Welfare Enhancing Direct Investment

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Abstract

We present a model in which an acceptance of direct investment is welfare enhancing for a developing country in a multi-commodity multi-factor framework. Contrary to the pessimistic conventional wisdom of Uzawa-Hamada-Brecher-Diaz proposition, this paper provides a justification for capital importation and the export-led growth policy in the ASEAN countries.

Keywords: Uzawa-Hamada-Brecher-Diaz proposition, Direct Investment, GDP Function
JEL Classification: F11, F21
1. Introduction

As the recent ASEAN countries experiences demonstrate, instead of the import substitution policy, many developing countries are trying to diversify the industrial structure and making export-led growth by accepting foreign direct investment. In order to analyze the implications of these policies, it is necessary to provide a model that can justify the acceptance of direct investment.

The analysis of capital importation and welfare was a hot issue in 1970s and many seminal papers have been written on this topic. Among them, Bhagwati(1973), Brecher and Diaz(1977), Brecher and Choudhri(1982), Brecher and Findlay(1983), and Srinivasan (1983) made important contributions to the analyses in this field. Specifically, Brecher and Diaz(1977) showed the possibility of immiserizing capital importation. In the two previous papers written in Japanese, Uzawa(1969) and Hamada(1971) also showed that the capital importation under a tariff is always welfare reducing for a small open economy. This is called as the Uzawa-Hamada proposition. As another extension, the welfare effects of the free trade zone have also attracted considerable attentions among trade theorists and papers such as Hamada (1974) and Miyagiwa (1986) have been written.

However all these previous results are rather pessimistic and do not explain the recent reality of many developing countries. Furthermore these papers have a limitation in dimensionality. The purpose of this paper is to provide a model that can justify the acceptance of foreign capital in a more general framework. We provide a model in which an acceptance of direct investment enhances welfare under some reasonable assumptions.

This paper is organized as follows. In section 2, we demonstrate the implications of the conventional wisdom of Uzawa-Hamada-Brecher-Diaz proposition in a two by two duality
approach and point out some related papers. In section 3, we identify salient features of
direct investment and its agreed definition. The section 4 develops our model and the
section 5 provides a sufficient condition for welfare enhancing direct investment. In section
6, we consider the same problem in a three commodities three factors model. The section 7
concludes the paper. In the appendix, we provide an analysis to justify our assumption in
general settings. In this paper, we present a model in which an acceptance of direct
investment is welfare enhancing in a general framework.

2. The Uzawa-Hamada-Brecher-Diaz proposition and related papers

In order to justify our approach, we first take up the Uzawa-Hamada-Brecher-Diaz
proposition in the standard Heckscher-Ohlin model. This proposition says that a capital
importation under a tariff is welfare reducing. The essence of this proposition is as follows.
Suppose a small open economy that imports capital intensive good under a tariff in the
Heckscher-Ohlin model. Then, an importation of foreign capital is welfare reducing if the
foreign capital takes out the fruits of protected high return. In other words, this proposition
says that the capital importation reduces welfare because the capital importing country
provides a subsidy to the imported foreign capital. In this section we demonstrate this
proposition by the use of a duality approach and point out the reasons for this pessimistic
result.

Suppose a small open economy that produces two commodities \((Y_1, Y_2)\) by the use of
two factors capital \((K)\) and labor \((L)\) under the usual assumptions. The first good is
exportable and the second good is importable. Let the relative price of the second good in
terms of the first be \(p (\equiv p_2 / p_1)\). We assume that the first good is numeraire and set
\( p_1 = 1 \). It is also assumed that the importable is capital intensive and exportable is labor intensive: \( k_2 > k_1 \), where \( k_j \equiv K_j / L_j, j = 1,2 \). Let the GDP or revenue function be \( G(p, K, L) = \max \left[ Y_1(K_1, L_1) + p Y_2(K_2, L_2) \right] \), where \( G(\bullet) \) is assumed to be convex in prices and concave in factor supply. We have: \( G_p = Y_2, G_{pp} = \partial Y_2 / \partial K > 0, G_{pp} > 0, G_K = r, G_L = w \). Define the expenditure function: \( E(p, u) = \min \left[ D_1 + p D_2 : U(D_1, D_2) = u \right] \), where \( D_j (j = 1,2) \) denotes the consumption of \( j \) th good and \( u \) is the level of utility.

