Does International Trade Stabilize Exchange Rate Volatility?

Hui-Kuan Tseng, Kun-Ming Chen, and Chia-Ching Lin *

Abstract

Since the early 1980s, major industrial countries have been suffering severe multi-lateral trade imbalance, accompanying with tremendously volatile exchange rates. This paper examines the relationship between trade balance and exchange rate volatility. A stochastic macroeconomic model with sticky price is developed. Our comparative statics and numerical simulation results indicate that increased trade balance (relative to domestic aggregate demand) tends to reduce exchange rate volatility when the domestic absorption shock perturbs the economy. In the presence of all other domestic and foreign shocks, however, increased trade balance tends to augment exchange-rate volatility, except for the case of disturbance of domestic real income in which the effect of increased trade balance is indeterminate. Our results suggest that whether trade imbalance has aggravated exchange rate volatility in many industrial countries is an open question, which needs to be solved through empirical investigation.

Keywords: Trade Imbalance, Exchange Rate Volatility, Stochastic Macroeconomic Model
JEL Classification: F17, F31, F47

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I. Introduction

Since the early 1980s, major industrial countries have been suffering severe multi-lateral trade disequilibrium – the U.S. has exhibited huge trade deficits against Japan, Canada, West Germany, the Asian newly industrializing countries, and more recently, China. Moreover, foreign exchange rates in many countries have been tremendously volatile since the breakdown of the Bretton Woods system in 1973. Are these two phenomena related? To the best of our knowledge, this issue is still not well investigated.

A popular view is that exchange rate volatility tends to reduce the volume of international trade, evidence is not unanimous. For instance, Abrahms [1980] and Thursby and Thursby [1987] found a large negative effect of exchange rate volatility on trade, whereas Hooper and Kohlgagen [1978] found no significant effects on trade volumes but a large effect on commodity prices. Later studies, including Frankel and Wei [1993], Eichengreen and Irwin [1996] and De Grauwe and Skudelny [2000] all reported small or insignificant negative effects.[1] Tenreyro [2003] argued that the estimation techniques used in previous studies of the impacts of exchange rate volatility on trade have multiple sources of problems that appear to bias their empirical findings.[2]

Most of existing studies, both theoretical and empirical, focus on the effect of exchange rate volatility on trade. Few papers had attempted to examine the reverse causality of whether international trade can dampen exchange rate volatility. In his optimal currency area hypothesis, Mundell [1961] firstly looked into this reverse direction of causality. He found that trade flows reduce real exchange rate volatility. This paper reexamines the same reverse direction of causality using numerical simulations.

[1] Cote' [1994] provides an extensive survey of the literature – both theoretical and empirical – on exchange rate volatility and trade.

[2] Another strand of research also emerged to explore whether central banks can somehow intervene in foreign exchange markets so as to reduce exchange rate volatility. For example, Kawai [1984], Eaton and Turnovsky [1983], Tseng

In this paper we develop a stochastic macroeconomic model to answer the central question: Does trade imbalance play a role in aggravating exchange rate fluctuations due to random disturbances originating in the home country and abroad? This study can supplement the literature by focusing on the causality relationship from international trade to exchange rate volatility rather than the way around. In addition, it can provide more evidence of whether international trade may help stabilize exchange rate movements, as seen in Mundell [1961].

The remainder of this paper proceeds as follows. Section II outlines the structure of our theoretical model. Section III first solves the model for the equilibrium exchange rate and its volatility under the rational expectations hypothesis, and then the impact of trade imbalance on the volatility of exchange rate is examined. Section IV conducts numerical simulations of the effect of trade imbalance on volatility of nominal exchange rate and real exchange rate with baseline parameter values. Sensitivity analysis on the results is also implemented. Brief concluding remarks are given in the final section.

II. The Model

The model of a small open economy is summarized by the following equations:

$$d_t = b_1 a_t + b_2 T_t \quad b_1 > 0\,,\; b_2 > 0 \tag{1}$$

$$a_t = \alpha_1 y_t - \alpha_2 \left(r_t - E_t\left[p_{t+1} - p_t\right]\right) + u_{1t} \quad 1 > \alpha_1 > 0\,,\; \alpha_2 > 0 \tag{2}$$

$$T_t = \beta_1 \left(e_t + p_t^* - p_t\right) + \beta_2 y_t^* - \beta_3 y_t\,,\; \beta_i > 0,\; i = 1,\; 2,\; 3 \tag{3}$$

$$p_t = p_{t-1} + \phi\left(d_{t-1} - y_{t-1}\right),\; \phi > 0 \tag{4}$$

$$m - p_t = \Omega_1 y_t - \frac{1}{\Omega_2} r_t + u_{2t} \quad ,\; \Omega_1 > 0\,, \Omega_2 > 0 \tag{5}$$

[1991] and Dominguez [1998].

$$r_t = r_t^* + E_t\left[e_{t+1}\right] - e_t \tag{6}$$

$$y_t = \overline{y} + u_{3t} \tag{7}$$

$$y_t^* = \overline{y}^* + u_{4t} \tag{8}$$

$$p_t^* = \overline{p}^* \tag{9}$$

$$r_t^* = \overline{r}^* + u_{5t} \tag{10}$$

where all variables except capital letters, r and r^* are measured in logarithm, subscript t denotes period t, and E_t [.] is an expectation operator based on all information available in period t. The definition of each variable is given in Table 1.

