THE DYNAMIC STRUCTURE OF OPTIMAL TARIFF ON TIMBER TRADE: A DIFFERENTIAL GAME APPROACH

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Abstract

The analysis of import tariffs on natural resource has been undertaken on the establishment of two modeling approaches for both nonrenewable resources and renewable resources. However, it is too hard to find the literature that analyzed the optimal tariffs on timber trade by modeling the biological characteristics of forest explicitly. This paper, hence, is designed to examine the dynamic structure of optimal tariffs on timber by specifying both the dynamic change of forest stock and the quantity of timber production in a realistic sense. For this purpose, we establish a Stackelberg differential game of timber trade where the buying country has a market power, and analyze the dynamic structure of optimal tariff on imported timber. We perform this procedure for two forms of cost function for timber production. One form is dependent on the stock of forest and the other is independent of it. In addition, we show that the costate variable for the stock of forest can be partitioned into the scarcity effect and the cost effect. On the basis of this result, we identified that the selling country takes the cost effect of the shadow value for the stock of forest as a rent.

Key words; Optimal tariff on timber, Stackelberg differential game, Dynamic inconsistency, Scarcity effect and cost effect

JEL Code: F12, H21, Q23

INTRODUCTION

Natural resource has a special and attractive property that the scarcity effect of natural resource increases as does the rate of extraction (or harvests) of it. This exhibits that there is a significant difference in imposing optimal tariffs between on ordinary produced goods and on imported natural resources. The analysis of import tariffs on natural resource has been undertaken on the establishment of two modeling approaches associated with the characteristics of natural resources. For non-renewable resource such as oil, Ulph and Folie(1980), Kemp and Long (1980), Newberry (1981), Karp (1984), Eckstein and Eichenbaum (1985), Maskin and Newberry (1990), Karp and Newberry (1991, 1992) investigated a theoretical basis of how the buying country imposes optimal tariff on imported non-renewable natural resource over time. On the other hand, Batabyal and Beladi (2006), and Batabyal (2006) developed a theoretical modeling approach to examine the structure of optimal tariff on renewable resource imported over time. For the import tariff on timber, however, it is too hard to find the literature that modeled the specific characteristics of the dynamic change for the stock of forest as well as the quantity of timber production explicitly. Renewable resources such as fish, ivory (from elephants), and horns (from rhinoceroses) have the characteristic that some portion of the stock of resource is instantaneously regenerated after it has been harvested. It, however, may take from several decades through more than one hundred years that young trees grow to become old-growth trees that are adequate for timber production. For example, it may take 100 years for oak trees and as few as 40 years for sitka spruce. As a result, the quantity of timber production can be calculated as the multiplication of the amount of trees harvested by the growth of tree. We, hence, specify the law of motion for the stock of forest and the quantity of timber production that approximates the biological characteristics of forest in a realistic sense. We include it in the model, and intend to identify the dynamic structure of tariff on imported timber.

For this purpose, we adapted a Stackelberg differential game of timber trade in which the buying country has a market power. We establish two modeling approaches to take into consideration the form of the cost function for timber production. One form is dependent on the stock of forest and the other is independent of it. In particular, we show that the shadow value (costate variable) for the stock of forest can be partitioned into the scarcity effect and the cost effect, and based on this result, examine that the selling country takes the cost effect of the shadow value for the stock of forest as a rent. Below, we first specify and formulate a Stackelberg differential game of timber trade with two forms of cost function. Second, we develop our procedures to identify the dynamic structure of optimal tariff on imported timber. Third, we observe the characteristics of the shadow value for the stock of forest, and provide a theoretical rationale that the selling country's rent is equivalent to the cost effect of the shadow value for it.

DYNAMIC STRUCTURE OF OPTIMAL TARIFF ON TIMBER

In this section, we establish a Stackelberg differential game of timber trade in which the buying country has a market power. This model is based on the tradition of Siman and Cruz (1973), Karp (1984), Brazee and Mendelsohn (1990), Karp and

Newberry (1993), Sohngen and Mendelsohn (1998), Lee and Lyon (2004), Batabyal (2006). Let us first examine the dynamic structure of optimal tariff on timber on the condition that the cost function for timber production is dependent on the stock of forest. Suppose that the buying country's utility of consuming imported timber x is given by a concave function $u(x)$ with two differentiable derivatives. The buying country's domestic market is competitive; hence, $u^{'}(x) = p(x)$, where p is the timber price that buying country's consumers pay. The buying country may impose a unit tariff by $\tau(t)$ for imported timbers, in which case the price received by the selling country is $p(x) - \tau(t)$. The buying county's economic surplus can be denoted as the sum of the discounted stream of the difference between the buying country's total utility of consuming $x(t)$ and the external payment, $(p(x) - \tau)x$.