The expenditure function is assumed to be homogeneous of degree one and concave in prices and increasing in utility. We also have: \( E_p = D_2, E_{pp} < 0 \), and \( E_{pu} > 0 \). The quantity of import is: \( M_z = E_p (p, u) - G_p (p, K, L) \). We now demonstrate that the welfare declines as the result of capital importation.

Let the tariff on import and imported foreign capital be \( t \) and \( K_f \) respectively. Then the budget equation is:

\[
E(p, u) = G(p, K, L) + t M_z - G_K (p, K, L) K_f,
\]

(1)

where, \( t M_z \) is the tariff revenue and \( G_K (p, K, L) K_f \) is the repatriation to foreign capital. From (1), we obtain:

\[
(E_u - t E_{pu}) du = - t G_{pk} dK_f - K_f G_{kk} dK_f.
\]

(2)

In (2), since \( E(\bullet) \) is homogenous of degree one in prices, we have: \( (E_u - t E_{pu}) > 0 \). Thus if we specify the signs of \( G_{pk} \) and \( G_{kk} \), we could see the welfare effects of capital importation. First, in the Heckscher-Ohlin model, any change in factor supply does not affect factor prices. Thus we have: \( G_{kk} = 0 \). Further, since the second sector is capital intensive, we have: \( G_{pk} > 0 \). Under these specifications, we obtain: \( du / dK_f < 0 \). This is the Uzawa-Hamada-Brecher-Diaz proposition in duality approach and we see that the welfare declines as the result of capital importation. It is clear that this pessimistic result
comes from the model and specifications: $G_{pk} > 0$ and $G_{kk} = 0$. The model and assumptions should be reconsidered in order to provide a justification for the capital importation.

In this connection, we point out some previous papers that are related to this proposition from three aspects. The first is a specific factor model. Srinivasan (1983) uses a specific factor model to analyze the effects of capital importation where foreign capital is different from the domestic one. If the foreign capital is specific to the import sector, the second term in the right hand side of (2) is $-\frac{K}{K}G_{kk}G_{K} > 0$. Thus the welfare can increase as the result of capital importation. The second is the free trade zone model. In this field, the classical paper is Hamada (1974). By the use of standard Heckscher Ohlin model, it shows that an exogenous increase in foreign capital within the zone decreases national welfare if the import sector is capital intensive. Mayagiwa (1986) also considers the impact of establishing an free trade zone by a government subsidy in order to promote the diversification of industry and export. It shows that the establishment of a free trade zone as the third sector can increase welfare regardless of the relative factor intensity of a zone based industry. The third is the export processing zone model. Beladi and Marjit (1992) considers the welfare effects of an expansion of an export processing zone by the use of a three sectors three factors (two capitals and one labor) model. It assumes that domestic capital is used in the traditional import and export sector but not in the export processing zone. Using such a model, it shows that if the economy imports capital intensive second good under a given tariff, an increase in foreign capital in the export processing zone decreases its welfare. It also shows that an increase in foreign capital increases the output of import and decreases that of export, making the anti-trade growth.
However the above models and results are not always consistent with the real world of developing countries. We need a model that can justify the capital importation and export-led growth strategy in developing countries.