Equation (1) is a log-linearized aggregate demand function specifying that domestic aggregate demand is composed of real domestic absorption, a_t, and real trade surplus, T_t, (or real trade deficit as T_t is negative) in period t.[3] The parameters b_1 and b_2 reflect the weights of domestic absorption and real trade balance in the economy's aggregate demand, respectively.

Equation (2) states that real domestic absorption a_t depends positively on real domestic output, y_t, and negatively on domestic real interest rate, $r_t - E_t[p_{t+1} - p_t]$. The current domestic absorption is also affected by a stochastic disturbance, u_{1t}, which contains random changes in either domestic fiscal policy or private consumption (investment).

Equation (3) indicates that real trade balance, T_t, is determined by current terms of trade ($e_t + p_t^* - p_t$), foreign income y^*, and domestic income y. Equation (4) stipulates the same price adjustment rule specified in Dornbusch's [1976] seminal sticky price model, where the goods price is predetermined at any point in time. The positive parameter, ϕ, represents the speed of price adjustment in response to excess demand ($d_{t-1} - y_{t-1}$) for domestic goods in period t-1. The greater

[3] In fact, this specification of domestic aggregate demand is in line with Bhandari [1983].

the value of parameter ϕ, the more flexible is the goods price. As ϕ goes to infinity, the goods market is continuously cleared along the time horizon. However, since ϕ is assumed to be finite, equilibrium in the domestic goods market is unlikely to be achieved in the short run.

Table 1 Variable Definitions

Variable	Definition
d	Domestic real aggregate demand
a	Domestic real absorption, which is the sum of private and government consumption and gross investment
T	Real aggregate trade balance
y	Domestic output level; $\bar{y}$ = long-run stationary level of y.
p	Domestic goods price
r	Domestic nominal rate of interest
m	Domestic nominal money supply
y^*	Foreign output level; $\bar{y}^*$ = long-run stationary level of y^*.
p^*	Foreign price level; $\bar{p}^*$ = long-run stationary level of p^*
r^*	Foreign nominal rate of interest; $\bar{r}^*$ = long-run stationary level of r^*
e	The nominal exchange rate (measured in terms of units of domestic currency per unit of foreign currency)
q	The real exchange rate
$E_t[X_{t+1}]$	Expectations of X in period t+1 conditional on information available in period t
u_1	Random disturbance in domestic absorption
u_2	Random disturbance in excess supply of domestic nominal money
u_3	Random disturbance in domestic real income
u_4	Random disturbance in foreign real income
u_5	Random disturbance in nominal foreign rate of interest

Equilibrium in the domestic money market is characterized by equation (5). The nominal stock supply of domestic money m is assumed to be fixed. The domestic real money demand is of the usual form in that it is positively associated with domestic real income and negatively associated with domestic nominal interest rate r. It is assumed that foreigners do not hold domestic money and the only opportunity cost of holding domestic money is domestic nominal interest rate. Like the goods market, the domestic money market is subject to a random disturbance, u_{2t}, representing random changes in domestic monetary policy or private money demand.

Equation (6) is an uncovered interest parity condition, linking domestic nominal interest rate r_t to foreign nominal interest rate r_t^* plus an uncovered risk premium $E_t[e_{t+1}] - e_t$. This condition implies agents are risk-neutral and political risk is non-existent.[4]

Equations (7), (8) and (10) state that domestic income y_t, foreign income y_t^* and foreign nominal interest rate r_t^* fluctuate around their own steady state equilibrium, respectively, subject to random disturbances u_{3t}, u_{4t} and u_{5t}. Equation (9) implies that foreign price level p_t^* is exogenous and fixed at the long-run stationary level. In fact, all foreign variables mentioned above are exogenous, reflecting the fact that the home country is a small open economy.

There are five random disturbances in the model – one domestic demand shock u_{1t}, one domestic monetary shock u_{2t}, one domestic income shock u_{3t}, one foreign income shock u_{4t} and one foreign monetary shock u_{5t}. These disturbances are white noises and are assumed to be independent of each other with mean $E_t[u_{jt+i}] = 0$ and have a bounded variance, $V_t[u_{jt+1}] < \infty$ for j = 1, 2, 3, 4 and 5 and $i \geq 1$.

