

**Firm-Specific Human Capital, Sunk Costs and Macroeconomic Adjustment in**

**Japan: Who Bears the Burden?**

by

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## I. Introduction

“Life-long labor” contracts, a seniority wage system, and firm-specific human capital investment constitute distinctive, but mutually reinforcing, characteristics of the Japanese labor market. The purpose of this research is to explore the extent to which particularities of the functioning of Japanese labor markets could explain macroeconomic adjustment, increasing unemployment for specific sub-populations, as well as low productivity growth in Japan, since the early 1990s. A central issue to be highlighted is how sunk costs related to firm-specific human capital formation define both short-run and long-run labor market adjustment processes and associated macroeconomic performance. More specifically, such firm-specific human capital tends to undermine the functioning of external labor markets, and hence the ability of firms to adapt to macroeconomic shocks. There are associated hysteresis effects, which, *ceteris paribus*, are larger than those that occur when human capital is more generic and worker specific.

In Japan, the most prominent factor affecting the labor market over the past decade has been the stagnation in growth of productivity and output. This has led to the prolonged stagnation in labor productivity growth, accompanied by a doubling of the unemployment rate over the decade (Figure 1). Many observers agree that the collapse of the asset price bubble of the late 1980s and mistakes in macro and micro policy have been the primary causes for why the slump has been prolonged. There are, however, a number of other explanations. In particular, the question remains as to whether the Japanese economic system including employment practices have also been responsible.

In recent literature, Hayashi and Prescott (2002) document that treating TFP (total factor productivity) as exogenous, growth theory accounts well for the Japanese lost decade of growth. They propose that the decline of TFP as well as a dramatic decline of labor hour was the source of the stagnation in output growth. Other recent studies, however, report a permanent productivity slowdown was moderate in Japan in the 1990s. In the following analysis, we explore how firm-specific human capital and the resulting hold-up problem magnify the negative impacts of the mild productivity slowdown.

In the model, there are skilled and unskilled workers. Workers become skilled workers if and only if the firm invests in firm-specific human capital. Turnover costs and firm-specific human capital investments generally drive a wedge between the lowest wage for which a skilled employee will work and the highest wage the employer will pay. They thus generate a rent to continued employment as well as a seniority wage. If the shocks were temporally, Japanese firms would have minimized the costs of the hold-up problem through adjusting hours of work and inventory. However, when the productivity slowdown is persistent, the hold-up problem can cause various losses in Japanese labor markets even under the mild shocks. In the short-run, the losses tend to arise as over-employment of skilled workers and under-employment of unskilled workers. In the long-run, the losses, however, eventually decrease the number of skilled workers and have serious negative impacts on total production. In particular, under endogenous labor supply, unemployment prevails only among young workers. In the 1990s, there were

salient empirical observations that young workers dramatically increased unemployment rates and reduced labor participation ratios. Our theoretical model implies that the observations during the past decade can be a bad symptom in predicting another prolonged stagnation in Japan.

In his seminal work, Becker (1975) argues that the firm will not be willing to invest in general training when labor markets are competitive but is willing to invest in specific training because it cannot be transferred to outside the firm. The provision of firm-specific training, however, causes a hold-up problem. In Japanese labor markets, the problem seems more serious under productivity slowdown because of their large amount of firm-specific human capital investment. The hold-up problem seems potentially promising for understanding aspects of labor markets in Japan.

There are a large number of studies that investigate the hold-up problem in labor markets (see, for example, Malcomson [1997] for their overview). The hold-up literature is concerned with economic relationships with the following characteristics: (1) because of turnover costs or specific investments, there are rents to continuing a relationship once started that are, in principle, available for the parties to bargain over; (2) there are problems in writing contracts contingent on all the future events that are important for the relationship; and (3) any contract that is made between the parties can be renegotiated by mutual consent. In this paper, we first discuss the first two of these that have been widely discussed in the literature on specific investments in labor markets. We then investigate the third, renegotiation. In particular, we explore interactions between skilled and unskilled workers in a dynamic general equilibrium framework. The model has various attractive features that are consistent with salient empirical observations in Japan during the 1990s.

In our model, the prolonged stagnations are initiated by exogenous external shocks. This paper thus cannot explain why the slump started in Japan in the 1990s. It, however, verifies why the slump has been so prolonged even if the exogenous shocks were moderate. Until the late 1980s, mutually reinforcing, characteristics of the Japanese labor market had served well in Japan. They, however, started magnifying the negative impacts under persistent productivity slowdown. The economic system which had served well during previous decades was then due for a rapid transformation.

The organization of the rest of this paper is as follows. In the next section, we propose a simplified two-period framework, in order to examine the implications of an unanticipated demand shock, when a firm makes an intertemporal investment in firm-specific human capital. Section III then extends this basic analysis to consider a multi-period overlapping-generations version of the model, which also allows for consumers' optimal labor-leisure decisions affected rate of labor force participation.

## II. Salient Characteristics of Japanese Labor Market and Macroeconomic Performance in the Nineties

### **Salient Empirical Observations:**

**1. The sharp increase in unemployment since 1992, which is particularly acute for young male workers. (Figure 2)**

Unemployment remained at low levels by international standards until the mid 1990s, when the rate climbed abruptly from 3 percent, reaching 5 percent in the summer of 1999. This was particularly pronounced in the case of young men (under the age of 25) as well as older men (age 60-64), both having rates over 10 percent. There has been an increase in the proportion of long-term unemployed with spells out of work of over 1 year, and the proportion of long-term unemployment among prime-aged workers between the ages of 30 and 50 now exceeds 20 percent.

**2. Average labor participation ratios of male workers were almost stable over the 1990s. There was, however, a dramatic downturn in labor participation for young male workers. (Figure 3)**

Although unemployment rates have risen, employment rates have not fallen. Average male participation rates have been stable over the 1990s under the two-point rise in the unemployment rate. There was, however, a dramatic decline in labor participation of young men (under the age of 25) over the 1990s. Labor participation ratios of older men (age 60-64), in contrast, showed only a marginal decline over the 1990s.

**3. The consequence of the foregoing was a striking fall in the number of male workers for specific young age groups. (Figure 4)**

Although unemployment rates have risen, the number of male participation rates has on average been stable over the 1990s. There was, however, a dramatic decline in the number of young male workers (under the age of 25) over the 1990s. The number of older workers (age 60-64) was, in contrast, stable over the 1990s.

**4. On the other hand, there was a large and sustained upswing in labor costs, as a share of total value added. (Figure 5)**

There was a large and sustained upswing in labor costs, as a share of total value added. Much of the increase in the share of labor costs can be attributed to a sharp increase in wages from the late 80s through the mid-90s.

The above salient empirical observations indicate that a dramatic structural change in Japanese labor markets in the 1990s occurred mainly for young workers (under the age of 25). In the following sections, we investigate how the hold-up problem can

explain these empirical observations. We also explore whether high unemployment rates and low labor participation of young workers that were observed during the past decade can be a bad symptom for another prolonged stagnation in Japan.