3. Features of Direct Investment

As these countries are accepting direct investment, we must identify some important features of direct investment. The analyses of foreign direct investment (FDI) started from Caves (1971) in 1970s. Since then many seminal papers such as Helpman (1984), Helpman and Krugman (1985), Wong (1995), Markusen (2002), Feenstra (2004), and Navarette and Venables (2004) have been written. These previous papers demonstrate that the direct investment should be differentiated from the movement of capital in the Heckscher-Ohlin model and that it has at least following features which differentiate it from the portfolio investment:

i) it is a starter of new production sector in the host countries

ii) it is made in order to use the firm specific advantages in the host countries

iii) it is cross-hauling within a industry

iv) it is a key vehicle of technology transfer

v) it is made by the non-financial multinational firms to control subsidiaries.

These features have been gathered into an agreed definition of direct investment: it is international transfers of firm specific managerial resources or assets by multinational firms and is a starter of new production sector in the host countries.

FDI is divided into two types: Vertical FDI (VFDI) and Horizontal FDI (HFDI). While the HFDI is the market-oriented investment and is popular among developed countries,
the VFDI is the cost-oriented investment and is popular between developed and developing countries. We are interested in the VFDI. In the case of VFDI, from the aspect of developed countries, direct investment is made in order to use these managerial resources to obtain cost advantages in the vertical production processes. On the other hand, from the aspect of developing countries, they accept direct investment in order to diversify their industrial structure and increase the level of employment of domestic factors of production. Specifically, in the case of ASEAN countries, they accept it in order to establish a new export sector in addition to the traditional export sectors in the economy. However in order to justify such a policy in developing countries, it is necessary to consider a condition under which an acceptance of direct investment is welfare enhancing. In the next section, we will provide a model that includes the two features of FDI: it is sector specific and it is a starter of new production sector in a host economy.

4. The Model

Suppose a small open developing country that produces \( n \) commodities \(( j = 1, \ldots, n)\) by the use of \( m \) factors \(( i = 1, \ldots, m)\) before a direct investment takes place. The numbers of \( n \) and \( m \) are arbitrarily chosen. The multi-dimensionality is the first crucial feature of our model.

The production function of each commodity is assumed to be twice-continuously differentiable, increasing, linearly homogeneous, and strictly-quasi-concave in all factors of production:

\[
y_j = f^j(x_{1j}, x_{2j}, \ldots, x_{mj}), \quad j = 1, \ldots, n.
\]  

(3)

It is assumed that the \( m \) factors are inelastically supplied and the full employment
condition holds for each of them:

$$\bar{x}_i = \sum_{j=1}^{n} x_{ij}, \ i = 1, ..., m,$$  \hspace{1cm} (4)

where, $\bar{x}_i$ is the domestic supply of $i$ th factor. All commodity and factor markets are competitive.

Let $p^T \equiv (p_1, p_2, ..., p_n)$ be the commodity price vector. The GDP function is defined as:

$$F(p, \bar{x}) \equiv \max \sum_{j=1}^{n} p_j f^j (x_{1j}, ..., x_{mj})$$

with respect to $x_{ij} (i = 1, ..., m, j = 1, ..., n)$ subject to (4), where $\bar{x}^T \equiv (\bar{x}_1, ..., \bar{x}_m)$ is the factor-endowment vector of the economy.

Now, suppose that the foreign capital $x_{00}$ comes into this country and that a new export sector, 0 th good, is started by accepting this foreign capital and using existing domestic $m$ factors of production. We assume that not only foreign capital $x_{00}$ but also existing $m$ domestic factors, $x^0$, are used in the production of new export good. This is the second crucial feature of our model. This assumption is reasonable because the expansion of employment of domestic factors and the diversification of industrial structure are the important targets of these countries. Since $x_{00}$ is specific to the 0 th sector and this new export sector is started by the acceptance of this foreign capital, we call it as foreign direct investment.

