# Education, Economic Growth, and Brain Drain

Kar-yiu Wong
University of Washington
and
The Chinese University of Hong Kong

Chong K. Yip
The Chinese University of Hong Kong

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**Abstract**

This paper constructs a two-sector overlapping-generations model of endogenous growth to study the effects of brain drain on growth, education and income distribution. The engine of growth is human capital accumulation through education and intergenerational spillover. Brain drain reduces both the economic growth rate and the wage rate of the unskilled, but raises the wage rate of the skilled. Brain drain, however, generally hurts the non-emigrants through the static income distributional effects and also the dynamic damage on economic growth and human capital accumulation. If the initial rate of human capital accumulation is relatively low, brain drain could deteriorate both the sum of discounted income and lifetime discounted utility of a representative non-emigrant. Finally, we show that the government can choose to spend more on education in order to lessen the detrimental impacts of brain drain on economic growth.

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# 1 Introduction

International migration of skilled and professional workers, commonly known as brain drain, has long been an important topic for economists and government planners in many countries.[1] Many governments, especially those that are losing these workers, have great concerns about the possible adverse effects of brain drain on economic growth, education, income distribution, and welfare. These concerns stem from the fact that brain drain is the outflow of one of the most scarce resources in many source countries: human capital. Over time, brain drain could adversely affect the formation of human capital in a source country, and this could hurt the growth and other important variables of the economy.

Surprisingly, there has been very little work in the literature that examines the linkage between brain drain and economic growth. While Rodriguez (1976), Findlay and Rodriguez (1981), and Blomqvist (1986) analyze the determination of the education level in the presence of international migration in a dynamic context, their models are characterized by fixed skilled levels of the educated workers, implying that the growth rate of the economy is not substained. Shea and Woodfield (1996) derive the optimal immigration policy in a dynamic context, but their economy still does not exhibit perpetual growth. Galor and Stark (1994) show how immigration may reverse the adjustment of an economy, but in their model human capital accumulation is bounded, and hence the growth of an economy eventually stops. Wong (1995) is more explicit about the choice of the types of migration by individuals, but he does not provide any model that generates growth in the long run. Wong (1997) develops a rigorous model on brain drain in the presence of endogenous growth. His concerns, however, are more on the endogenous choice of the type of labor migration, but less on economic growth. Moreover, he does not examine the welfare effects of brain drain and the effect of a change in the education policy.

In this paper, we construct an endogenous growth model of overlapping generations to analyze the effects of brain drain on growth, education, income distribution, and welfare. The advantage of this model is that it allows an explicit examination of the dynamic and welfare effects of brain drain. We

[1]See Bhagwati (1976) for a summary and discussion of some of the arguments.

also study the effects of an education policy on labor movement, factor prices, and economic growth.

The source of growth of the economy considered in the present paper is the accumulation of human capital through education and intergeneration spillover. To model education and endogenous growth, we extend the model of Uzawa (1965) and Lucas (1988). In the Uzawa-Lucas model, education is the only channel through which human capital accumulates, and the rate of accumulation of human capital depends on, among other things, the time an agent spent on education. However, unlike what is assumed in the Uzawa-Lucas model, the marginal rate of substitution between skilled and unskilled workers is assumed to be diminishing in the present production process. This allows an explicit consideration of the effects of brain drain and education policies on the wage rates of unskilled and skilled workers. Endogeneity of growth in the present model means that generally there are some policies, such as a change in the education expenditure or brain drain, that have growth effects.

Analyzing the features of brain drain is the main focus of this paper. In addition to deriving the balanced growth path of brain drain and analyzing its properties, this paper also examines the dynamic adjustment of the economy as well as the welfare effects of brain drain. The interesting thing here is that brain drain has effects on not just income distribution in any period, which have been analyzed in the literature,[2] it also has dynamic effects as it affects the growth rate of human capital.

As will be argued below, brain drain hurts the growth rate of the economy. This leads to some argument that the government should intervene, possibly restricting emigration to protect the economy's growth. This paper analyzes the growth effect of a more aggresive education policy: a higher educator-student ratio. It is shown that such a policy can be used to maintain the autarkic growth rate in the presence of brain drain. This policy would require, however, a higher income tax rate if the government budget is to be balanced.

The paper is organized as follows. Section 2 describes the main properties of the present model. The educational sector is described in details. Section 3 derives the autarkic balanced growth path of an economy and also examines the effects of an increase in the labor endowment and a more aggressive education policy. Section 4 analyzes the important properties of brain drain. Section 5 examines the properties of the balanced growth path while Section

[2]See, for example, Grubel and Scott (1996), Berry and Soligo (1969), and Wong (1986).

6 investigates the transitional dynamics of the model and its global stability. Section 7 analyzes the welfare effects of brain drain, and Section 8 derives the education policy that may counter the adverse effect of brain drain on economic growth. Section 9 concludes.

# 2 The Model

Consider a Diamond-type overlapping-generations economy with two sectors: commodity and education production. A homogeneous commodity is produced by competitive firms and consumed by local residents.

## 2.1 Technology

Competitive firms hire two types of inputs, unskilled labor and skilled labor, to produce the homogeneous good. In period $t$, $t = 0, 1, \ldots, \infty$, the sectoral production function is represented by

$$Q_t = F(L_t^s, L_t^u), \tag{1}$$

where $Q_t$ is the output, $L_t^s$ is the input of skilled labor measured in efficiency units, and $L_t^u$ is the input of unskilled labor, also measured in efficiency units. The production function, which is stationary over time, is increasing, differentiable, linearly homogeneous and concave in inputs. Let $\ell_t \equiv L_t^u/L_t^s$ be the ratio of unskilled labor input to skilled labor input; for simplicity, it is called the unskilled-skilled-labor (USL) ratio. Linear homogeneity means that the production function can be written in an intensive form:

$$Q_t = L_t^s f(\ell_t), \tag{2}$$

where $f(\ell_t) \equiv F(\ell_t, 1)$. The production function further satisfies the following conditions:

$$f' > 0,\ f'' < 0,\ f'(0) = \infty,\ f'(\infty) = 0.$$

Perfect competition and cost minimization of firms imply that the wage rate per efficiency unit of skilled labor and that of unskilled labor are respectively given by:

$$w_t^s = f(\ell_t) - \ell_t f'(\ell_t), \quad (3)$$
$$w_t^u = f'(\ell_t). \quad (4)$$

## 2.2 Workers

Each worker lives for two periods, which, with respect to a particular individual, are labeled young ($y$) and old ($o$). In period $t$, after an individual has been born, he inherits the general knowledge level in the previous period, which is denoted by $x_{t-1}$. This knowledge level also represents the amount of labor efficiency he supplies when working for one unit of time. With the inherited knowledge, he can work as an "unskilled" worker and/or spend time on receiving education to further improve his human capital. Assuming that each individual is endowed with one unit of nonleisure time in each period, denote the amount of time a representative individual chooses to spend on education by $\tau_t \in [0,1]$. The amount of time that the individual has for working as an unskilled worker is therefore $(1-\tau_t)$, which can provide an income of $(1-\tau_t)x_{t-1}w_t^u$.

Education is provided for free by the government. This means that the cost of education for an individual is only the forgone time spent on receiving education. If an individual chooses to receive education when young, he will get a higher level of knowledge in the next period, which is denoted by $x_{t+1}$. Since receiving education when old can be ruled out, an individual when working as a skilled worker will supply $x_{t+1}$ units of efficiency labor, generating an income of $x_{t+1}w_{t+1}^s$, which is subject to an income tax of ad valorem rate $\phi_{t+1}$. It is assumed that both $x_{-1}$ and $x_0$ are given as the initial conditions of the economy, with $x_0 > x_{-1}$.

For simplicity, bond markets are not considered, meaning that each individual has to balance the budget in each period. Consider a representative consumer who is born in the beginning of period $t$. Denoting his consumption when young in period $t$ and that when old in period $t+1$ by $c_t^y$ and $c_{t+1}^o$, respectively, his budget constraints in the two periods are:

$$c_t^y \leq (1-\tau_t)x_{t-1}w_t^u, \quad (5)$$
$$c_{t+1}^o \leq (1-\phi_{t+1})x_{t+1}w_{t+1}^s. \quad (6)$$

Let the intertemporal utility function of the representative individual be denoted by

$$u_t = \ln c_t^y + \rho \ln c_{t+1}^o, \tag{7}$$

where $\rho \in (0, 1)$ is the discount factor. The utility function is uniform among all individuals. The representative individual, who is assumed to have rational expectation (perfect foresight), chooses the consumption bundles in the two periods, subject to the budget constraints in (5) and (6), to maximize his intertemporal utility.

Denote the numbers of unskilled workers and skilled workers available for production in period $t$ by $N_t^u$ and $N_t^s$, respectively. Therefore the total supplies of unskilled and skilled labor, which are the inputs to the firms in equilibrium, are

$$L_t^u = (1 - \tau_t) x_{t-1} N_t^u, \tag{8}$$

$$L_t^s = x_t N_t^s. \tag{9}$$

It is assumed that in periods $-1$ and $0$, there are equal sizes of each type of workers, $\overline{N}$, and that starting from period 0 on average each individual gives birth to one child before he or she dies, implying that if economy remains closed, its population is equal to $2\overline{N}$ in each period.

## 2.3 Human Capital Accumulation

Education is the channel through which an individual improves the knowledge he possesses. The new knowledge he acquires in the next period, $x_{t+1}$, depends on three factors: the current general knowledge level $x_t$, the amount of time he spends on receiving education $\tau_t$, and the number of educators $E_t$, who are hired and provided by the government. More specifically, the education production function is assumed to be:

$$x_{t+1} = x_t A(\tau_t) B(E_t). \tag{10}$$

The education production function satisfies the following properties:

1. $A(0) = 1$, $A(\tau_t) > 1$ for $\tau_t > 0$, $A'(\tau_t) > 0$, $A'(0) > 1/\rho$, and $A''(\tau_t) < 0$ for $\tau_t \geq 0$;

2. $B(0) = 1$, and $B'(E_t) > 0$.

The specification of our education production function implies that existing knowledge can be improved by either self-learning of the students or research of the educators.

