The co-movement of inflation and the real growth of output

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Abstract

In order to find out whether there really are co-movements between inflation and the real growth of output for the post-WWII period, this paper adopts FIML Markov-Switching Model to solve the equation of Phillips Curve and the equation of Okun's Law together.

The findings of this paper are as follows: inflation is procyclical in movement with the real growth of output during the Korean War and the two Oil Shock periods; and there is relatively little evidence of co-movement between inflation and real growth of output with the exceptions of the Korean War and the two Oil Shock periods.

Keyword: FIML Markov-Switching Model, Hamilton filter, Phillips Curve, Okun's Law, co-movement, Korean War, Oil Shock, inflation

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1. Introduction

The relationship between inflation and the real growth of output has been assumed to be positive and stable. Mills (1946), Mitchell (1951), Kuznets (1930) found a strong but not perfect conformity in the movements of prices and output both for short term contraction and longer term cycles. For the post-WWII period Lucas (1972) observed that inflation and unemployment rate are negatively correlated, thus indicating a stable Phillips Curve. However, Cooley and Ohanian (1991) argued that with the exceptions of the two world wars, particularly during the period of the Great Depression (1928-1946) and part of the late 19th century, there was relatively little evidence of procyclical prices over the last century and a half including the post-WWII period. Cooley and Ohanian's (1991) findings are quite different from Mankiw (1989) who claimed that in the absence of identifiable real shocks such as OPEC oil price changes, inflation tends to rise during booms and fall during recessions. Cooley and Ohanian's findings are also different from Den Haan (2000) who found that the co-movement between output and prices is positive in the short run and negative in the long run using a VAR model.

Thus the purpose of this paper is to find out whether there really are co-movements between inflation and real growth of output for the post-WWII period.

To establish the relationship between inflation and the real growth of output, we adopt FIML Markov-Switching Model by Yoon (2004) and Spagnolo, F., Psaradakis, Z. and Sola M. (2005) to solve the equation of Phillips Curve and the equation of Okun's Law together. The merit of the FIML Markov-Switching Model is that we can deal with the problem of simultaneous equations based on the Hamilton filter (1989).

The findings of this paper are as follows: inflation is procyclical in movement with the real growth of output during the Korean War and the two Oil Shock periods for the post-WWII period; and there was relatively little evidence of co-movement between inflation and the real growth of output with the exceptions of the Korean War and the two Oil Shock periods.

The paper has been divided in 4 sections. Section 2 presents FIML Markov-Switching Model. Section 3 summarizes the empirical results. Section 4 concludes this paper.

2. FIML Markov-Switching Model

In order to get the consistent estimation of the parameters of the Markov-switching model in the simultaneous equations, we consider the following FIML Markov-Switching Model.

$$YBs_t + Z\Gamma s_t = U_{St} \quad , \qquad\qquad U_{St} \sim i.i.d.N(0, \Sigma_{St} \otimes I_T) \qquad (1)$$

where

$$Y = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1M} \\ Y_{21} & Y_{22} & \cdots & Y_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{T1} & Y_{T2} & \cdots & Y_{TM} \end{bmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{pmatrix}, \quad Bs_t = \begin{bmatrix} \beta_{11,S1t} & \beta_{12,S2t} & \cdots & \beta_{1M,SMt} \\ \beta_{21,S1t} & \beta_{22,S2t} & \cdots & \beta_{2M,SMt} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{M1,S1t} & \beta_{M2,S2t} & \cdots & \beta_{MM,SMt} \end{bmatrix}$$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1K} \\ Z_{21} & Z_{22} & \cdots & Z_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{T1} & Z_{T2} & \cdots & Z_{TK} \end{bmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_T \end{pmatrix}, \quad \Gamma s_t = \begin{bmatrix} \gamma_{11,S1t} & \gamma_{12,S2t} & \cdots & \gamma_{1M,SMt} \\ \gamma_{21,S1t} & \gamma_{22,S2t} & \cdots & \gamma_{2M,SMt} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{K1,S1t} & \gamma_{K2,S2t} & \cdots & \gamma_{KM,SMt} \end{bmatrix}$$

