Incentive Mechanisms for International Public Goods under Productivity Uncertainty

Jin Kim [a] and Seung-jin Shim [b]

a) Division of Fiscal Analysis, Korea Institute of Public Finance, Seoul, Korea

b) School of Economics and Trade, Kyungpook National University, Taegu, Korea.

Abstract

We investigate incentive mechanisms for the optimal supply of and demand for international public goods under uncertainty of production costs among nations in a region. We formalize an incentive mechanism as an international economic system (IES), which contains the decision rules about the supply of and demand for international public goods (IPG) under productivity uncertainty, i.e., incomplete information on productivity differentials. Ihori (1996) analyzes the impact of productivity differentials on welfare within a non-cooperative game setup. By using expenditure minimization behavior he could formalize indirect utility functions as valuation functions. We use indirect utility functions as valuation functions in order to formalize the setup of incentive mechanisms. Based on the mechanism design theory of the Groves mechanisms, we analyze the possibility of incentive mechanisms with monetary transfers in the case that there is uncertainty in production costs of IPG. We finally characterize a necessary and sufficient condition for the existence of the mechanism. That is, we find the incentive

mechanism exists iff the nation-wise income as well as the global income would be greater than the critical values determined by productivity uncertainty.

Key words: International Public Goods, International Economic Systems, Mechanism Design, Transfers, Groves Mechanisms

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Corresponding Author: Seung-jin Shim, School of Economics and Trade, Kyungpook National University, 1370 Sangyuk-dong, Puk-gu, Daegu, Korea.

Tel: +82-53-950-5437. Fax: +82-53-950-5429. e-mail : shimsj@mail.knu.ac.kr

1. Introduction

This paper investigates an incentive mechanism as an international economic system (IES), which contains the decision rules about the supply of and demand for international public goods (IPG) under cost uncertainty, i.e., incomplete information on productivity differentials. Ihori (1996) analyzes the impact of productivity differentials on welfare within a non-cooperative game setup. By using expenditure minimization behavior he could formalize indirect utility functions as valuation functions.

We use indirect utility functions as valuation functions in order to formalize the setup of incentive mechanisms. Based on the mechanism design theory of the Groves mechanisms, we analyze the possibility of incentive mechanisms with monetary transfers in the case that there is uncertainty in production costs of IPG. We finally characterize a necessary and sufficient condition for the existence of the mechanism.[1]

2. Basic Model

We assume that n countries are interested in producing and consuming two goods; an international public good G and a private good c. G consists of each country i's contribution g_i, which is transformed from the private good c_i; $G = g_1 + g_2 + \ldots + g_n$. By assuming that c_i is a numeraire, country i's budget constraint would be $c_i + \theta_i g_i = Y_i$, where θ_i denotes country i's unit cost for producing g_i, and Y_i income, respectively. We restrict our attention to quasi-linear utility functions; $u^i(c_i, G) = c_i + b_i \ln(G)$ for country i.

We assume that b_i's and Y_i's are given fixed and public information, and that θ_i's are private information called types. Let $\Theta_i = [\underline{\theta}, \bar{\theta}]$ be the common set of types with $\underline{\theta} > 0$. Let us decompose the state set $\Theta = \prod_i \Theta_i$ into i subsets; for each i, $\Theta_i^* = \{ \theta \in \Theta \mid \theta_i = \min_j \theta_j \}$ is the set of the states where country i has the lowest

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[1] Laffont and Martimort (2005) recently analyze the design of incentive mechanisms for the provision of transnational public goods under asymmetric information among countries.

production cost. Since each country i knows her cost parameter θ_i and is only aware of the distribution of the other country's cost parameters, one of the important roles of the IES would be how to obtain the true information about θ_i 's from the member countries.