In period $t$, the government hires $E_t$ educators to provide free education. Denote the ratio of educators to students by $\alpha_t$, which can be interpreted as the degree of education subsidization. For the time being, $\alpha_t$ is treated as a parameter. If all unskilled workers spend a positive amount of time on education (we shall prove this point later), then the number of unskilled workers in period $t$ is the same as the number of students, and the number of educators is equal to

$$E_t = \alpha_t N_t^u. \tag{11}$$

Free education is financed by an income tax of ad valorem rate $\phi_t$ on all skilled workers, including the educators. A balanced government budget implies that

$$x_t E_t w_t^s = \phi_t x_t [N_t^s + E_t] w_t^s, \tag{12}$$

where $N_t^s + E_t$ is the total number of skilled people, including skilled workers and educators, in the economy in period $t$. Because of condition (12), a balanced government budget requires that the ad valorem income tax rate be equal to:

$$\phi_t = \frac{\alpha_t N_t^u}{N_t^s + \alpha_t N_t^u}. \tag{13}$$

# 3 Autarkic Equilibrium

To solve for the utility maximization problem of a representative individual, we substitute (5), (6) and (10) into (7) to get

$$\begin{aligned} u_t &= \ln[(1-\tau_t)x_{t-2}A(\tau_{t-1})B(E_{t-1})w_t^u] \\ &\quad + \rho \ln[(1-\phi_{t+1})x_{t-1}A(\tau_t)B(E_t)w_{t+1}^s]. \end{aligned} \tag{14}$$

We then differentiate (14) with respect to $\tau_t$ to give

$$\frac{\partial u_t}{\partial \tau_t} = -\frac{1}{1-\tau_t} + \frac{\rho A'(\tau_t)}{A(\tau_t)},$$

which implies the following first-order condition:

$$\frac{(1-\tau_t)A'(\tau_t)}{A(\tau_t)} = \frac{1}{\rho}. \tag{15}$$

The second-order condition is satisfied, as

$$\frac{\partial^2 u_t}{\partial \tau_t^2} = -\frac{1}{(1-\tau_t)^2} + \rho\frac{A(\tau_t)A''(\tau_t) - [A'(\tau_t)]^2}{A(\tau_t)} < 0, \tag{16}$$

where the properties of the function $A(\tau_t)$ have been used. Equation (15) can be solved for the optimal time spent on education, $\overline{\tau}_t$.

**Lemma 1** $\overline{\tau} \in (0,1)$ and is unique.

**Proof.** If $\tau_t \to 0$, then by the properties of the function $A(\tau_t)$, $\partial u_t/\partial \tau_t > 0$, and if $\tau_t \to 1$, then $\partial u_t/\partial \tau_t < 0$. By continuity of the utility function, there exists an interior solution. Since the second-order derivative of the function with respect to $\tau_t$ is always negative, the solution is unique. ■

Note that the optimal time of education chosen by individuals with fixed $\rho$ is independent of the number of educators and is stationary over time. This property is partly due to the functional form of the utility function, and partly due to the multiplicative form of the education production function.

Since for this closed economy the number of each type of workers in any period is equal to $\overline{N}$, we have

$$N_t^u = \overline{N} \tag{17}$$

$$N_t^s + E_t = \overline{N}. \tag{18}$$

Combining equations (11), (17) and (18), we get

$$N_t^s = (1-\alpha)\overline{N}. \tag{19}$$

Substituting (19) into (13) to give the autarkic income tax rate that balances the government budget, we have $\overline{\phi}^a = \alpha$, where the superscript "$a$" denotes the autarkic value of a variable.

A balanced growth path (BGP) of the economy is one in which all intensive variables (e.g., rates or ratios) such as $\tau_t$, $\ell_t$, and factor prices are stationary, while all extensive variables (e.g., levels) such as $x_t$, $Q_t$, and factor supplies grow at constant rates. To analyze the existence of a BGP, let us denote the (gross) growth rate of human capital by

$$g_t = \frac{x_{t+1}}{x_t}, \tag{20}$$

and define a new variable:

$$n_t = \frac{(1-\tau_t)N_t^u}{N_t^s}. \tag{21}$$

This variable $n_t$, which is called the unskilled-skilled-worker (USW) ratio, can be interpreted as the ratio of "effective" number of unskilled workers to the number of skilled workers, and is related to the unskilled-skilled-labor (USL) ratio in the following way:

$$n_t = g_{t-1}\ell_t. \tag{22}$$

We can now analyze the BGP. By lemma 1, each individual spends a fraction of non-leisure time $\overline{\tau} \in (0,1)$ on education when young. Assuming that the government sets a constant education parameter $\alpha$ in all periods, (10) and (20) imply that the BGP (gross) growth rate of human capital is equal to

$$\overline{g}^a = A(\overline{\tau})B(\alpha\overline{N}), \tag{23}$$

where the "overline" denotes a BGP value. Using (17) and (18), the BGP value of $n_t$ is equal to

$$\overline{n}^a = \frac{1-\overline{\tau}}{1-\alpha}. \tag{24}$$

Equations (23) and (24) can be combined to give the BGP value of $\ell_t$,

$$\overline{\ell}^a = \frac{\overline{n}^a}{\overline{g}^a} = \frac{1-\overline{\tau}}{A(\overline{\tau})B(\alpha\overline{N})(1-\alpha)}. \tag{25}$$

Note that the USL ratio is constant over time along a BGP and so are the factor prices.

The autarkic BGP can be described by conditions (23) to (25). Its existence can easily be seen from these three conditions, and can be illustrated in Figure 1. Ray OG from the origin has a slope equal to $\overline{g}^a$, while line MN has a vertical height of $(1-\overline{\tau})/(1-\alpha)$. Continuity of the two lines means that they meet once and only once, and the intersecting point, H, gives $\overline{\ell}^a$. This proves the existence and uniqueness of the BGP.

Another property of the BGP is that from any initial point and when given the education parameter $\alpha$, it is achieved in at most one period, meaning that this economy when closed has no transitional dynamics. The reason is that in period 0, when the individuals are given the knowledge levels in this and the previous periods, as well as the education parameter $\alpha$, the unskilled workers will choose the optimal education time $\overline{\tau}$, which yields the growth rate of human capital as given by (23), and at the same time $n_t$ achieves its BGP value.

The BGP growth rate of output is equal to

$$\frac{Q_{t+1}}{Q_t} = \frac{L^s_{t+1} f(\overline{\ell}^a)}{L^s_t f(\overline{\ell}^a)} = \frac{x_{t+1}}{x_t},$$

which is the same as that of human capital.

The above results are summarized by the following proposition:

**Proposition 1** *A balanced growth path of the economy exists and is unique.*

Along a BGP, the welfare of a representative individual in period $t$ is given by

$$u^a_t = \ln(1-\tau)x_{t-1}\overline{w}^{ua} + \rho\ln(1-\overline{\phi}^a)x_{t+1}\overline{w}^{sa}. \tag{26}$$

Because of the accumulation of human capital through education, condition (26) implies that the welfare of individuals increases over time. For example, the change in welfare level over one period is equal to

$$u^a_{t+1} - u^a_t = (1+\rho)\ln\overline{g}^a. \tag{27}$$

Note that because $u^a_t$ is expressed in logarithmic form, $u^a_{t+1} - u^a_t$ is interpreted as the (gross) growth rate of welfare of individuals (over two consecutive generations).

Before we turn to brain drain, we perform two comparative statics exercises. First, suppose that there is a sudden increase in population while the government maintains the same educator-student ratio. More specifically, suppose that in each period both the number of unskilled workers and that of skilled individuals, including the skilled workers and educators, increase to $\overline{N}' > \overline{N}$. With a larger population, equation (23) implies that the growth rate of the economy increases to

$$\overline{g}^{a\prime} = A(\overline{\tau})B(\alpha\overline{N}') > \overline{g}^{a}.$$

This growth rate is represented in Figure 1 by a steeper ray OG′ from the origin. Line MN, however, remains unchanged. Therefore the new intersecting point, H′, represents the new BGP equilibrium. With this new BGP, the USL ratio is lower, meaning that there is relatively more skilled labor in the economy. This implies, by (3) and (4), that the increase in population would cause a drop in the skilled workers' wage rate but a rise in the unskilled workers' wage rate per efficiency unit of labor.

The changes in factor wages due to an increase in population demonstrate the roles of educators in education and the education policy. As there are more unskilled workers, the government would hire more skilled workers to educate the students. This improves the growth rate of human capital. Since the USW ratio remains constant at $(1-\overline{\tau})/(1-\alpha)$, there is a drop in $\overline{\ell}^{a}$ and thus the corresponding changes in factor prices.

The next exercise about the autarkic BGP is to determine the effects of a change in the education policy. Suppose that the government chooses to increase $\alpha$, i.e., providing more educators to any given number of students. The effects of the new education policy on the growth rate and USW ratio are

$$\frac{d\overline{n}^{a}}{d\alpha} = \frac{(1-\overline{\tau})}{(1-\alpha)^2} > 0 \tag{28}$$

$$\frac{d\overline{g}^{a}}{d\alpha} = A(\overline{\tau})B'(\alpha\overline{N})\overline{N} > 0. \tag{29}$$

These effects are illustrated in Figure 2. Condition (28) implies a rise of line MN to M′N′, while condition (29) represents a rotation of ray OG to OG′. The new equilibrium point, point H′, is the intersecting point between M′N′ and OG′. Depending on by how much each line is shifted, the autarkic USL ratio $\overline{\ell}^{a}$ may go up or down. The ambiguous direction of change of $\overline{\ell}^{a}$ can also be confirmed by direct differentiation:

$$\frac{d\overline{\ell}^a}{d\alpha} = \frac{1}{(\overline{g}^a)^2}\left[\frac{(1-\overline{\tau})\overline{g}^a}{(1-\alpha)^2} - \overline{n}^a A(\overline{\tau})B'(\alpha\overline{N})\overline{N}\right] \gtrless 0.$$

To interpret the ambiguity of an increase in education subsidy on the USL ratio, note that an increase in $\alpha$ would improve the growth rate, which means that the skilled labor supply tends to rise. On the other hand, an increase in $\alpha$ also raises the ratio of effective unskilled workers to skilled workers. This tends to encourage the supply of unskilled labor. Finally, if the change in the USL ratio is ambiguous, then so are the factor prices.

# 4 Brain Drain

Suppose that in addition to the above economy, which from now on is called the source country, there is another country, which is called the host country. Both countries have similar economic structures as described in the previous section, but they may have different labor endowments, preferences, production and education technologies, and income tax policies. Brain drain, which is the outflow of skilled workers from the source country to the host country, is allowed. To simplify our analysis, the following assumptions are made.

1. The source country is a small one as compared with the host country in the sense that the economic conditions such as prices and policies in the host country are not affected by any labor movement. In other words, the source country takes the economic conditions in the host country as given.

2. Before any labor movement, both countries are on their own autarkic balanced growth paths.

3. The host government has a free immigration policy.

4. The host government would not accept unskilled immigrants. This means that brain drain is the only type of labor migration considered. An emigrant will work in the host country as a skilled worker until he dies.

5. The amount of human capital possessed by a skilled worker is country invariant. This means that a skilled emigrant with human capital given by $x_t$ is able to supply the same amount of human capital in the host country.

