Dynamic Patterns of Trade Imbalances with Recursive Preference

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Abstract

Based on the recursive preference approach, the dynamic and global properties of the two-country open economy are examined with one good and inputs of labor and capital, with capital being freely traded internationally. First by showing that the world's consumption increases (resp. decreases) with an increase (resp. decrease) in the world's capital, the global stability of the economy is obtained. Second, the non-monotonicity of consumption between the impatient country 1 and the patient country 2 is established. Third, with the Cobb-Douglas type production function and the country 1 's technological superiority, the dynamic trade patterns and the asset-debt position are derived.

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I. Introduction

This paper shows the global properties of the two country open economy where the representative consumer maximizes not additive but recursive utility over time with perfect foresight and free capital mobility between the two countries.

Our first contribution is the exhibition of the global stability of a two country open model with recursive preference and the global properties of aggregate consumption as an increasing function of the world's capital stock with fewer sufficient conditions than those used by Epstein(1987b). According to Epstein(1987b), the global stability of the recursive preference model intuitively stems from the following introspective grounds; the expected future consumption stream changes the rate of time preference (the discount rate). Hence if future utility is above (resp. below) the steady state, the agent will discount the future more (resp. less) heavily to reduce (resp. increase) expected utility down (resp. up) to the steady state level and reduce (resp. increase) investment[1/].

While Epstein(1987b) employs the Lyapounov function to show global stability, we derive the aggregate consumption function as an increasing function of the world's total capital stock and show indirectly the existence of the globally stable saddle point path of aggregate consumption c and the world's capital stock k with $c'(k) > 0$.

Our second contribution is indebted to Devereux and Shi(1991)'s two country open growth model with recursive preference. We generalize their local results into global ones. They show that each country's consumption near the stationary state is non-monotonistic in the sense that one country's consumption increases while the other country's consumption decreases, because the distributional effects(i.e., the foreign asset-debt position) and the world's capital stock effects(i.e., growth effects) exert conflicting effects on a country's saving and hence on consumption.[2/] We generalize their results by showing that the patient country 2's consumption is always larger than that of the impatient country 1 not only near the stationary state but also globally when the world's capital stock increases, and the same holds eventually when the world's capital stock decreases.

Furthermore, as for the global trade patterns and asset-debt positions, given the specification of the felicity and production functions, we show in a unified manner, that the patient country 2 remains both an importer of the good and a creditor throughout the transitional period in the world's capital accumulation where the patient country 2's consumption is always higher than that of the impatient country 1, and the same holds in the world's capital stock decumulation as the patient country 2's consumption eventually becomes higher than that of the impatient country 1. In short both the trade-deficit position and creditor position of the patient country 2 reinforces its higher consumption along the transitional period.

The significance of global analysis compared to local analysis is seen most vividly by checking the consumption dynamics of the two countries as shown in Fig.2. For the case of the world's capital accumulation the patient country 2's consumption c_2 is always higher than that of the impatient country 1, c, c_1 and c_2 keep increasing with the increase in the world's capital stock k, while c_1 increases at the initial low level of k but starts decreasing as k approaches its stationary value $\bar{k}$. Similar conclusions hold for the case of the world's capital decumulation ; c_2 keeps decreasing as k decreases, but c_1 decreases as k decreases from a very high initial level of k, then starts increasing as k approaches $\bar{k}$. These global characteristics of consumption dynamics cannot be inferred from the local analysis. For example, it is possible for both c_1 and c_2 to move in the same direction when these are far from their stationary values in the global analysis, which is impossible in the local analysis. If this happens in reality, local analysis casts the recursive preference assumption in doubt, but of course global analysis fully explains this situation.

The local stability of (a one good, two input open economy with) many heterogeneous agents with recursive preferences has been analyzed by Epstein (1987a), and its global stability also by Epstein (1987b). To the best of our knowledge, not many authors have analyzed the global properties of recursive preference model. One exception is Palivos, Wang and Zhang (1997) who obtained the global stability of the balanced growth path and its characteristics of the closed economy, employing an *Ak* model.