The production function of the new export good is:

$$y_0 = f^0 (x_{00}, x_0),$$  \hspace{1cm} (6)

where, $x_0^T \equiv (x_{10}, ..., x_{m0})$ is the domestic factors used in the new export sector and it has
We assume that this function also satisfies all the standard properties as a neo-classical production function. Integrating the pre-FDI GDP function with the production function of the 0th good, we can formulate the post-FDI GDP function as follows:

$G(p_0, p, \bar{x}, x_{00}) \equiv \max\left[p_0 f^0(x_{00}, x_0) + F(p, \bar{x} - x_0)\right]$  \hspace{1cm} (7)

with respect to $x_0$. It is assumed that $G(\bullet)$ is concave with respect to $\bar{x}$ and $x_{00}$. Assuming the existence of the interior solution to this maximization problem, we can write the first order condition as follows:

$p_0 f^0_{x_0}(x_{00}, x_0) = F_x(p, \bar{x} - x_0)$, \hspace{1cm} (8)

where, $f^0_{x_0}(x_{00}, x_0)^T \equiv \left(\frac{\partial f^0(x_{00}, x_0)}{\partial x_{10}}, ..., \frac{\partial f^0(x_{00}, x_0)}{\partial x_{m0}}\right)$,

$F_x(p, \bar{x} - x_0)^T \equiv \left(\frac{\partial F(p, \bar{x} - x_0)}{\partial (\bar{x}_1 - x_{10})}, ..., \frac{\partial F(p, \bar{x} - x_0)}{\partial (\bar{x}_m - x_{m0})}\right)$.

The left hand side of (8) is the value of marginal product of each existing factor of production in the 0th sector, while the right hand side is its factor price. Therefore, $m$ equations in (8) are the profit maximization conditions in the 0th sector. Assume that there exists a unique $x_0$ that satisfies (8) and we denote it by $x_0^*$.

Now, let us turn to the demand side of the model. Denoting the expenditure function of the whole residents of the country by $E(p_0, p, u)$ and assuming that the government of this country imposes an import tariff and transfers the whole tariff revenue to the residents in a lump-sum manner, we can write the budget constraint as follows:

$E(p_0, p, u) = G(p_0, p, \bar{x}, x_{00}) + \Gamma^T \left[E_p(p_0, p, u) - G_p(p_0, p, \bar{x}, x_{00})\right]$

$- G_{x_0}(p_0, p, \bar{x}, x_{00}) x_{00}$, \hspace{1cm} (9)
where, \( \Gamma^T \equiv (0,0,...,0,t_{h+1},t_{h+2},...,t_n) \) is the import tariff vector. We assume that the first \( h \) sectors including the 0th sector are exportable and from \( h+1 \) to \( n \) sectors are importable. We assume that there exits no non-traded goods. The second term of the right hand side of (9) is the tariff revenue and the third term is the repatriation to the foreign direct investment. Since \( p_0 \) and \( p \) are the domestic prices, by denoting the foreign prices by \( p^*_0 \) and \( p^* \), we have following relationships:

\[
p_0 = p^*_0,
\]

\[
p = \Gamma + p^*.
\]

We assume that (9) determines the welfare level \( u \) uniquely and it is denoted by \( u^e \).

5. The Analyses

Now, we derive our main result. The total differentiation of (9) with respect to \( u^e \) and \( x_{00} \) yields,

\[
\begin{align*}
\left[ E_u (p_0, p, u^e) - \Gamma^T E_{p_0} (p_0, p, u^e) \right] du^e \\
= G_{x_0} (p_0, p, \bar{x}, x_{00}) dx_{00} - \Gamma^T G_{x_0} (p_0, p, \bar{x}, x_{00}) dx_{00} \\
- G_{x_0} (p_0, p, \bar{x}, x_{00}) dx_{00} - G_{x_0 x_0} (p_0, p, \bar{x}, x_{00}) x_{00} dx_{00} \\
= -\left[ \Gamma^T G_{x_0 x_0} (p_0, p, \bar{x}, x_{00}) + G_{x_0 x_0} (p_0, p, \bar{x}, x_{00}) x_{00} \right] dx_{00}.
\end{align*}
\]

From which we obtain:

\[
\frac{du^e}{dx_{00}} = -\frac{\left[ \Gamma^T G_{x_0 x_0} (p_0, p, \bar{x}, x_{00}) + G_{x_0 x_0} (p_0, p, \bar{x}, x_{00}) x_{00} \right]}{E_u (p_0, p, u^e) - \Gamma^T E_{p_0} (p_0, p, u^e)}.
\]