III. Derivation of Rational Expectations Equilibria and Comparative Statics

1. Derivation of rational expectations equilibria

We can reduce the model to a system of two equations with two endogenous variables, e_t and p_t. In what follows we proceed to solve the reduced-form system under the rational expectations hypothesis. The rational-expectations solution for each endogenous variable will then be expressed in terms of all random disturbances and structural parameters. The first step is to obtain a set of

[4] Eaton and Turnovsky [1983] showed that covered interest parity would collapse in the presence of political risk.

reduced form equations. For simplicity, we set the values of those exogenous variables such as $\bar{y}$, $\bar{y}^*$, $\bar{p}^*$ and $\bar{r}^*$ equal to zero. Thus, from equation (5), the domestic nominal interest rate r_t is given by:

$$r_t = \Omega_2 p_t + \Omega_2 u_{2t} + \Omega_1 \Omega_2 u_{3t} \ . \tag{5'}$$

Using equations (1), (2), (3), (5'), (7), (8) and (9), the domestic price adjustment equation (4) becomes,

$$p_t = \gamma_1 \, p_{t-1} + \gamma_2 \, e_{t-1} + \gamma_3 \, u_{1t-1} - \gamma_4 \, u_{2t-1} + \gamma_5 \, u_{3t-1} + \gamma_6 \, u_{4t-1} \tag{11a}$$

where

$$\gamma_1 = \frac{1 - b_1\alpha_2\phi - \phi b_1\alpha_2\Omega_2 - \phi b_2\beta_1}{1 - b_1\alpha_2\phi}, \qquad \gamma_2 = \frac{\phi b_2\beta_1}{1 - b_1\alpha_2\phi},$$

$$\gamma_3 = \frac{\phi b_1}{1 - b_1\alpha_2\phi}, \qquad \gamma_4 = \frac{\phi b_1\alpha_2\Omega_2}{1 - b_1\alpha_2\phi},$$

$$\gamma_5 = \frac{\phi\left(b_1\alpha_1 - b_1\alpha_2\Omega_1\Omega_2 - b_2\beta_3 - 1\right)}{1 - b_1\alpha_2\phi}, \qquad \gamma_6 = \frac{\phi b_2\beta_2}{1 - b_1\alpha_2\phi}.$$

Substituting equation (5') for r_t in equation (6), we rewrite the uncovered interest parity condition as

$$E_t\left[e_{t+1}\right] = e_t + \Omega_2 p_t + \Omega_2 u_{2t} + \Omega_1\Omega_2 u_{3t} - u_{5t} \ . \tag{11b}$$

Equations (11a) and (11b) represent the reduced-form system mentioned above. To solve this system, we use a two-step procedure under the assumed Muthian rational expectations.[5] First, we find the solution for $E_t\left[e_{t+1}\right]$, a conditioned expectation of the exchange rate, by taking expectations for (11a) and (11b) in period i ($i > 0$) conditional on period 0 (Note $X_{i,0} = E_0\left[X_i\right]$). In so doing, all the disturbance terms are washed out for their conditioned means equal zero and therefore we can obtain.

$$p_{i,0} = \gamma_1 p_{i-1,0} + \gamma_2 e_{i-1,0} \tag{12a}$$

$$e_{i,0} = \Omega_2 p_{i-1,0} + e_{i-1,0} \tag{12b}$$

Equations (12a) and (12b) can be rewritten in the matrix form as

$$\begin{bmatrix} p_{i,0} \\ e_{i,0} \end{bmatrix} = \begin{bmatrix} \gamma_1 & \gamma_2 \\ \Omega_2 & 1 \end{bmatrix} \begin{bmatrix} p_{i-1,0} \\ e_{i-1,0} \end{bmatrix}$$

or

$$X_{i,0} = AX_{i-1,0}.$$

The characteristic equation of the system of (12a) and (12b) can be derived by setting the determinant of $(\lambda \otimes I - A)$ equal to zero, where λ denotes the characteristic vector and I a 2x2 identity matrix. Assuming the characteristic roots are distinct, we solve the characteristic equation for $p_{i,0}$ and $e_{i,0}$:

$$p_{i,0} = B_1 \lambda_1^i + B_2 \lambda_2^i, \tag{13a}$$

$$e_{i,0} = B_1 Z_1 \lambda_1^i + B_2 Z_2 \lambda_2^i, \tag{13b}$$

where B_1 and B_2 are two arbitrary coefficients determined by two initial conditions, and Z_1 and Z_2 are the elements of a normalized characteristic matrix of which the first row is an unit vector. The characteristic roots, λ_1 and λ_2, are

$$\lambda_1, \lambda_2 = \frac{(1+\gamma_1) \pm \sqrt{(1+\gamma_1)^2 - 4(\gamma_1 - \gamma_2 \Omega_2)}}{2}. \tag{14}$$

As is well known, λ_1 and λ_2 must satisfy the following:

$$\lambda_1 \cdot \lambda_2 = \gamma_1 - \gamma_2 \Omega_2, \tag{15a}$$

$$\lambda_1 + \lambda_2 = 1 + \gamma_1. \tag{15b}$$

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[5] See Muth [1961].