$$U_{St} = \begin{bmatrix} u_{11,S1t} & u_{12,S2t} & \cdots & u_{1M,SMt} \\ u_{21,S1t} & u_{22,S2t} & \cdots & u_{2M,SMt} \\ \vdots & \vdots & \ddots & \vdots \\ u_{T1,S1t} & u_{T2,S2t} & \cdots & u_{TM,SMt} \end{bmatrix} = (u_{S1t} \quad u_{S2t} \quad \cdots \quad u_{SMt})$$

$$E(U_{St}^{'} U_{St}) = E\left(\begin{pmatrix} u_{S1t} \\ u_{S2t} \\ \vdots \\ u_{SMt} \end{pmatrix} (u_{S1t} \quad u_{S2t} \quad \cdots \quad u_{SMt}) \right)$$

$$= \begin{pmatrix} \sigma_{S1t,S1t} I_T & \sigma_{S1t,S2t} I_T & \cdots & \sigma_{S1t,SMt} I_T \\ \sigma_{S2t,S1t} I_T & \sigma_{S2t,S2t} I_T & \cdots & \sigma_{S2t,SMt} I_T \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{SMt,S1t} I_T & \sigma_{SMt,S2t} I_T & \cdots & \sigma_{SMt,SMt} I_T \end{pmatrix} = \Sigma_{St} \otimes I_T$$

Y is the T x M matrix of jointly dependent variables, B_{St} is an M x M matrix and nonsingular. Z is the T x K matrix of predetermined variables, Γs_t is K x M matrix and *rank*(Z) = K. Us_t is T x M matrix of the structural disturbances of the system. Thus, the model has M equations and T observations. The structural errors are assumed as a nonsingular M-variate normal (Gaussian) distribution. σ is the covariance of the error terms. Σ_{St} is an M x M matrix and positive definite and no restrictions are placed on it. It is assumed that all equations satisfy the rank condition for identification. Also if lagged endogenous variables are included as predetermined variables, the system is assumed to be stable. An orthogonality assumption, E(Z'U_{St})=0, between the predetermined variables and structural errors is required and, we assume the presence of contemporaneous correlation but no intertemporal correlation in (1). If we assume that the single Markov-switching variable S_t has an N-state, first-order Markov process, then we can write the transition probability matrix in the following way:

$$p = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1N} \\ p_{21} & p_{22} & \cdots & p_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ p_{N1} & p_{N2} & \cdots & p_{NN} \end{pmatrix}$$

where $p_{ij} = \Pr(S_t = j \mid S_{t-1} = i)$ with $\sum_{j=1}^{N} p_{ij} = 1$ for all i

If our model involves only two unobserved two-state first order Markov-switching variables such as S_{1t} and S_{2t}, The dynamics of Markov-switching variables can be represented by a single Markov-switching variable S_t in the following manner:

$S_t = 1$ if $S_{1t} = 0$ and $S_{2t} = 0$
$S_t = 2$ if $S_{1t} = 0$ and $S_{2t} = 1$
$S_t = 3$ if $S_{1t} = 1$ and $S_{2t} = 0$
$S_t = 4$ if $S_{1t} = 1$ and $S_{2t} = 1$

with $p_{ij} = \Pr(S_t = j \mid S_{t-1} = i)$, $\sum_{j=1}^{4} p_{ij} = 1$

To derive the FIML Markov-Switching Model in the simultaneous equations, we can obtain $\Pr(S_t = j \mid \psi_t)$ by applying a Hamilton filter (1989) as follows:

Step 1 :

At the beginning of the t^th iteration, $\Pr(S_{t-1} = i \mid \psi_{t-1})$, $i = 0,1,\cdots,N$ is given. And, we calculate

$$\Pr(S_t = j \mid \psi_{t-1}) = \sum_{i=1}^{N} \Pr(S_{t-1} = i, S_t = j \mid \psi_{t-1})$$

$$= \sum_{i=1}^{N} \Pr(S_t = j \mid S_{t-1} = i) \Pr(S_{t-1} = i \mid \psi_{t-1})$$

where $\Pr(S_t = j \mid S_{t-1} = i)$, $i = 0,1,\cdots,N$, $j = 0,1,\cdots,N$ are the transition probabilities.

