Property Markets and Public Policy - Spillovers through Collateral Effect*  

*(preliminary and incomplete)*

Nan-Kuang Chen and Charles Ka Yui Leung†

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Abstract

This paper investigates the spillover effect which transmits shocks between commercial and residential real estate markets. Specifically, as a negative productivity shock sets in, entrepreneurs earn lower revenues and may have to liquidate their collateralized assets to pay back their debts. The productivity shock thus spills over to the household sector by way of lowering their wage incomes. The reverse channel of the spillover kicks in when the households are forced to default on their mortgages. This further depresses property prices and thus feedbacks to the production sector by exacerbating collateral damage and triggering more liquidation of productive assets. We then study how land use regulations affect the property prices and the likelihood of triggering feedback effect. We show that both the residential and the commercial property price can be lower given either one of land use regulations and that both types of land uses shrink due to the collateral effect. Moreover, households may be more vulnerable to bad shocks.

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†Nan-Kuang Chen, Department of Economics, National Taiwan University, 21 Shuchow Road, Taipei 10021, TAIWAN; e-mail: nankuang@ccms.ntu.edu.tw. Charles Ka Yui Leung, Department of Economics, Chinese University of Hong Kong, Shatin, Hong Kong; e-mail: charlesl@cuhk.edu.hk.
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1 Introduction

Are the property sector and the macroeconomy related? Recent evidence seems to be affirmative. For instance, studies of the Savings and Loans debacle in the U.S. have the following findings. First, regions where had higher loan-to-price ratios at the beginning experienced a larger decline in house prices and more defaults on bank loans. Second, increasing commercial real estate lending exposes banks to an adverse exogenous shock. Declining economic activity and property prices resulted in housing mortgage defaults. As non-performing loans mounted, many S&Ls bankrupted and lead to even lower property prices. Third, the boom-bust cycle of commercial real estate market is more sensitive to economic fluctuations than that of the residential real estate market (Peek and Rosengren (1992), Browne and Case (1992), and Litan (1992)). Dehesh and Pugh (2000) find that in several Asian countries, the significance of the interdependency between the property sector and the macroeconomy has increased after the breakdown of the Bretton-Woods system. Green (1997) finds that housing leads and other types of investment lag the business cycle in the U.S. Coulson and Kim (2000) show further that, in a mutli-variate VAR system, the residential investment shock is more important in determining the GDP than other shocks. Iacoviello (2000, 2004) also find evidence that in the U.S. and Europe, house prices, monetary policy and the business cycles are all closely related. Gan (2003) provides micro-evidence of the “collateral channel”, that with a sudden drop in collateral value, the capacity for firms to borrow and invest shrink and that further depress their ability to repay.

Since the property sector and the macroeconomy are related, a policy that affects the property sector could have an implication to the macroeconomy. In fact, Edelstein and Paul (2000), Mera (2000a, b) argue that several “urban economic policies,” such as land use regulation, are partially responsible to the formation of the land price bubble in Japan, which leads to a financial crisis later. Adachi and Patel (1999) also argues that the inheritance tax can affect the timing of land development. To formally investigate the relationships among the property sector, the macroeconomy and public policy, this paper builds a dynamic general equilibrium model which highlights the importance of collateral
in determining business loans and housing loans.\footnote{The idea that the amount of loan a firm can borrow will depend on the net worth of itself is far from being new. For instance, Tsiang (1951, p.332-3) remarked that “...It is far from true, however, to say that the supply curve of capital for each individual firm at any particular moment is infinitely elastic... For the borrowing capacity of a firm with a given amount of entrepreneurial capital is certainly not unlimited... One reason that this is so is that lenders will rely primarily on the net value of the business of a going concern for their security, and will be unwilling to lend to any firm more than a given proportion of its equity capital...”} In the model, both residential and commercial properties are able to serve as collateral to borrow either for consumption, houses purchases, or investment. (Throughout this paper, we will use the term “land” and “property” interchangeably). We investigate the spillover effect of a shock from one sector to the other through collateral value, which may also trigger a feedback effect back to the sector which origianates the shock.

Specifically, we construct a three-period model to investigate the mechanism of this spillover effect. The households derive utility from consuming non-durable consumption goods and housing services. At date 0 each household purchases a house with a down payment, a fixed fraction of the house’s value, and borrow the rest from the creditors. \textit{We consider creditors are in foreign countries who are willing to lend at the given interest rate $\tau$.} The entrepreneurs pledge housing property as collateral, borrow an amount up to the collateral’s future value, and invest in a project which yields a stochastic return. Both households and entrepreneurs have to pay back their outstanding debts at date 1.

As a negative shock, such as a productivity shock, a tax policy change, an sharp depreciation in foreign exchange rate, an increase in the oil price for oil-importing countries, sets in, the production sector is the first to be affected.\footnote{Hendershoot and Hu (1983) investigate how capital is allocated between the residential and non-residential uses, and how capital-market constraints would offset the bias in favorable tax treatment.} By construction, the commercial property market is more sensitive than the residential property market. This is because entrepreneurs earn lower revenues and may have to liquidate of their collateralized assets to pay back their debts. As asset liquidation accumulates, asset prices begin to decline, which means entrepreneurs have to liquidate more assets to repay the same amount of debts. The productivity shock spills over to household sector by way of lowering their wage incomes which thus lessens their ability to pay for their mortgages.
Since the amounts of mortgages are pre-determined, the net worth of households shrinks more as the productivity shock hits harder.

The reverse channel of the spillover takes effect when the slowdown of the economic activity takes its toll on the households. When households’ incomes are so low that they are forced to default on their mortgages, more collateralized assets are auctioned off on the market and this further depresses asset prices. This feedbacks to the production sector by exacerbating collateral damage and thus triggers more liquidation of productive assets. The model is able to produce several key features between commercial and residential real estate markets mentioned above.

It should be noticed that we explicitly separate the use of property for commercial and residential purposes. There are reasons to do so. By definition, the residential property does not have a direct production value. While both kinds of property can be used for collateral, the terms of borrowing may be different as some firms only have limited liability. As a result, some banks demand a higher down payment requirement for firms than household for collateralized borrowing. Empirical works also suggest that they might have different cyclical behavior (for instance, see Wheaton (1999) for the U.S. data). On the other hand, the two property markets are highly correlated. Without government intervention, the same piece of land can be used for commercial as well as residential purposes. Clearly, there is to some extent of substitutability between the two types of properties; and both markets are similarly affected by general economic conditions and government regulations. In fact, “mixed land use” is commonly observed in many developing countries (for instance, see Tipple (1993)). Even in developed countries, when there are obvious land use regulation, the prices of different types of land can have significant co-movement. For instance, table 1 shows that the prices of commercial, industrial and residential land in 6 major cities in Japan exhibit pair-wise two-way causality (see figure 1 for a visualization). Ball et al. (1998) show in detail the co-movements of the two markets in the U.K. data. Kan et al. (2004) study a panel of 50 cities in the U.S. and find that each of lagged, contemporary, and forward commercial property prices is positively correlated with the residential property price, and also that the contemporaneous covariance between the two property prices is larger than the lagged covariance. Thus, it seems appropriate to study the joint determination of the two property prices.
Furthermore, in the theoretical part of Kan et al. (2004), they show that, under perfect capital market and with only exogenous technological shock, the prices of commercial and residential property are significantly positively correlated; however, quantitatively the model demands a much stronger output growth-property price correlation than the data to generate realistic correlations (contemporary, lead-lag) between the property prices. In other words, a transmission mechanism is missing. Thus, bridging the two property market through the collateral channel seems to be a natural modelling strategy to us.