### III. Labor Market Adjustments in the Short Run with Firm-Specific Human Capital

#### A. The General Framework

Let us consider a single representative firm's optimal long-term investment in human capital in a two-period framework. In this initially simplified framework, the firm does not produce in the first period, but merely undertakes the training of workers, in order to upgrade their skills. Thus, there are two categories of workers, unskilled and skilled, which will be distinguished by, respectively,  $u$  and  $s$  superscripts. The firm, which produces a single homogeneous good,  $Y$ , hires,  $N_t^U$  and  $N_t^S$ , at wages,  $w_t^U$  and  $w_t^S$ , where the subscript  $t$  is used to designate a given period, which will be understood to be either 0, or 1, in this initial two-period version of the model.<sup>1</sup>

The production function, in a given period, will be understood to depend on the two types of labor inputs, and on a physical capital stock,  $K_t$ , where  $\Delta_t^K$  represents the price of capital goods. Both  $K_t$  and  $p_t^K$  will, however, be understood to be predetermined constant values in this initial formulation of the model. Similarly, a constant coefficient,  $A_T$ , reflects technological productivity, so that:

$$(1) Y_t = A_t f(N_t^S, N_t^U, K_t)$$

In the first period, although the firm does not produce, it decides on a total number of unskilled “young” workers to be hired, as well as a total level of firm-specific investment in human capital,  $I_0^H$ .<sup>2</sup> Such investment entails a cost, which is represented by a general cost function  $c(N_0^U)$ . However, in this initial formulation, this function is assumed, for simplicity, to be linear, with  $\gamma$  representing the cost per-worker of training unskilled workers, who are upgraded to have a skilled status in the subsequent period. As a consequence of this investment, the quality (productivity) of each skilled worker improves to a value of  $q_1$  when they are older. More generally,  $q_t > 1$ , in this and the subsequent analysis, while the quality of an unskilled worker is set equal to 1, as numeraire. The supply of initially unskilled workers, who receive training, is understood

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<sup>1</sup> Although there are only two periods in this initial version of the model, a more general formulation representing a time period,  $t$ , is introduced here, in order to facilitate the subsequent analysis of the multi-period overlapping-generations version of the model, which will be analyzed in the next section.

<sup>2</sup> For simplicity, it is assumed that the unskilled workers are not paid during their “training” period. In the more general formulation of the model presented in the next section, the firm will be understood to initially hire a proportion,  $\lambda^S$ , of unskilled workers, who will be upgraded to a skilled status, when they are older, and another set of unskilled workers, who will remain unskilled. The two categories of unskilled workers who either become skilled through training, or remain unskilled, can be considered to be “genetically” different. More specifically, the underlying supply-elasticity conditions for initially hiring these two sets of unskilled workers are taken to be inherently different and, as such, need to be analyzed in terms of distinct labor markets.

to be elastic, but not perfectly so. Consequently, the firm faces an upward sloping supply of labor for this particular category of workers.

In the second period, the firm also hires unskilled workers, whose skills will not be upgraded, at a constant wage,  $w_t^U$ , and undertakes production and sales, at a price,  $P_t$ , which depends on its level of output. In light of the foregoing specifications, its profits in period  $t$  are given by:

$$(2) \pi_t = P_t A_t f(N_t^S, N_t^U, K_t) - w_t^U N_t^U - w_t^S N_t^S - \gamma N_{t-1}^S - \rho^K K_t$$

where the total quality-adjusted labor input,  $N_t$ , in the production function equals  $q_t N_t^S + N_t^U$ .

### B. The *Ex Ante* Analysis

The point of a departure for the analysis is a scenario of “boom”, where the firm is hypothesized to expect, with certainty, continuously high levels of economic demand and growth, such that its ongoing demand for skilled is at least as great as what it has known in the past.<sup>3</sup>

Without loss of generality, a specific formulation for the production function is introduced, while the firm is understood to face a competitive price,  $P_t$ . More specifically, the production function is taken to have additively separable components, which reflect constant returns for unskilled labor, but decreasing returns for the skilled labor and physical capital inputs. It is given by the following expression,

$$(3) Y_t = \alpha^U (N_t^U)^\theta + \alpha^S (N_t^S)^\beta + \alpha^K (K_t)^\sigma$$

where  $\beta < 1$ , and  $\sigma < 1$ .

Using backward induction, the firm’s demands for unskilled and skilled labor are obtain by first maximizing the expression for profit, in the second period, with regard to the number of unskilled workers and then choosing optimally the number of skilled workers,  $N_t^S$ , while anticipating that optimal value for  $N_t^U$ . The following first-order condition and demand function for unskilled labor reflect the standard proposition that the marginal value product of labor equals the wage rate:

$$(4a) \theta P_t \alpha^U (N_t^U)^{\theta-1} = w_t^U$$

$$(4b) N_t^U = (\theta P_t \alpha^U / w_t^U)^{1/(1-\theta)}$$

However, a consideration of the first-order condition and associated demand for skilled worker in the first period offers the insight that the firm internalizes the cost of firm-specific human capital formation by setting the marginal value product of skilled workers equal to the sum of the wage rate for skilled workers and the cost of training per worker,  $\gamma$ , such that:

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<sup>3</sup> In light of the two-period paradigm, these expectations can be taken as an initial state variable.

$$(5a) \beta P_t \alpha^S (N_t^S)^{\beta-1} = w_t^S + \gamma$$

$$(5b) N_t^S = \beta P_t \alpha^S / (w_t^S + \gamma)^{1/(1-\beta)}$$

Note, furthermore, that if the skilled workers are heterogeneous, in the sense that certain workers have higher marginal productivities than that of the least most productive one hired, equation 5a becomes an inequality.

The associated *ex ante* reduced form expression for the firm's profits is readily obtained:

$$(6) \pi_t = P_t [\alpha^U (\theta P_t \alpha^U / w_t^U)^{\theta/(1-\theta)} + \alpha^S (N_t^S)^\beta + \alpha^K (K_t)^\sigma] \\ - w_t^U N_t^U - w_t^S N_{t-1}^S - \rho^K K_t - \gamma N_{t-1}^S$$

### C. The *Ex Post* Analysis

Let us now assume that at the moment separating the two periods there is an unexpected structural change entailing a substantial fall in demand for the homogeneous product, which results in a lower price  $P_t'$ . Thus, instead of there being a “boom” period, as was initially anticipated when the firm-specific human capital investments were undertaken by the firm, the second period is now characterized by a “bust”. As a consequence, following this unexpected demand shock the firm will choose a lower number of unskilled workers, given by:

$$(7) N_t^{U'} = (\theta P_t' \alpha^U / w_t^U)^{1/(1-\theta)}$$

However, if the new, lower gross marginal value product of skilled labor remains above the wage rate,  $w_t^S$ , both the demand and supply for skilled workers is perfectly inelastic in the short run.<sup>4</sup> A crucial insight is that because of the firm-specific human capital investment there is a “lock in” effect for both the employer and employees. For the firm, the existing expenditures on training workers are a sunk cost, which in this instance equals,  $\gamma \beta P_t \alpha^S / (w_t^S + \gamma)^{1/(1-\beta)}$ . Given this bygone, the *ex post* incentive for the firm is to keep its skilled workers, provided that the value of their gross marginal product is greater than the *ex post* wage rate, which could fall depending on the relative bargaining power of the firm and its skilled workers. To the extent that there is hysteresis in the determination of the wage rate, a greater share of the sunk cost burden is borne by the firm. For the skilled workers, the lock-in effect may even be stronger, since to the extent their human capital is firm specific, the outside wage that they face is the same as for unskilled workers. Furthermore, that outside wage rate has been depressed by the negative labor demand shock for unskilled workers.