In the case of two countries with $\theta_1 < \theta_2$, the Cournot-Nash equilibrium according to Ihori (1996) would be G= $g_1 = \frac{b_1}{\theta_1}$, and each country's indirect utility from Nash equilibrium is

$$U_N^1(\theta_2; Y, b) = b_1 \ln(\frac{b_1}{\theta_1}) + Y_1 - b_1 \quad \text{with} \quad g_1(\theta_2) = \frac{b_1}{\theta_1} \tag{1}$$

$$U_N^2(\theta_1; Y, b) = b_2 \ln(\frac{b_1}{\theta_1}) + Y_2 \quad \text{with} \quad g_2(\theta_1) = 0\,, \tag{2}$$

respectively. Here, country 2 is the free-rider and the IPG is under-produced. Thus, under incomplete information, the first-best allocation would not be implemented even through Nash equilibrium behaviors.[2]

In order to implement the first-best allocation, we use the Groves mechanism as the optimal IES. Thus, we assume that countries can install the IES of a central agency that collects the reports on types and decides allocations and transfers.

When the IES can obtain the true information, it may decide that country 1 (additionally) produces $g_2 = \frac{b_2}{\theta_2}$ instead of country 2 and that country 2 pays the cost

[2] See Ihori (1996) for details.

$\theta_1\, g_2 = (\frac{\theta_1}{\theta_2})\, b_2$ to country 1. Then, each country's indirect utility from the IES is

$$U_M^1(\theta; Y, b) = b_1 \ln\left(\frac{b_1}{\theta_1} + \frac{b_2}{\theta_2}\right) + Y_1 - \theta_1\left(\frac{b_1}{\theta_1} + \frac{b_2}{\theta_2}\right) + \left(\frac{\theta_1}{\theta_2}\right) b_2, \tag{3}$$

$$U_M^2(\theta; Y, b) = b_2 \ln\left(\frac{b_1}{\theta_1} + \frac{b_2}{\theta_2}\right) + Y_2 - \left(\frac{\theta_1}{\theta_2}\right) b_2, \tag{4}$$

respectively. One important observation is that for some parameters, the utility of country 2 from the direct mechanism in (4) is greater than that from the free-riding at Nash equilibrium in (2).

By denoting the reports as $\hat{\theta} = (\hat{\theta}_1, \ldots, \hat{\theta}_n)$ and with $\theta^* \equiv \min_j \hat{\theta}_j$, we can express the Pareto allocation, $g_i(\hat{\theta}) = \frac{b_i}{\theta^*}$ and $G(\hat{\theta}) = \sum_i (\frac{b_i}{\theta^*})$. Only the most efficient country would produce the IPG and the other countries would pay costs. Let A be the set of all feasible outcomes with $(c_1, \ldots, c_n, g_1, \ldots, g_n) \in A$. Then, by using indirect utility functions from the above-mentioned method, we may set up a valuation function $v_i(\cdot, \theta_i)$ over A for each type θ_i. Specifically, the payoff of country i with type θ_i from the reports $\hat{\theta}$ is

$$v_i((c(\hat{\theta}), g(\hat{\theta})), \theta_i) = b_i \ln(\sum_k (\frac{b_k}{\theta^*})) + Y_i + \Gamma_{\Theta_i^*}(\hat{\theta})[-\theta_i \sum_k (\frac{b_k}{\theta^*}) + \sum_k b_k] - b_i \tag{5}$$

where the index function $\Gamma_{\Theta_i^*}(\hat{\theta})$ has value 1 if $\hat{\theta} \in \Theta_i^*$, 0 otherwise. We assume that non-producing countries exploit the whole gains of trade. We can verify that the previously-mentioned valuation functions satisfy the convexity condition of Holmström

(1979). Thus, by following Makowski and Mezzetti (1994) we can apply the Groves mechanism into our setup.

3. Incentive mechanism design under uncertainty of θ_i

A direct mechanism is denoted by (Θ,<s, t>). Θ is the message space of the type reports. <s, t> is an outcome function which consists of a decision rule s : $\Theta \rightarrow$A and a transfer scheme t=(t_1,..., t_n) with $t_i : \Theta \rightarrow \mathbb{R}$. Given <s,t>, country i's payoff with type θ_i from a report $\hat{\theta}$ is $v_i(s(\hat{\theta}), \theta_i) + t_i(\hat{\theta})$. We will use the notation <s,t> for a direct mechanism.