If T is the date at which consumption terminates and r is the discount rate, the buying country's economic surplus can be expressed as

$$W_B = \int_0^T e^{-rt}[u(x) - (p(x) - \tau)x]\,dt \tag{1}$$

According to timber supply models that Brazee and Mendelsohn (1990), Sedjo and Lyon (1990), Sohngen and Mendelsohn, and Lee and Lyon (2004) have developed, the quantity of timber production is defined as the multiplication of the amount of old-growth tree harvested, $h(t)$ by the growth of tree, $V(t)$. Thus, the quantity of timber produced is expressed as $x(t) = h(t)V(t)$, where the growth function of tree has the property such that $V^{'}(t) > 0$, and $V^{''}(t) < 0$. In addition, in order to model the fact that the cost function for timber production is stock dependent, let us suppose that $c(z)$ is the average cost of producing a unit of the timber for the stock of forest z ; then $c(z)x$ is the instantaneous cost of producing x [1]. Assume that the average cost function for timber production has the characteristics such that $c^{'}(z) < 0$, and $c^{''}(z) > 0$. Then, the selling country's economic profit would be

$$W_S = \int_0^T e^{-rt}[p(x) - \tau - c(z)]x \, dt \tag{2}$$

The buying country controls $\tau(t)$ and the selling country does $x(t)$ to maximize the economic surplus and profit of each country. In a Stackelberg differential game, the buying country plays a role as the leader and the selling country does a role as the follower. Thus, if the buying country announces the time path of tariffs to be imposed on imported timber, then the selling country takes this as given, and maximizes her economic profit by choosing $x(t)$. Both the buying country and the selling county are constrained by the stock of forest $z(t)$. The dynamic change of forest stock is given by

$$\frac{dz(t)}{dt} = -h(t) + g(t) , \qquad z(0) = z^o \quad \text{given,} \tag{3}$$

where $g(t)$ denotes the amount of young trees that is naturally regenerated. Equation (3) expresses the change of the total size of forest stock at each time period; it is the

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[1] For the simplicity, both regeneration and forest management cost are not considered in this model. When they are included in the model, the result of this result may not change.

difference between the amount of old-growth trees harvested and the amount of young trees regenerated. In this game, when the buying country determines what time path for timber tariff, it takes the reaction of the buying country into account. If the selling country takes $\tau(t)$ as given, then the selling country maximizes Equation (2) subject to Equation (3) and $x = hV$. The selling country's optimization problem can be summarized as

$$Max \int_0^T e^{-rt} \, [p(x) - \tau - c(z)] x dt$$

$$s.t \quad \frac{dz}{dt} = -h + g \qquad\qquad z(0) = z^o$$

$$x = hV$$

The necessary conditions of this problem can be derived as follows;

$$e^{-rt}[P(x) - \tau - c(z)]V - \lambda = 0 \tag{4}$$

$$\frac{d\lambda}{dt} = e^{-rt} c^{'}(z) x \tag{5}$$

$$\frac{dz}{dt} = -h + g \, , \qquad\qquad z(0) = z^o \tag{6}$$

where $\lambda(t)$ denotes the costate variable for the stock of forest. The buying country takes these conditions as constraints on his maximization problem. The buying country's optimization problem can be

$$Max W_B = \int_0^T (e^{-rt}[u(x) - c(z)x] - \frac{\lambda x}{V})\, dt \tag{1-1}$$

$$s.t \quad \frac{d\lambda}{dt} = e^{-rt} c^{'}(z) x$$

$$\frac{dz}{dt} = -h + g \qquad z(0) = z^o$$

$$x = hV$$

We can get the necessary conditions for optimization and arrange them as follows;

$$e^{-rt}[p(x) - c(z)]V - \lambda - \psi_1 + \psi_2 e^{-rt} c^{'}(z) V = 0 \tag{7}$$

$$\frac{d\psi_1}{dt} = e^{-rt}[c^{'}(z) - \psi_2 c^{''}(z)]x \tag{8}$$

$$\frac{d\psi_2}{dt} = h \tag{9}$$

$$\frac{d\lambda}{dt} = e^{-rt} c^{'}(z) x \tag{10}$$

$$\frac{dz}{dt} = -h + g, \qquad\qquad z(0) = z^o \tag{11}$$

where $\psi_1(t)$ and $\psi_2(t)$ are the costate variables for $z(t)$ and $\lambda(t)$, respectively. Because $\lambda\,(0)$ is free, the initial condition of $\psi_2(t)$ is zero[2]. With this condition, the solution of differential equation, Equation (9), is

$$\psi_2(t) = z^o - z(t) + \omega(t) \tag{12}$$

[2] Equation (10) describes a jump state constraint, and a jump state variable has a specific property that the initial value of the costate variable for it is zero. For more detail on jump state constrains, refer to Siman and Cruz(1973) and Karp and Newberry(1993).

where $\omega(t)$ denotes the accumulation of young trees that are naturally regenerated between [0, *t*], i.e. $\omega(t) = \int_0^t g(s)ds$. Equation (12) implies $\psi_2(t) > 0$ for all *t*.