To ensure the system to be stable, it must be parameterized such that the inequality condition holds: $1-(1+\Omega_2)(b_2\beta_1\phi+b_1\alpha_2\phi)>0$.[6] We then employ the general solutions for expectations variables in equations (13a) and (13b) to obtain

$$E_t[e_{t+1}] = B_1Z_1\lambda_1^{t+1} + B_2Z_2\lambda_2^{t+1} \tag{16}$$

Second, we substitute equation (16) back into (11b) and divide the latter equation by Ω_2. This procedure yields

$$p_t = -\Omega_2^{-1}e_t - u_{2t} - \Omega_1 u_{3t} + \Omega_2^{-1}u_{5t} + \Omega_2^{-1}\left(B_1Z_1\lambda_1^{t+1} + B_2Z_2\lambda_2^{t+1}\right). \tag{17}$$

Now, substituting (17) into (11a), the whole system further reduces to one first-order difference equation:

$$\left[1-(\gamma_1-\gamma_2\Omega_2)L\right]e_t = -\Omega_2 W_t, \tag{18}$$

where L is a lag operator such that $LX_n=X_{n-1}$,and

$$\begin{aligned} W_t &= u_{2t} + \Omega_1 u_{3t} - \Omega_2^{-1}u_{5t} + \gamma_3 u_{1t-1} - (\gamma_1+\gamma_4)u_{2t-1} + (\gamma_5-\gamma_1\Omega_1)u_{3t-1} + \gamma_6 u_{4t-1} + \gamma_1\Omega_2^{-1}u_{5t-1} \\ &\quad - \Omega_2^{-1}\left(B_1Z_1\lambda_1^{t+1} + B_2Z_2\lambda_2^{t+1}\right) + \gamma_1\Omega_2^{-1}\left(B_1Z_1\lambda_1^{t} + B_2Z_2\lambda_2^{t}\right) \end{aligned}$$

We can easily solve equation (18) for e_t :

$$e_t = -\Omega_2\sum_{k=0}^{\infty}(\gamma_1-\gamma_2\Omega_2)^k W_{t-k} + D(\gamma_1-\gamma_2\Omega_2)^t \tag{19a}$$

where D is an arbitrary coefficient determined by initial conditions. Moreover, we obtain the solution of real exchange rate q_t, or $e_t + p_t^* - p_t$, as

$$\begin{aligned} q_t &= -(1+\Omega_2)\sum_{k=0}^{\infty}(\gamma_1-\gamma_2\Omega_2)^k W_{t-k} + u_{2t} + \Omega_1 u_{3t} - \Omega_2^{-1}u_{5t} \\ &\quad + \left(1+\Omega_2^{-1}\right)D(\gamma_1-\gamma_2\Omega_2)^t \end{aligned}. \tag{19b}$$

[6] $1-(1+\Omega_2)(b_2\beta_1\phi+b_1\alpha_2\phi)>0$ implies $1>\lambda_1\cdot\lambda_2>0$ since $\lambda_1\cdot\lambda_2=[1-(1+\Omega_2)(b_2\beta_1\phi+b_1\alpha_2\phi)](1-b_1\alpha_2\phi)^{-1}$. It also implies $\gamma_1>0$. Therefore, we have $1>\lambda_1\cdot\lambda_2>0$ and $\lambda_1+\lambda_2>1$. In other words, $\lambda_1>1>\lambda_2>0$ or $1>\lambda_1>\lambda_2>0$.

2. Comparative statics

We are now ready to derive the variance of the exchange rate, which measures the degree of exchange rate variability. For the purpose of exposition, we assume that the variance of each random disturbance equals unity. We also assume that all disturbances do not jointly impinge on the economy. Under these assumptions, the use of equation (19a) allows us to derive the variance of the nominal exchange rate e_t that fluctuates due to disturbance *i* as follows:

$$\sigma_{e,i}^2 = \frac{\Omega_2^2}{1-\xi^2}\left\{M_{e,i}\right\},\ i = 1, 2, 3, 4, 5 \tag{20a}$$

where

$\xi = (\gamma_1 - \gamma_2\Omega_2)$,

$M_{e,1} = \gamma_3^2$,

$M_{e,2} = \left[1 + (\gamma_1 + \gamma_4)^2 - 2(\gamma_1 + \gamma_4)\xi\right]$,

$M_{e,3} = \left[\Omega_1^2 + (\gamma_5 - \gamma_1\Omega_1)^2 + 2\Omega_1(\gamma_5 - \gamma_1\Omega_1)\xi\right]$,

$M_{e,4} = \gamma_6^2$,

$M_{e,5} = \left[\Omega_2^{-2} + \Omega_2^{-2}\gamma_1^2 - 2\Omega_2^{-2}\gamma_1\xi\right]$.