The global gain function from the Pareto allocation is

$$g(\theta) \equiv \sum_i v_i((c(\theta), g(\theta)), \theta_i) \equiv [\sum_i b_i \ln(\sum_k (\frac{b_k}{\tilde{\theta}})) + \sum_i Y_i - \sum_i b_i], \tag{6}$$

where $\tilde{\theta} = \min_i \theta_i$.

As a direct mechanism is installed and a state is realized, countries face a direct revelation game. A mechanism <s,t> is dominant-strategy incentive compatible if every country has the incentive to report her own type honestly regardless of the others' report schemes at any state, i.e., for all i, for all θ_{-i}, for all θ_i, and for all θ'_i,

$$v_i(s(\theta_{-i}, \theta_i), \theta_i) + t_i(\theta_{-i}, \theta_i) \geq v_i(s(\theta_{-i}, \theta'_i), \theta_i) + t_i(\theta_{-i}, \theta'_i). \tag{7}$$

A decision rule s is outcome-efficient if $\sum v_i(s(\theta), \theta_i) = g(\theta)$ for all θ, that is, if it always realizes the global gain. A mechanism <s,t> is a first-best dominant-strategy mechanism if it is outcome-efficient and dominant-strategy incentive compatible.

Since our setup satisfies the convexity condition in Holmström (1979), we can use his result that a mechanism is a first-best dominant-strategy if and only if it is a Groves mechanism. Following Makowski and Mezzetti (1994), we can define the participation charge on country i at state θ as the difference of i's payoff from the global gain; $h_i(\theta) \equiv g(\theta) - v_i(s(\theta),\theta_i) - t_i(\theta)$ for all i and θ. A mechanism <s,t> is a Groves mechanism if it is outcome-efficient and its participation charges on country i are independent of i's type for each i. Then, country i's payoff from the participation in a Groves mechanism at state θ is

$$v_i(s(\theta),\theta_i) + t_i(\theta) = g(\theta) - h_i(\theta_{-i}) . \tag{8}$$

Since each country's participation charges are non-distortionary lump-sum in Groves mechanisms, there is no incentive for any country to lie in the direct revelation game. One simple Groves mechanism is a mechanism with zero participation charges; $h_i(\theta) = 0$ for all i and for all θ. Then each country's payoff would be equal to the global gain $g(\theta)$ at each θ, and by using (8) we know that the zero-charge Groves mechanism incurs a deficit $g(\theta) - v_i(s(\theta),\theta_i)$ for country i at state θ. The (*ex ante*) expected budget deficit for country i in the zero-charge Groves mechanism is

$$B_i \equiv E[g(\theta) - v_i(s(\theta), \theta_i)] = E[\sum_{j \neq i} b_j \ln(\sum_k \frac{b_k}{\tilde{\theta}}) + \sum_{j \neq i}(Y_j - b_j)]. \tag{9}$$

A mechanism <s,t> is *ex post* individual rational (EPIR) if its payoff is not negative for any country at any state. [3]

Since the IES does not observe country i's type, the maximal amount that the IES can charge on country i without violating country i's EPIR condition is, by using (8), $c_i(\theta_{-i}) = \min_{\theta_i}\{g(\theta)\}$ for all θ_{-i}. Then, the (*ex ante*) expected lump-sum charge without violating country i's EPIR condition is

$$C_i \equiv E[c_i(\theta_{-i})] = E[\sum_j b_j \ln(\sum_k \frac{b_k}{\tilde{\theta}_{-i}}) + \sum_j Y_j - \sum_j b_j] \tag{10}$$

where $\tilde{\theta}_{-i} = \min_{j \neq i} \theta_j$.

(9) and (10) might be interpreted as two edges of a `benefit-charge' analysis, in that for each country the IES measures the benefit from the zero-charge Groves mechanism and levies the corresponding lump-sum charge for it.

In plain terms, an annoying problem in the Groves mechanism literature is how to fairly divide the expected surplus from the mechanism. We introduce two surplus-division methods; equal division and proportional division. The former is related with *ex ante* budget balancedness (EABB), $E[\sum_i^n t_i(\theta)] = 0$. The latter is related with zero expected net transfer (ZENT), $E[t_i(\theta)]=0$ for each i.