Applying optimal control theory, it can be shown that the costate variable for the selling country's rent at time *t* is the same as the shadow value for it at time *t*, i.e. $\psi_2(t) = \partial W_B(t) / \partial \lambda(t)$ [3]. From this, $\psi_2(t) > 0$ implies that as the selling country's rent increases, the buying country's welfare would improve. In addition, Equation (4) provides the information on what a size of optimal tariff the buying country ought to impose on imported timber at each time period. With these, for any $t > 0$, the buying country would like to revise the trajectory of import tariff on timber announced at initial time rather than to keep on it. In doing so, it could increase the seller's rent, and therefore, improve her welfare level. Therefore, we significantly suggest that the buying country ought to impose a dynamically inconsistent open loop tariff on imported timber in order to improve her welfare.

Second, let us examine the dynamic structure of optimal tariff on imported timber on the condition that cost function is independent of the stock of forest. Suppose that the cost for timber production be the function of timber harvested, $c(x(t))$, and the cost increases at an increasing rate with harvests, $c'(x) > 0$, and $c''(x) > 0$. Then, Equation (2), which denotes the selling country's economic profit, can be changed into

[3] For the rigorous proof that the costate variable for the state variable is the same as the shadow value for it at each time period, see Luenberg(1969, pp. 239-257) and Clark(1990, pp.102-107).

$$W_S = \int_0^T e^{-rt}[(p(x) - \tau)x - c(x)]\,dt \tag{2-1}$$

The selling country's objective is to maximize Equation (2-1) subject to Equation (3) and $x = hV$. Necessary conditions of this problem are

$$e^{-rt}[(p(x) - \tau) - c^{'}(x)]V - \lambda = 0 \tag{4-1}$$

$$\frac{d\xi}{dt} = 0 \tag{5-1}$$

$$\frac{dz}{dt} = -h + g, \qquad z(0) = z^o \tag{6-1}$$

where $\xi(t)$ is the costate variable for the stock of forest. It is obvious that the solution of the differential equation for the dynamics of the costate variable, Equation (5-1), is $\xi(t) = 0$. This implies that the selling country's rent is zero to be paid. With these constraints, the buying country's optimization problem can be expressed as

$$Max\, W_B = \int_0^T e^{-rt}[u(x) - c^{'}(x)x]dt \tag{1-2}$$

$$s.t \qquad \frac{dz}{dt} = -h + g \qquad z(0) = z^o$$

$$x = hV$$

We can get the necessary conditions for optimization and arrange them as follows;

$$e^{-rt}[p(x)-c''(x)x-c'(x)]V-\varsigma(t)=0$$

$$\frac{d\varsigma}{dt}=0$$

$$\frac{dz}{dt}=-h+g, \qquad z(0)=z^{o}$$

Comparing Equation (1-2) with Equation (1-1), we observe that the selling country's rent does not affect the buying country's economic surplus at all. Therefore, there is no reason that the buying country alters the trajectory of optimal tariff on imported timber announced at the time of beginning as discussed above. This suggests that the buying county ought to impose a dynamically consistent open loop tariff on imported timber to improve her welfare.

THE CHARACTERISTIC OF PRODUCTION RENT

As discussed above, we identified that the dynamic structure of tariff on timber trade is dependent on whether or not the selling country's rent restricts the importing country's economic surplus. In this section, we, hence, examine the characteristic of the selling country's rent in detail that plays a critical role in determining the dynamic structure of tariff on timber. For this purpose, we first investigate the property of the shadow value for the stock of forest that can be decomposed into the scarcity effect and the cost effect. As one of the necessary conditions for the selling country's optimization problem, Equation (5) denotes the dynamics of the costate variable for the stock of forest. With the terminal value of $\lambda(T)$, this differential equation has the general solution as

$$\lambda(t) = \lambda(T) - \int_t^T e^{-rs} c^{'}(z(s))x(s) \; ds \tag{13}$$

This is generated by treating $d\lambda/dt$ as a linear first order differential equation with a constant coefficient and variable terms. For the derivation of Equation (13) see the Appendix. This costate variable denotes the shadow value for stock of forest at time t, and has the role in Equation (4) of rationing the forest stock over time. It is also the current value rate of change in the solution value of Equation (2) per unit change in the resource stock at time t. For time *t*, this can be stated as $\lambda(t) = \partial W_S(t)/\partial z(t)$.