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<sup>4</sup> The terminology, gross marginal value product of skilled labor, is understood to reflect the rate of return to the firm, whereas a net marginal value product refers to the return after the human capital investment per worker,  $\gamma$ , is deducted.

The interaction between *ex post* labor demand, labor supply and curves reflecting the relative positions of the *ex ante* and *ex post* marginal value products of labor market are presented in the follow graph.

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Insert Graph 1

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The foregoing analysis of the lock in effect of firm-specific human capital formation is analytical akin to the well-known “hold up” problem in the theory of bargaining between two agents. There, one party principally bears the costs of investment, for which the value is of specific use to the other party.<sup>5</sup> The hold-up problem offers a rationale for vertical integration in the face of a specific investment and bilateral bargaining. In comparison, the lock-in effect, identified here, entails in effect a form of vertical integration with the labor market, in order that a firm may internalize the benefits from specific investments, which constitute a mutually beneficial exchange between itself and workers. However, the unanticipated shock generates an *ex post* sub-optimality in this human capital investment for the investing firm. If it had anticipated such a “bust” scenario, the firm would have undertaken less human capital investment and hired fewer “skilled” workers.

There is also an apparent relation between the lock-in effect of firm-specific human capital investment and insider-outsider models of unemployment. In the latter, it is a labor union, which leads to a wage gap that favors workers within the firm, who receive more than the marginal value product of their labor input. In contrast, to these protected insiders, equally productive outsiders are disadvantaged in the external labor market. In the case of firm-specific human capital, the skilled workers may receive wages (depending on their bargaining power), which are equal to their net marginal value product within the firm. However, those wages are higher than the outside wage they face on the labor market and the potential external marginal value product of their labor supply.

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<sup>5</sup> See the discussion in Tirole (1998), pages 23-24.



#### IV. An Overlapping Generations Model of Labor Market Adjustments with Firm-Specific Human Capital

##### 1. A Representative Firm

In the following analysis, we consider an overlapping-generations model with productive capital. In the overlapping generations model, individuals live for three periods: young, middle-age, and old. They supply labor in the young and in the middle-age, but retire in the old. They are genetically identical within and across generations. The young workers will, however, upgrade to skilled workers in the middle-age if and only if they work for the same firm in the middle-age.<sup>6</sup> The quality of each skilled middle-age worker depends on the firm's human capital investment in the previous period.

In the analysis, we consider a competitive world where economic activity is performed over infinite discrete time. There is a single final good, taken as a numeraire; it is competitively produced and can be either consumed or invested. Labor and capital are combined with a constant-return-to-scale technology in each firm. The firm hires three types of workers: young workers  $N_{0,t}$ , unskilled middle-age workers  $N_t^{us}$ , and skilled middle-age workers  $N_t^s$ . The skilled middle-age workers have higher quality than the other types of workers in the sense that their effective labor supply is larger. Denoting output in period  $t$  by  $Y_t$  and the input of capital in period  $t$  by  $K_t$ , the production function in period  $t$  is then specified as

$$(1) Y_t = A F(N_{0,t}, q_{t-1} N_t^s, N_t^{us}, K_t).$$

where  $q_{t-1} (> 1)$  is the quality of middle-age skilled worker in period  $t$ .

The profit in period  $t$  is equal to output in period  $t$  minus the costs of production in period  $t$ . The costs of production are the sum of labor costs, capital costs, and costs of human capital investment. The costs of human capital investment are the training costs that enhance unskilled young workers to skilled middle-age workers. We assume that the costs of human capital investment in period  $t$  are a convex and increasing function of  $q_t$ . Denoting the wages for young, skilled middle-age, and unskilled middle-age workers by  $w_{0,t}$ ,  $w_t^s$ , and  $w_t^{us}$  respectively, the profit in period  $t$  is written as

$$(2) \pi_t = A F(N_{0,t}, q_{t-1} N_t^s, N_t^{us}, K_t) - w_{0,t} N_{0,t} - w_t^s N_t^s - w_t^{us} N_t^{us} - p_t^I K_t - c(q_t) N_{0,t}.$$

where  $p_t^I$  is price of capital goods in period  $t$ . In each period, the firm maximizes the present value of current and future profits as follows

$$(3) \text{Max } \Pi_t = \sum_{i=0}^{\infty} \delta^i \pi_{t+i}$$

where  $\delta$  is discount factor such that  $0 < \delta < 1$ .

Under the profit maximization, the wages for young and unskilled middle-age workers are determined by the marginal products of labor. The wage for skilled middle-age

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<sup>6</sup> Human capital theory distinguishes between training in general-usage and firm-specific skills. For simplicity, the following analysis focuses on firm-specific skills.

workers,  $w_t^s$ , is, however, determined by a bargaining game between the firm and the skilled workers. This is because the skilled workers face a hold-up problem when their skill is firm-specific. In our model, the skilled workers only receive the competitive wage of unskilled middle-age workers when they quit the firm. Denoting the marginal products of the skilled workers in period  $t$  by  $w_t^*$ , it thus holds that

$$(4) \quad w_t^s = \gamma w_t^{us} + (1-\gamma) w_t^*, \quad \text{where } 0 < \gamma < 1.$$

where  $\gamma$  denotes the bargaining power of the firm that lies between zero and one. One may argue that the hold-up problem becomes more serious for the skilled workers as  $\gamma$  increases.

Since  $\gamma$  is less than one,  $w_t^s$  is smaller than the marginal product of the skilled workers  $\partial Y_t / \partial N_t^s$ . The firm thus always hires all of the existing skilled middle-age workers. On the other hand, to the extent that  $\gamma > 0$ , the skilled workers never have an incentive to quit the firm. This implies that the number of the skilled middle-age workers is equal to the number of the young workers in the previous period, that is,

$$(5) \quad N_t^s = N_{0,t-1}.$$

In period  $t$ , the firm decides the number of young workers  $N_{0,t}$  and human capital investment  $q_t$  so as to maximize the present value of profits under the constraint that  $N_{t+1}^s = N_{0,t}$ . The first-order conditions  $\partial \Pi_t / \partial N_{0,t} = 0$  and  $\partial \Pi_t / \partial q_t = 0$  thus lead to

$$(6) \quad A F_1(N_{0,t}, q_{t-1} N_{0,t-1}, N_{t-1}^{us}, K_t) = w_{0,t} + \delta [w_{t+1}^s - q_t (\partial w_{t+1}^s / \partial q_t)] + [c(q_t) - q_t c'(q_t)],$$

$$(7) \quad A F_3(N_{0,t}, q_{t-1} N_{0,t-1}, N_{t-1}^{us}, K_t) = w_{t-1}^{us},$$

$$(8) \quad \delta A F_2(N_{0,t+1}, q_t N_{0,t}, N_{t+1}^{us}, K_{t+1}) = c'(q_t) + \delta (\partial w_{t+1}^s / \partial q_t),$$

where  $F_1 \equiv \partial Y_t / \partial N_{0,t}$ ,  $F_2 \equiv \partial Y_{t+1} / \partial N_{t+1}^s$ , and  $F_3 \equiv \partial Y_t / \partial N_{t-1}^{us}$ .