6. Unless stated otherwise, the government of each country maintains the same education policy as that under autarky.

7. Movement of skilled workers is costless, instantaneous, and risk free. This means that in any period, local skilled workers will move out until a marginal skilled workers will get the same welfare whether he moves or not. This is a temporary equilibrium. However, a temporary equilibrium may or may not be a permanent migration. Furthermore, welfare gap is the only motive for labor movement.

Several properties of brain drain deserve discussion. When skilled workers emigrate permanently out, not only they, but also their descendants, are permanent residents of the host country. This means that the source country will lose population through brain drain. The evolution of population is governed by the following equation:

$$N_t^u = N_{t-1}^s + E_{t-1}, \tag{30}$$

which means that the number of unskilled workers in any period is equal to the total number of remaining skilled workers and educators in the previous period.

The outflow of skilled workers from the source country has several implications. First, the educator-student ratio does not change, the USL ratio, $\ell_t$, will increase. Second, there will be less people to support the free education. Third, the decrease in the tax base for financing the free education, the government is forced to either (a) provide an inferior education with a smaller educator-student ratio, or (b) raise the income tax rate, or (c) find other revenue sources to finance the free education.

Let the after-tax wage rate per efficiency unit of labor in the host country be $F^*$. Since the host country is in a BGP and since the income tax rate is constant, the after-tax wage rate $F^*$ is stationary. To simplify the notation, relabel the periods so that period 0 is the first period when brain drain is allowed. The following condition is assumed

$$F^* > (1 - \overline{\phi}^a)\overline{w}^{sa}, \tag{31}$$

where $\overline{w}^{sa}$ is the autarkic BGP wage rate. Condition (31) implies an incentive for brain drain to the host country in period 0 and possibly beyond.

Since emigration is assumed to be costless, instantaneous, and risk free, an temporary equilibrium in period $t$, $t = 0, 1, \ldots \infty$, is characterized by

$$F^* = (1 - \phi_t)w_t^s. \tag{32}$$

The income tax rate that balance the government budget is given by condition (13), which, by making use of condition (21), reduces to:

$$\phi_t = \frac{\alpha n_t}{(1 - \overline{\tau}) + \alpha n_t}. \tag{33}$$

By condition (3), the skilled workers' wage rate depends on the USL ratio, i.e.,

$$w_t^s = w^s(\ell_t). \tag{34}$$

Conditions (33) and (34) can be substituted into the temporary equilibrium condition (32) to give:

$$F^* = \frac{(1 - \overline{\tau})w^s(\ell_t)}{(1 - \overline{\tau}) + \alpha n_t}. \tag{35}$$

Condition (35) is illustrated by schedule FF in Figure 3, which shows the locus of temporary equilibria as skilled workers are flowing out. The slope of schedule FF is obtained by direct differentiation of condition (35):

$$\left.\frac{dn_t}{d\ell_t}\right|_{\mathrm{FF}} = \frac{(1 - \overline{\tau})w^{s\prime}}{\alpha F^*} > 0, \tag{36}$$

where $w^{s\prime} = \mathrm{d}w^s/\mathrm{d}\ell_t > 0$.

The growth rate of human capital depends on the current level of the number of unskilled workers, i.e.,

$$g_t = A(\overline{\tau})B(\alpha N_t^u). \tag{37}$$

Note that with a multiplicative education function, the optimal time chosen by individuals for education is not affected by brain drain.

# 5 Balanced Growth Path

We now analyze the balanced growth path (BGP) under brain drain of the source country. Such a BGP is defined in the same way as that under autarky, that is, all intensive variables are stationary while all extensive variables are growing with constant rates. This requires that the temporary equilibrium condition (35) be satisfied in all periods. Furthermore, because the emigrants and their descendants do not return back and because the autarkic population in the source country is constant, labor movement ceases along a BGP.

We distinguish the values of variables along a BGP under brain drain by an overline together with a superscript "$b$." The equilibrium condition along a BGP is characterized by:

$$F^* = \frac{(1-\overline{\tau})w^s(\overline{\ell}^b)}{(1-\overline{\tau})+\alpha\overline{n}^b}. \tag{38}$$

Note that in the present model, the education time $\overline{\tau}$ is independent of labor movement, and for the time being the education parameter $\alpha$ is fixed by the government. Since the population of the source country is constant along a BGP, the analysis in the previous section implies that the USW ratio reduces to

$$\overline{n}^b = \frac{1-\overline{\tau}}{1-\alpha}. \tag{39}$$

Conditions (38) and (39) give the BGP values of $\overline{n}^b$ and $\overline{\ell}^b$. Because the optimal time for education chosen by individuals is stationary over time, condition (39) implies that the BGP value of the USW ratio $\overline{n}^b$ is not affected by brain drain, and condition (33) implies the same income tax rate to balance the government budget.

The BGP under brain drain can be derived graphically in Figure 3. Horizontal line MN, which has a vertical intercept equal to $(1-\overline{\tau})/(1-\alpha)$, illustrates condition (39). For comparison purpose, the autarkic BGP is shown to be at point A at which line MN and ray OG of a slope of $\overline{g}^a$ meet. Schedule FF is the locus of temporary equilibrium points. By condition (31), which is satisfied if brain drain occurs in period 0, point A is on the left-hand side of schedule FF, as the diagram shows. The BGP under brain drain is depicted by the point of intersection, point B, between FF and MN.

Through point B let us draw ray OG′ from the origin. The slope of OG′

gives the (gross) growth rate of human capital under brain drain, $\overline{g}^b$. This growth rate is given by

$$\overline{g}^b = A(\overline{\tau})B(\alpha\overline{N}^b), \tag{40}$$

where $\overline{N}^b$ is the population size of the source country along a new BGP.

**Proposition 2** *The BGP of the economy under brain drain exists and is unique. Assuming stability, brain drain causes a rise in the USL ratio and the skilled workers' wage rate and a drop in the unskilled workers' wage rate and the economic growth rate.*

**Proof.** We first prove the existence and uniqueness of the BGP. We showed earlier that schedule FF in Figure 3 is strictly positively sloped. For schedule FF, as $n_t \to 0$, condition (35) suggests that $\ell_t$ is finite, while as $n_t \to \infty$, $\ell_t \to \infty$. This means that schedule FF must cut line MN once, and only once. Let this point be denoted by B in the diagram. Next, note that the shrinkage of population due to the outflow of labor movement implies $\overline{N}^b < \overline{N}$. This result together with conditions (23) and (40) imply that $\overline{g}^b < \overline{g}^a$. If the BGP under brain drain is stable, then, as will be shown later, schedule FF must be steeper than ray OG at their intersecting point, point T, and they must cut each other once. In other words, point A is to the left of point B. What this implies is that brain drain lowers the BGP value of the USL ratio, $\overline{\ell}^b$. By conditions (3) and (4), there is a drop in $\overline{w}^{ub}$ but a rise in $\overline{w}^{sb}$. ■

The effects of brain drain as described in the above proposition requires stability, which will be analyzed in the next section.

# 6 Transitional Dynamics and Stability

So far we have been focusing on the features of the new BGP. We now analyze the transitional dynamics under brain drain. Such an analysis can be simplified by using the fact that temporary equilibrium in each period is represented by equation (35), meaning that beginning from period 0 the adjustment of the economy is represented by movemnts along schedule FF in Figure 3.

Let us begin with period 0, the time when brain drain is allowed to occur. In this period, some of the skilled workers, but not unskilled workers, in the source country flow out, implying that at the temporary equilibrium, both $n_0$ and $\ell_0$ rise. These two values can be illustrated in Figure 3. First, by condition (22), $n_0 = g_{-1}\ell_0$, meaning that they can be represented by a point on ray OG, which has a slope of $g_{-1} = \overline{g}^a$. Second, the temporary equilibrium is a point on schedule FF. As a result, the point $(n_{0,}\ \ell_0)$ is at point T, the intersecting point between ray OG and schedule FF.

We can now turn to period 1 and derive its temporary equilibrium. First, we note that it must be at a point on schedule FF. Second, we have $n_1 = g_0\ell_1$ at a new temporary equilibrium. Note that $g_0 = \overline{g}^a$ because the growth rate depends on education time and the current population of unskilled workers, both of which are not affected by brain drain. As a result, $(n_1,\ \ell_1)$ is still at a point on ray OG. Combining these results, we can conclude that the temporary equilibrium in this period is still represented by point T.

Starting from period 1, the number of unskilled workers drops. This also lowers the growth rate of human capital accumulation, $g_t$. As this growth rate drops, ray OG in Figure 3 rotates in a clockwise direction until it has a slope equal to the new BGP growth rate. Since the temporary equilibrium continues to be represented by the intersecting point between schedule FF and the new ray OG, as the latter rotates in a clockwise direction, the equilibrium point shifts downward along schedule FF. This means that both $n_t$ and $\ell_t$ decrease after period 1.

The outflow of skilled workers stops when the BGP is reached. As explained earlier, this occurs when the USW ratio is equal to $n_t = (1-\overline{\tau})/(1-\alpha)$, with the number of unskilled workers the same as the number of skilled workers plus educators. This means that in Figure 3, as skilled workers flow out, ray OG rotates in a clockwise direction until it cuts through point B, the intersection point between line MN and schedule FF.

The adjustment of the economy under brain drain can be summarized as follows: As brain drain is allowed, there will be a jump from the autarkic equilibrium point A to point T in one period, and stays there for one more period. Then it shifts down along schedule FF until point B is reached.

Figure 3 can also be used to examine the global stability of the BGP under brain drain. We want to derive the conditions under which starting from period 1 point T will move down, not up, along schedule FF.

Before we derive the stability condition in a rigorous way, we can first provide some economic intuition. This will help our derivation below. As explained earlier, when skilled workers flow out, both the population of skilled workers and that of unskilled workers shrink. This lowers the growth rate, and ray OG rotates in a clockwise direction. Therefore to have point T shifting down, schedule FF have to be steeper than any ray from the origin. This is the case shown in Figure 3.

We now formally derive this stability condition. Differentiation of condition (35), which is represented by schedule FF, gives

$$\mathrm{d}n_t = \frac{(1-\overline{\tau})w^{s\prime}}{\alpha F^*}\mathrm{d}\ell_t. \tag{41}$$

Similarly, differentiation of the growth rate condition (37) gives

$$\mathrm{d}n_t = g_{t-1}\mathrm{d}\ell_t + \ell_t g'_{t-1}\mathrm{d}N^u_{t-1}, \tag{42}$$

where $g'_{t-1} = A(\overline{\tau})B'(E_{t-1}) > 0$. Solve conditions (41) and (42) to give

$$\mathrm{d}n_t = \frac{1}{D}\frac{(1-\overline{\tau})\ell_t g'_{t-1}w^{s\prime}}{\alpha F^*}\mathrm{d}N^u_{t-1}, \tag{43}$$

where

$$D = \frac{(1-\overline{\tau})w^{s\prime}}{\alpha F^*} - g_{t-1},$$

which is the slope of schedule FF minus that of ray OG. Since starting from period 1 $N^u_t$ drops continuously as long as outflow of skilled workers occurs. From Figure 3, it is clear that for stability of the BGP, $n_t$ drops over time until the BGP is reached. This requires that $D > 0$, or that schedule FF be steeper than ray OG. A technical appendix on the stability condition in terms of the Jabobian matrix of the dynamic system is provided.