Similarly, from equation (19b) we can derive the variance of the real exchange rate q_t that fluctuates due to disturbance *i* as follows:

$$\sigma_{q,i}^2 = \frac{(1+\Omega_2)^2}{1-\xi^2}\left\{M_{q,i}\right\},\ i = 1, 2, 3, 4, 5 \tag{20b}$$

where

$M_{q,1} = M_{e,1}$,

$M_{q,2} = \left[4 - 3\xi^2 + (\gamma_1 + \gamma_4)^2 - 2(\gamma_1 + \gamma_4)\xi\right]$,

$M_{q,3} = \left[\left(4 - 3\xi^2\right)\Omega_1^2 + (\gamma_5 - \gamma_1\Omega_1)^2 + 2\Omega_1(\gamma_5 - \gamma_1\Omega_1)\xi\right]$,

$M_{q,4} = M_{e,4}$,

$M_{q,5} = \left[\left(4 - 3\xi^2\right)\Omega_2^{-2} + \Omega_2^{-2}\gamma_1^2 - 2\Omega_2^{-2}\gamma_1\xi\right]$.

Equations (20a) and (20b) are of great interest to the purpose of the paper for it measures exchange rate volatility. Based on these equations, we will proceed to examine how increased trade balance affects exchange rate volatility. Differentiating equations (20a) and (20b) with respect to b_2, we have the following proposition (proof is provided in Appendix):

Proposition:

1. When the domestic absorption shock (u_1) perturbs the economy, increased trade balance (relative to domestic aggregate demand) reduces nominal (and real) exchange rate volatility; that is, $\frac{\partial \sigma_{e,1}^2}{\partial b_2} < 0$, $\frac{\partial \sigma_{q,1}^2}{\partial b_2} < 0$.

2. In the presence of foreign real income disturbance (u_4) and foreign nominal rate of interest disturbance (u_5)], increased trade balance augments nominal (and real) exchange rate volatility; that is, $\frac{\partial \sigma_{e,4}^2}{\partial b_2} > 0$, $\frac{\partial \sigma_{e,5}^2}{\partial b_2} > 0$, $\frac{\partial \sigma_{q,4}^2}{\partial b_2} > 0$, and $\frac{\partial \sigma_{q,5}^2}{\partial b_2} > 0$.

3. If $\Omega_2 > 1$, and when domestic nominal money disturbance (u_2) disturbs the economy, increased trade balance augments nominal (and real) exchange rate volatility; that is,

$$\frac{\partial \sigma_{e,2}^2}{\partial b_2} > 0 \text{ , and } \frac{\partial \sigma_{q,2}^2}{\partial b_2} > 0 .$$

The above proposition indicates that increased trade balance might aggravate or alleviate nominal (and real) exchange rate, depending on the source of the disturbances. As for the effect of increased trade balance on exchange rate volatility when domestic real income disturbance (u_3) disturbs the economy, the comparative statics result is too complicated to determine its sign, which we resorts to numerical simulations in the following section.

IV. Numerical Simulations

1. Parameter values

For the purpose of numerical simulations, the second column of Table 2 presents a set of baseline parameter values and the third column says the range for a parameter to change. The explanations of baseline parameter values are in order. First, the share of domestic absorption in aggregate demand b_1 is set equal to 0.90, and the parameter b_2 reflecting the importance of international trade balance in aggregate demand is calibrated under the assumption that d_t is equal to zero and trade balance relative to aggregate demand ranges from 0.01 to 0.30. It turns out that the value of b_2 is between 0.9 and 1.07 (see Table 3 for details.)

Second, the value of α_1 measuring the elasticity of domestic income to aggregate demand is set equal to 0.3 and is allowed to range from 0.1 to 0.8, while the value of α_2 measuring the real interest rate semi-elasticity of domestic absorption is set at 0.30. Third, the semi-elasticity of trade balance with respect to current terms of trade β_1 is set equal to 0.1, while the semi-elasticity of trade balance with respect to foreign income β_2 and that with respect to domestic income β_3 are equally set at 0.3 and both are allowed to range from 0.1 to 0.8.

Fourth, the income elasticity of real money demand Ω_1 is set at 0.3 and is allowed to change from 0.1 to 0.8. Most empirical estimates indicate that the interest-rate elasticity of real money demand is around 0.02. It thus turns out that the interest-rate semi elasticity $1/\Omega_2$ is 0.6667, or Ω_2 approximates 1.5. Finally, the baseline parameter value of the degree of price flexibility ϕ is set equal to 0.5, but it is allowed to change in a wide range from 0.001 to 1; that is, from being inelastic to unitary elastic. Note that the parameter values chosen in Table 2 ensure that the system exists at least one stable characteristic root (see Footnote 6).

Table 2 Parameter Values

Parameter Set	Baseline Values	Variants				
b_1	0.9					
b_2	0.9	0.92	0.95	0.98	1.00	1.07
α_1	0.3	0.1	0.2	0.4	0.6	0.8
α_2	0.3					
β_1	0.1					
β_2	0.3	0.1	0.2	0.4	0.6	0.8
β_3	0.3	0.1	0.2	0.4	0.6	0.8
Ω_1	0.3	0.1	0.2	0.4	0.6	0.8
Ω_2	1.5					
ϕ	0.5	0.001	0.01	0.05	0.1	1
$1-(1+\Omega_2)(b_2\beta_1\phi+b_1\alpha_2\phi)$	0.55	0.9991	0.9909	0.9540	0.9075	0.0575
$\lambda_1\lambda_2=\gamma_1-\gamma_2\Omega_2$	0.64	0.9994	0.9936	0.9671	0.9327	0.0788