In the next section, first the consumption of the world is shown to increase globally with the world's capital accumulation (Theorem 1), which is predicted by Epstein(1987b). Second, by restricting our model basically to Devereux and Shi (1991)'s simplified case, the consumption of country 1, which is more impatient than county 2, is shown to be less than that of country 2 in the case of the world's capital accmulaton (Theorem 2), while in the world's capital decumulation case the possibility of the reversed ranking is shown (Theorem 3). Next, assuming the Cobb-Douglas type production function, country 1 is shown to export the good and remain a debtor throughout the transitional periods when the world's capital accumulates, while when the world's capital decumulates either the same trade patterns and asset-debt position prevail from the beginning, or do so after a certain period. (Theorem 4).

As for recent empirical discussions on the non-convergence or non-monotonicity of the world's growth rates, the differences in per capita GDP growth rates among developing countries and developed countries remain unchanged and hence per capita income among these countries remain. (See, e.g., Barro and Sala-i-Martin (1995), Chapt.1, or Durlauf and Quah(1999).)

In this respect, Theorem 2 explains such a non-convergence in terms of the difference in per capita consumption between the two countries based on the recursive preference approach.

Furthermore, the recent rapid economic growth of such Asian countries as China (including Hong Kong), Korea and Thailand is accompanied by trade surpluses, or exports as an engine of growth and a low average propensity to consume. (See, e.g., International Financial Statistics(2003) and Statistical Yearbook(2003).) Theorem 3 tries in a unified manner, to explain these accompanying characteristics of Asian countries' economic growth by depicting them as the more impatient countries that consume less than the other countries and enjoy a trade surplus while remaining debtors. [3/]

As is well known, quite a few authors have contributed to establish and elaborate the concept and significance of recursive utility. Here we just mention a few of them. Uzawa (1968) first established this concept. Then Epstein (1983), (1987a) further extended it. Obstfeld (1981) explained the significance of recursive preference by developing models of exchange-rate and current account determination of a small open economy, and Epstein and Hynes (1983) created several applications addressing macro economic issues. Becker, Boyd and Sung (1989) provided the existence of the optimal capital accumulation paths of the recursive preference model. Obstfeld (1990) explained the significance of recursive preference very heuristically and showed the global stability of the closed model.

Relating to our present work, Becker (1980) derived Ramsey's conjecture that in the long run steady state the income distribution is determined by the lowest discount rate. Buiter (1981) analyzed the asset-debt positions of a two country overlapping generation model. Lipton and Sachs (1983) analyzed the saving and capital accumulation of a two good, two country model with time-additive preference employing a simulation approach.

In the next section, the basic framework of our model is introduced and in the third section, the global stability of the two country open economy is derived. Basically we follow the framework of Epstein (1987a) and Devereux and Shi(1991).

II. Basic Framework

II. 1 Social Planner's Optimum in Open Economy

First we investigate the global stability of the two country open economy. Following Devereux and Shi (1991), we assume there is one consumer in each country who supplies one unit of labor. First we analyze the case of the social planner's optimum. Let $C=(C_1,C_2)$ be the streams of the consumption of the country 1 and the country 2 with $C_i=(c_i(t))_{t=0}^{\infty}$, $i=1,2$ and c_i is the per capita consumption of country *i* at time *t*. Let $c=c_1+c_2$ be the world's per capita consumption and let $k=k_1+k_2$ be the capital of the world where k_i is the capital of country *i*, $i=1,2$. Capital is freely and costlessly traded between the two countries implying that the marginal products of capital of both countries are equalized. Since efficient production implies $f'_1(k_1)=f'_2(k_2)$, i.e., the equality of both countries' marginal products of capital given $k=k_1+k_2$, we can express $f(k)=f_1(k_1)+f_2(k_2)$ with $f'(k)=f'_1(k_1)=f'_2(k_2)$ where $f_i(k_i)$ is the production function of country *i*, $i=1,2$. f_i satisfies the Inada condition. $f'(k)=f_i'(k_i)$, $i=1,2$ implies that the capital of each country moves in the same direction, i.e., $\dot{k}_1>0 \Leftrightarrow \dot{k}_2>0 \Leftrightarrow \dot{k}>0$. The social planner maximizes the following utility;