## 2. The Short-run Analysis

The goal of our theoretical analysis is to explore what impacts a small productivity slowdown has in our model. Specifically, we investigate what impacts a permanent decline of the productivity parameter “A” has on employment and total production in the economy. The impacts, however, depend on what time span we are interested in. This section first explores the short-run impacts. We define “the short-run” where physical capital stock is fixed, that is,  $K_t = K$ . We also assume that wages do not adjust instantaneously in the short-run.

The first-round impacts of the productivity slowdown are the declines of the marginal products of labor for all types of workers. However, to the extent that the decline of productivity is small, the wage for skilled middle-age workers still remains smaller than the marginal product of the skilled workers. Even when the wage is rigid, the firm therefore hires all of the existing skilled middle-age workers. In contrast, the declined marginal products of labor reduce the demand for young and unskilled middle-age workers. As a result, the number of employment declines for young and unskilled middle-age workers under the wage rigidity.

The algebraic impacts on the number of employments are obtained by using the first-order conditions (6) and (7). We assume that unexpected productivity slowdown occurred when  $t = T$ . Since  $N^s_T = N_{0, T-1}$ ,  $w_{0, T} = w_{0, T-1}$ ,  $w^{us}_T = w^{us}_{T-1}$ , and  $w^s_{T+1} = w^s_{1, T-1}$ , equations (6) and (7) lead to

$$(9) \quad A F_1(N_{0, T}, q_{T-1} N_{0, T-1}, N^{us}_T, K) = w_{0, T-1} + \delta w^s_{T-1},$$

$$(10) \quad A F_3(N_{0, T}, q_{T-1} N_{0, T-1}, N^{us}_T, K) = w^{us}_{T-1},$$

In equations (9) and (10), the number of employed skilled workers, the amount of human capital investment, and the wages are predetermined in period T. A decline of A, however, affects the employment of young and unskilled middle-age workers. Equations (9) and (10) lead to

$$(11) \quad \Delta N_{0, T} = \frac{-F_1 F_{33} + F_3 F_{13}}{A(F_{11} F_{33} - F_{13}^2)} \Delta A,$$

$$(12) \quad \Delta N^{us}_T = \frac{-F_3 F_{11} + F_1 F_{13}}{A(F_{11} F_{33} - F_{13}^2)} \Delta A.$$

Under the standard assumption of the production function, it holds that  $F_1 > 0$ ,  $F_3 > 0$ ,  $F_{13} > 0$ ,  $F_{11} < 0$ , and  $F_{33} < 0$ . Equations (11) and (12) thus state that a decline of A always has a negative impact on  $N_{0, T}$  and  $N^{us}_T$ . The impact is particularly large when either  $F_1$  or  $F_{13}$  is large or when  $F_{11}$  is small. Under wage rigidity, the productivity slowdown would have a negative impact on the number of employments even in a standard model. However, in our model, the decline of young and unskilled middle-age workers is large because the number of skilled workers remains constant. The model therefore has over-employment of skilled middle-age workers and under-employment of young and unskilled middle-age workers. The decline of employment would have been diversified to skilled workers if their demand for labor was determined by their marginal products of labor. Such diversification is not profitable for the firm because of the hold-up problem,

The productivity slowdown, however, has different impacts on employment of skilled workers after period T+1. This is because the decline of young workers reduces the number of skilled middle-age workers in the next period. Since  $N^s_{T+1} = N_{0, T}$ , the number of skilled middle-age workers definitively declines in period T+1. The impacts on employment of young and unskilled middle-age workers, on the other hand, depend on how large the wages adjust in period T+1. When the wages are still rigid in period T+1, the large decline of young and unskilled middle-age workers would remain. In contrast, when the wages decline in period T+1, the decline of young and unskilled middle-age workers would be moderate.

### 3. The Long-run Analysis

The purpose of this section is to investigate what impacts a permanent decline of the productivity parameter “A” has on employment and total output in the long-run. In the long-run, both wages and physical capital stock are fully adjusted to the equilibrium

level. The consumers, on the other hand, choose their labor supply and the amount of savings so as to maximize their life-time utility.

(i) The Equilibrium in the Long-run

For analytical simplicity, we assume that the young worker, the unskilled middle-age worker, and the quality-adjusted skilled middle-age worker are perfectly substitutable in production. The production function (1) is then written as

$$(13) \quad Y_t = A f(N_{0,t} + q_{t-1} N_t^s + N_t^{us}, K_t).$$

Since the marginal product of the skilled workers in period  $t$  is  $q_t w_t^{us}$  in (13), it holds that  $w_t^* = q_t w_t^{us}$ . Equation (4) thus leads to

$$(14) \quad w_t^s = [\gamma + (1-\gamma) q_{t-1}] w_t^{us}, \quad \text{where } 0 < \gamma < 1.$$

By using the first-order conditions (6), (7), and (8), we obtain

$$(15) \quad A f_1(N_{0,t} + q_{t-1} N_{0,t-1} + N_t^{us}, K_t) = w_t^{us},$$

$$(16) \quad w_{0,t} = w_t^{us} - \delta \gamma w_{t+1}^{us} - [c(q_t) - q_t c'(q_t)],$$

$$(17) \quad \delta \gamma w_{t+1}^{us} = c'(q_t).$$

Under the production function (13), the marginal products of labor are equalized among all types of workers. The wages are, however, equal to the marginal products of labor only for the middle-age unskilled workers. Equation (14) states that the wages of the middle-age skilled workers are less than the marginal products of labor. This reflects the fact that the skilled workers face the hold-up problem when their skill is firm specific. Equation (16), on the other hand, implies that the young workers receive the wages that are less than the marginal products of labor. This is because the young workers can upgrade to the middle-age skilled workers in the next period.

Under the life-time utility maximization that is discussed in the Appendix, each middle-age worker always supplies the fixed amount of labor. The equilibrium condition for the middle-age workers is thus described by  $N_t^s + N_t^{us} = N^*$ , or equivalently

$$(18) \quad N_{0,t-1} + N_t^{us} = N^*.$$

where  $N^*$  is the total number of population of each generation. In contrast, under the life-time utility maximization in the Appendix, each young worker is indifferent between employed and unemployed under the reservation wage. The equilibrium condition for the young workers is  $w_{0,t} = J (w_{t+1}^{us} / w_{t+1}^s)^{\alpha(1+\beta)}$ , or equivalently

$$(19) \quad \ln w_{0,t} = \ln J - \alpha(1+\beta) \ln [\gamma + (1-\gamma) q_t].$$

where  $J \equiv \exp[w(L^*) - w(0)]/L^*$ . The condition (19) states that the equilibrium wage for the young workers is decreasing in human capital investment  $q_t$ . This reflects the fact that the gap between  $w_t^s$  and  $w_t^{us}$  increases as  $q_t$  increases. Larger gap between skilled and unskilled wages, more likely the young accept lower wage.