The effects of an acceptance of direct investment on welfare are demonstrated by (10). Let us check the signs of (10). First, the denominator of (10) has to be positive. Since the partial derivative of the expenditure function with respect to welfare \( u \) is linearly homogeneous
in $p_0$ and $p$, we have the Euler condition: $E_u = p_0 E_{wp_0} + p^T E_{wp}$. It follows that $E_u = p_0 E_{wp_0} + (p^* + \Gamma)^T E_{wp} > \Gamma^T E_{wp}$. Thus the signs of (10) depend on the numerator.

First, due to the concavity of the GDP function with respect to $\bar{x}$ and $x_{00}$, 

$$G_{wp}(p_0, p, \bar{x}, x_{00}) \leq 0.$$ 

It is clear that $G_{wp}$ is the remuneration to direct investment. Thus as the supply of $x_{00}$ increases, the rate of remuneration must decline. This is the economic reason why $G_{wp}(p_0, p, \bar{x}, x_{00}) \leq 0$. Thus what remains to analyze is the term 

$$\Gamma^T G_{wp}(p_0, p, \bar{x}, x_{00}) \text{ in the numerator.}$$

Recalling the definition of the GDP function (7), we see that 

$$G_p(p_0, p, \bar{x}, x_{00}) = F_p(p, \bar{x} - x_0).$$  \hfill (11) 

Therefore, we have, 

$$G_{wp}(p_0, p, \bar{x}, x_{00}) = -F_{px}(p, \bar{x} - x_0) \frac{dx_0}{dx_{00}}.$$ \hfill (12) 

Thus the sign of $G_{wp}(p_0, p, \bar{x}, x_{00})$ depends on two terms: $F_{px}(p, \bar{x} - x_0)$ and $\frac{dx_0}{dx_{00}}$.

First, $F_{px}(p, \bar{x} - x_0)$ is the change in the output vector of domestic sectors as the result of the movement of domestic factors between the new export sector and domestic sectors and it is positive. Thus if the vector $\frac{dx_0}{dx_{00}}$ is positive, a FDI is welfare enhancing. For this purpose, we introduce a following assumption:

**Assumption 1.** There is a positive value $\alpha$, such that $\frac{dx_0}{dx_{00}} = \alpha \bar{x}$.

This assumption implies that all domestic factors move into the new export sector in equal proportion to the endowments of factors. When an additional FDI takes place, the rewards
of existing factors in the new export sector increase, so that the existing factors move from the existing industries into the new export sector. The assumption 1 says that all factors move into the new export sector in equal proportion to the endowments of factors. We think that it is a reasonable assumption.

Under this assumption, we see from (12) that $G_{px_{00}}(p_0, p, \bar{x}, x_{00})$ is a negative vector, which implies that $\Gamma^T G_{px_{00}}(p_0, p, \bar{x}, x_{00}) < 0$. From (10), we obtain following result.

**Result 1.** An acceptance of foreign direct investment is welfare enhancing under the assumption 1.

The intuition of this result is as follows. At given tariff rates, suppose an additional direct investment is accepted. An increase in direct investment increases the rewards of domestic factors in the new export sector, which induce the domestic factors to move from previous sectors into the new export sector. This reduces the level of outputs of all previous sectors, increasing imports and tariff revenue. An increase in tariff revenue increases the welfare of this country. We have a following remark.

**Remark 1.** Let the output vector of previous sectors before FDI as $y^T \equiv (y_1, y_2, ..., y_n)$. Under the assumption 1, we have: $dy^T / dx_{00} < 0$. There is a reciprocity relation between quantities and prices.\(^5\) We have: $dw_0 / dp^{n-h} < 0$ and an increase in welfare.