Step 2 :

Consider the joint conditional density of y_t and unobserved $S_t = j$ variable, which is the product of the conditional and marginal densities:

$$f(y_t, S_t = j \mid \psi_{t-1}) = f(y_t \mid S_t = j, \psi_{t-1}) \Pr(S_t = j \mid \psi_{t-1})$$

from which the marginal density of y_t is obtained by:

$$\begin{aligned} f(y_t \mid \psi_{t-1}) &= \sum_{j=1}^{N} f(y_t, S_t = j \mid \psi_{t-1}) \\ &= \sum_{j=1}^{N} f(y_t \mid S_t = j, \psi_{t-1}) \Pr(S_t = j \mid \psi_{t-1}) \end{aligned}$$

where the conditional density $f(y_t \mid S_t = j, \psi_{t-1})$ is obtained from (2) :

$$\begin{aligned} & f(y_t \mid S_t = j, \psi_{t-1}) \\ &= (2\pi)^{-M/2} \det(\Sigma_{St})^{-1/2} \mid \det(Bs_t) \mid \cdot \exp(-\frac{1}{2}(y_t Bs_t + z_t \Gamma s_t) \Sigma_{St}^{-1} (y_t Bs_t + z_t \Gamma s_t)') \quad (2) \end{aligned}$$

where $\Sigma_{St} = \frac{1}{T}(YBs_t + Z\Gamma s_t)'(YBs_t + Z\Gamma s_t)$, y_t is the t^{th} row of the Y matrix. z_t is the t^{th} row of the Z matrix. Bs_t and Γs_t is obtained from (1).

Step 3 :

Once y_t is observed at the end of time t, we update the probability terms:

$$\begin{aligned} & \Pr(S_t = j \mid \psi_t) \\ &= \Pr(S_t = j \mid \psi_{t-1}, y_t) \\ &= \frac{f(S_t = j, y_t \mid \psi_{t-1})}{f(y_t \mid \psi_{t-1})} \end{aligned}$$

$$= \frac{f(y_t \mid S_t = j, \psi_{t-1}) \Pr(S_t = j \mid \psi_{t-1})}{f(y_t \mid \psi_{t-1})}$$

As a byproduct of the above filter in Step 2, we obtain the log likelihood function:

$$\ln L = \sum_{t=1}^{T} \ln f(y_t \mid \psi_{t-1})$$

which can be maximized in respect to the parameters of the model.

3. Empirical Results

Let's consider the Phillips Curve. An OLS regression for *t*=1949 to 2004 using annual data for inflation (π_t) and unemployment rate u_t in year t is given by equation (3)

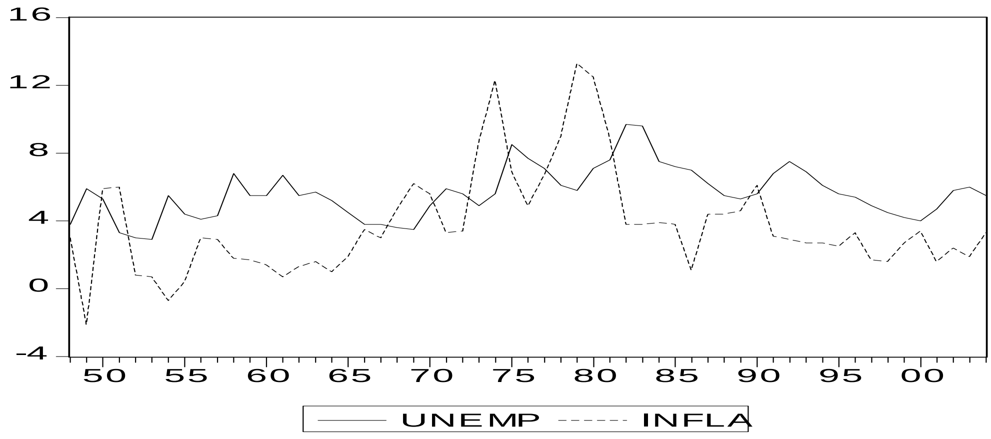
$$\pi_t = 2.55 - 0.28 u_t + 0.75 \pi_{t-1} + e_t \qquad (3)$$