As an application of the simple theory developed here, we concentrate on the effects of land use regulations, and how changes in land use regulations affect the likelihood of triggering feedback effect, which can be interpreted as a “housing-related financial crisis”. Empirical works have long established that land use regulations can lead to significant distortions. This paper seems to be an appropriate framework to study land use regulation. Recall that in the baseline model, properties can be used for both commercial or residential purposes and hence the price discrepancy between commercial and residential property price is zero at the equilibrium. Once the land use regulation is imposed, a price differential between the two types of property arises. Furthermore, under certain conditions, we show that both the residential and the commercial property price can be lower upon the imposition of the regulation either for commercial land use or for residential land use. All these are due to the collateral effect. The likelihood of a “crisis” can also be affected by land use regulations. We show that, given a land use regulation, households may be more vulnerable to bad shocks so as to trigger feedback effect.

Recently several papers have studied the implications of collateral in an equilibrium setting. This paper builds on the insights of Hart and Moore (1994, 1998) to incorporate the possibility of endogenous default and asset liquidation, and also the contributions of Kiyotaki and Moore (1997) where the interactions of credit constraints and asset prices generate a powerful transmission mechanism. What is different here is that ours is a two-

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sector model in which the durable asset provides housing services to households and, on the other hand, serves as productive input and collateral for borrowing to entrepreneurs. As in Kiyotaki and Moore, the asset prices plays a key role in triggering spillovers across sectors. This paper is also related to the literature which emphasizes on the role of collateralizable asset and financial crisis, such as Caballero and Krishnamurthy (2001), and Schneider and Tornell (2004). They are typically sophisticated models and this paper attempts to complement their efforts by explicitly distinguishing the commercial and residential property, and also investigating the macroeconomic implications of “urban policies” such as land use regulations.\^4

(to be added)

The rest of the paper is organized as follows. Section 2 outlines the environment of the model and analyzes collateralized financial contracting and discuss conditions for loan default and asset liquidation. Section 3 discusses the spillover across sectors and how it triggers feedback effect. Section 4 discusses the impact of land use regulations on land prices and feedback effect. Section 5 concludes.

2 The Environment

Consider a small open economy with three periods, \( t = 0, 1, \) and 2. There are two types of goods: a durable asset and a non-durable consumption good. The aggregate supply of the durable asset is assumed to be fixed at \( H \). The durable asset can be considered as real estate or land, which serves either as a residential house, collateral for loans, or a productive input. There are three types of agents: households, entrepreneurs and landlords. The population of each group is normalized to be unity.

Each household is initially endowed with one unit of house and an amount of outstanding debt, and also one unit of labor each period which is supplied inelastically to the firms. The household consumes housing services \((h_0^H, h_1^H)\) and a non-durable consumption good \(c\):

\[
U(h_0^H, h_1^H, c) = E_0 \left\{ v(h_0^H, h_1^H) + (1 - \alpha)\delta c_2 \right\},
\]

\(\^4\)Schneider and Tornell (2004) also envision a credit game in their infinite horizon model, while this paper focuses on the case of perfectly competitive markets.
where \( v(h_0^H, h_1^H) = \alpha \ln(h_0^H + \beta h_1^H), \) \( 0 < \delta < 1 \). We assume that the housing stock is indivisible. Since the households are not to adjust the size of their houses in period 1, \( h_1^H = h_0^H \) if no default and \( h_1^H = 0 \) if default. Thus,

\[
v(h_0^H, h_1^H) = \begin{cases} 
\alpha \ln(h_0^H) & \text{if default in period 1} \\
\alpha \ln(h_0^H) + \xi & \text{if no default in period 1}
\end{cases}
\]

where \( \xi = \alpha \ln(1 + \beta) > 0, \alpha \in (0, 1) \) and \( E_0 \) denotes the expectation operator conditional on time 0 information.

At date 0 the household trades in the asset market, repays mortgage debt, and purchases a new house. The budget constraint for the household at date 0 is

\[
\psi q_0 h_0^H \leq [q_0 h_{-1}^H - d_{-1}^H] - s_0, \tag{2}
\]

where \( \psi, 0 < \psi < 1 \), denotes the percentage of the current value of the house which must be paid out of the buyer’s pocket and \( q_0 \) is the date 0 house price. Equation (2) says that the down payment \( \psi q_0 h_0^H \) for \( h_0^H \) units of housing is financed by the household’s net worth minus his holding of risk-free debt (deposit) \( s_0 \). The household’s initial net worth comes from the sale of the old house \( q_0 h_{-1}^H \) net of his initial debt \( d_{-1}^H \).

We take the down payment ratio \( \psi \) as given and considers it as a policy tool. We also assume that the households can borrow from creditors at most the amount \( (1 - \psi)q_0 h_0^H \), but not the down payment \( \psi q_0 h_0^H \). This means that \( s_0 \) must be non-negative.\(^5\)

At date 1, the household receives his labor income \( w_1 \) and savings income \( r^H s_0 \), and then repays the outstanding mortgage debt \((1 - \psi)r^H q_0 h_0^H\) when the household does not default:

\[
(1 - \psi)r^H q_0 h_0^H + s_1 \leq w_1 + r^H s_0, \tag{3}
\]

where the gross interest rate \( r^H \) is taken as given by the households and creditors. Households also cannot borrow at date 1, and thus \( s_1 \) is also nonnegative.

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\(^5\)Apart from being a policy tool, the fraction of down payment \( \psi \) needs not to be a constant. It can be justified as a device to mitigate adverse selection problem. In a more general setup, the ratio may depend on interest rates and the size of the down payment. See Stein (1995) for details.
The household defaults if (3) is reversed. In this case, the creditor seizes the house and sell it off, and thus $h^H_1 = 0$. If the household does not default, the household stays at the same house, $h^H_1 = h^H_0$. At date 2 the household receives the deterministic wage and interest income, and then consumes:

$$c_2 \leq w_2 + r^H s_1.$$  \hspace{1cm} (4)

Since $s_1$ acts as a residual whose value is contingent on the realization of the random variable $z$ (which will be specified below), households’ choice variables are only $\{h^H_0, s_0\}$.

At date 0, entrepreneurs are each endowed with a project which combines land and labor to yield a stochastic return at date 1,

$$y_1(z) = z \left( h^E_0 \right)^\beta \left( n_0 \right)^{1-\beta},$$

where $h^E_0$ and $n_0$ are the durable asset and labor respectively employed by the entrepreneur, and $0 < \beta < 1$. The random variable $z$ represents the borrower’s productivity. The realization of $z$ at date 1 is publicly observable and independent of borrowers. On the other hand, the technology between date 1 and date 2 is non-stochastic. The output $y_2(z) = \left[ h^E_1(z) \right]^\beta \left[ n_1(z) \right]^{1-\beta}$ depends on the date 1 realization of $z$. The random variable $z$ takes on values in the set $[0, Z]$ according to the probability distribution function $F(\cdot)$, with density $f(\cdot)$. We assume that projects are not reversible but can be divisible at date-1.

Entrepreneurs are risk neutral and consume only at the end of date 2. Each entrepreneur is endowed with a piece of property $h^E_{-1}$ by which he pledges this durable asset as collateral and borrows to purchase more durable asset for investment. The budget constraint at date 0 is

$$q_0 h^E_0 + d^E_{-1} \leq q_0 h^E_{-1} + d^E_0,$$  \hspace{1cm} (5)

where $q_0$ is date 0 price of the durable asset and $h^E_0$ is the durable asset used for production, $d^E_{-1}$ is the entrepreneur’s initial debt, and $d^E_0$ is the date 0 borrowing. We will explain how $d^E_0$ is determined later.

When it comes to date 1, the entrepreneur repays debt, plans for reinvestment, and faces the following budget constraint

$$q_1(z) \left[ h^E_1(z) - h^E_0 \right] \leq \left[ y_1(z) - w_1(z) n_0 \right] - r^E d^E_0,$$  \hspace{1cm} (6)
We assume that the creditors are competitive, so that borrowers have all the bargaining power when renegotiating their debt with their creditors, which implies that creditors earn an expected zero profit.