Finally, the equilibrium capital stock in period  $t$  is equal to the aggregate saving in period  $t-1$ . Since the aggregate income in the middle-age is  $w_t^s N_t^s + w_t^{us} N_t^{us}$ , the saving function in the Appendix implies that the equilibrium capital stock is

$$(20) \quad K_t = [\beta/(1+\beta)][w_{t-1}^s N_{t-1}^s + w_{t-1}^{us} N_{t-1}^{us}], \\ = [\beta/(1+\beta)][(1-\gamma)(q_{t-1}-1) N_{0,t-2} + N^*] w_{t-1}^{us}.$$

The equilibrium capital stock in period  $t$  is increasing in  $q_{t-1}$ ,  $N_{0,t-2}$ , and  $w_{t-1}^{us}$ .

(ii) The dynamic equilibrium and the steady state

Among the equilibrium conditions, four equations (14), (16), (17), and (19) determine four endogenous variables:  $q_t$ ,  $w_{0,t}$ ,  $w_t^{us}$ , and  $w_t^s$ . It is noteworthy that these five equations are independent of the parameter  $A$ . This implies that the equilibrium wages and human capital investment do not change when the productivity shock is the only shock in the economy. In the steady state, all endogenous variables are constant. Assuming that these endogenous variables are at the steady state, the equilibrium wages and human capital investment therefore remain constant even after the productivity shock occurred, that is,  $q_t = q$ ,  $w_{0,t} = w_0$ ,  $w_t^{us} = w^{us}$ ,  $w_t^s = w^s$ .

Under these circumstances, equations (15), (18), and (20) lead to the equilibrium dynamics as follows

$$(21) \quad A f_1(N_{0,t} + (q-1) N_{0,t-1} + N^*, K_t) = w^{us}, \\ (22) \quad K_t = [\beta/(1+\beta)][(1-\gamma)(q-1) N_{0,t-2} + N^*] w^{us}.$$

(iii) The impacts of a permanent productivity slowdown

In the steady state, it holds that

$$(23) \quad A f_1(q N_0 + N^*, K) = w^{us}, \\ (24) \quad K = [\beta/(1+\beta)][(1-\gamma)(q-1) N_0 + N^*] w^{us},$$

where  $N_{0,t} = N_0$  and  $K_t = K$  in the steady state. By taking total differentiation of (23) and (24), we obtain

$$(25) \quad \Delta K = \Psi \Delta N_0, \\ (26) \quad \Delta N_0 = [(f_1/A)/(-f_{11} q N_0 - f_{12} \Psi)] \Delta A,$$

where  $\Psi \equiv [\beta/(1+\beta)](1-\gamma)(q-1) > 0$ .

It is easy to see that  $\Delta w^{us} / \Delta A > 0$  if and only if

$$(27) \quad -f_{11} q N_0 > f_{12} \Psi.$$

The inequality holds if either  $\Psi$  or  $f_{12}$  is small. Assuming the inequality, the following analysis explores the impacts of a permanent productivity slowdown on the employment and total output.

When  $\Delta N_0 / \Delta A > 0$ , the productivity slowdown has a negative impact on the employment of the young workers and reduces the number of skilled middle-age workers. It also reduces the physical stock. Since  $N^{us} = N^* - N_0$ , it in turn increases the number of unskilled middle-age workers.

The total impacts of the productivity slowdown on output is as follows

$$(28) \quad \Delta Y = f \Delta A + Af_1 q \Delta N_0 + Af_2 \Delta K, \\ = f \Delta A + (Af_1 q + Af_2 \Psi) (\Delta N_0 / \Delta A).$$

The above equation indicates that when (27) holds, the negative impacts of the productivity slowdown on output are magnified by the decline of the number of skilled workers  $N_0$  as well as the decline of physical capital stock  $K$ .

It is noteworthy that the hold-up problem made the decline of output more serious. This is because without the hold-up problem, the decline of output would have been mitigated by the decline of the wages. Because the productivity slowdown has no impact on the wages, the decline of output increases the labor cost shares under the productivity slowdown.

## V. Simulation Results

In our model, equations (21) and (22) determine the dynamics of  $N_{0,t}$ . Assuming that the production function takes the Cobb-Dougllass form, that is,

$$(29) \quad Y_t = A (N_{0,t} + q_{t-1} N_t^s + N_t^{us})^{1-\eta} K_t^\eta, \quad (0 < \eta < 1),$$

equations (21) and (22) thus lead to the second-order difference equation as follows

$$(30) \quad N_{0,t} = - (q-1) N_{0,t-1} + \phi(A) (1-\gamma)(q-1) N_{0,t-2} + (\phi(A)-1) N^*,$$

where  $\phi(A) \equiv A^{1/\eta} (w^{us})^{1-1/\eta b}$ .

By setting  $N_{0,t} = N_{0,t-1} = N_{0,t-2} = N_0$ , equation (30) leads to the steady state equilibrium  $N_0$  as follows

$$(31) \quad N_0 = \frac{\phi(A) - 1}{1 + (q-1)[1 - \phi(A)(1-\gamma)]} N^*.$$

Now, suppose that the unexpected permanent slowdown occurred in period 1. For simplicity, we assume that all endogenous variables are at the steady state before period 1. Figures 6(i) – 6(iii) show the dynamic paths of the number of young workers  $N_{0,t}$ , the number of effective workers  $N_{0,t} + q_{t-1} N_t^s + N_t^{us}$ , capital stock  $K_t$ , and total output  $Y_t$  when the productivity parameter  $A$  had a permanent decline by 5%. To make them

comparables, all variables are normalized to be one before the productivity slowdown occurred.

In the figures, the number of young workers shows significant declines. It declines by 2.65% in period 1 and shows up and down convergence to the steady state (Figure 6(i)). The number of unskilled workers, in contrast, rises up dramatically. It increases more than 15% in period 1 and remains at the high level before converging to the steady state (Figure 6(ii)). Because of the mixed impacts on the number of workers, the number of effective workers declines gradually. It, however, shows 1% decline in period 1 and approximately 1.5% decline after a few periods, accompanied by up and down convergence to the steady state (Figure 6(i)).

Compared to the decline in the number of effective workers, the decline of capital stock is rather moderate. It declines by 0.32% in period 2 and converges to the steady state that is approximately 0.42% lower than the original steady state level (Figure 6(iii)). This implies that the decline of capital stock is smaller than the decline of the productivity slowdown. The impacts on the total output are, however, relatively dominated by the declines in the number of effective workers. The total output declines by 1% in period 1 and converges to the steady state that is approximately 1.4% lower than the original steady state level (Figure 6(iii)).