The implications of this remark are as follows. Suppose the tariff rate increases at given direct investment. As the result of an increase in tariff, the price of imports and thus the
rewards of domestic mobile factors increase, which in turn, at given $p_0$, reduces the reward of specific factor $w_0$. The reduction of $w_0$ reduces the repatriations to foreign country, increasing the welfare of this country.

It is necessary to stress the differences between the Uzawa-Hamada-Brecher-Diaz proposition and our result. In the former, it is a two sectors two factors model and the importable good is capital intensive. As the result of tariff, the price of imports increases, so that the repatriation to foreign capital increases. This reduces the welfare. In contrast, we assume a multi-sectors multi-factors model. Suppose the prices of imports increase by tariffs. Then the rewards of domestic mobile factors increase. However, as $p_0$ is fixed, the reward to the specific factor $w_0$ must decline. This reduces the repatriation to direct investment and increases domestic welfare.

Following remarks must be necessary. First, the desirability of the acceptance of direct investment depends on the fact that domestic factors are also used in the new export sector. This assumption is reasonable and plausible in many developing countries because an increase in employments is an important target in these countries. Second, in our model, the desirability of direct investment depends on the fact that the acceptance of direct investment reduces the level of outputs of all previous sectors including importable sectors.

6. Three Commodity Three Factor Case

In this section, we consider a condition under which an acceptance of direct investment increases welfare in a three sectors three factors model. Assume that sector 0 is the new export sector and sector 1 and 2 are the previous two sectors. Specifically, similar to the previous section, we specify that sector 1 is the traditional export sector and sector 2 is the
import sector. Assume three factors where factor $x_0$ is the foreign capital specific to sector and factors $x_1$ and $x_2$ are domestic factors used in all three sectors. Assuming the factor rewards of three factors be $w_0, w_1, w_2$ respectively, the zero profit condition is:

$$p_0 = c_0^0(w_0, w_1, w_2), \quad (13)$$

$$p_1 = c_1^1(w_1, w_2), \quad (14)$$

$$p_2 = c_2(w_1, w_2). \quad (15)$$

Differentiating these three equations totally assuming $p_0$ to be fixed, we obtain:

$$dw_0 = -\frac{1}{c_0^0} [c_0^0 dw_1 + c_0^0 dw_2],$$

$$dw_1 = \frac{c_2^1}{\Delta} dp_1 - \frac{c_1^1}{\Delta} dp_2, \quad (16)$$

$$dw_2 = -\frac{c_1^2}{\Delta} dp_1 + \frac{c_1^1}{\Delta} dp_2,$$

where, $\Delta \equiv c_1^1 c_2^2 - c_1^2 c_2^1 - c_1^1 c_1^2 \left[\frac{c_2^2}{c_1^2} - \frac{c_1^2}{c_1^1}\right]$. From (16), we obtain:

$$\frac{dw_0}{dp_1} = c_0^1 c_1^1 \left[\frac{c_0^2}{c_1^2} - \frac{c_1^2}{c_1^1}\right], \quad \frac{dw_0}{dp_2} = c_0^0 c_1^2 \left[\frac{c_0^1}{c_1^1} - \frac{c_0^2}{c_1^1}\right], \quad (17)$$

In (17), $\frac{c_0^0}{c_1^1}, \frac{c_1^1}{c_1^1},$ and $\frac{c_1^2}{c_1^1}$ are the factor intensity of the second factor relative to the first factor in the new export sector, first sector, and second sector respectively. The parenthesis in $\Delta$ implies the difference in the factor intensity between the second sector and the first.
sector. If the second sector is intensive in second factor relative to the first sector, the parenthesis in $\Delta$ is positive. On the other hand, the parenthesis in numerator of (17) is the differences in the factor intensity between new export sector and second sector or first sector. From (17), we obtain:

\[
\text{Result.2. If } \min \left[ \frac{c_2^1}{c_1^1}, \frac{c_2^2}{c_1^2} \right] < \frac{c_2^0}{c_1^0} < \max \left[ \frac{c_2^1}{c_1^1}, \frac{c_2^2}{c_1^2} \right], \text{ we have } \frac{dw_0}{dp_1} = \frac{dy_1}{dx_0} < 0 \text{ and } \frac{dw_0}{dp_2} = \frac{dy_2}{dx_0} < 0.
\]