Landlords, who are the “deep pockets” in this model, hold the rest of the asset $h^l_t$ which is not occupied by households or employed by the entrepreneurs. Their utility function is given by $U^L(h^0_L, h^1_L) = G(h^0_L) + G(h^1_L)$, where the $G(\bullet)$ satisfies the usual neoclassical assumptions, $G'(0) = \infty$, $G'(\infty) = 0$, and $G'(x) > 0$, $G''(x) < 0$ for all $x$. Therefore, the per period user cost of the asset for alternative uses is given by $G'(h^L_t)$.

We assume that the investment technology is specific to each entrepreneur: if the entrepreneur who has invested at date-0 abandon his project before date-2, the project produces nothing and is left only with its liquidation value; and the entrepreneur is free to walk away from the project with date-1 project proceeds. Foreseeing the possibility that entrepreneurs can threaten to walk away from production at date-1, and also that the asset becomes valueless at the end of date-2, creditors do not lend more than the date-1 expected value of the collateral discounted by the (gross) loan rate of interest:

$$d^E_0 \leq \frac{E(q_1)h^E_0}{r^E}, \tag{7}$$

where $E(q_1)$ is the expected date-1 asset price. The borrowing constraint implicitly assumes that entrepreneurs can not divert fixed asset for their own benefit after the initial investment has been in place. Since the collateral becomes valueless at date-2, there is no way to enforce borrowers to repay at date-2. Thus, borrowers must repay their debts at date-1.

### 2.1 Liquidation and Default at Date 1

We first take as given the date 0 determined financial contract and solve for the decision rules of creditors at date 1. We then study the date 0 lending decision of creditors.

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6See Hart and Moore (1994, 1998) and Kiyotaki and Moore (1997) for more details in the analysis of renegotiation and debt repayment. Empirically, this is consistent with the literature of corporate finance, stating that a weak contractual enforcement is closely associated with credit market constraint. See, for example, La Porta et al. (1997), whereby weak enforcement of shareholder rights explains a great deal of the variation in how firms are funded and owned across countries.
Define \( z^* \) to be such that

\[
y_1(z^*) = w_1(z^*) n_0 + r^E d_0^E.
\] (8)

Recall the entrepreneur’s date 1 budget constraint (6). If \( z \) is realized to be lower than \( z^* \), then the entrepreneur’s investment return falls below his debt, \( y_1(z) - w_1(z) n_0 - r^E d_0^E < 0 \), and thus the entrepreneur has to liquidate a certain fraction of asset to repay debt. For \( z < z^* \), the quantity of asset liquidation is

\[
\Delta h^E(z) = h_0^E - h_1^E(z),
\]

which depresses date 1 house price to

\[
q_1(z) = G'(h_1^L(z)),
\]

where \( h_1^L(z) = h_0^L + \Delta h^E(z) > h_0^L \).

We next denote \( z^{**} \) to be such that \( h_1^E(z^{**}) = 0 \) and \( h_1^H = h_0^H \). Using (6), \( z^{**} \) satisfies

\[
-q_1(z^{**}) h_0^E = y_1(z^{**}) - w_1(z^{**}) n_0 - r^E d_0^E,
\] (9)

where

\[
q_1(z^{**}) = G'(h_0^L + h_0^E).
\] (10)

As for the household sector, by (3), households default when \( a_1(z) < (1 - \psi) r^H q_0 h_0^H \), where \( a_1(z) = w_1(z) + r^H s_0 \). Denote \( \varpi \) to be such that

\[
w_1(\varpi) = (1 - \psi) r^H q_0 h_0^H - r^H s_0.
\] (11)

In the following, we concentrate on the case where

\[
\varpi < z^*,
\] (12)

which says that a productivity shock affects the investment sector harder than does the household sector. We will show that the condition (12) can be satisfied under reasonable parameter range.\(^7\)

\(^7\)When \( \varpi > z^* \), the household sector is more sensitive to exogenous shocks than production sector, which is less likely according to the empirical evidence mentioned in the introduction.
2.2 Maximization Problems of Entrepreneurs and Households

Given the critical value \( \{z^*, z^{**}, \pi\} \) and prices \( \{r^E, r^H, w_1(z), w_2(z), q_0, q_1(z)\} \), the date 0 optimization problem of a representative entrepreneur is given by

\[
\text{Max. } E \left[ y_2(z) - w_2(z) n_1(z) \right],
\]

subject to the entrepreneur’s budget constraints, (5) and (6), borrowing constraint (7), and the creditor’s participation constraint on loans to production sector,

\[
[1 - F(z^*)] r^E d^E_0 + \int_0^{z^*} \left\{ [y_1(z) - w_1(z) n_0] + (h^E_0 - h^E_1(z)) q_1(z) \right\} dF(z) \geq \tau d^E_0, \tag{13}
\]

On the left-hand side, the first term is project returns accrued to the creditor when there is no liquidation, and the second term is the revenue from the project as well as from liquidating the asset when the productivity falls below the critical value \( z^* \). The right-hand side is the opportunity cost of the creditor.

Now turn to the maximization problem of households. Given the above critical value and prices, the representative household’s expected utility function, (1), can be written as

\[
U(h^H_0, h^H_1, c_2) = \alpha \left[ \ln h^H_0 + \xi (1 - F(\pi)) \right] + (1 - \alpha) \delta \left\{ \int_0^1 c_2(z) dF(z) + \int_\pi^\infty w_2(z) dF(z) \right\}, \tag{14}
\]

where \( F(\pi) \) is the probability of default. The constraints include (2), (3), (4), and the creditor’s participation constraint on loans to the households,

\[
[1 - F(\pi)] r^H d^H_0 + \int_0^\pi \left\{ w_1(z) + q_1(z) h^H_0 \right\} dF(z) \geq \tau d^H_0,
\]

where \( d^H_0 = (1 - \psi) q_0 h^H_0 \).

To close the model, the housing market and labor market clearing conditions respectively, for \( t = 0, 1 \), are given by

\[
h^H_t + h^E_t + h^L_t = H, \tag{15}
\]

\[
n_t = 1. \tag{16}
\]
2.3 Solving the Maximization Problems and Critical Values

Using (5) and (7), it is straightforward to derive the entrepreneur’s date 0 asset demand

\[ h_E^0 = \frac{a_E^0}{q_0 - E(q_1)/r} \]  

(17)

where \( a_E^0 \) is the net asset value at time 0, \( a_E^0 \equiv q_0 h_{E-1} - d_{E-1} \). We assume that the initial debt \( d_{E-1} \) is large enough so that the asset demand is increasing in date 0 price, \( dh_0^E/dq_0 > 0 \), and also that \( d_{E-1} \) is not too large so that the initial net worth is positive:

\[ E(q_1) h_{E-1}^E/r < d_{E-1} < h_{E-1} q_0 \]

(18)

Together with labor market clearing condition, the date 1 wage rate is given by

\[ w_1(z) = z (1 - \beta) (h_0^E)^\beta. \]

(19)

Then, the critical value \( z^* \) can be solved by using (8), (17), and (19):

\[ z^* = \frac{E(q_1)}{\beta} (h_0^E)^{1-\beta}. \]

(20)

The other critical value \( z^{**} \), at which all investment assets are liquidated, can be solved by using (9), (10), (17), and (19):

\[ z^{**} = \frac{[E(q_1) G^r(h_0^L + h_0^E)]}{\beta} (h_0^E)^{1-\beta}, \]

(21)

which is clearly lower than \( z^* \). Note that \( z^* \) are \( z^{**} \) are both functions of \( q_0 \) and \( E(q_1) \) and parameters. The value \( z^{**} \) may also be a function of \( \tau \), depending on whether households’ demand for houses is constrained or not.