## VI. Conclusion

Many labor markets cannot be adequately described as spot markets. Turnover costs and firm-specific investments drive a wedge between the lowest wage for which a skilled employee will work and the highest wage the employer will pay. They thus generate a rent to continued employment of skilled workers that is the source of the hold-up problem in our model.

When the productivity slowdown is persistent, the hold-up problem causes various losses such as high unemployment rates and low labor participation of young workers in Japan even under the mild shock. In the short-run, the losses are reflected mainly in high unemployment rates of unskilled workers. The losses, however, eventually decrease the number of skilled workers and have serious negative impacts on total output in the long-run.

## Appendix. Consumer behavior

In this appendix, we consider the consumer behavior in our overlapping-generations model. In the overlapping generations model, individuals live for three periods: young, middle-age, and old. They are genetically identical within and across generations. They supply labor in the young and in the middle-age but retire in the old. Let  $C_{0,t}$ ,  $C_{1,t}$  and  $C_{2,t}$  denote the consumption of young, middle-age, and old individuals who are born in period  $t$ . Individuals born in period  $t$  are characterized by the following life-time utility function

$$(1) U_t \equiv \ln C_{0,t} - v(L_{0,t}) + \alpha [\ln C_{1,t} - v(L_{1,t}) + \beta \ln C_{2,t}],$$

where  $v(\cdot)$  are increasing convex functions that denote disutility of labor.

The parameter  $\alpha$  and  $\beta$  are discount factors. For analytical simplicity, the young and the middle-age are assumed to have the same temporal utility function. Following Hansen (1985), we also assume that labor is indivisible so that  $L_{j,t}$  ( $j = 1, 2$ ) takes either  $L^* > 0$  or zero. They earn the competitive market wage  $w_{j,t} L^*$  ( $j = 1, 2$ ) when they work, where  $w_{0,t}$  and  $w_{1,t}$  are the wage rates for the young and for the middle-age respectively. They, however, receive only the small amount of transfer  $W$  when they do not work.

In our model of human capital accumulation, the wage of the middle-age is always higher than that of the young. To the extent that the wage difference is large, only the middle-age therefore has an incentive to save for the old. The middle-age divides the labor income  $w_{1,t+1} L_{1,t}$  between consumption  $C_{1,t}$  and saving  $S_t$ . The returns from the saving enable the cohort to consume in the old. The intertemporal budget constraints are then written as

$$(2) C_{0,t} \leq w_{0,t} L_{0,t} \text{ (no borrowing constraint),}$$

$$(3) C_{1,t} + S_t \leq w_{1,t+1} L_{1,t},$$

$$(4) C_{2,t} = r_{t+1} S_t.$$

where  $r_{t+1}$  is returns for the saving from period  $t+1$  to  $t+2$ .

The budget constraints are based on two assumptions. One is that the young faces the borrowing constraint. Consumer can save or dissave but cannot borrow. However, to the extent that the young does not save, the young's borrowing constraint is always binding. The other is that the wage in the middle-age depends on the labor supply in the young. It holds that

$$(5) w_{1,t+1} \equiv w_{t+1}^s \text{ when } L_{0,t} = L^* > 0 \\ \equiv w_{t+1}^{us} \text{ when } L_{0,t} = 0.$$

where  $w_{t+1}^s$  is the wage rate for skilled workers and  $w_{t+1}^{us}$  is the wage rate for unskilled workers in period  $t+1$ . Since the workers become skilled if and only if they worked when young, the middle-age workers receive  $w_{t+1}^s$  when they supplied labor in the young but receive  $w_{t+1}^{us}$  when they did not supply labor in the young. Because of skill formation,  $w_{t+1}^s$  is always higher than  $w_{t+1}^{us}$ .

The consumer maximizes the life-time utility function (1) subject to the budget constraints (2)-



(4). The first-order conditions lead to:

$$(6) \quad S_t = [\beta/(1+\beta)][w_{1,t+1} L_{1,t}].$$

Equation (6) is the saving function of the middle-age. Because of the log utility, the saving of the middle-age is proportional to the wage income in the middle-age.

Under indivisible labor,  $L_{1,t}$  takes either  $L^* > 0$  or zero. The saving function (6) thus implies that the middle-age workers always supply their labor regardless of skill formation, that is,  $L_{1,t} = L^*$  if

$$(7) \quad v(L^*) - v(0) < (1+\beta) [\ln w_{t+1}^{us} L^* - \ln W]$$

In the following analysis, we assume that this inequality always holds. When  $L_{1,t} = L^*$ , the life-time utility function of the young is

$$(8) \quad U_t^{us} \equiv H - v(0) + \alpha (1+\beta) \ln w_{t+1}^{us}, \quad \text{when } L_{0,t} = L^* > 0,$$

$$(9) \quad U_t^s \equiv H + \ln (w_{0,t} L^*) - v(L^*) + \alpha (1+\beta) \ln w_{t+1}^s, \quad \text{when } L_{0,t} = 0,$$