The costate variable for the stock of forest can be separated into two components according to Equation (13):

	$\lambda(T)$	is the Scarcity Effect
and	$-\int_t^T e^{-rs} c^{'}(z(s))x(s) \; ds$	is the Cost Effect

The scarcity effect is simply the terminal scarcity value at the optimal stopping time of timber harvest, and the cost effect is the present value of the cost saving associated with the marginal unit of increment of forest stock at time *t*. For this costate variable, we have the following three observations: (1) If the forest stock is not exhausted there will be no scarcity effect (i.e. $\lambda(T) = 0$). (2) The scarcity effect can occur only if the forest stock is exhausted[4]. (3) The cost effect approaches zero as t approaches T, which implies that $-\int_t^T e^{-rs} c^{'}(z(s))x(s)\ ds \to 0$ as $t \to T$. For the case where the forest stock is exhausted and $c^{'}(z) < 0$, the shadow value contains the cost savings associated with the marginal increment of forest stock and the scarcity effect evaluated at the terminal time of timber harvest. However, if the forest stock is not exhausted but optimal harvests take place over a positive time period, then the shadow value is due solely to the cost savings, but the scarcity effect does not occur. Referring to the characteristics of the costate variable for the stock of forest discussed above, let us investigate the property of the selling country's rent being paid. If we transform Equation (1-1) by using Equation (13), the buying country's objective function can be expressed as

$$W_B = \int_0^T (e^{-rt}[u(x) - c(z)x] - [\lambda(T) - \int_t^T e^{-rs} c^{'}(z(s))x(s)ds]\frac{x}{V})\ dt \qquad (14)$$

This equation tells us that the buying country's objective functional is maximized by setting $\lambda(T) = 0$. So, the buying country drives the selling country's rent to zero at the end of game. With this proviso, the costate variable for the stock can be expressed as

$$\lambda(t) = -\int_t^T e^{-rs} c^{'}(z(s))x(s)\ ds$$

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[4] The transversality condition for the selling country's optimization, $\lambda(T) \geq 0, \ \ \lambda(T)z(T) = 0$,provides information on the existence of the scarcity effect of the costate variable for the stock of forest.

This shows that the costate variable for the stock of forest does not contain the scarcity effect, but contains solely the cost effect. Hence, if the cost function for timber production is dependent on the forest stock, the selling country takes the cost effect of the shadow value for the stock of forest as a rent. In connecting this result with Equation (1-1), we identify that the cost effect of the costate variable for the stock of forest does restrict the buying country's economic surplus, and causes the optimal tariff on imported timber to be dynamically inconsistent.

CONCLUSIVE REMARKS

In order to identify the dynamic structure of optimal tariff on imported timber, we specified the law of motion for the stock of forest and the quantity of timber production that reflects the biological characteristics of forest in a realistic sense. We adapted a Stackelberg differential game of timber trade in which the buying country has a market power, and established two modeling approaches to take into consideration the form of the cost function for timber production. We identified that if the cost function is dependent on the stock of forest, the buying country ought to impose a dynamically inconsistent open loop tariff on imported timber in order to improve her welfare level. On the contrary, if the cost function is independent of it, the buying country has to impose a dynamically consistent tariff announced at an initial time period. We also observed the characteristics of the shadow value for the stock of forest that can be separated into the scarcity effect and the cost effect, and as a result, identified that the selling country takes the cost effect of the shadow value for the stock of forest as a production rent. In this context, our findings would provide significant policy implications in the following two aspects. One is that our policy suggestion would help the buying country establish trade policies of how to impose optimal tariffs on imported timber over time in order to improve her welfare level. The other is that our policy suggestion would affect the selling country's decision on what amount of timber she would produce to export abroad. Unless the buying country levies an optimal tariff at each time period on timber imported, the selling country might overexploit forest, and result in a Pareto inefficient timber production. As the concern about global warming that adversely impacts on the economic system has increased, emphasis has been put on the sustainable forest management to promote the conservation of forest that plays a role in sequestering atmospheric carbon. In this regard, our policy suggestion would contribute to the selling country's establishment of an effective forest conservation policy to improve the value of forests that produce a positive externality on the economic system.

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APPENDIX

We start with Equation (6)

$$\frac{d\lambda(t)}{dt} = e^{-rt} c^{'}(z(t))x(t)$$

With the terminal value $\lambda(T)$, this differential has general solution:

$$\lambda(t) = A + \int e^{-rt} c^{'}(z(t))x(t)) \ dt \qquad \text{(A1)}$$

where A is the constant of integration. Define

$$F(t) := \int e^{-rt} c^{'}(z(t))x(t) \ dt$$

Thus, Equation (A1) can be written

$$\lambda(t) = A + F(t)$$

and $$\lambda(T) = A + F(T)$$

Solving for A yields,

$$A = \lambda(T) - F(T)$$

Using this result we get,

$$\lambda(t) = \lambda(T) - [F(T) - F(t)]$$

and $$\lambda(t) = \lambda(T) - \int_t^T e^{-rs} c^{'}(z(s))x(s) \ ds$$