We next solve the maximization problem of households. Some remarks deserve emphasis. First, in the case with \( s_0 > 0 \), the demand for housing

\[ h_0^H = \frac{\alpha}{(1 - \alpha) \delta H q_0 [1 - F(\tau)]^{-1}}, \]

is decreasing in date 0 house price and is independent of down payment ratio; while in the case where \( s_0 = 0 \), where households are credit constrained, the demand for housing

\[ h_0^H = \frac{a_0^H}{\psi q_0}, \]
is increasing in date 0 house price and decreasing in down payment ratio, where $a_0^H \equiv q_0 h_{-1}^H - d_{-1}^H$. We show in the appendix that if the household’s initial debt is not too large, then we can concentrate on credit-constrained case $s_0 = 0$. Specifically, we assume that

$$q_0 h_{-1}^H - \frac{\alpha \psi}{(1 - \alpha) \delta r^H} < d_{-1}^H < q_0 h_{-1}^H,$$

where the former inequality says that the household is credit-constrained, and the latter ensures the initial net worth to be positive. Given that the entrepreneur is credit-constrained, the critical value $z$ at which households default is given by

$$z = \frac{r^H (1 - \psi)}{\psi (1 - \beta) (h_0^E)^\beta}.$$

Note that $\partial z/\partial \psi < 0$ when households are credit-constrained. This means that if mortgage debt is larger given an amount of net worth (that is, $\psi$ is lower), then the likelihood of default is higher. This is consistent with the evidence that mortgages with higher leverages are more likely to default (Peek and Rosengren (1992), Browne and Case (1992), and Litan (1992)).

## 3 Spillover and Feedback Effect

There exists a two-way spillover effect between the land uses of the household sector and that of the entrepreneur sector. First, when there is an adverse productivity shock, an entrepreneur may be forced to liquidate asset to cover their debts. This in turn shrinks date 1 investment, reducing the employment of fixed asset in investment sector and the next period’s output. This initial impact spills over to the household sector because they receive a lower wage rate, $w_1(z)$, which reduces the households’ date 1 wealth $a_1(z)$, and thus their consumption and housing demand.

Second, the feedback effect kicks in if households default, i.e., $z < \bar{z}$, and their houses are auctioned off. This drives the house prices even lower and feedbacks to the entrepreneurs’ net worth so that they are forced to liquidate more asset to repay debts than when households do not default.

Figure 2 depicts the case in which the spillover takes place only from production sector to household sector, at which $\bar{z} < z^{**} < z^*$. The horizontal axis indicates the level of $z$
and the vertical axis records the total quantity of asset to be liquidated. We denote the locus of entrepreneurs’ asset liquidation \( \triangle h(z) \). As long as \( z > \overline{z} \), only entrepreneurs’ assets will be liquidated. In the range \( (z^{**}, z^*) \) where households are solvent, by (6), the locus \( \triangle h(z) \) can be expressed as

\[
q_1(z) \triangle h(z) = r^E d_0^E - [y_1(z) - w_1(z)],
\]

which says the shortfall of debt repayment is supplemented by liquidating assets. Together with (7) and (19), we have

\[
\triangle h(z) = \frac{1}{q_1(z)} \left[ E(q_1) h_0^E - z \beta (h_0^E)^\beta \right],
\]

where \( q_1(z) = G'(H - h_0^E - h_0^H + \triangle h(z)) \). Note that \( E(q_1) h_0^E - z \beta (h_0^E)^\beta \) is the net debt of the entrepreneur. When the asset price drops lower, an entrepreneur has to liquidate more asset to cover his debt.

The slope of the locus \( \triangle h(z) \) is thus given by\(^8\)

\[
\frac{d \triangle h(z)}{dz} = - \left[ \frac{\beta (h_0^E)^\beta}{q_1(z)} + \triangle h(z) \frac{dq_1(z)}{dz} \right] < 0,
\]

where \( dq_1(z)/dz > 0 \). When \( z \) drops to \( z^{**} \), all assets that had been employed for investment are liquidated, i.e., \( \triangle h(z) = h_0^E \) for \( z < z^{**} \). Thus, the asset price drops to \( q_1(z^{**}) = G'(H - h_0^H) \).

Finally when \( z \) drops below \( \overline{z} \), households default, and the total quantity of liquidation rises to \( h_0^H + h_0^E \). The house price falls to its lowest level \( q_1(\overline{z}) = G'(H) \).

[Insert Fig. 2 here]

Figure 3a shows the case when \( z^{**} < \overline{z} < z^* \). In this case, the two-way spillover triggers more liquidation of entrepreneurs’ productive assets. To see this, suppose now \( z \) drops slightly below \( \overline{z} \), and thus households default. We consider what will happen in the neighborhood of \( \overline{z} \). The immediate effect is to raise the total quantity of liquidation

\(^8\)The second order derivative of \( \triangle h^E(z) \) with respect to \( z \) depends on the magnitude of \( d^2 q_1(z) / dz^2 \). The slope of \( \triangle h^E(z) \) is increasing in \( z \) if \( d^2 q_1(z) / dz^2 \) is negative or not too large when positive.
up to $\triangle h(z) + h_0^H$. Since more asset is auctioned off in the market, the asset price drops even further. But the liquidation will not stop there. It triggers a feedback effect such that entrepreneurs have to liquidate more asset than when households do not default. Thus, the feedback effect effectively shifts the locus $\triangle h(z)$ upward, depicted as the bold line, $\triangle h'(z)$, in Figure 2. Similar to (25), the equilibrium locus of liquidation taken into account of the feedback effect, $\triangle h'(z)$, given $z^{**} < z < \overline{z}$, satisfies

$$\triangle h'(z) = \frac{1}{q_1(z)} \left[ E(q_1) h_0^E - z \beta (h_0^E)^\beta \right],$$

where the house price is now $q_1(z) = G'(H - h_0^E + \triangle h'(z))$.

Figure 2a clearly shows that the locus of liquidation not only shifts upward, but also hit the upper bound at a higher realization of $z$, denoted as $\bar{z}$, which satisfies $z^{**} < \bar{z} < \overline{z} < z^{*}$ and

$$q_1(\bar{z}) \triangle h'(\bar{z}) = E(q_1) h_0^E - \bar{z} \beta (h_0^E)^\beta,$$

when $q_1(\bar{z}) = G'(H - h_0^E + \triangle h'(\bar{z}))$ and $\triangle h'(\bar{z}) = h_0^E$. That is,

$$\bar{z} = \frac{1}{\beta} \left[ E(q_1) - G'(H) \right] (h_0^E)^{1-\beta}.$$ (27)

It can be immediately checked that the intercept as well as slope (in absolute value) of $\triangle h'(z)$ are higher than $\triangle h(z)$, i.e.,

$$\left| \frac{d \triangle h'(z)}{dz} \right| > \left| \frac{d \triangle h(z)}{dz} \right|.$$ (28)

This says the feedback effect raises the sensitivity of asset liquidation with respect to a change in the realization of $z$, more likely to trigger a widespread crisis.

Figure 3b depicts the locus of house prices $q_1(z)$ against the amount of asset liquidation when feedback effect is present. The immediate effect of the feedback effect around the neighborhood of $\overline{z}$ is that the house price drops to $q_1(\overline{z})$. The house price then continues to drop, at a faster rate, as liquidation accumulates, and ends up at an even lower level $q_1(\overline{z}) = G'(H)$.

[Insert Fig. 3a, b here]
Together with (12), the condition that feedback effect takes place, that is \( z^{**} < \bar{z} < z^* \), is given by

\[
\frac{1}{\beta} E(q_1) - G^*(H - h_0^H) h_0^E < \frac{1}{1 - \beta} (1 - \psi) q_0 h_0^H < \frac{1}{\beta} E(q_1) h_0^E.
\] (28)

Given (28), for the condition that the production sector is more sensitive to shocks than the household sector, i.e., \( \bar{z} < z^* \), the amount of borrowing adjusted for the income share by entrepreneurs is larger than that by households. The following proposition summarizes the discussion:

**Proposition 1** (a) If \( \bar{z} < z^{**} < z^* \), there will be a one-way spillover from production sector to household sector when \( z \) falls below \( z^* \),

(b) If \( z^{**} < \bar{z} < z^* \), the spillover from production sector to household sector feedbacks to cause more asset liquidation, driving the property price to decline even more when \( z \) falls below \( \bar{z} \).