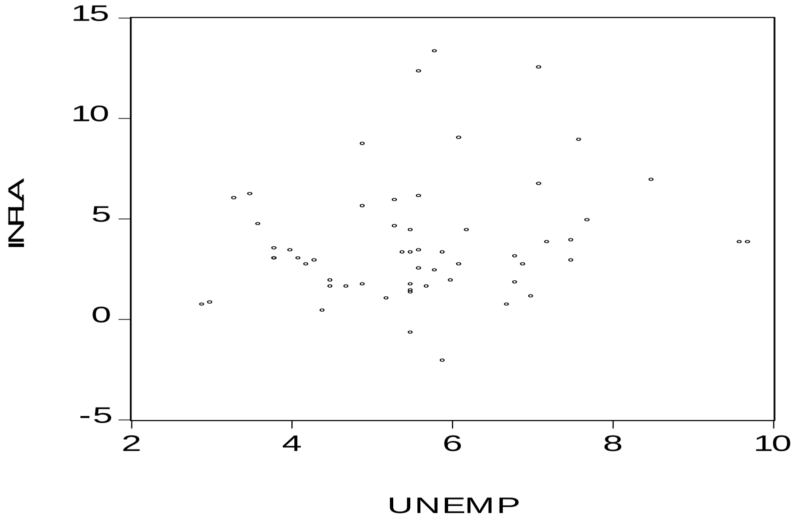
(1.20) (0.22) (0.11)

where standard errors of the parameters estimates are reported in the parentheses.

The OLS regression (3) reveals statistically insignificant evidence of an inflation-unemployment trade-off because the parameter of unemployment rate u_t in the equation (3) reveals statistically insignificant. *Figure 1* depicts the relationship between inflation π_t and unemployment rate u_t .

Figure 1. inflation π_t and unemployment rate u_t





From the equation (3) and *Figure 1*, we can find that the short run Phillips Curve does not remain stable. These results suggest us to utilize a Markov-Switching Model for the unstable Phillips Curve for the post WWII period.

To find out the relationship between unemployment and the real growth of output, the equation of Okun's Law for t=1949 to 2004 is given by equation (4)

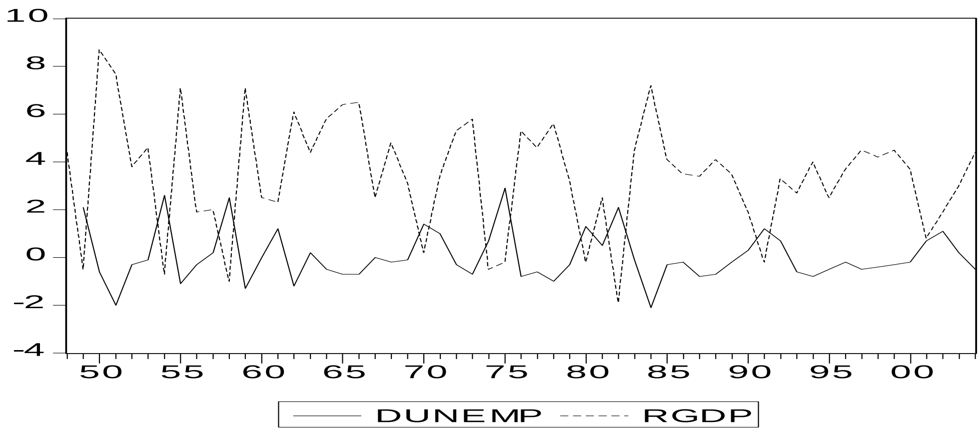
$$\Delta u_t = 1.35 - 0.38 y_t + \nu_t \quad (4)$$