This result shows that if the factor intensity in the new export sector lies between the two previous sectors, an acceptance of direct investment reduces the outputs of two previous sectors, increasing imports, tariff revenue, and welfare. This condition is weaker than the assumption 1, because here we just need that $c_2^0 / c_1^0$ lies between that of other two sectors. It should be noted that the result 2 is a generalization of Beladi and Marjit (1992), because here the domestic factors are also used in the new export sector and the conditions for increasing in welfare is weaker than that of Beladi and Marjit (1992).

7. Conclusions

This paper considered the welfare effects of direct investment in a multi-dimensional framework and derived a sufficient condition for an acceptance of direct investment in the new export sector to be welfare enhancing. Our result is not only more general and optimistic but also contrary to the pessimistic conventional wisdom of Uzawa-Hamada-Brecher-Diaz proposition. The rationale of our result comes from the two assumptions: the domestic factors are used in the new export sector and the price of new export good is fixed.
Under these reasonable assumptions, we showed that an acceptance of direct investment not only increases welfare but also provides a new export sector in developing countries. This paper justifies the export-led growth strategy in many developing countries. We supplemented our analysis by adding a three commodities three factors model.

A number of topics suggest themselves for further research. First, in the analyses we assumed that the price of new export good is fixed. This assumption is reasonable for many ASEAN countries. However if the commodity price changes as the result of direct investment, additional effects must be considered. Second, the level of technology may change as the result of direct investment and the full repatriation of direct investment may not be the case. In such cases, the results of our analyses will change. These aspects are the topics for the further research.

Appendix

In this appendix, we consider why the assumption \( \frac{dx_0}{dx_{00}} \approx 0 \) holds in two general cases: one is when the number of existing commodity \( n \) is equal to that of factor \( m \) and the other is when they are different.

Case 1. \( n = m \)

Denote the cost function of the new export sector by \( c^0 (w_0, w) y_0 \), where \( y_0 \) is the output of that sector and \( w \) is the \( m \) dimensional vector of factor prices which is determined by

\[
p_i = c^i (w_1, \ldots, w_m), \quad i = 1, \ldots, n
\]

Let \( w(p) \) be the solution vector to this system of equations. Using this solution vector, we
can write

\[ p_0 = c_0^0(w_0, w(p)), \]  
\[ x_{00} = c_0^0(w_0, w(p))y_0, \]  
\[ x_0 = (c_1^0(w_0, w(p))y_0, ..., c_m^0(w_0, w(p)))^T. \]

The first equation determines \( w_0 \) as \( w_0(p_0, p) \), and the rest of equations determine \( y_0 \) and \( x_0 \). Thus we see that

\[
x_0 = \frac{x_{00}}{c_0^0(w_0(p_0, p), w(p))} \left( c_1^0(w_0(p_0, p), w(p)), ..., c_m^0(w_0(p_0, p), w(p)) \right)^T.
\]

Therefore, if the \( m \) -dimensional vector

\[
\left( c_1^0(w_0(p_0, p), w(p)), ..., c_m^0(w_0(p_0, p), w(p)) \right)^T
\]

is proportional to \( \bar{x} \), so is \( x_0 \), i.e., \( x_0 \approx \alpha \bar{x}x_{00} \). In this case we have:

\[
\frac{dx_0}{dx_{00}} \approx \alpha \bar{x}.
\]

Case 2. \( n \neq m \)