Furthermore, we can translate the relationship between property price and the extent asset liquidation into that between property price and output. As depicted in Figure 4, there is a discontinuity in the output-property price relationship. Our model may then potentially generate some empirical implications. Now consider that we repeat this artificial economy for infinitely many times, and hence obtain observations for all possible output-property price combinations for different realizations of \( z \). (Alternatively, we can imagine that there is a continuum of identical economies and the distribution of shock is i.i.d. Thus, by the Law of Large Numbers, we would have all possible realizations of the productivity shock \( z \).\(^9\)) Figure 4 displays all these combinations graphically, compare and contrast to the case with and without two-way spillover. Clearly, the two-way spillover introduces a discontinuity in the output-property price relationship. This may shed light on the large empirical literature which attempts to investigate the relationship between property price and the “economic fundamentals”. Taking the model literally, it implies that it is inappropriate to use conventional, linear VAR type model to investigate the relationship between the aggregate output and property price. Researchers should employ

\(^9\)For instance, see Uhlig (1996).
more flexible specification such as allowing for structural breaks or regime switching in their empirical investigations.\textsuperscript{10}

[Insert Fig. 4 here]

\section*{3.1 An Example}

In this sub-section, we present a simple example for better understanding of the model. To do so, we first assume that \( F(z) \) is uniformly distributed and the productive function is given by \( G(H - x) = \theta \ln(H - x) \). Furthermore, we concentrate on the case where \( s_0 = 0 \), that is, households are credit constrained; and also \( z^{**} < \underline{z} < \overline{z} < z^{*} \). We can then obtain closed form solutions for the variables and summarize the date-1 asset liquidation and asset prices for each range of \( z \),

\begin{align*}
q_1(z) &= \begin{cases} 
G'(H - h_0^H - h_0^E) = \frac{\theta}{H - h_0^H - h_0^E} & z > z^* \\
G'(H - h_0^H - h_0^E + \Delta h'(z)) = \frac{\theta - D(z)}{H - h_0^H - h_0^E} & \underline{z} < z < z^* \\
G'(H) = \frac{\theta}{H} & z < \underline{z}
\end{cases}, \\
\Delta h'(z) &= \begin{cases} 
0 & z > z^*, \\
\frac{(H - h_0^H - h_0^E)}{\theta - D(z)} D(z) & \underline{z} < z < z^*, \\
\frac{(H - h_0^E)}{\theta - D(z)} D(z) & \overline{z} < z < \overline{z}, \\
h_0^E & z < \underline{z},
\end{cases},
\end{align*}

where \( D(z) \equiv (h_0^E)^\beta \left[ E(q_1) (h_0^E)^{1-\beta} - z^\beta \right] \) is the net amount of debt.

\textsuperscript{10}See, for example, Girouard and Blondal (2001). A related line of literature studies the observed instability of the empirical relation between oil prices and output. Studies find that oil shocks affect short-run economic activity by temporarily disrupting purchases of consumer durables and investment goods and by triggering an allocative effect between sectors, which generates a nonlinear relation between oil prices and GDP (Hooker (1996), Davis and Haltiwanger (2001), and Balke et al. (1999)).
4 Land Use Regulation

In the previous sections, we have built a simple equilibrium framework for property price spillover. Now we are ready to extend it to include more realistic features. In particular, we attempt to use the model as a lens to study the land use regulation issue in the presence of the property price spillover effect. Regulatory restrictions on land uses for either residential or commercial and industrial developments are widely imposed, either intended to protection of neighborhood environment or to reserve for agricultural purposes. There are at least two kinds of land regulations, the regulation for commercial development and that for residential housing use, respectively.

4.1 Regulation for Commercial Land Use

Suppose the land used for commercial development is restricted to be

$$h_0^E \leq \bar{h}. \quad (29)$$

Recall (22), we concentrate on the case in which households are initially credit constrained. Intuitively, since the total amount of land for commercial purposes is limited, it will drive up the price of commercial land. On the other hand, since the total amount of land available for households and landlords now increases, the price of residential land decreases. In other words, a price differential between commercial and residential land price now emerges. The existence of credit constraints on collateralizable asset, however, introduces an additional effect. As in the literature in which the credit constraint for durable assets binds, the demand for commercial land by firms is upward sloping.\(^{11}\) Now, with less property, the firms have less collateral and other things being equal, they will borrow less. This will affect how much they can produce. As a result, the commercial property price can decrease. Theoretically, there can be various possibilities. To fix the idea, we adopt uniformly distributed \(F(z)\) and the functional form \(G(H-x) = \theta \ln(H-x)\) from the earlier example. Equipped with these assumptions, it is possible to prove the following proposition and figure 5 simply provides a visualization of this case (the details are in the appendix):

\(^{11}\text{See, for example, Stein (1995) and Kiyotaki and Moore (1997).}\)
Proposition 2: If \( f(z) \) is a uniform distribution and the utility function for landlord is in log form, \( G(H - x) = \theta \ln(H - x) \), then the following results can be proved: (a) The demand for property by entrepreneurs is increasing in property price,

\[
\frac{dh^E_0}{dq_0} > 0.
\]  

(b) The demand for property by households and landlords jointly can be positive or negative, depending on whether the value of initial debt is too large,

\[
\frac{d(h^L_0 + h^H_0)}{dq_0} \geq 0 \iff d^{-1} - \psi \theta \leq 0.
\]

[Insert Fig.5a,b here]

Figure 5 illustrates the impact on land uses and property prices when land commercial land use (29) is imposed. Figure 5a depicts the case when the joint demand \( h^L_0 + h^H_0 \) is increasing in property price. Here the residential house price turns out to be higher than the commercial property price and also higher than the equilibrium property price in the absence of regulation, which is clearly against the evidence. Figure 5b, where the joint demand by landlords and households is decreasing in property price, shows that the residential and the commercial property price are both lower given the regulation for commercial land use and that the commercial property price is still higher than the residential property price. In this case, both land uses for commercial and residential properties shrink due to the collateral effect: the initial decline of asset price due to the regulation for commercial land use lessens the net worth of both entrepreneurs and households and thus lowers the demand for commercial and residential property.

To further explain the impact of land use regulation, the corresponding critical values given the restriction (29) are derived:

\[
z^* = \frac{E(q_1)}{\beta} \left( \frac{q_0}{a_0^H} \right)^{1-\beta}, \quad z^{**} = \frac{[E(q_1) - G'(h^L_0 + h)]}{\beta} \left( \frac{q_0}{a_0^H} \right)^{1-\beta}, \quad z = \frac{r^H}{\psi(1-\beta)} \frac{a_0^H}{(h)}^{\frac{1}{\beta}}, \quad \tilde{z} = \frac{1}{\beta} \left[ E(q_1) - G'(H) \right] \left( \frac{q_0}{a_0^H} \right)^{1-\beta},
\]

where \( a_0^H \equiv q_0 h^H_1 - d^{-1}_H \). Since the property prices are lower, the critical values \( z^* \), \( z^{**} \), and \( \tilde{z} \) will decrease, while \( \tau \) may increase or decrease. Note that the feedback effect is more.
likely when \( z \) becomes higher. The condition for \( z \) to be higher is that when the extent of land use regulation is more restrictive relative to the drop in residential property price. This occurs when the absolute value of the price elasticity of joint demand by landlords and households is larger.

The intuition is that given the commercial land use regulation, the wage incomes of households decline due to a smaller scale of production. On the other hand, the total quantity of land available for households and landlords now increases, and the price of residential land should decline. If the price of residential land, however, does not fall enough, due to a higher price elasticity of joint demand by landlords and households, the total expenditure on residential housing will now increase relative to households’ wage incomes, thereby raising households’ outstanding debt that can be supported by their incomes. Therefore, households will be more vulnerable to bad shocks.

The following proposition provides a characterization of the regulation on commercial land use.