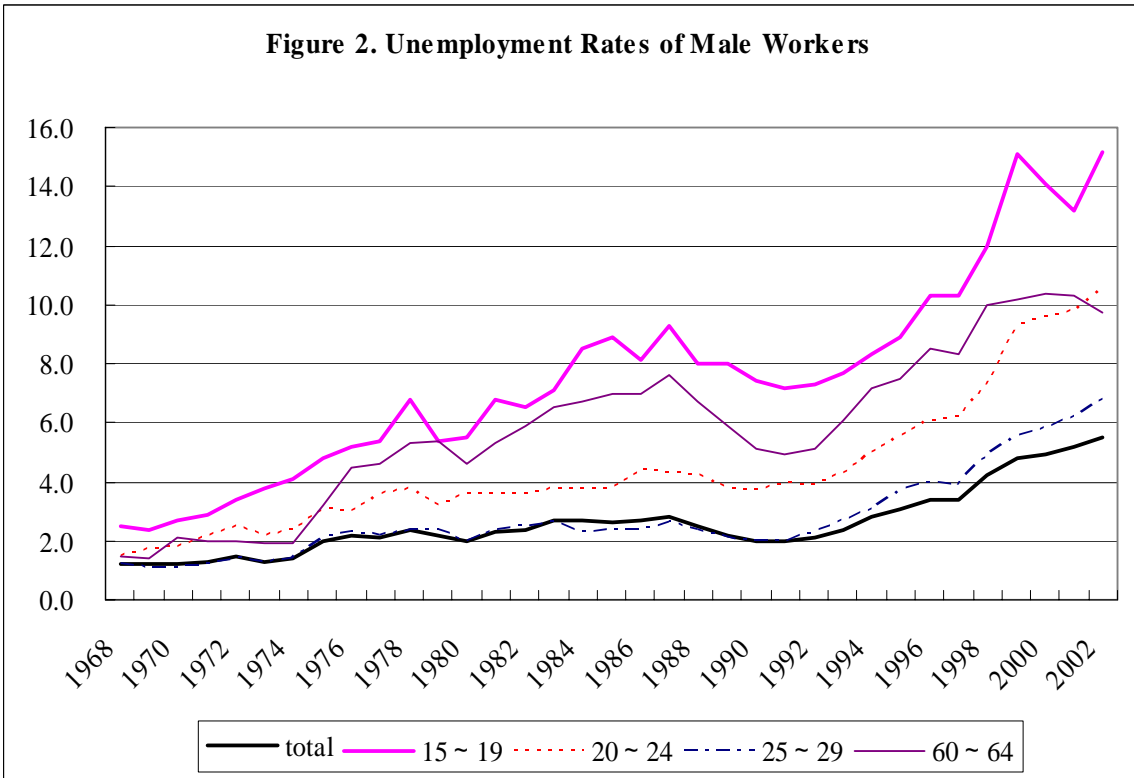
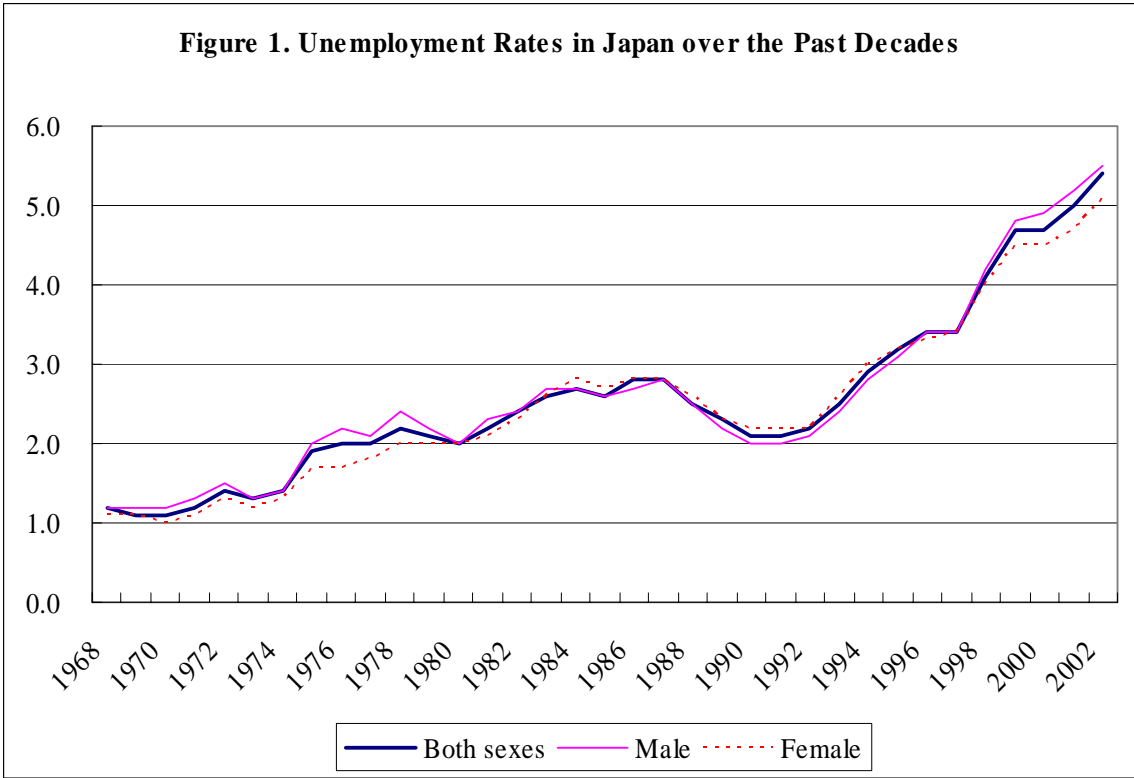
where  $H$  is a common component. Because of indivisibility, labor supply of the young is equal to either  $L^*$  or zero. The condition that  $U_t^{us} = U_t^s$  thus leads to the reservation wage for the young as follows

$$(10) \quad w_{0,t} = J (w_{t+1}^{us} / w_{t+1}^s)^{\alpha(1+\beta)}.$$

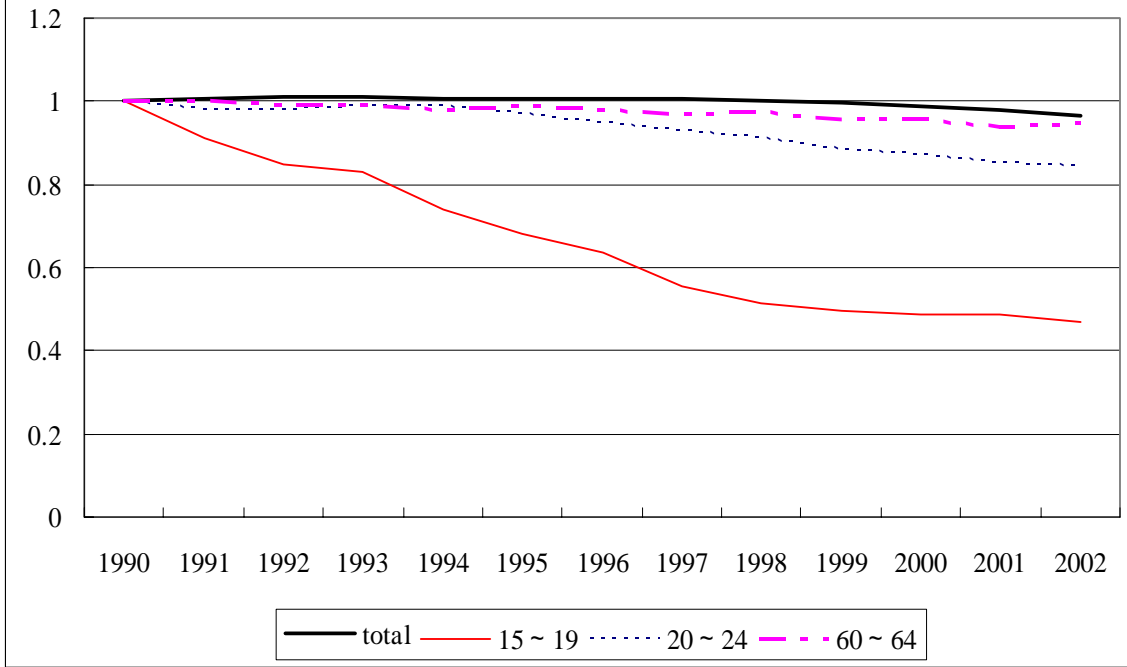
where  $J \equiv \exp[v(L^*) - v(0)] / L^*$ . Under the reservation wage, consumer in the young is indifferent between employed and unemployed. The reservation wage declines as the gap between  $w_{t+1}^s$  and  $w_{t+1}^{us}$  increases. This is because the middle workers become skilled workers if and only if they work when young. As a result, larger gap between skilled and unskilled wages in the middle age makes the young's reservation wage lower.