$$\sum_i \alpha_i(0)U_i(C_i)=\sum_i \alpha_i(0)\int_0^{\infty} v_i(c_i(t))e^{-\int_0^t u_i(c_i(\tau))d\tau}dt \tag{1}$$

subject to the law of motion of capital

$$\dot{k}=f(k)-\sum_i c_i \tag{2}$$

and

$$\dot{\alpha}_i=-\alpha_i(t)u_i(c_i(t)) \tag{3}$$

where $\alpha_i(t)=\alpha_i(0)e^{-\int_0^t u_i(c_i(\tau))d\tau}$, $i=1,2$ and $\alpha_i(0)$ is the weight of the utility of country i at $t=0$. As seen by definition $\alpha_i(t)$ changes continuously overtime. $U_i(C_i)=\int_0^{\infty} v_i e^{-\int u_i d\tau}dt$, where $v_i(c_i)<0$ and $u_i(c_i)>0$ are the instantaneous felicity functions of country *i*, $i=1,2$. Recursive preferences are expressed by the endogenously determined intertemporal substitution rate of consumption $\int_0^t u_i(c_i)d\tau$. v_i and u_i are assumed to satisfy the regularity conditions for the existence of the optimal path (C_1,C_2); $v_i'(c)\geq 0$, $v_i''<0$, $-\infty<\inf_{c_i>0} v_i(c)\leq \sup_{c_i>0} v_i(c_i)<0$, $u'_i(c_i)>0$, $u''_i(c_i)<0$ and $\inf_{c_i>0} u_i(c_i)>0$, $i=1,2$. (See Epstein (1987a), Lemma 1.) u_i and v_i, $i=1,2$ are thrice continuously differentiable. This is required to employ Lemma 1 as shown in Appendix I. Furthermore $u_i'(c_i)\to+\infty$ as $c_i\to 0$, $i=1,2$ is assumed. This is required for the positiveness of per capita consumption c_i, $i=1,2$ along the optimal path.

Let H be the Hamiltonian

$$H = \sum_i \alpha_i v_i(c_i) + \lambda(f(k) - \sum_i c_i) - \sum_i \phi_i \alpha_i u_i(c_i) \tag{4}$$

and we obtain the first order conditions,

$$\dot{\lambda} = -\lambda f'(k), \tag{5}$$

$$\dot{\phi}_i = -v_i + \phi_i u_i \ , i = 1,2 \tag{6}$$

$$\alpha_i v_i'(c_i) - \lambda - \phi_i \alpha_i u_i'(c_i) = 0,\ i = 1,2 \tag{7}$$

and the transversality conditions

$$\lambda k \to 0 \quad \text{as} \quad t \to \infty \tag{8}$$

and

$$\phi_i \alpha_i \to 0 \quad \text{as} \quad t \to \infty \ , \ i = 1,2 . \tag{9}$$

Here by letting $\mu_i = \alpha_i / \lambda$, (3) and (5) are rewritten as

$$\dot{\mu}_i = \mu_i(f'(k) - u_i(c_i)),\ i = 1,2 \tag{10}$$

and (7) as

$$\mu_i(v_i' - \phi_i u_i') = 1,\ i = 1,2 . \tag{11}$$

By differentiating (11) with help of (6) and (10), we obtain

$$\dot{c}_i = -(v_i' - \phi_i u_i')(v_i'' - \phi_i u_i'')^{-1}(f' - \rho_i) \tag{12}$$

where ρ_i is the rate of time preference and expressed by

$$\rho_i = (u_i v_i' - v_i u_i')(v_i' - \phi_i u_i')^{-1} > 0 . \tag{13}$$

It is known that equation (6) implies that ϕ_i is expressed as

$$\phi_i(t) = \int_t^{\infty} v_i(c_i(\tau)) e^{-\int_t^{\tau} u_i(c_i(s))ds} d\tau < 0,\ i = 1,2 \tag{14}$$

which is the utility of the i country's consumer starting from initial time $t \geq 0$.