**Proposition 3** *A sufficient condition for global stability of the BGP is that*

$$D = \frac{(1-\overline{\tau})w^{s\prime}}{\alpha F^*} - g_{t-1} > 0,$$

*i.e., in the space bounded by OG and OG′, schedule FF is steeper than the ray from the origin that represents the prevailing growth rate of human capital.*

We now derive explicitly the adjustment of some of the more important variables. Figure 4 illustrates a possible adjustment path of $\ell_t$, which is based on the above analysis. Once this path is known, the changes in factor prices can easily be obtained. Figures 5 and 6 show the possible adjustment paths of the wages, $w_t^s$ and $w_t^u$, respectively. Figure 7 shows the changes in $N_t^s$ and $N_t^u$.

One feature of the adjustment of these variables is that they may overshoot or undershoot with respect to their new BGP values. For example, as shown in Figure 4, when brain drain is allowed $\ell_t$ first shoots up to a high level before it gradually decreases down to the new BGP level, which is higher than the initial one. Such an overshooting is due to the fact that in each period only skilled workers are allowed to move out but unskilled workers are not. Thus the initial outflow of skilled workers will substantially raise the USL ratio.

# 7 Welfare Analysis

We now analyze the effects of brain drain on the welfare levels of individuals in different generations. Brain drain in the present model has impacts on the welfare levels different from those recognized in the literature. The reason is that it affects not only wage rates, but also the income tax rates and the rate of human capital accumulation. Furthermore, the fact that the welfare of every individual is defined over two periods makes our analysis more difficult.

We first consider the welfare impacts along the balance growth path under brain drain, along which the wage rates and the growth rate of the economy are stationary over time. By Proposition 2, brain drain causes a drop in the BGP value of $\overline{w}^u$ but a rise in $\overline{w}^s$.

Under brain drain, the welfare of a representative individual born in period $t$ along a BGP is given by

$$u_t^b = \ln(1-\tau)x_{t-1}\overline{w}^{ub} + \rho\ln(1-\overline{\phi}^b)x_{t+1}\overline{w}^{sb}. \tag{44}$$

This condition implies that the increase in the welfare of individuals over one period is equal to

$$u_{t+1}^b - u_t^b = (1+\rho)\ln\overline{g}^b. \tag{45}$$

As before, $u_{t+1}^b - u_t^b$ is interpreted as the gross growth rate of the welfare of individuals over two consecutive generations. Thus, conditions (27) and (45) mean that brain drain slows down the welfare improvement over time between two consecutive generations due to the resulting lower rate of human capital accumulation ($\overline{g}^b < \overline{g}^a$).

A more direct way of measuring the welfare impacts of brain drain is to compare the actual welfare a non-emigrant receives under brain drain with the hypothetical welfare he receives should brain drain not be allowed starting from period 0. This approach requires the estimation of the welfare of the individual in the hypothetical case with no brain drain.

Let us begin with period 0, right after the policy of free brain drain is allowed. There are two groups of individuals in that period (as in other periods as well): the skilled workers plus educators, and the unskilled workers. We first consider the skilled workers and educators. The second-period welfare of a representative skilled worker/educator is given by[3]

$$u_0^o = \ln(1 - \phi_t^b) x_0 w_t^{sb}. \tag{46}$$

Substitute condition (32) into condition (46) to give

$$u_0^o = \ln x_0 F^*. \tag{47}$$

Because of condition (31), i.e., $F^* > (1 - \overline{\phi}^a)\overline{w}^{sa}$, and because $x_0$ is given as an initial condition, condition (47) implies that the welfare of skilled workers in period 0 is improved by the outflow of some skilled workers.

This result is not surprising. The welfare of the remaining skilled workers must have been raised to the level of the welfare enjoyed by the emigrants at the temporary equilibrium, or they will move out. The unskilled workers, who do not have the option of moving out, have a lower utility in that period, however. This is due to the decrease in the unskilled-worker wage rate.

Since there are gainers and losers in the economy in the period casued by the outflow of skilled workers, the question is whether the gainers can compensate the losers. To answer this question, we denote the number of emigranting skilled workers in period $t$ by $N_t^{se}$ and that of remaining skilled workers by $N_t^{sm}$. Because the level of human capital in period $t$ is equal to $x_t$, the inputs of skilled labor and unskilled labor are equal to, respectively, $L_t^{se} = x_t N_t^{se}$ and $L_t^{sm} = x_t N_t^{sm}$. By condition (12), the tax on skilled workers is equal to the after-tax income of the educators. This means that the national

[3]The first-period welfare of a representative skilled worker depends on his human capital and the unskilled worker's wage rate in the previous period, both of which are independent of brain drain.

income of those left behind in period $t$, including skilled workers, unskilled workers, and educators, is equal to

$$I_t = F(L_t^{sm} + L_t^{se}, L_t^u) - w_t^s L_t^{se}. \tag{48}$$

Note that in condition (48), $w_t^s L_t^{se}$ is subtracted from the value of output to get the national income of those left behind.

The effect of brain drain is obtained by differentiating both sides of (48) to give[4]

$$\frac{\partial I_t}{\partial L_t^{se}} = -L_t^{se}\frac{\partial w_t^s}{\partial L_t^{se}} \geq 0, \tag{49}$$

where $\partial w_t^s/\partial L_t^{se} < 0$. Condition (49) suggests that (a) if initially there is no labor movement, i.e., $L_t^{se} = 0$, a marginal brain drain does not affect $I_t$;[5] (b) if already a certain number of skilled workers have left, outflow of an additional skilled worker hurts those left behind in the sense that the gain of the gainers is not big enough to compensate for the loss of the losers in the same period. For a discrete outflow of skilled workers, condition (49) can be written alternatively as

$$(1-\tau)x_{-1}\overline{N}w_0^{ub} + (1-\phi_0^b)x_0 N_0^{sb} w_0^{sb} < (1-\tau)x_{-1}\overline{N}\overline{w}^{ua} + (1-\overline{\phi}^a)x_0 N_0^{sb}\overline{w}^{sa}, \tag{50}$$

meaning that an outflow of a finite number of skilled workers will hurt those left behind.

These results are not surprising, and is well known from the static welfare analysis of factor movement.[6] This approach to measuring the welfare effects of brain drain is based on the existence of an interpersonal lumpsum compensation scheme. Suppose instead no such scheme exists. Then the welfare levels of different groups of individuals have to be measured separately. In particular, even though individuals are hurt when they are young as unskilled workers, they may gain when old as skilled workers. What we want to determine is how brain drain may affect the intertemporal welfare of a representative individual.

[4]Because in a given period $x_t$ is given, $\partial I_t/\partial L_t^{si} = \partial I_t/\partial N_t^{si}$, $i = e, m$.

[5]The reason is that the marginal skilled worker takes with him his contribution to the economy, but the economy saves the payment to the worker that is equal to his contribution.

[6]See, for example, Gruebel and Scott (1966), Berry and Soligo (1969), and Wong (1986).

**Lemma 2** If $N_0^{sb} > \overline{N} g_0$, then for a discrete outflow of skilled workers,

$$(1-\tau)x_{-1}w_0^{ub}+\sigma(1-\phi_0^b)x_1 w_0^{sb} < (1-\tau)x_{-1}\overline{w}^{ua}+\sigma(1-\overline{\phi}^a)x_1\overline{w}^{sa}, \quad (51)$$

where $0 < \sigma \leq 1$.

**Proof.** Condition (50) can be rearranged to give

$$(1-\tau)x_{-1}\left[w_0^{ub} - \overline{w}^{ua}\right] < \frac{x_0 N_0^{sb}}{\overline{N}}\left[(1-\overline{\phi}^a)\overline{w}^{sa} - (1-\phi_0^b)w_0^{sb}\right]. \quad (52)$$

By condition (31), the right-hand side of condition (52) is negative. If $N_0^{sb} > \overline{N} g_0$, then $x_0 N^{sb}/\overline{N} > x_1 \geq \sigma x_1 > 0$. Thus we have

$$\frac{x_0 N_0^{sb}}{\overline{N}}\left[(1-\overline{\phi}^a)\overline{w}^{sa} - (1-\phi_0^b)w_0^{sb}\right] < x_1\left[(1-\overline{\phi}^a)\overline{w}^{sa} - (1-\phi_0^b)w_0^{sb}\right]. \quad (53)$$

Combining conditions (52) and (53), and using the fact that $(1-\phi_0^b)w_0^{sb} = (1-\phi_1^b)w_1^{sb}$, we get the lemma. ■

What does Lemma 2 imply? We note that under brain drain, the growth rates of human capital remain the same as under autarky in periods 0 and 1. This means that the human capital levels, $x_0$ and $x_1$, are not affected by brain drain. Furthermore, $\sigma$ can be interpreted as the discount factor. As a result, the left-hand side of condition (51) is the sum of discounted income of a representative non-emigrant born in the beginning of period 0 in the presence of brain drain, while the right-hand side is his welfare under autarky. Thus Lemma 2 implies that when the initial rate of human capital accumulation is relatively low (i.e., $N_0^{sb} > \overline{N} g_0$), brain drain amplifies the adverse situation and causes a drop in the sum of discounted income of a non-emigrant.

In terms of the welfare of an individual, the lemma implies that the sum of the discounted income of a representative non-emigrant born in the beginning of period 0 is smaller under brain drain than under autarky. In other words, if a perfect bond market exists before and after brain drain so that a representative non-emigrant can borrow or lend in the two periods to maximize his own welfare, then there will be a drop in his welfare due to brain drain. In the absence of such a bond market, the change in the welfare of the non-emigrant is still ambiguous. However, we can apply Lemma 2 to conclude that when the initial rate of human capital accumulation is relatively low (i.e., $N_0^{sb} > \overline{N} g_0$), brain drain has an adverse welfare effect.