**Proposition 3** Suppose that the commercial land use is regulated, and that the household’s initial debt is not too large, then

1. a price differential between commercial and residential land price emerges: the residential and the commercial property price are both lower than the equilibrium property price level in the absence of regulation, and the commercial property price is higher than the residential property price;

2. both land uses for commercial and residential properties shrink due to collateral effect; and

3. the feedback effect becomes stronger (i.e., \( z \) is higher) when the absolute value of the price elasticity of joint demand by landlords and households is larger.
4.2 Regulation for Residential Land Use

Suppose now the land used for residence is restricted to be

\[ h_0^H \leq \bar{h}, \]  

(32)

As a result, the demand for residential land use is lower than its equilibrium level. We concentrate on the case that joint demand by landlords and households is decreasing in property price, then the graphic result of residential land use regulation is the same as in Figure 5b: the residential and the commercial property price are both lower, the commercial property price is higher than the residential property price, and also both land uses for commercial and residential properties shrink.

Furthermore, we can compute the corresponding critical values given the restriction

\[ z^* = \frac{E(q_1)}{\beta} (h_0^E)^{1-\beta}, \quad z^{**} = \frac{[E(q_1) - G'(h_0^L + h_0^E)]}{\beta} (h_0^E)^{1-\beta}, \]

\[ z = \frac{r^H (1-\psi) q_0 \bar{h}}{(1-\beta) (h_0^E)^{\beta}}, \quad \bar{z} = \frac{1}{\beta} \left[ E(q_1) - G'(H) \right] (\bar{h})^{1-\beta}. \]

The same mechanism applies: the initial decline of asset price due to the regulation for residential land use lessens the net worth of entrepreneurs and eventually brings down both land uses for commercial and residential properties through collateral effect. Similar to the above case, the critical values \( z^*, z^{**}, \) and \( \bar{z} \) decrease, while \( \bar{z} \) may increase or decrease. Note that \( \bar{z} \) becomes higher when the drop in entrepreneurs’ land use \( h_0^E \) due to collateral effect is relatively larger than the lower housing expenditure \( q_0 \bar{h} \). Thus, the feedback effect becomes more likely (1) when the absolute value of the price elasticity of joint demand by landlords and households is larger, which is the same as the case when the regulation for commercial land use is imposed; or (2) when the (positive) price elasticity of demand by entrepreneurs is larger.

The intuition for the latter condition is that when the price elasticity of entrepreneurs’ demand is larger, a small decline in property price leads to a larger contraction in land use for production. This contributes to a larger cut in households’ wage incomes and raises households’ debt-income ratio, making them more vulnerable to bad shocks.

**Proposition 4** Suppose that the residential land use is regulated, and that the household’s initial debt is not too large, then the feedback effect becomes stronger (i.e., \( \bar{z} \) is higher) when
either the absolute value of price elasticity of joint demand by landlords and households is larger, or the (positive) price elasticity of demand by entrepreneurs is larger.

5 Concluding Remarks

This paper attempts to contribute to both the “urban economics” literature as well as the “financial crisis” literature. Specifically, we investigate the spillover effect which transmits shocks between commercial and residential real estate markets and how it exacerbates an exogenous shock to the economy. The mechanism that links these two markets is the role of collateral value: both residential and commercial properties serve as collateral to borrow either for consumption, purchasing houses, or investment. We show that a change in collateral value may trigger a feedback effect to the sector which originates the shock, and this exacerbates the extent of liquidation of productive assets. The model thus conforms with several key features between commercial and residential real estate markets.

We then study land use regulations on residential and non-residential uses. We examine how land use regulations affect the house prices and the likelihood of triggering feedback effect. We show that both land uses for commercial and residential properties may shrink due to the collateral effect, and that both the residential and the commercial property price can be lower given either one of the land use regulations. In this case, households are more vulnerable to bad shocks and thus it becomes more likely to trigger feedback effect.

One may object that this discontinuity is due to the artifact that all households are homogenous, therefore they either do not default, and they default together. To tackle this shortcoming, we can in fact replace the assumption by a more moderate one. Suppose that there is an idiosyncratic unemployment risk, and thus a lower realization of productivity shock means a higher unemployment rate (or a larger fraction of the population losing their jobs). In this case, we can still show that there exist a certain threshold above which no household will sell their properties. When the shock drops below a certain
level, some households will start to liquidate. This pushes more pressure to the firms who are partially liquidating their properties, and resulting in a lower equilibrium property price. As the shock goes down further below a certain level, all households liquidate and hence the original output-property price relationship would be restored. This means that there are now two “kinks” on the output-property price curve and the curve is hence not differentiable. Again, applying linear VAR would obtain a biased estimation. The bottom line is that, to investigate how property price is related to the economic fundamental, it is advised to use more flexible methodology which allows for possible non-linear relationship.

(to be added)
References


[35] La Porta


A Appendix

A.1 Households’ maximization problem

Given prices \( \{r^E, r^H, w_1(z), w_2(z), q_0, q_1(z)\} \), the representative household maximizes his expected utility, (14), subject to (2), (3), and (4). Then we have

\[
L = \alpha \ln (h_0^H) + \delta [1 - F(\bar{z})] + (1 - \alpha)\delta^2 \int \{w_2(z) + r^H [w_1(z) + r^H s_0 - (1 - \psi)r^H q_0h_0^H] \} dF(z) + \lambda_1 \{a_0^H - \psi q_0h_0^H - s_0\} + \lambda_2 s_0,
\]

where \( a_0^H \equiv q_0h_{-1}^H - d_{-1} \) is the household’s net worth at the beginning of date 0. The first order conditions are

\[
\frac{\alpha}{h_0^H} - (1 - \alpha)(1 - \psi)\delta^2 r^H q_0 [1 - F(\bar{z})] - \psi q_0\lambda_1 = 0, \tag{33}
\]

\[
(1 - \alpha)\delta^2 r^H [1 - F(\bar{z})] - \lambda_1 + \lambda_2 \leq 0, \quad \text{if } s_0 > 0, \tag{34}
\]

\[
s_0 \geq 0, \quad \text{if } \lambda_2 > 0. \tag{35}
\]

We consider the following two cases

Case 1 \( \lambda_2 > 0 \)

This means that \( s_0 = 0 \). The date 0 housing demand is constrained by (2),

\[
h_0^H = \frac{a_0^H}{\psi q_0} = \frac{1}{\psi} \left[ h_{-1}^H - \frac{d_{-1}}{q_0} \right].
\]

By (33), we can solve for \( \lambda_1 \). Plugging \( h_0^H \) into (11), the critical value \( \bar{z} \) satisfies

\[
w_1(\bar{z}) = \frac{1 - \psi}{\psi} r^H a_0^H, \tag{36}
\]

where \( a_0^H = q_0h_{-1}^H - d_{-1} \).

Finally, using (3) and (4), we can solve for the date 1 saving and consumption

\[
s_1(z) = w_1(z) - \frac{1 - \psi}{\psi} r^H a_0^H > 0, \text{ if } z > \bar{z},
\]

\[
= 0, \text{ if } z \leq \bar{z},
\]

\[
c_2(z) = w_2 + r^H w_1(z) - \frac{1 - \psi}{\psi} r^H a_0^H > 0 \text{ if } z > \bar{z},
\]

\[
= w_2, \text{ if } z \leq \bar{z}.
\]

To solve for \( \bar{z} \), note that, imposing \( n_0 = 1 \), \( w_1(z) = z(1 - \beta)(h_0^E)^\beta \) and \( h_0^E = q_0h_{-1}^E/(q_0 - E(q_1)/r^E) \). Using (36), we get

\[
\bar{z} = \frac{r^H (1 - \psi) a_0^H}{\psi (1 - \beta)} (h_0^E)^{-\beta} = \frac{r^H (1 - \psi) a_0^H}{\psi (1 - \beta) (h_0^E)^{\beta}}.
\]
Case 2 $\lambda_2 = 0$