[3] We assume that the outside option payoff of any country i at any state is zero.

Makowski and Mezzetti (1994) obtain a necessary and sufficient condition for the existence of the efficient dominant-strategy mechanism with EPIR and EABB; $\sum_i C_i \geq \sum_i B_i$. Now, we propose a necessary and sufficient condition for the existence of an efficient dominant-strategy mechanism with EPIR and ZENT.

Proposition: There exists an IES which is first-best dominant-strategy incentive compatible, ex post individual rational (EPIR), and zero-expected net-transferred (ZENT) iff $\mathrm{E}[\mathrm{c}_\mathrm{i}(\theta_{-\mathrm{i}})] \geq \mathrm{E}[\mathrm{g}(\theta) - \mathrm{v}_\mathrm{i}(\mathrm{s}(\theta), \theta_\mathrm{i})]$ for all i.

Proof: (If) Define a transfer scheme t by $\mathrm{t}_\mathrm{i}(\theta)=\mathrm{g}(\theta) - \mathrm{v}_\mathrm{i}(\mathrm{s}(\theta), \theta_\mathrm{i}) - \mathrm{c}_\mathrm{i}(\theta_{-\mathrm{i}}) + \mathrm{K}_\mathrm{i}$ for all i and θ, where $\mathrm{K}_\mathrm{i} = \mathrm{E}[\mathrm{c}_\mathrm{i}(\theta_{-\mathrm{i}})] - \mathrm{E}[\mathrm{g}(\theta) - \mathrm{v}_\mathrm{i}(\mathrm{s}(\theta), \theta_\mathrm{i})] \geq 0$. Then, <s, t> is a Groves mechanism. It's trivial to check out EPIR and ZENT.

(Only if) By the result of Makowski and Mezzetti (1994), it suffices to show that E[$\mathrm{t}_\mathrm{i}(\theta)$]=0 for all i. By definition, $E[t_i(\theta_i)] = 0$ for all i. Q.E.D.

4. Implications

The above conditions in the proposition bring forth the range of the income level for the existence of the incentive mechanism in the two-country case; for EABB with (11) and for ZENT with (12), respectively.

$$Y_1 + Y_2 \geq N \equiv E[\ln[(\frac{\theta_1\theta_1}{\tilde{\theta}(b_1+b_2)})^{b_2}(\frac{\theta_2\theta_2}{\tilde{\theta}(b_1+b_2)})^{b_1}] + b_1 + b_2, \quad (11)$$

$$Y_i \geq N_i \equiv E[\ln[(\frac{\theta_i}{\tilde{\theta}})^{b_j}(\frac{\theta_j}{b_1+b_2})^{b_i}]] + b_i, \text{ where i, j=1, 2 and } i \neq j. \quad (12)$$

<<Fig 1> here>

Not only the global income is important, but also each country's income must be large enough to match the existence of the IES. On the other hand, the critical values representing the range of income levels are determined by the parameters of utility functions. Under cost uncertainty, the absolute level of income is an important criterion for establishing an efficient IES with incentive compatibility and individual rationality.

References

Dasgupta, P.S., P.J. Hammond, and E.S. Maskin, 1979, The implementation of social choice rules: Some general results on incentive compatibility, Review of Economic Studies 46, 185-216.

Groves, T. and M. Loeb, 1975, Incentives and public inputs, Journal of Public Economics 4, 211-226.

Holmström, B., 1979, Groves schemes on restricted domains, Econometrica 47, 1137-1144.

Ihori, T, 1996, International Public Goods and Contribution Productivity Differentials, Journal of Public Economics 61, 139-154.

Laffont J.J. and D. Martimort, 2005, The design of transnational public good mechanisms for developing countries, Journal of Public Economics 89, 159-196.

Makowski, L. and C. Mezzetti, 1994, Bayesian and weakly robust first best mechanisms: Characterizations, Journal of Economic Theory 64, 500-519.

<Fig. 1>