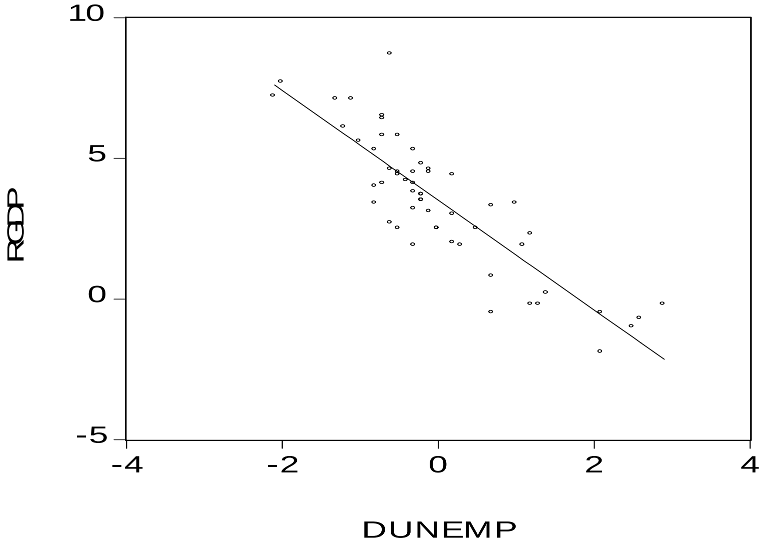
(0.13) (0.03)

where y_t, is the annual real growth of GDP, $\triangle u_t$ is changes in the unemployment rate in year t.

The equation of Okun's Law in (4) reveals statistically significant evidence between changes in the unemployment rate and real growth of GDP.

Figure 2. changes in the unemployment rate and real growth of GDP





From the equation (4) and *Figure 2*, we can find that the stable Okun's Law for the post-WWII period.

For the estimation of Phillips Curve and Okun's Law together, the proposed FIML Markov-Switching Model was applied, which adopts a simple two-state Markov switching parameters in the simultaneous equations.

$$\pi_t = \alpha_{St} + \beta_{St} u_t + \gamma \pi_{t-1} + e_{St} \quad (5)$$

$$\Delta u_t = \phi_1 + \phi_2 y_t + v_t$$

$$u_t = u_{t-1} + \phi_1 + \phi_2 y_t + v_t \quad (6)$$

where $\alpha_{St} = \alpha_1 S_t + \alpha_0 (1 - S_t)$, $\beta_{St} = \beta_1 S_t + \beta_0 (1 - S_t)$,

$\Pr(S_t = 0 \mid S_{t-1} = 0) = q$, $\Pr(S_t = 1 \mid S_{t-1} = 1) = p$,

$p = \begin{pmatrix} p & 1-q \\ 1-p & q \end{pmatrix}$, $\begin{pmatrix} e_{St} \\ v_t \end{pmatrix} \sim i.i.d.N(0, \Sigma_{St} \otimes I_T)$,

To solve the equation (5) and (6) together, we can rewrite it as follows:

$$[\pi_t \quad u_t] * \begin{bmatrix} 1 & 0 \\ -\beta_{St} & 1 \end{bmatrix} - [\pi_{t-1} \quad y_t \quad u_{t-1}] * \begin{bmatrix} \gamma & 0 \\ 0 & \phi_2 \\ 0 & 1 \end{bmatrix} - [\alpha_{St} \quad \phi_1] = \begin{pmatrix} e_{St} \\ v_t \end{pmatrix}$$

where $\begin{pmatrix} e_{St} \\ v_t \end{pmatrix} \sim i.i.d.N(0, \Sigma_{St} \otimes I_T)$,