Table 3 Trade Balance and the Value of b_2

Scenario	$\overline{D}=\overline{Y}$	$\overline{A}$	$\overline{T}$	$\overline{d}$	$\overline{a}$	b_1	b_2
1	1	0.99	0.01	0	-0.0101	0.9	0.90

2	1	0.95	0.05	0	-0.0513	0.9	0.92
3	1	0.90	0.10	0	-0.1054	0.9	0.95
4	1	0.85	0.15	0	-0.1625	0.9	0.98
5	1	0.80	0.20	0	-0.2231	0.9	1.00
6	1	0.70	0.30	0	-0.3567	0.9	1.07

2. Simulation results with baseline parameter values

Given the baseline parameter values, the first part of numerical simulations is conducted by allowing b_2 to take on values ranging from 0.90 to 1.07. The variance of the nominal exchange rate (equation 20a) and the variance of the real exchange rate (equation 20b) are then computed for each chosen value of b_2. The numerical results of the effects of trade balance on nominal exchange rate volatility and real exchange rate volatility are reported in Table 4 and Table 5, respectively, and their qualitative outcomes are summarized in Table 6. A striking finding is that an increase in the weight of trade balance in aggregate demand (b_2) tends to decrease both nominal and real exchange rate volatility when domestic absorption disturbance (u_1) disturbs the economy. However, in the presence of all other disturbances (domestic nominal money disturbance (u_2), domestic real income disturbance (u_3), foreign real income disturbance (u_4), or foreign nominal rate of interest disturbance (u_5)), increased trade balance weight tends to be destabilizing rather than stabilizing both nominal and real exchange rate volatility.

Some other findings from Table 4 and Table 5 also deserve attention. First, the variance of real exchange rate volatility is much larger than the variance of nominal exchange rate is in all cases. Second, in the presence of foreign real income disturbance (u_4), the variance of the exchange rate turns out to be far below unity. But the exchange rate variance exceeds unity in the presence of all other disturbances in most scenarios. Third, the domestic monetary shock is seen to have the most powerful destabilizing effect on exchange rate movements, for the nominal (real) exchange -rate variance is more than two (twenty-six) times as much as the variance of u_2 in all cases, which accord with the overshooting phenomenon firstly formulated in Dornbusch [1976].

Table 4 Variance of Nominal Exchange Rate: Baseline Parameter Values

b_2	u_1	u_2	u_3	u_4	u_5
0.90	1.0222	2.6180	1.9065	0.0920	1.0102
0.92	1.0160	2.6198	1.9162	0.0955	1.0106
0.95	1.0068	2.6226	1.9310	0.1010	1.0112
0.98	0.9978	2.6254	1.9458	0.1065	1.0118
1.00	0.9920	2.6272	1.9557	0.1102	1.0123
1.07	0.9722	2.6338	1.9909	0.1237	1.0137

Table 5 Variance of Real Exchange Rate: Baseline Parameter Values

b_2	u_1	u_2	u_3	u_4	u_5
0.90	2.8395	26.0222	6.9833	0.2556	11.1395
0.92	2.8221	26.0273	7.0104	0.2654	11.1406
0.95	2.7966	26.0349	7.0513	0.2804	11.1423
0.98	2.7717	26.0426	7.0924	0.2958	11.1440
1.00	2.7554	26.0478	7.1200	0.3062	11.1451
1.07	2.7006	26.0662	7.2177	0.3435	11.1493

Table 6 Summary of Qualitative Results with Baseline Parameter Values

Source of Disturbance	Effect on Exchange Rate Volatility
domestic absorption, u_1	-
domestic nominal money, u_2	+
domestic real income, u_3	+
foreign real income, u_4	+
nominal foreign rate of interest, u_5	+

3. Sensitivity analysis

The second part of numerical simulations is sensitivity analysis, which is to see whether the above numerical results are robust to changes in the values of some parameters including α_1, β_2, β_3, Ω_1 and ϕ. Since the qualitative results for all disturbance except for domestic real income disturbance (u_3) are known from our comparative statics results, our sensitivity analysis thus focuses on the case of domestic real income disturbance. Tables 7-8 report the results of sensitivity analysis. It is found that the exchange rate variance is sensitive to the parameter values of α_1, β_3 and Ω_1 in the presence of u_3. When the values of α_1, or β_3, or Ω_1, are small, increased trade balance b_2 tends to *decrease* exchange rate volatility if price adjustment is inelastic, but it turns out to be destabilizing if price adjustment is unit elastic. However, when the values of α_1, β_3 and Ω_1 are set at 0.4 or higher, increased trade balance tends to *increase* exchange rate volatility whether price adjustment is inelastic or unit elastic.