**Lemma 3** Consider four positive numbers, $a$, $b$, $a'$, and $b'$, where $a > a'$, and $a + b \geq a' + b'$. For $0 \leq \sigma \leq 1$, we get

$$\ln a + \sigma \ln b > \ln a' + \sigma \ln b'.$$

**Proof.** By the concavity of the logarithms function, for $\delta > 0$,

$$\ln a - \ln a' > \ln(a + \delta) - \ln(a' + \delta). \tag{54}$$

Let $\delta = b - a'$ and $b'' = a + b - a'$. Condition (54) reduces to

$$\ln a - \ln a' > \ln b'' - \ln b \geq \ln b' - \ln b, \tag{55}$$

where the last inequality comes from the fact that $b'' \geq b'$ since $a + b \geq a' + b'$. If $b' > b$, condition (55) reduces to

$$\ln a - \ln a' > \sigma(\ln b' - \ln b). \tag{56}$$

If $b' \leq b$, condition (56) is also satisfied because its right-hand side is non-positive while the left-hand side is positive. Rearranging the terms in the condition gives the lemma. ■

We now make use of Lemmas 2 and 3 to prove the following proposition.

**Proposition 4** *If $N_0^{sb} > \overline{N} g_0$, then brain drain reduces the welfare of a representative non-emigrant who is born in period 0.*

**Proof.** Along a BGP, the welfare of a non-emigrant born in period 0 if brain drain is not allowed is

$$u_0^a = \ln[(1 - \tau)x_{-1}\overline{w}^{ua}] + \rho \ln[(1 - \overline{\phi}^a)x_1\overline{w}^{sa}]. \tag{57}$$

His welfare in the presence of brain drain is

$$u_0^b = \ln[(1 - \tau)x_{-1}\overline{w}^{ub}] + \rho \ln[(1 - \overline{\phi}^b)x_1\overline{w}^{sb}]. \tag{58}$$

Because $\overline{w}^{ua} > \overline{w}^{ub} = w_0^{ub}$, Lemmas 2 and 3 can be combined to give the proposition. ■

By this proposition, brain drain hurts the non-emigrants who are born in the beginning of period 0 if $N_0^{sb} > \overline{N} g_0$. Brain drain also tends to hurt later generations since it lowers the growth rate of human capital.

# 8 Education and Brain Drain

The above analysis shows that brain drain has detrimental effects on the growth rate and welfare of the source country. The analysis thus raises the concerns about what the government can do to protect the country's growth and welfare. The simple answer is to prohibit brain drain. This policy, however, is not considered to be realistic. First, many people regard out-migration as an important human right, and nearly all countries allow at least some number of emigrants each year. Second, prohibiting brain drain could be difficult politically. In particular, the professional people may be influential in the political system, and they are against any regulations on outflow of skilled workers.

In this section, we suggest a policy that can be used to protect the growth rate of the economy in the presence of brain drain. The policy is to increase the educator-student ratio, $\alpha$. We note from Section 3 that for a closed economy an increase in the educator-student ratio will raise the growth rate of the economy. We now examine how this policy can be used in the presence of brain drain to counter the detrimental effect of brain drain on economic growth.

The objective of the government considered here is to choose the right education policy to maintain the growth rate of the economy at the autarkic level when free brain drain is allowed. We first focus on a new BGP in the presence of this policy and brain drain. Let us denote the BGP value of a variable under this new education policy by a "tilde."

Under the objective that the growth rate remains the same under autarky, condition (40) reduces to

$$\overline{g}^a = A(\overline{\tau})B(\widetilde{\alpha}^b \widetilde{N}^b), \tag{59}$$

where $\widetilde{N}^b$ is the new population along a new BGP. In condition (59), we have made use of the fact that the optimal education time chosen by the unskilled workers is not affected by the education policy. Furthermore, with the existence of brain drain, $\widetilde{N}^b < \overline{N}$. This implies that to maintain the same growth rate as before, the government has to increase the educator-student ratio, $\widetilde{\alpha}^b > \alpha$. We have to find out what this new educator-student ratio should be.

The temporary equilibrium under brain drain is charaterized by

$$F^{*} = \frac{(1-\overline{\tau})w^{s}(\widetilde{\ell}^{b})}{(1-\overline{\tau})+\widetilde{\alpha}^{b}\widetilde{n}^{b}}. \tag{60}$$

The relationship between $\widetilde{\ell}^{b}$ and $\widetilde{n}^{b}$ is given by

$$\widetilde{n}^{b} = \overline{g}^{a}\widetilde{\ell}^{b}, \tag{61}$$

whereas $\widetilde{n}^{b}$ is related to the educator-student ratio

$$\widetilde{n}^{b} = \frac{1-\overline{\tau}}{1-\widetilde{\alpha}^{b}}. \tag{62}$$

Conditions (60) to (62) can then be solved for $\widetilde{\alpha}^{b}$, $\widetilde{n}^{b}$, and $\widetilde{\ell}^{b}$.

As analyzed above, with the presence of brain drain, the new education policy will raise $\widetilde{\alpha}^{b}$. By conditions (61) and (62), the policy will also increase $\widetilde{n}^{b}$ and $\widetilde{\ell}^{b}$. Because of the change in $\widetilde{\ell}^{b}$, there will be an increase in the skilled-worker wage rate but a decrease in the unskilled-worker wage rate.

These results mean that the new education policy does not reverse the directions of change of $\widetilde{\ell}^{b}$ and factor prices. They are not surprising because an equilibrium with brain drain requires an increase in the welfare of the skilled workers, which depends on the after-tax income when they are old. The question is whether these changes with this new education policy can be compared with the changes with the initial education policy.

These changes can indeed be compared. To do that, we first determine the change in the income tax rate. We note that the new education policy increases the term $\widetilde{\alpha}^{b}\widetilde{n}^{b}$, which is equal to

$$\widetilde{\alpha}^{b}\widetilde{n}^{b} = \frac{\widetilde{\alpha}^{b}(1-\overline{\tau})}{1-\widetilde{\alpha}^{b}} = \frac{1-\overline{\tau}}{1/\widetilde{\alpha}^{b}-1}.$$

The new income tax rate that balances the government budget, by condition (33), is equal to

$$\widetilde{\phi}^{b} = \frac{\widetilde{\alpha}^{b}\widetilde{n}^{b}}{(1-\overline{\tau})+\widetilde{\alpha}^{b}\widetilde{n}^{b}}. \tag{63}$$

By condition (63), an increase in $\widetilde{\alpha}^{b}\widetilde{n}^{b}$ means an increase in the income tax rate. Intuitively, such an increase in the income tax rate is needed to finance a higher educator-student ratio under the new policy.

We are now ready to compare the factor prices with and without the more aggressive education policy. In equilibrium, skilled workers that remaining behind have the same after-tax wage rate, the same as what the emigrants get in the host country. A higher income tax rate therefore must be balanced by a higher wage rate for skilled workers, i.e., $\widetilde{w}^{sb} > \overline{w}^{sb}$. This implies $\widetilde{w}^{ub} < \overline{w}^{ub}$, and $\widetilde{\ell}^b > \overline{\ell}^b$. By condition (61), $\tilde{n}^b > \overline{n}^b$.

The above results can be illustrated graphically. In Figure 8, ray OG represents the autarkic growth rate of the economy; i.e., its slope is equal to $\overline{g}^a$. In the presence of brain drain, the government chooses a new education policy to maintain the same growth rate as before. As analyzed above, the new education policy requires an increase in the educator-student ratio, $\alpha$. By equation (60), this new policy will shift schedule FF down and to the right. Let the new schedule be at F′F′, and let it cut ray OG at point S. At the same time, an increase in $\alpha$ implies a rise in the equilibrium value of $n$, as shown by condition (62). To have point S as an equilibrium point, it is required that the horizontal line with an intercept on the vertical axis equal to $(1-\overline{\tau})/(1-\widetilde{\alpha}^b)$, shown as line M′N′ in the diagram, passes through point S.

Figure 8 can be used to derive the transitional adjustment of the economy under brain drain and the new education policy. Point A is the autarkic point. When brain drain occurs in period 0, the outflow of skilled workers immediately raises the USL ratio, $\ell_0$. The simultaneous rise in the educator-student ratio, $\alpha$, shifts the temporary equilibrium schedule FF to F′F′ and raise the equilibrium value of the USW ratio, $n_0$. Since $n_0 = g_{-1}\ell_0$, the temporary equilibrium in this period must be on ray OG and schedule F′F′, i.e., at point S. In other words, the economy jumps up from point A to point S in period 0.

Once the economy is at point S, no more changes in the economy will be observed because all the three BGP conditions, (60) to (62), are satisfied. This means that the economy reaches the new BGP in one period.

The above results are summarized in the following proposition:

**Proposition 5** *When the government increases the educator-student ratio in the presence of brain drain in order to maintain the autarkic growth rate of human capital, there will be an increase in (a) the USL and USW ratios, (b) the wage rate of the non-emigrating skilled workers, and (c) the income tax rate; but a drop in the wage rate of the unskilled workers. The economy reaches the new BGP in one period.*

# 9 Conclusion

In this paper we have developed a two-sector, endogenous growth model to examine the relationship between economic growth and brain drain. The engine of growth in the present model is the accumulation of human capital through education. After proving existence, uniqueness and stability of the equilibrium of a closed economy, we examined several properties of brain drain. One of the more important results is that brain drain has an adverse effect on the wage rate of the unskilled workers but it improves the wage rate of the skilled workers. Brain drain also tends to hurt the growth of the source country.

Even though brain drain benefits the remaining skilled workers in the period when it is first allowed, it in general is bad for other non-emigrants. Some of the detrimental effects of brain drain are static, hurting those left behind in any period in the sense that there is a drop in the national income. As a result, no compensatory policy is available to make sure that all non-emigrants are not hurt in any period. In fact, despite the positive effect of skilled workers' wage rates, the generation that is born just before brain drain could also be hurt. The more damaging effects brain drain brings to the non-emigrants are those on growth rate. In the present model when the economy grows with human capital, a drop in the growth rate of human capital hurts future generations.

From a policy perspective, we have demonstrated how the government may use a more aggressive education policy to counter the detrimental effect of brain drain on the economy's growth rate. This requires an increase in the educator-student ratio. As a consequence, the USL and USW ratios are both raised, making the skilled workers' wage rate higher but hurting the unskilled workers' wage rate. The interesting feature of this policy is that the economy reaches the new BGP in one period, even though without the change in the education policy, it may take much longer to reach a new BGP.

## APPENDIX

With brain drain, the population size in each period is governed by (30). Since $E_t = \alpha N_t^u$, we can rewrite (30) by using the definition of $n_t$ as

$$\frac{N_t^u}{N_{t-1}^u} = \alpha + \frac{1-\overline{\tau}}{n_{t-1}}. \tag{64}$$

Next, combining (37) and (35) to eliminate $\ell_t$, we have

$$F^* = \frac{(1-\overline{\tau})w_t^s(n_t/g_{t-1}(N_{t-1}^u))}{(1-\overline{\tau})+\alpha n_t} \tag{65}$$

where $g'_{t-1} = \alpha A(\overline{\tau})B'(\alpha N_{t-1}^u) > 0$. This provides a $2 \times 2$ system to study the transitional dynamics under brain drain.