$$\hat{\Sigma}_{St} = \frac{1}{T}(Y\hat{B}s_t + Z\hat{\Gamma}s_t)'(Y\hat{B}s_t + Z\hat{\Gamma}s_t) = \begin{pmatrix} \hat{\sigma}_{S1t,S1t} & 0 \\ 0 & \hat{\sigma}_{2,2} \end{pmatrix},$$

$$\alpha_{St} = \alpha_1 S_t + \alpha_0(1-S_t), \quad \beta_{St} = \beta_1 S_t + \beta_0(1-S_t),$$

$$\Pr(S_t = 0 \mid S_{t-1} = 0) = q, \quad \Pr(S_t = 1 \mid S_{t-1} = 1) = p, \quad p = \begin{pmatrix} p & 1-q \\ 1-p & q \end{pmatrix}$$

Table 1 reports estimation results using annual data for 1949-2004.

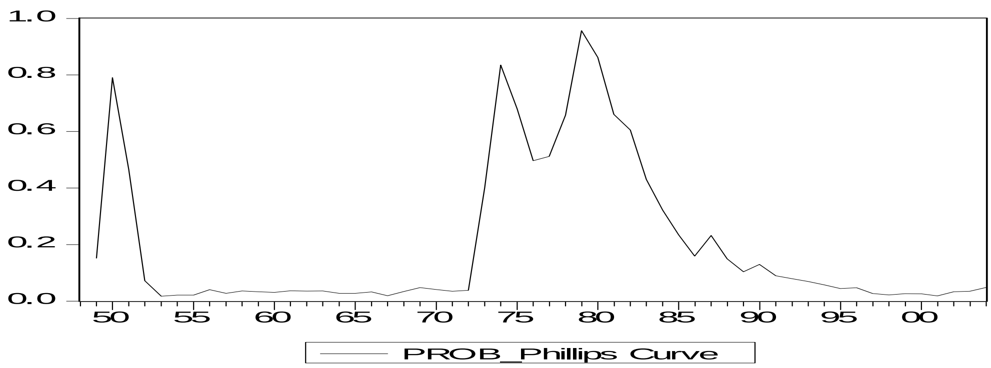
TABLE 1: MAXIMUM LIKELIHOOD ESTIMATION OF THE MODEL: (1949~2004)

$\pi_t = 8.58S_t + 2.00(1-S_t) + \{-0.95S_t - 0.19(1-S_t)\}u_t + 0.68\pi_{t-1} + e_{St}$	
(3.37) (1.11) (0.57) (0.21) (0.11)	
$u_t = u_{t-1} + 1.35 - 0.381y_t + \nu_t$	
(0.13) (0.030)	
$\Pr(S_t = 0 \mid S_{t-1} = 0) = 0.96$, $\Pr(S_t = 1 \mid S_{t-1} = 1) = 0.85$	
(0.04) (0.14)	
Log Likelihood	-165.02

Standard errors of the parameters estimates are reported in the parentheses

The coefficient $\beta_1 = -0.95$ is negative during regime 1 period. However, $\beta_0 = -0.19$ is negative, statistically insignificant during the regime 0 period.

Figure 3. Probabilities of regime 1 $\Pr(S_t = 1 \mid S_{t-1} = 1) = p$ for 1949~2004



From *Figure 3*, the inferred probabilities $\Pr(S_t = 1 \mid S_{t-1} = 1) = p$ accord quite well with the Korean War (1950) and the two Oil Shock periods (1974-1975, 1978-1982).

Although the results of β_1 in *Table 1* seems to be statistically meaningful during regime 1 period using annual data, we estimate the model again with quarterly data because quarterly data has more precise information than annual data. We can identify regime switching probabilities more concisely with quarterly data, which may be missed by annual data for the unstable Phillips Curve.

We obtained seasonally adjusted quarterly data of the unemployment rate and the consumer price index for the U.S. from the Bureau of Labor Statistics, and seasonally adjusted quarterly GDP percent change based on the chained 2000 dollars from the Bureau of Economic Analysis. Inflation rates are calculated from the log differenced consumer price index. The sample period is from 1949:I to 2004:IV.