Then, we have $s_0 > 0$. By (34), $\lambda_1 = (1 - \alpha)\delta^2 r H^2 [1 - F(\bar{z})]$. Plugging into (33), we have

$$h_0^H = \frac{\alpha}{(1 - \alpha) \delta^2 r H^2 q_0} [1 - F(\bar{z})]^{-1}.$$ 

Then, by (2), we can solve for the date 0 saving

$$s_0 = a_0^H - \frac{\alpha}{(1 - \alpha) \delta^2 r H^2} [1 - F(\bar{z})]^{-1}.$$ 

Using (3) and (4), date 1 saving and consumption are

$$(s_0) = w_1(z) + r^H a_0^H - \frac{\alpha}{(1 - \alpha) \delta^2 r H} [1 - F(\bar{z})]^{-1} > 0, \text{ if } z > \bar{z},$$

$$= 0, \text{ if } z \leq \bar{z},$$

$$(c_2(z) = w_2 + r^H w_1(z) + r^H a_0^H - \frac{\alpha}{(1 - \alpha) \delta^2 r H} [1 - F(\bar{z})]^{-1}, \text{ if } z > \bar{z},$$

$$= w_2, \text{ if } z \leq \bar{z}.$$ 

Plug $h_0^H$ and $s_0$ into (11), the critical value $\bar{z}$ satisfies

$$w_1(\bar{z}) = \frac{\alpha}{(1 - \alpha) \delta^2 r H} [1 - F(\bar{z})]^{-1} - r^H a_0^H.$$

Note that since

$$s_0 = a_0^H - \frac{\alpha}{(1 - \alpha) \delta^2 r H^2} [1 - F(\bar{z})]^{-1}$$

$$< a_0^H - \frac{\alpha \psi}{(1 - \alpha) \delta^2 r H^2},$$

given assumption (22), we need only to concentrate on the case $s_0 = 0$.

A.2 Slopes of Demand Curves for Property (30), (31)

To derive the slopes of the demand curve for different kinds of economic agents, recall that:

$$h_0^E = \frac{a_0^E}{q_0 - E(q_1)/r^E}, h_0^H = \frac{a_0^H}{\psi q_0}, \text{ where } a_0^E \equiv q_0 h^E_{-1} - d^E_{-1}, a_0^H \equiv q_0 h^H_{-1} - d^H_{-1},$$ 

Slopes of the housing demands of entrepreneurs and households can be easily derived and they are

$$\frac{dh_0^E}{dq_0} = \frac{-h^E_{-1} E(q_1) / r^E + d^E_{-1}}{q_0 - E(q_1) / r^E} > 0, \quad \frac{dh_0^H}{dq_0} = \frac{d^H_{-1}}{\psi (q_0)^2} > 0.$$ 

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respectively, where the assumption (18) has been made used of. Clearly,
\[
\frac{d^2 h^0 H}{dq_0^2} = \frac{-2d_{-1}}{\psi(q_0)^3} < 0.
\]
In other words, the entrepreneur’s demand is concave in property price.

Now we turn to the demand of land/property by the landlords. Suppose that \( G(H - x) = \theta \ln(H - x) \). Then we have \( q_t(z) = G'(h^L_t) = \theta / h^L_t \). The demand for housing asset by landlords is then
\[
h^L_0 = \frac{\theta}{q_0}.
\]
(40)
The slope of joint demand by landlords and households is
\[
\frac{d(h^L_0 + h^H_0)}{dq_0} = \frac{dh^L_0}{dq_0} + \frac{dh^H_0}{dq_0} = \frac{-\theta}{(q_0)^2} + \frac{q_0 h^H - (q_0 h^H - d^H)}{\psi(q_0)^2}
\]
\[
= \frac{d^H - \psi \theta}{\psi(q_0)^2} \leq 0,
\]
(41)
depending on whether \( d^H - \psi \theta \leq 0 \) (the denominator is always positive), which gives rise to (31). It means that the derivative of the demand is positive when the household initial debt is large enough. Clearly,
\[
\frac{d^2 (h^L_0 + h^H_0)}{dq_0^2} = \frac{-2(d^H - \psi \theta)}{\psi(q_0)^3} = \left(\frac{-2}{q_0}\right) \frac{d(h^L_0 + h^H_0)}{dq_0},
\]
which is of the opposite sign as the \( \frac{d(h^L_0 + h^H_0)}{dq_0} \). It means that the joint demand curve for the household and landlord is concave.

A.3 Conditions for Commercial Property Price Higher Than Residential Property Price

Recall that under the commercial land use regulation, \( h^E_0 = \bar{h} \). We are interesting in the case where the regulation is binding, i.e., \( \bar{h} \) is smaller than the equilibrium level but larger than the endowment \( \bar{h} > h^E_{-1} \). By (17), we have
\[
q^E_0 = \frac{\bar{h}E(q_1) / r^E - d^E_{-1}}{\bar{h} - h^E_{-1}},
\]
Since the denominator is positive, the numerator must also be positive.

Now by (??), (??), and (15), we have
\[
q^H_0 = \frac{\theta \psi - d^H_{-1}}{(H - \bar{h}) - h^H_{-1}}.
\]
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Notice that by assuming the joint demand for property by landlord and household being negative in price, we have effectively assume that
\[ \theta \psi - d_{-1}^H > 0, \]
and we simply assume that
\[ (H - \bar{h}) - h_{-1}^H > 0. \]
Now, to ensure that \( q_0^E > q_0^H \), we need
\[
\frac{q_0^E}{\bar{h}E(q_1)/r^E - d_{-1}^E} > \frac{q_0^H}{H - H_{-1}},
\]
\[
\Leftrightarrow \left( \bar{h}E(q_1)/r^E - d_{-1}^E \right) \left( (H - \bar{h}) - h_{-1}^H \right) > \left( \theta \psi - d_{-1}^H \right) \left( \bar{h} - h_{-1}^E \right),
\]
\[
\Leftrightarrow (H - \bar{h}) - h_{-1}^H > (\theta \psi - d_{-1}^H) \left( \bar{h} - h_{-1}^E \right) / \bar{h}E(q_1)/r^E - d_{-1}^E, \]
\[
\Leftrightarrow H > \bar{h} + h_{-1}^H + d_{1}^E,
\]
where \( d_{1}^E = \left( \theta \psi - d_{-1}^H \right) \left( \bar{h} - h_{-1}^E \right) / \bar{h}E(q_1)/r^E - d_{-1}^E \).

### A.4 Condition for Entrepreneurs Demand Flatter Than Joint Demand by Households and Landlords at the Unregulated Equilibrium
Recall from (??), (??), (??), (??) that the demand for property by different types of agents, and the corresponding slopes are
\[
h_0^E = \frac{q_0h_{-1}^E - d_{-1}^E}{q_0 - E(q_1)/r^E}, \quad h_0^H = \frac{q_0h_{-1}^H - d_{-1}^H}{\psi q_0} + \frac{\theta}{q_0},
\]
\[
dh_0^E = \frac{-h_{-1}^E E(q_1)/r^E + d_{-1}^E}{q_0 - E(q_1)/r^E}, \quad dh_0^H = \frac{\theta h_{-1}^H}{\psi q_0} = \frac{d-h_{-1}^E + d_{-1}^H}{\psi (q_0)^2}. \]

Notice that \( \frac{dh_0^E}{dq_0} > 0 \). Thus, \( \left| \frac{dh_0^E}{dq_0} \right| = \frac{dh_0^H}{dq_0} \), and \( \left| \frac{dh_0^E}{dq_0} \right| > \left| \frac{d(h_{-1}^E + h_{-1}^H)}{dq_0} \right| \) if \( \frac{dh_0^E}{dq_0} > \frac{d(h_{-1}^E + h_{-1}^H)}{dq_0} > 0 \), or \( \frac{dh_0^E}{dq_0} > -\frac{d(h_{-1}^E + h_{-1}^H)}{dq_0} \) and \( \frac{d(h_{-1}^E + h_{-1}^H)}{dq_0} < 0 \).