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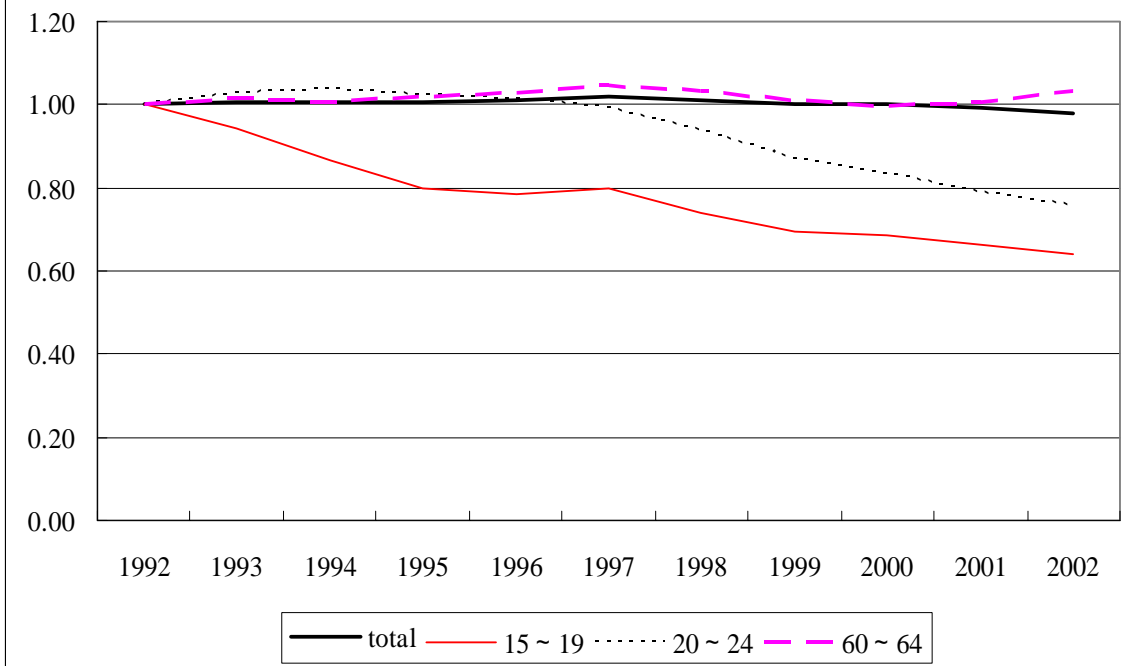
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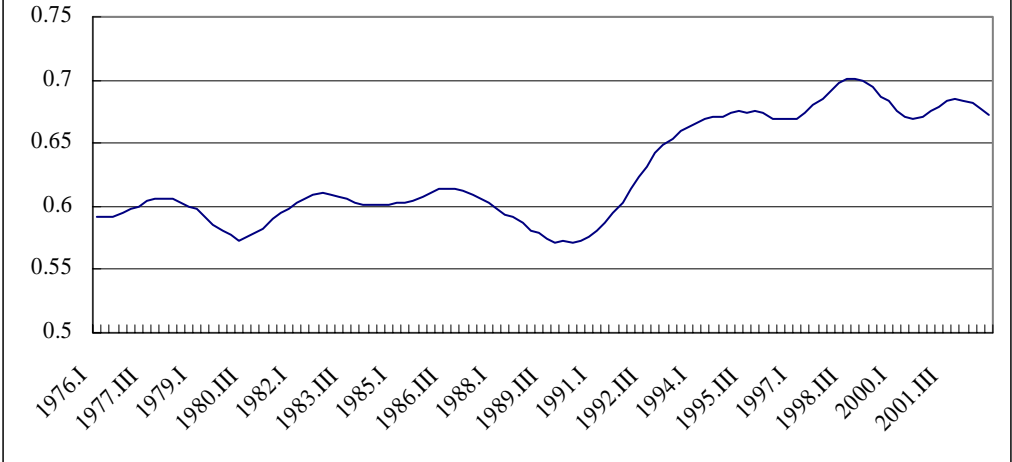
**Figure 3. Labor Participation Ratios (Male)**



**Figure 4. The Number of Male Workers**



**Figure 5. Labor Cost Shares during the Past Decades**



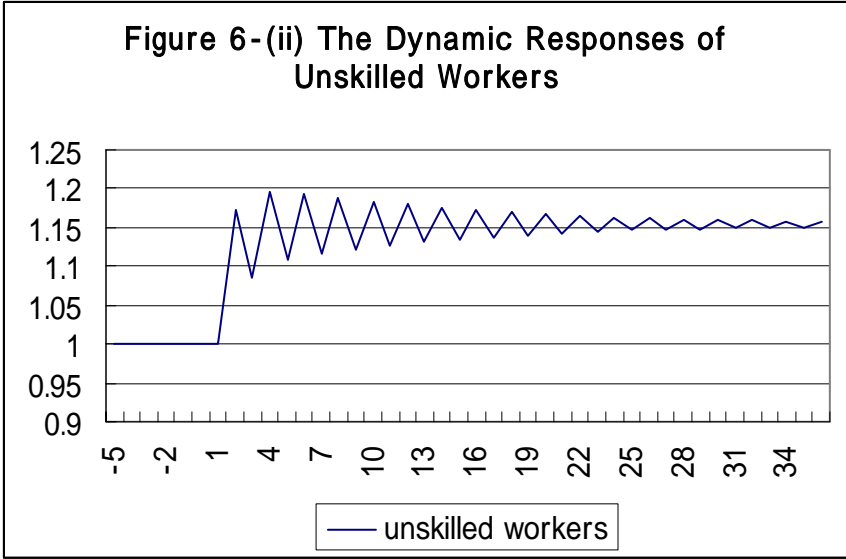
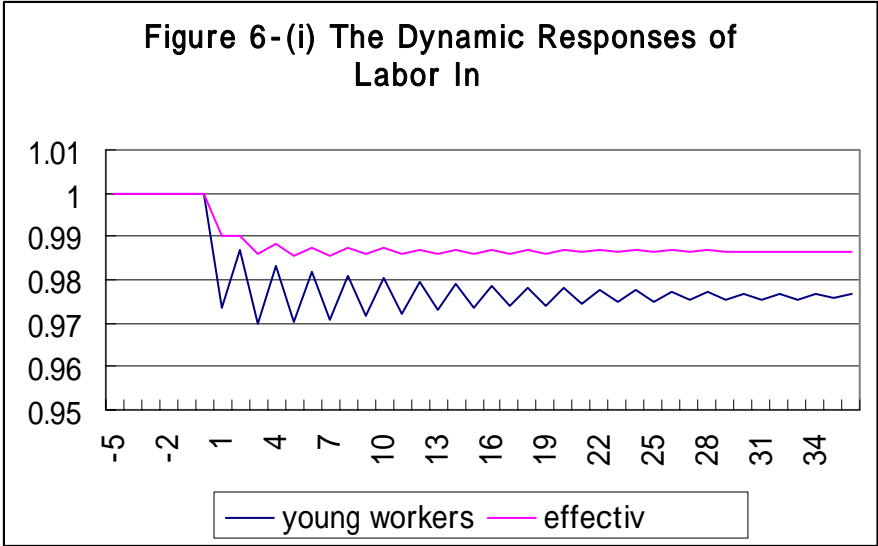


Figure 6-(iii) The Dynamic Responses of Capital Stock and Output