We define $\widehat{z}_t \equiv z_t - \overline{z}$ for any variable $z$, where the "overline" denotes the balanced-growth equilibrium value of the variable. Linearization of (64) and (65) yields the following matrix equation:

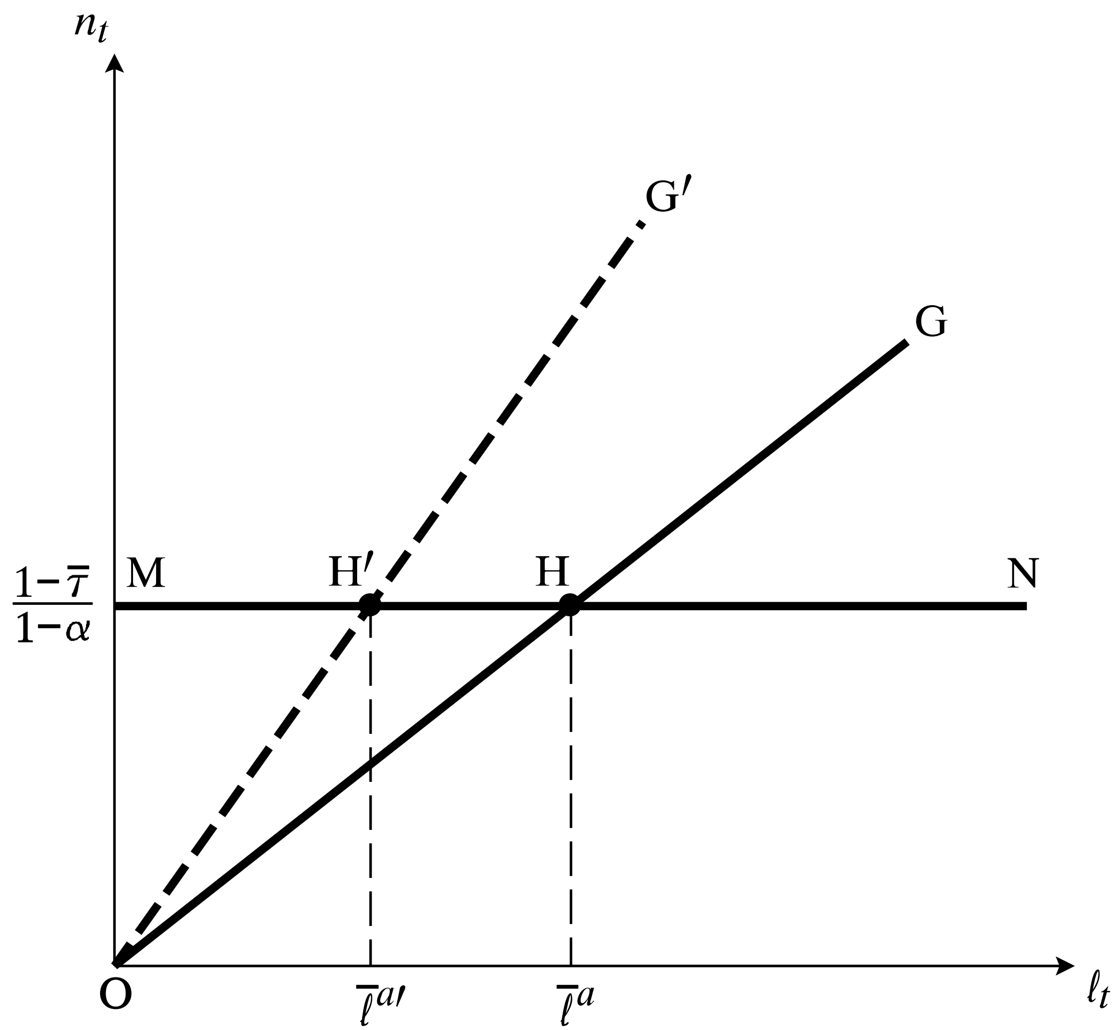
$$\begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} \widehat{n}_t \\ \widehat{N}_t^u \end{bmatrix} = \begin{bmatrix} 0 & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \widehat{n}_{t-1} \\ \widehat{N}_{t-1}^u \end{bmatrix} \tag{66}$$

where $a_{11} \equiv \alpha F^* - (1-\overline{\tau})w^{s\prime}/\overline{g}^b$, $a_{22} \equiv 1/\overline{N}^b > 0$, $b_{12} \equiv -\overline{n}^b\overline{g}^{b\prime}/(\overline{g}^b)^2 < 0$, $b_{21} \equiv -(1-\overline{\tau})/(\overline{n}^b)^2 < 0$, $b_{22} \equiv 1/\overline{N}^b > 0$. Inverting the matrix on the LHS, we get

$$\begin{bmatrix} \widehat{n}_t \\ \widehat{N}_t^u \end{bmatrix} = \begin{bmatrix} 0 & b_{12}/a_{11} \\ \overline{N}^b b_{21} & 1 \end{bmatrix} \begin{bmatrix} \widehat{n}_{t-1} \\ \widehat{N}_{t-1}^u \end{bmatrix}. \tag{67}$$

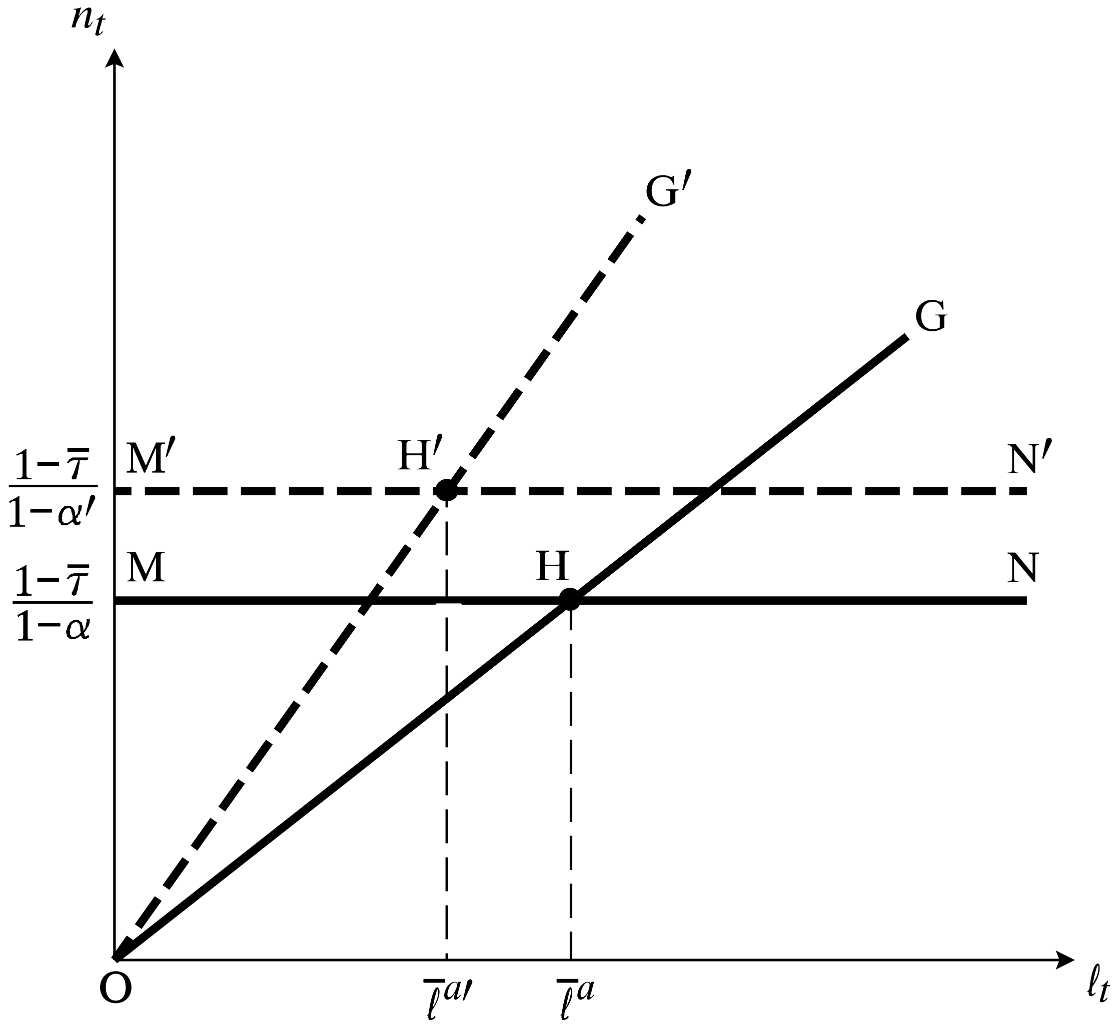
The trace ($Tr$) of the Jacobian is 1 and the determinant ($Det$) is $-\overline{N}^b b_{21}b_{12}/a_{11}$. In order to have stability so that the BGP equilibrium is a *sink* (i.e., the eigenvalues of the dynamic system must lie inside the unit circle), it is necessary and sufficient to have $Det > 0$.[7] This is equivalent to the condition of $D > 0$ in the paper.

[7] In general, the condition that $Det > 0$ is only necessary but not sufficient for stability. However, in our special case that $Tr = 1$, this condition is also sufficient.