Table 7 Sensitivity Analysis of Nominal Rate Variance against Domestic Real Income Disturbance

$\alpha_1, \beta_3, \Omega_1$	0.1			0.4			0.8		
b_2 \ ϕ	0.001	0.05	1	0.001	0.05	1	0.001	0.05	1
0.90	0.02448	0.1247	4.8027	0.36264	0.4960	6.7226	1.44366	1.6286	10.2637
0.92	0.02448	0.1243	4.8146	0.36266	0.4970	6.8072	1.44373	1.6320	10.4750
0.95	0.02447	0.1238	4.8334	0.36269	0.4985	6.9364	1.44383	1.6371	10.7977
0.98	0.02446	0.1233	4.8532	0.36272	0.5000	7.0673	1.44393	1.6422	11.1273
1.00	0.02445	0.1230	4.8670	0.36274	0.5010	7.1562	1.44399	1.6456	11.3509
1.07	0.02443	0.1219	4.9190	0.36280	0.5045	7.4761	1.44422	1.6576	12.1591

Notes: Robust to changes in the values of β_2.

Table 8 Sensitivity Analysis of Real Rate Variance against Domestic Real Income Disturbance

$\alpha_1, \beta_3, \Omega_1$	0.1			0.4			0.8		
b_2 \ ϕ	0.001	0.05	1	0.001	0.05	1	0.001	0.05	1
0.90	0.25551	0.5338	13.528	4.00734	4.3778	21.674	16.0102	16.524	40.510
0.92	0.25549	0.5329	13.562	4.00739	4.3806	21.909	16.0104	16.533	41.097
0.95	0.25547	0.5314	13.614	4.00747	4.3847	22.267	16.0106	16.547	41.994
0.98	0.25544	0.5301	13.669	4.00755	4.3889	22.631	16.0109	16.562	42.909
1.00	0.25542	0.5292	13.707	4.00760	4.3916	22.878	16.0111	16.571	43.530
1.07	0.25536	0.5261	13.851	4.00779	4.4013	23.767	16.0117	16.605	45.775

Notes: Robust to changes in the values of β_2.

V. Conclusion

This paper examines the relationship between trade balance and exchange rate volatility, using a stochastic small open economy model with sticky price. Several random disturbances that perturb the economy and cause exchange rate fluctuations are considered. These disturbances include shocks to domestic absorption, domestic money stock, domestic income, foreign income, and foreign nominal interest rate. Both Comparative statics and numerical simulations with sensitivity analysis are conducted.

This paper has found that increased trade balance (relative to domestic aggregate demand) tends to reduce exchange rate volatility when the domestic absorption shock perturbs the economy. In the presence of all other domestic and foreign shocks, however, increased trade balance tends to augment exchange-rate volatility, except for the case of disturbance of domestic real income in which the effect of increased trade balance is indeterminate. Our results suggest that whether trade imbalance has aggravated exchange rate volatility in many industrial countries is an open question. Further research to solve this question through empirical investigation seems warranted.

The paper therefore calls into question the role of international trade in dampening exchange rate fluctuations in an international environment affected simultaneously by multiple sources of random disturbances, be it domestic or foreign.

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Appendix: Comparative Statics

A.1 The volatility of the nominal exchange rate

Differentiating equation (20a) with respect to b_2, we have

$$\frac{\partial \sigma_{e,1}^2}{\partial b_2} = \frac{2b_1^2\beta_1\phi\Omega_2^2\left(1+\Omega_2\right)\Psi_1\left(b_1,b_2,\beta_1,\alpha_2,\Omega_2,\phi\right)}{\left[\Gamma\left(b_1,b_2,\beta_1,\alpha_2,\Omega_2,\phi\right)\right]^2}, \tag{A1a}$$

$$\frac{\partial \sigma_{e,2}^2}{\partial b_2} = \frac{2\beta_1\phi\Omega_2^4\left(b_1\alpha_2+b_2\beta_1\right)\Psi_2\left(b_1,b_2,\beta_1,\alpha_2,\Omega_2,\phi\right)}{\left[\Gamma\left(b_1,b_2,\beta_1,\alpha_2,\Omega_2,\phi\right)\right]^2}, \tag{A1b}$$

$$\frac{\partial \sigma_{e,4}^2}{\partial b_2} = \frac{2b_2\beta_2^2\phi\Omega_2^2\Psi_3\left(b_1,b_2,\beta_1,\alpha_2,\Omega_2,\phi\right)}{\left[\Gamma\left(b_1,b_2,\beta_1,\alpha_2,\Omega_2,\phi\right)\right]^2}, \tag{A1c}$$

$$\frac{\partial \sigma_{e,5}^2}{\partial b_2} = \frac{2b_2\beta_1^2\phi\Omega_2^2\Psi_3\left(b_1,b_2,\beta_1,\alpha_2,\Omega_2,\phi\right)}{\left[\Gamma\left(b_1,b_2,\beta_1,\alpha_2,\Omega_2,\phi\right)\right]^2}, \tag{A1d}$$

