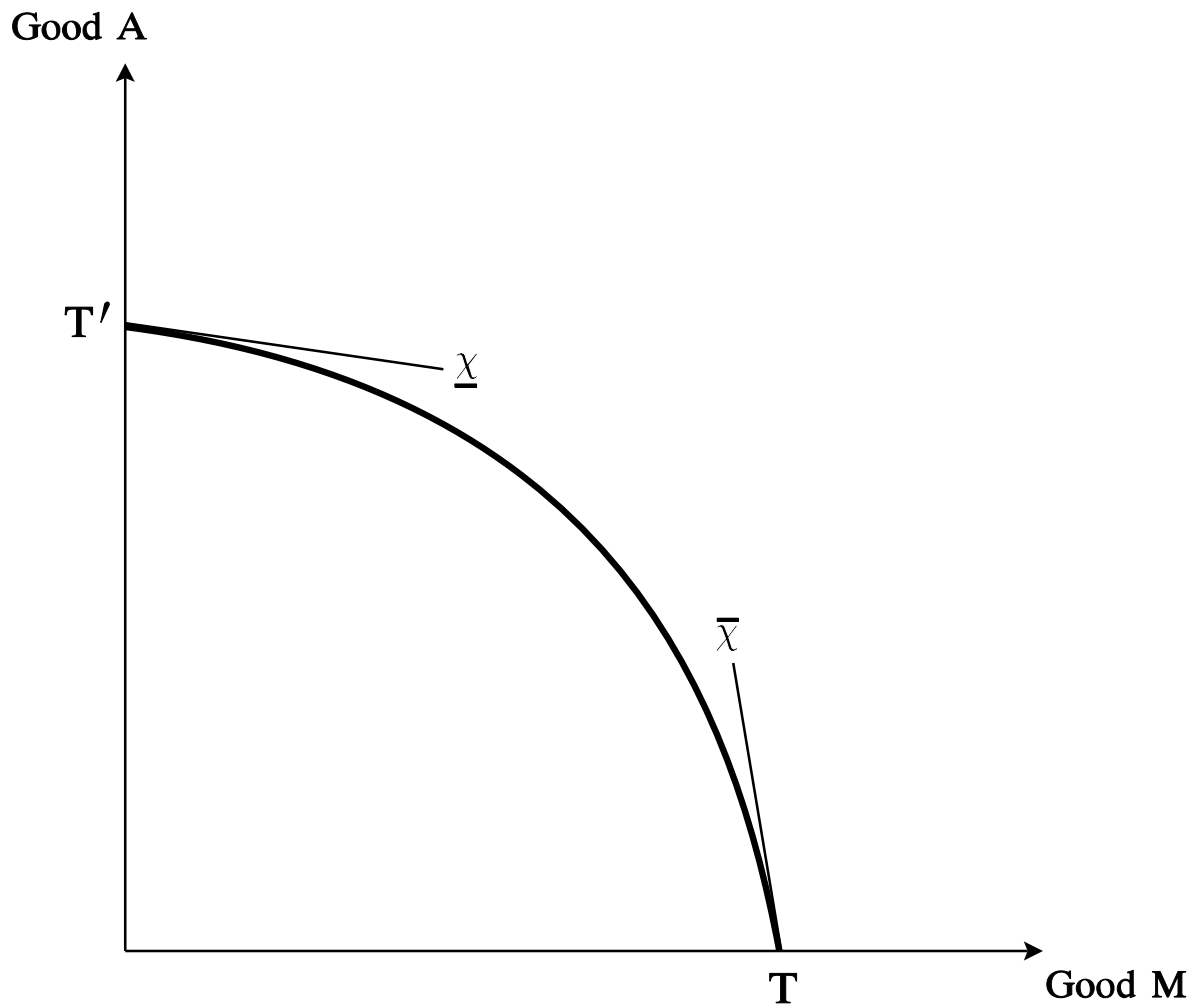


Figure 2

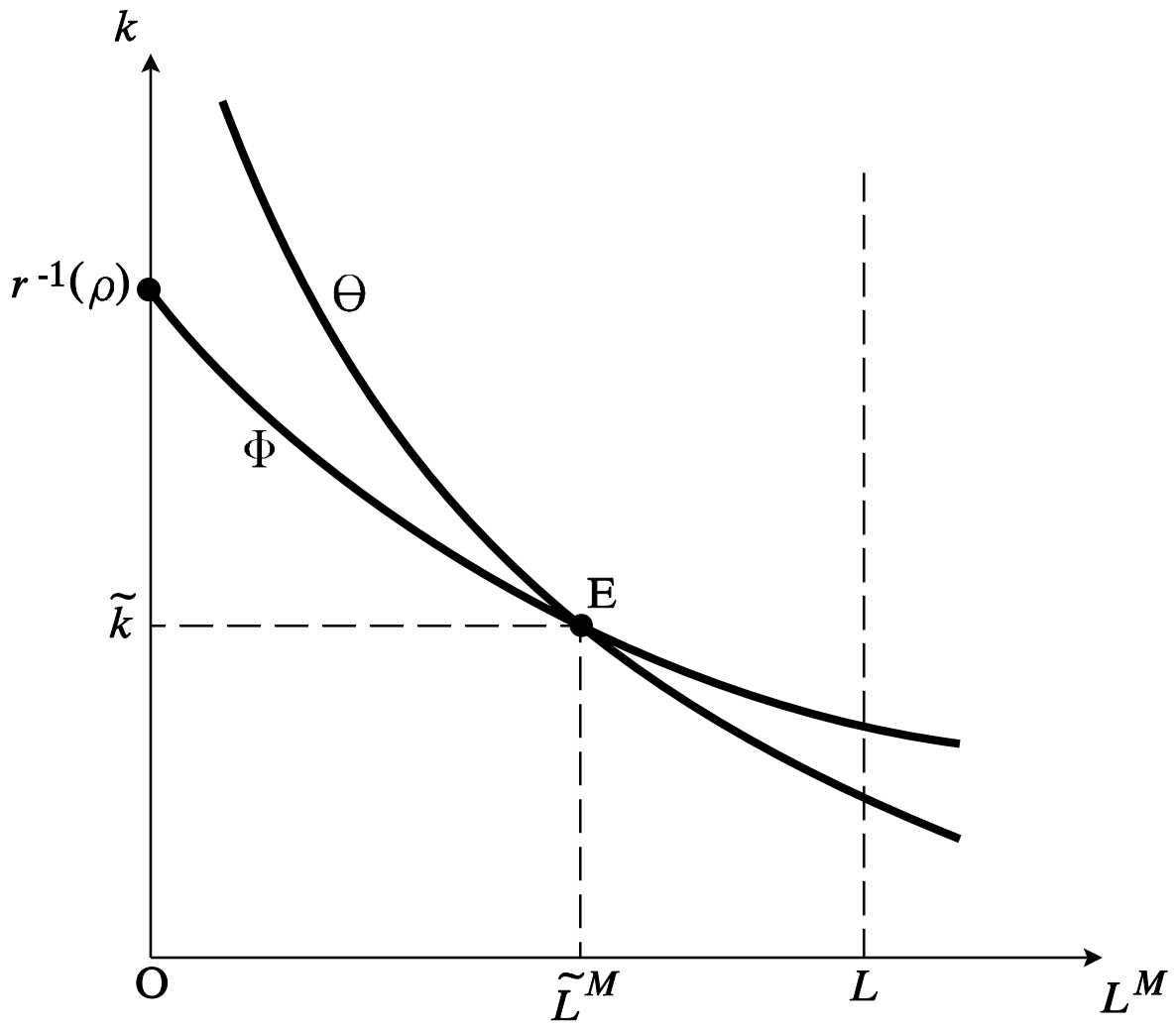


Figure 3

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