In this case, \( w(= F_\mathbb{x}) \) generally depends not only on \( p \) but also on \( \bar{x} - x_0 \). Thus (a1) - (a3) can be rewritten as

\[ p_0 = c_0^0(w_0, w(p, \bar{x} - x_0)), \]  
\[ x_{00} = c_0^0(w_0, w(p, \bar{x} - x_0))y_0, \]  
\[ x_0 = (c_1^0(w_0, w(p, \bar{x} - x_0))y_0, ..., c_m^0(w_0, w(p, \bar{x} - x_0)))^T. \]

(a4) determines \( w_0 \) as \( w_0(p_0, p, \bar{x} - x_0) \) for given \( p_0, p \), and \( \bar{x} - x_0 \). Then from (a5), we can determine \( y_0 \). By the use of these, (a6) is written as

\[
x_0 = x_{00}H(p_0, p, \bar{x} - x_0),
\]

where,

\[
H(p_0, p, \bar{x} - x_0) \equiv (H_1(p_0, p, \bar{x} - x_0), ..., H_m(p_0, p, \bar{x} - x_0))^T
\]
\[ \equiv \left( c_0^0(w_0(p_0, p, \bar{x} - x_0), w(p, \bar{x} - x_0)), \ldots, c_m^0(w_0(p_0, p, \bar{x} - x_0), w(p, \bar{x} - x_0)) \right)^T. \]

Totally differentiating (a7) with respect to \( x_0 \) and \( x_{00} \), we obtain:

\[ dx_0 = H(p_0, p, \bar{x} - x_0)dx_{00} + x_{00} \nabla[H(p_0, p, \bar{x} - x_0)]dx_0 \]

or

\[ \frac{dx_0}{dx_{00}} = H(p_0, p, X) - x_{00} \nabla[H(p_0, p, X)]\frac{dx_0}{dx_{00}} \]

(a8)

where, \( X \equiv (X_1, \ldots, X_m)^T = (\bar{x}_1 - x_{10}, \ldots, \bar{x}_m - x_{m0})^T \) and

\[ \nabla[H(p_0, p, X)] \equiv \begin{bmatrix} \frac{\partial H_1(p_0, p, X)}{\partial X_1} & \ldots & \frac{\partial H_1(p_0, p, X)}{\partial X_m} \\ \frac{\partial H_2(p_0, p, X)}{\partial X_1} & \ldots & \frac{\partial H_2(p_0, p, X)}{\partial X_m} \\ \vdots & \ldots & \vdots \\ \frac{\partial H_m(p_0, p, X)}{\partial X_1} & \ldots & \frac{\partial H_m(p_0, p, X)}{\partial X_m} \end{bmatrix} \]

From (a8), considering the definition of \( H(\bullet) \), we have:

\[ \frac{dx_0}{dx_{00}} \bigg|_{x_0 = 0} = H(p_0, p, \bar{x}) \]

\[ = \left( c_1^0(w_0(p_0, p, \bar{x}), w(p, \bar{x})), \ldots, c_m^0(w_0(p_0, p, \bar{x}), w(p, \bar{x})) \right)^T \] (a9)

Since our basic assumption is that the coefficient vector of the new export sector is approximately proportional to the endowment vector of the existing factors, i.e., since we assume

\[ \left( c_1^0(w_0(p_0, p, \bar{x}), w(p, \bar{x})), \ldots, c_m^0(w_0(p_0, p, \bar{x}), w(p, \bar{x})) \right)^T = \alpha \bar{x}. \]
We obtain \((a0)\) as

\[
\frac{dx_0}{dx_{00}} \bigg|_{x_{00}=0} \approx \alpha_0.
\]

Footnotes

1. On the analyses of industrialization by the acceptance of foreign technology, see Chen and Shimomura(1998).

2. In what follows, each vector is a column vector. The superscript \(T\) attached to a vector means the transpose of the vector.

3. It is reasonable to assume that \(x_{00}\) comes into this country as long as its marginal product is positive.

4. The acceptance of direct investment may introduce some monopolistic elements into the market. However we assume that perfect competition still prevails.

5. See, for example, Chang(1979).

References


Markusen James (2002), Multinational Firms and the Theory of International Trade, MIT