where

$$\Gamma\left(b_1,b_2,\beta_1,\alpha_2,\Omega_2,\phi\right) = \left[b_1\alpha_2\Omega_2 + b_2\beta_1\left(1+\Omega_2\right)\right]\left[-2 + b_2\beta_1\phi\left(1+\Omega_2\right) + b_1\alpha_2\phi\left(2+\Omega_2\right)\right]$$

$$\Psi_1\left(b_1,b_2,\beta_1,\alpha_2,\Omega_2,\phi\right) = -1 + \phi\left(1+\Omega_2\right)\left(b_1\alpha_2 + b_2\beta_1\right),$$

$$\Psi_2\left(b_1,b_2,\beta_1,\alpha_2,\Omega_2,\phi\right) = b_1^2\alpha_2^2\phi + b_1\alpha_2\left(\Omega_2 - 1\right) + b_2\beta_1\left(1+\Omega_2\right),$$

$$\Psi_3\left(b_1,b_2,\beta_1,\alpha_2,\Omega_2,\phi\right) = b_2\beta_1\left(1+\Omega_2\right)\left[1 - b_1\alpha_2\phi\left(1+\Omega_2\right)\right] + b_1\alpha_2\Omega_2\left[2 - b_1\alpha_2\phi\left(2+\Omega_2\right)\right].$$

The stability condition, $1-\phi\left(1+\Omega_2\right)\left(b_1\alpha_2 + b_2\beta_1\right) > 0$,implies $\Psi_1\left(b_1,b_2,\beta_1,\alpha_2,\Omega_2,\phi\right) < 0$. Thus ,

$$\frac{\partial \sigma_{e,1}^2}{\partial b_2} < 0 \ . \tag{A2a}$$

Moreover, since

$$\frac{1}{\left(1+\Omega_2\right)b_1\alpha_2} - \frac{1}{\left(1+\Omega_2\right)\left(b_1\alpha_2 + b_2\beta_1\right)} = \frac{b_2\beta_1}{b_1\alpha_2\left(1+\Omega_2\right)\left(b_1\alpha_2 + b_2\beta_1\right)} > 0$$

and

$$\frac{2}{\left(2+\Omega_2\right)b_1\alpha_2}-\frac{1}{\left(1+\Omega_2\right)\left(b_1\alpha_2+b_2\beta_1\right)}=\frac{b_1\alpha_2\Omega_2+2b_2\beta_1\left(1+\Omega_2\right)}{b_1\alpha_2\left(1+\Omega_2\right)\left(2+\Omega_2\right)\left(b_1\alpha_2+b_2\beta_1\right)}>0,$$

these imply $\Psi_3\left(b_1,b_2,\beta_1,\alpha_2,\Omega_2,\phi\right)>0$, or

$$\frac{\partial\sigma_{e,4}^2}{\partial b_2}>0, \tag{A2b}$$

and

$$\frac{\partial\sigma_{e,5}^2}{\partial b_2}>0. \tag{A2c}$$

Finally, if $\Omega_2>1$, we have

$$\frac{\partial\sigma_{e,2}^2}{\partial b_2}>0. \tag{A2d}$$

A.2 The volatility of the real exchange rate

Differentiating equation (20b) with respect to b_2 and applying similar reasoning as we use in proving the change in the volatility of nominal exchange rate in A.1, we have

$$\frac{\partial\sigma_{q,1}^2}{\partial b_2}=\frac{2b_1^2\beta_1\phi\Omega_2^2\left(1+\Omega_2\right)^3\Psi_1\left(b_1,b_2,\beta_1,\alpha_2,\Omega_2,\phi\right)}{\left[\Gamma\left(b_1,b_2,\beta_1,\alpha_2,\Omega_2,\phi\right)\right]^2}<0, \tag{A3a}$$

$$\frac{\partial\sigma_{q,2}^2}{\partial b_2}=\frac{2\beta_1\phi\Omega_2^2\left(1+\Omega_2\right)^2\left(b_1\alpha_2+b_2\beta_1\right)\Psi_2\left(b_1,b_2,\beta_1,\alpha_2,\Omega_2,\phi\right)}{\left[\Gamma\left(b_1,b_2,\beta_1,\alpha_2,\Omega_2,\phi\right)\right]^2}>0,\ \text{if } \Omega_2>1 \tag{A3b}$$

$$\frac{\partial\sigma_{q,4}^2}{\partial b_2}=\frac{2b_2\beta_2^2\phi\left(1+\Omega_2\right)^2\Psi_3\left(b_1,b_2,\beta_1,\alpha_2,\Omega_2,\phi\right)}{\left[\Gamma\left(b_1,b_2,\beta_1,\alpha_2,\Omega_2,\phi\right)\right]^2}>0, \tag{A3c}$$

$$\frac{\partial\sigma_{q,5}^2}{\partial b_2}=\frac{2b_2\beta_1^2\phi\left(1+\Omega_2\right)^2\Psi_3\left(b_1,b_2,\beta_1,\alpha_2,\Omega_2,\phi\right)}{\left[\Gamma\left(b_1,b_2,\beta_1,\alpha_2,\Omega_2,\phi\right)\right]^2}>0. \tag{A3d}$$