Table 2 reports estimation results using quarterly data for 1949:I~2004:IV.

TABLE 2: MAXIMUM LIKELIHOOD ESTIMATION OF THE MODEL: (1949:I~2004:IV)

$$\pi_t = \underset{(0.426)}{1.689}S_t + \underset{(0.207)}{0.314}(1-S_t) + \{\underset{(0.068)}{-0.142}S_t - \underset{(0.036)}{0.038}(1-S_t)\}u_t + \underset{(0.06)}{0.36}\pi_{t-1} + \underset{(0.06)}{0.23}\pi_{t-2} + e_{St}$$

$$u_t = u_{t-1} + \underset{(0.027)}{0.248} - \underset{(0.005)}{0.069}y_t + \nu_t$$

$$\Pr(S_t = 0 \mid S_{t-1} = 0) = \underset{(0.01)}{0.99}, \quad \Pr(S_t = 1 \mid S_{t-1} = 1) = \underset{(0.03)}{0.96}$$

Log Likelihood -276.31

Standard errors of the parameters estimates are reported in the parentheses

From *Table 2*, the coefficient $\beta_1 = -0.142$ is negatively, significant during the regime 1 period. From this result, we can find that there is an inflation-unemployment trade-off during the regime 1 period. $\beta_0 = -0.038$ is negatively and statistically insignificant during the regime 0 period.

Figure 4. Probabilities of regime 1 $\Pr(S_t = 1 \mid S_{t-1} = 1) = p$ for 1949:I~2004:IV

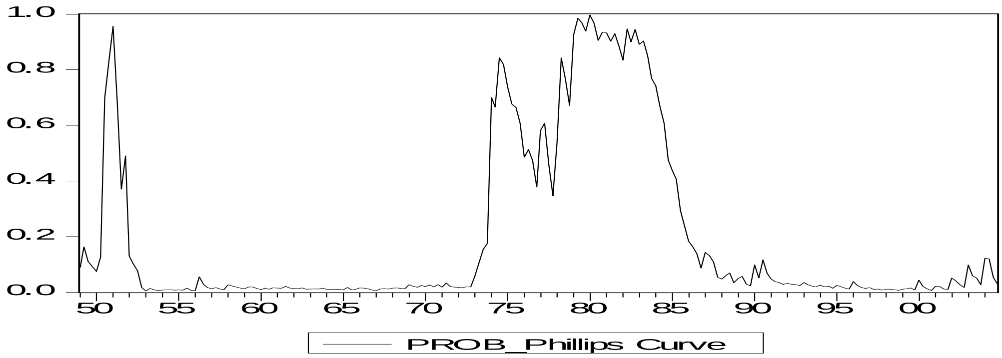


Figure 4 show that the inferred probabilities $\Pr(S_t = 1 | S_{t-1} = 1) = p$ accord quite well with the Korean War (1950:III-1951:II) and the two Oil Shock periods (1974:I-1975:IV, 1978:II-1984:III).

From $\beta_1 = -0.142$ in the *Table 2* and *Figure 4,* we can conclude that inflation is procyclical in movement with the real growth of output during the Korean War and two Oil Shock periods.

As the results of β_0 in the *Table 1* and *Table 2* are statistically insignificant, there was relatively little evidence of co-movement between inflation and the real growth of output with the exception of the Korean War and the two Oil Shock periods.

4 Conclusion

As the equations of the Phillips Curve with Okun's Law was applied to the FIML Markov-Switching Model for the post-WWII period, The findings of this paper are as follows: inflation is procyclical in movement with the real growth of output during the Korean War and the two Oil Shock periods for the post-WWII period; and there was relatively little evidence of co-movement between inflation and the real growth of output with the exceptions of the Korean War and the two Oil Shock periods.

These results suggest another explanation that when there are extremely large shocks such as big wars or oil shocks, inflation is procyclical in movement with the real growth of output, and the procyclical movement occurs in conjunction with the big shocks, exists not only pre-WWII period but also post-WWII period, which is the different result from Cooley and Ohanian's findings.

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