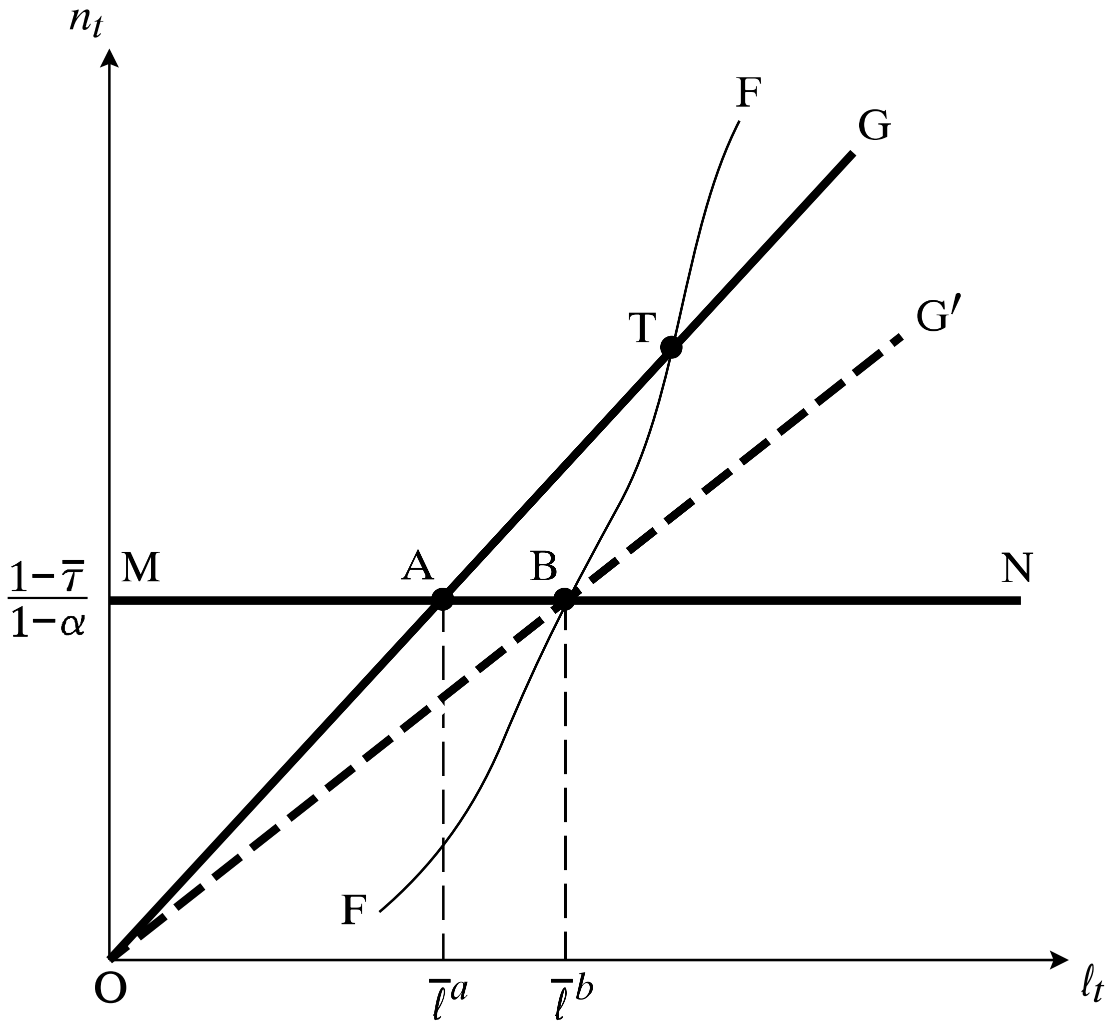
$n_t$
G′
G
$\frac{1-\overline{\tau}}{1-\alpha}$
M
H′
H
N
O
$\overline{\ell}^{a\prime}$
$\overline{\ell}^{a}$
$\ell_t$


Figure 1



$n_t$
G′
G
$\frac{1-\overline{\tau}}{1-\alpha'}$
M′
H′
N′
$\frac{1-\overline{\tau}}{1-\alpha}$
M
H
N
O
$\overline{\ell}^{a\prime}$
$\overline{\ell}^{a}$
$\ell_t$


Figure 2



$n_t$
F
G
T
G′
$\frac{1-\overline{\tau}}{1-\alpha}$
M
A
B
N
F
O
$\overline{\ell}^a$
$\overline{\ell}^b$
$\ell_t$