If \( \frac{dh_0^E}{dq_0} > 0 \), or \( (d_{-1}^H - \psi \theta) > 0 \),
\[
\Leftrightarrow -\frac{h_{-1}^E E(q_1)/r^E + d_{-1}^E}{q_0 - E(q_1)/r^E} d_{-1}^H - \psi \theta \]
\[
\Leftrightarrow \left[ \psi d_{-1}^E q_0 - (d_{-1}^H - \psi \theta) \right] q_0 - \left[ \psi \theta + (q_0)^2 h_{-1}^E - d_{-1}^H \right] E(q_1)/r^E > 0, \]
\[
\Leftrightarrow \left[ \psi d_{-1}^E q_0 - (d_{-1}^H - \psi \theta) \right] q_0 - \left[ \psi \theta + (q_0)^2 h_{-1}^E - d_{-1}^H \right] E(q_1)/r^E > 0, \]
31
as by \((18), (q_0 - E(q_1)/r^E) > 0\).

If \(\frac{d(h_H^E + h_L^E)}{dq_0} < 0\), or \((d_{-1}^H - \psi \theta) < 0\),

\[
\begin{align*}
\frac{db^E}{dq_0} & > \frac{d(h_H^E + h_L^E)}{dq_0} \\
\iff & \frac{-h_E^E(q_0)/r^E + d_{-1}^E}{\psi(q_0)^2} > 0 \\
\iff & \frac{\psi d_{-1}^E(q_0 - \psi \theta)}{\psi(q_0)^2(q_0 - E(q_1)/r^E)} > 0 \\
\iff & \left[q_0 d_{-1}^E(q_0 - (\psi \theta - d_{-1}^H)) - (d_{-1}^H + \psi (q_0)^2 h_{-1}^E - \psi \theta) E(q_1)/r^E \right] > 0,
\end{align*}
\]

as by \((18), (q_0 - E(q_1)/r^E) > 0\).

A.5 Existence and Uniqueness of the Unregulated Equilibrium

In this section, we want to explore the existence and uniqueness of the unregulated equilibrium of the model. By \((??), (??),\) we know that the total demand of property is

\[
h_t^E + h_t^I + h_L^I = \frac{q_0 - E(q_1)/r^E}{q_0 - E(q_1)/r^E} + \frac{a_0^E}{q_0} + \frac{H}{q_0}
\]

\[
= \psi q_0 q_0 (q_0 h_{-1}^E - d_{-1}^E) + (q_0 - E(q_1)/r^E) \left(q_0 h_{-1}^H - d_{-1}^H + \psi \theta\right)
\]

\[
= \frac{(\psi h_{-1}^E + h_{-1}^H)(q_0)^2 + (-\psi d_{-1}^E - d_{-1}^H - E(q_1) h_{-1}^H/r^E + \psi \theta) q_0 + (d_{-1}^H - \psi \theta) E(q_1)/r^E}{\psi q_0 (q_0 - E(q_1)/r^E)},
\]

which is equated to the total supply \(H\) at equilibrium, by \((15).\) It implies that

\[
(q_0 h_{-1}^E + h_{-1}^H - \psi H)(q_0)^2 + (-\psi d_{-1}^E - d_{-1}^H - E(q_1) h_{-1}^H/r^E + \psi \theta + \psi E(q_1) H/r^E) q_0 + (d_{-1}^H - \psi \theta) E(q_1)/r^E = 0.
\]

Notice that it is a quadratic equation in \(q_0\). To have real roots, we need \(D \geq 0\), where \(D\) is

\[
D = \left(\psi d_{-1}^E - d_{-1}^H - E(q_1) h_{-1}^H/r^E + \psi \theta + \psi E(q_1) H/r^E\right)^2
\]

\[
-4 \left(\psi h_{-1}^E + h_{-1}^H - \psi H\right) \left(d_{-1}^H - \psi \theta\right) E(q_1)/r^E,
\]

and

\[
q_0 = \frac{-\left(\psi d_{-1}^E - d_{-1}^H - E(q_1) h_{-1}^H/r^E + \psi \theta + \psi E(q_1) H/r^E\right) \pm \sqrt{D}}{2 \left(\psi h_{-1}^E + h_{-1}^H - \psi H\right)}.
\]

Notice that in the main text we assume \((d_{-1}^H - \psi \theta) < 0\), where the joint demand by landlords and households is decreasing in property price. Also, since \(0 < \psi < 1\), and it
must be that \( (h_{-1}^E + h_{-1}^H) < H \), it seems reasonable to assume that \( (\psi h_{-1}^E + h_{-1}^H - \psi H) < 0 \). It implies that
\[
-4 \left( \psi h_{-1}^E + h_{-1}^H - \psi H \right) \left( d_{-1}^H - \psi \theta \right) E (q_1) / r^E < 0.
\]

Thus,
\[
\sqrt{D} < \left| -\left( -\psi d_{-1}^E - d_{-1}^H - E (q_1) h_{-1}^H / r^E + \psi \theta + \psi E (q_1) H / r^E \right) \right|.
\]

In fact, if \( (\psi h_{-1}^E + h_{-1}^H - \psi H) < 0 \), we can re-write the formula as
\[
q_0 = \frac{-\psi d_{-1}^E - d_{-1}^H - E (q_1) h_{-1}^H / r^E + \psi \theta + \psi E (q_1) H / r^E}{2 \left| (\psi h_{-1}^E + h_{-1}^H - \psi H) \right|} \pm \sqrt{D}.
\]

Therefore, it is possible to have two positive solutions for \( q_0 \). The figures A1-A3 show the cases of (1) unique equilibrium, (2) multiple equilibrium, and (3) non-existence of equilibrium.

[insert figure A1-A3 here]
Table 1: Two-way Causality of land prices in 6 major cities of Japan

(Sample: 1959 2002; Lags: 3)

<table>
<thead>
<tr>
<th>Null Hypothesis:</th>
<th>Obs</th>
<th>F-Statistic</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
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<td>41</td>
<td>18.5377</td>
<td>2.7E-07</td>
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<tr>
<td>COMM6 does not Granger Cause IND6</td>
<td></td>
<td>10.6094</td>
<td>4.5E-05</td>
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<tr>
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<td>4.92302</td>
<td>0.00602</td>
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<tr>
<td>COMM6 does not Granger Cause RES6</td>
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<td>24.2828</td>
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</table>

Source: Japan Statistical Yearbook 2003, Statistical Research and Training Institute, http://www.stat.go.jp/english/data/nenkan/1431-17.htm. The indexes adopt March 2000 as their base period and are computed semianually (in March and September). Land prices incorporated in the index calculation are surveyed in 223 cities, classified according to use of the land (commercial, residential and industrial) and according to grade of the land (high, medium and low). The indexes are calculated as the simple average of the price relatives of the land prices surveyed.

Figure 1: Land Price of 6 major cities in Japan, 1959-2002.
Figure 2  One-way Spillover from Production Sector to Household Sector

\[ \Delta h(z) \]

\[ h_0^E \]

\[ \Delta h(z) \]

\[ 0 \]

\[ \bar{z} \]

\[ z^{**} \]

\[ z^* \]

\[ 1 \]
Figure 3a  Asset Liquidation When Feedback Effect is Present

Figure 3b  Asset Prices When Feedback Effect is Present
Figure 4  The Implied Relationship between the Property Prices and Output with Feedback Effect

Note: The bold line represents the relationship between the date-1 house prices and outputs given a specific shock when feedback effect is present.
Figure 5a  Demand Schedules for Properties When the Joint Demand by Landlords and Households Is Increasing in Property Price
(E: entrepreneurs; H: households; L: landlords)

Figure 5b  Demand Schedules for Properties When the Joint Demand by Landlords and Households Is Decreasing in Property Price
Figure A3  Negative Equilibrium Holding or Price

Negative equilibrium holding

Negative equilibrium price

$h^E$  

$h^L + h^H$  

$D (E)$  

$D (H+L)$