Figure 3

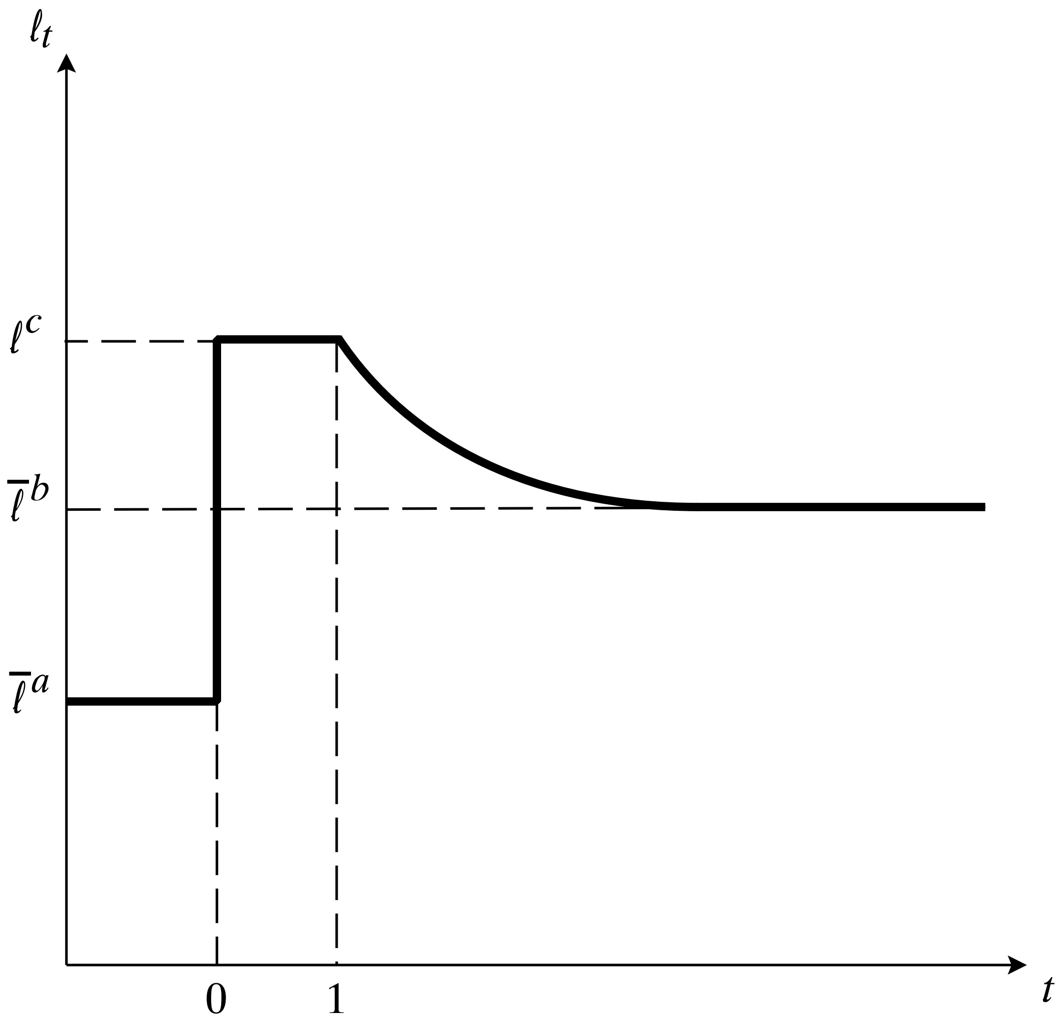



Figure 4

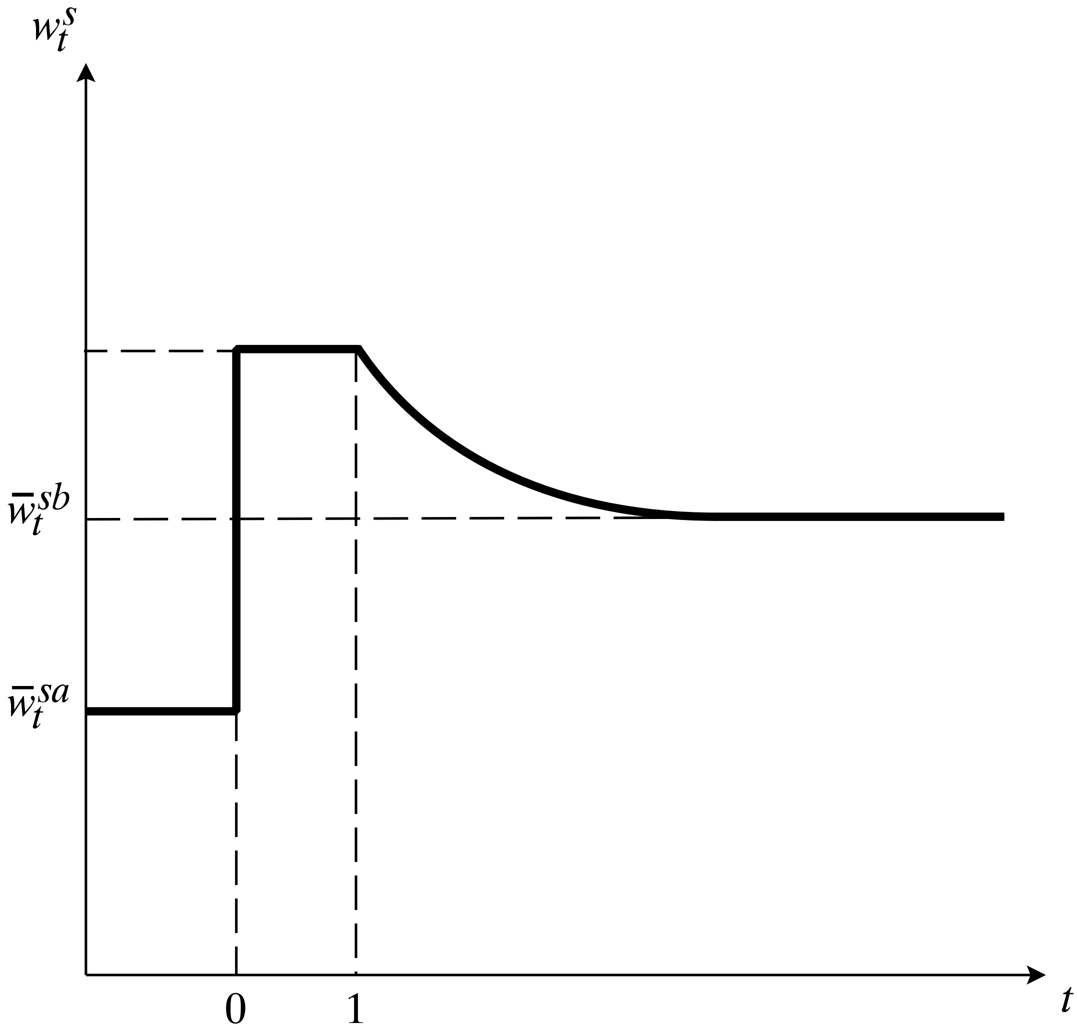



Figure 5

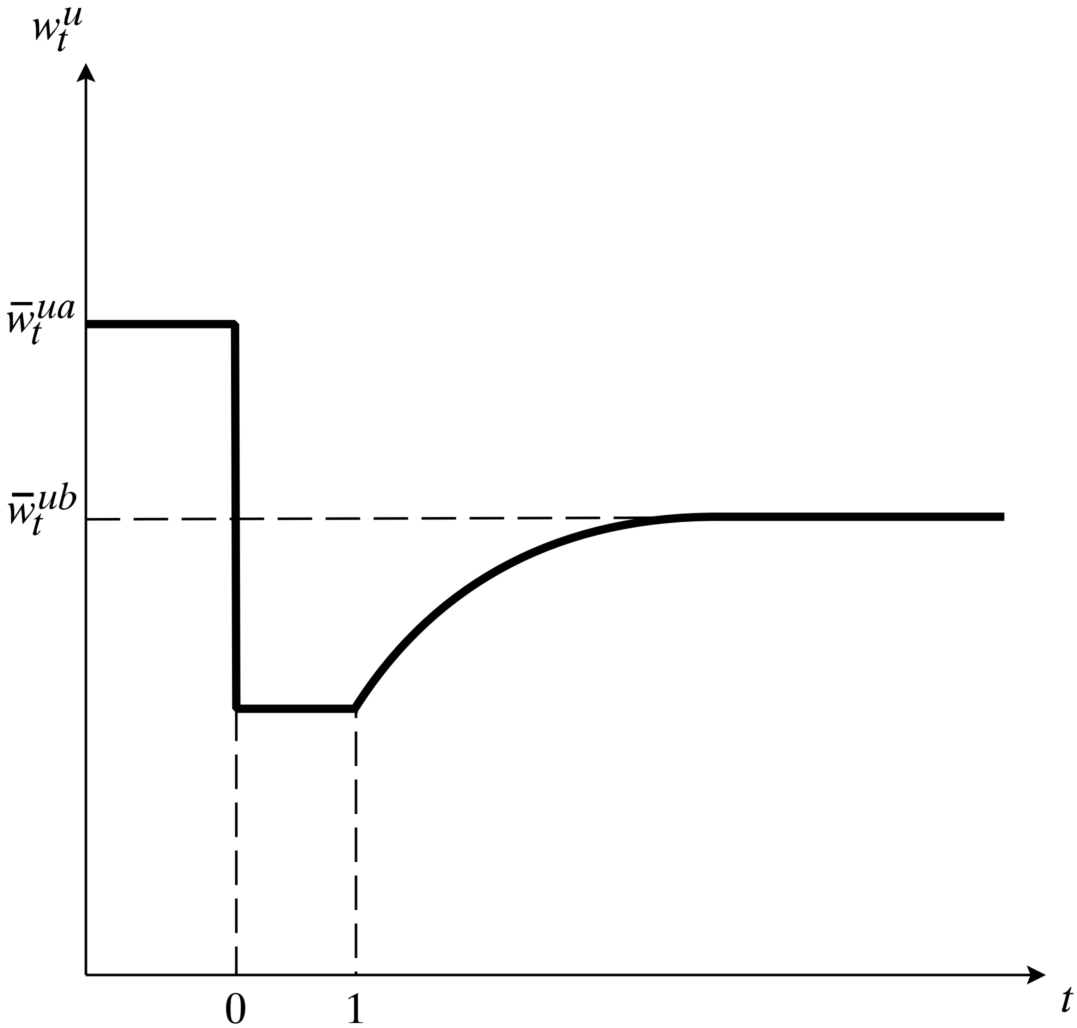



Figure 6

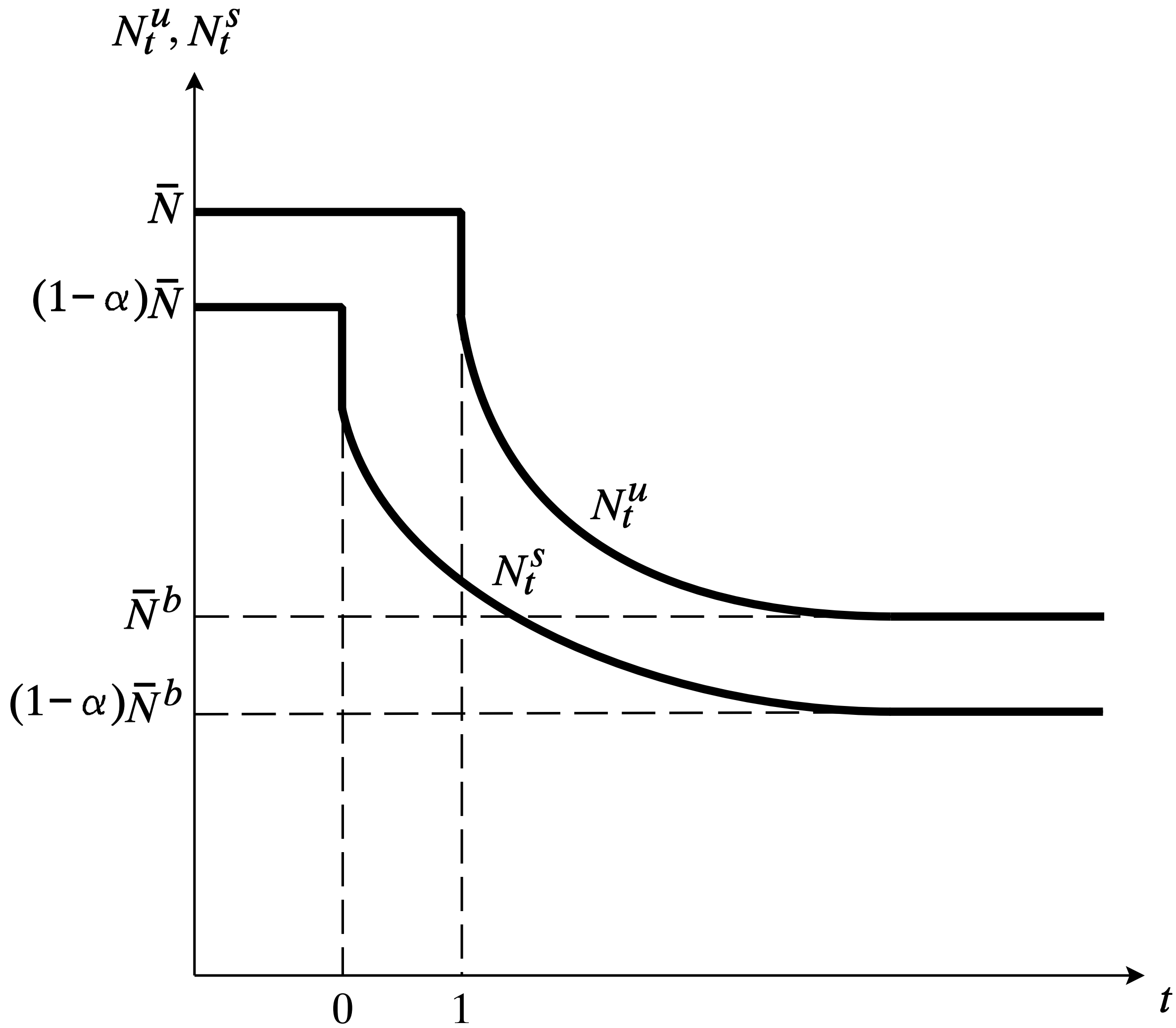



Figure 7

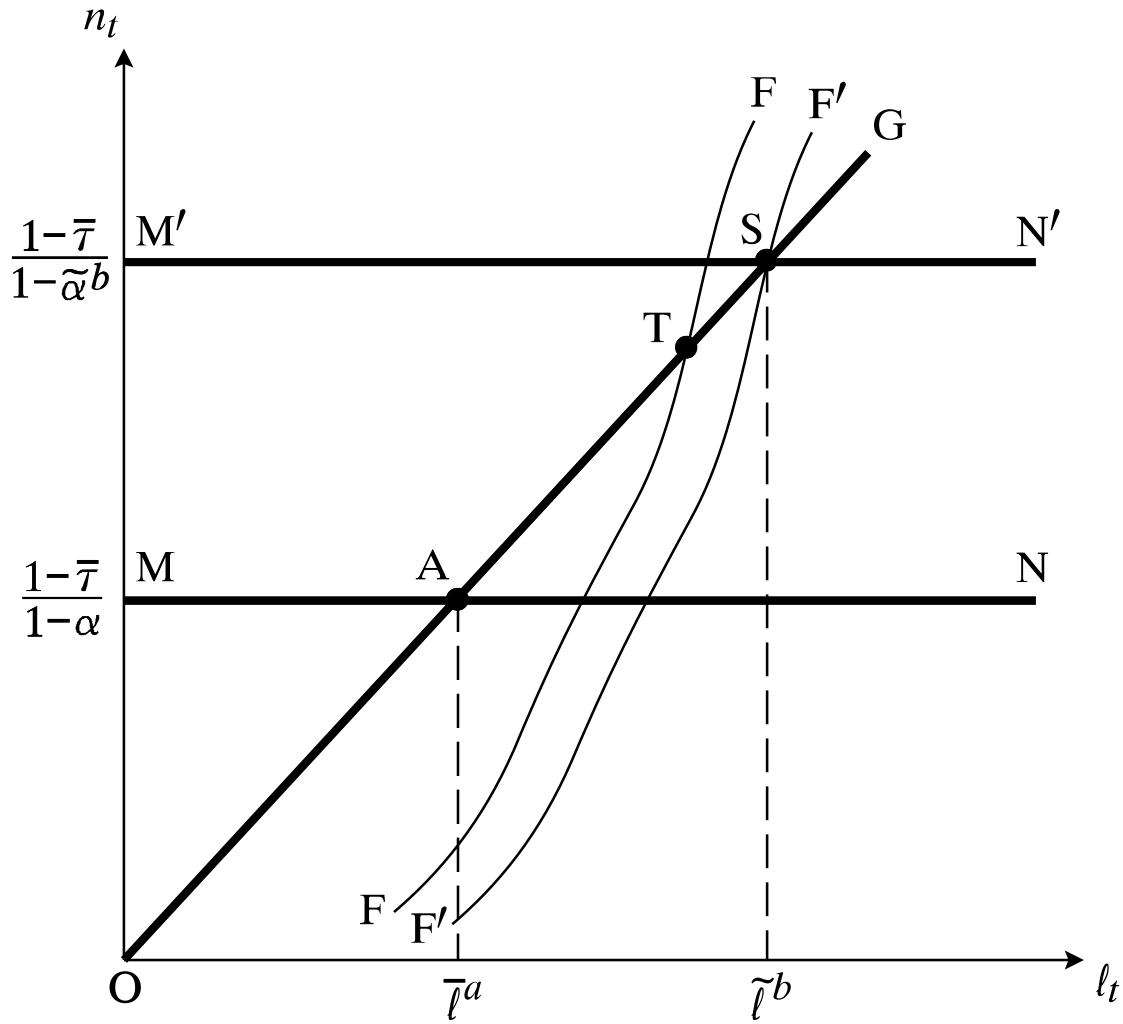

$n_t$
F
F′
G
$\frac{1-\overline{\tau}}{1-\widetilde{\alpha}^b}$
M′
S
N′
T
$\frac{1-\overline{\tau}}{1-\alpha}$
M
A
N
F
F′
O
$\overline{\ell}^a$
$\widetilde{\ell}^b$
$\ell_t$

Figure 8

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