II. 2 Equivalence between Competitive Equilibrium and the Social Planner's Optimum

Here in II. 2 we define competitive equilibrium in the open economy and reconfirm the well known equivalence between competitive equilibrium and social planner's optimum. In the competitive equilibrium, the consumer of country i tries to maximize

$$U_i(C_i) = \int_0^{\infty} v_i(c_i(t)) e^{-\int_0^{t} u_i(c_i(s))ds} dt, \quad i = 1,2 \tag{15}$$

subject to (3) and the budget constraint

$$\dot{m}_i = r m_i + w_i - c_i, \quad i = 1,2 \tag{16}$$

where $m_i > 0$ is the non-human wealth (abbreviated wealth) held by consumer i which is an equity claim on capital. Equities are traded internationally so that their interest rate is equal to the rental price of capital, r by arbitrage conditions. In other words, $m_i = k_i + F_i$ where $F_i = F_i(t) = m_i - k_i > 0\ (< 0)$ is the net foreign asset (debt) holdings by country i's consumer. Then from the flow budget constraint (16) and $F_i = m_i - k_i$, we obtain

$$\dot{F}_i = \dot{m}_i - \dot{k}_i = rF_i + ex_i \,,$$

in short

$$\dot{F}_i = rF_i + ex_i \tag{17}$$

where

$$ex_i = f_i(k_i) - c_i - \dot{k}_i \,,\; i = 1,2 \tag{18}$$

is the net export (i.e., balance of payments) of country i, which is the excess of domestic production over domestic absorption. (17) implies that country *i*'s foreign asset F_i increases by the amount of its current account, which is the sum of its interest earnings rF_i and the amounts of its net export ex_i. Here we define the home country to be a creditor (debtor) if $F_1 > 0 \; (< 0)$. From the profit maximization of the firm

$$r = f_i'(k_i) = f'(k) \,,\; i = 1,2 \tag{19}$$

and

$$w_i = f_i(k_i) - k_i f_i'(k_i) \,,\; i = 1,2 \tag{20}$$

hold, where w_i is the wage rate of country *i*. (19) implies that the capital of each country moves in the same direction, i.e., $\dot{k}_1 > 0 \Leftrightarrow \dot{k}_2 > 0$. Here

$$\sum_i m_i = \sum_i k_i = k \tag{21}$$

holds by definition.

II. 3 Existence and Uniqueness of the Stationary State of Competitive Equilibrium

The stationary state of the competitive equilibrium is obtained by letting $\dot{k} = 0,\; \dot{\mu}_i = 0,\; \dot{\phi}_i = 0,\; \dot{c}_i = 0$ and $\dot{m} = 0$. Hence we obtain at the stationary state *E*,

$$f(k) = \sum_i c_i = c, \tag{22}$$

$$u_i(c_i) = f'(k) = \rho_i, \;\; i = 1,2 \tag{23}$$

$$\phi_i = v_i(c_i)/u_i(c_i), \;\; i = 1,2 \tag{24}$$

$$m_i = (c_i - w_i)/r, \;\; i = 1,2 \tag{25}$$

and

$$F_1 = (1/2r)(c_1 - c_2 - f_1 + f_2). \tag{26}$$

Fig. 1

From (23), we observe $c_i = c_i(k)$ with $c_i'(k) < 0$, $i = 1,2$ and then from (22), the existence and the uniqueness of the stationary state *E* is immediate. We denote $c_i = \overline{c}_i,\; \phi_i = \overline{\phi}_i,\; m_i = \overline{m}_i,\; i = 1,2$ and $k = \overline{k}$ to be their respective values at the stationary state *E*. (The bar sign is attached to denote the value at the stationary state *E*.)

III. Global Stability of Competitive Equilibrium

III. 1 Global Properties of the Consumption Path *C*

Next, we show global stability. To show the global properties of the optimal path, we employ the following results;

We note that

ϕ_1 and ϕ_2 are bounded

from the regularity conditions of the instantaneous felicity functions u_i and v_i, i = 1,2.

Next we analyze the path $Y = (k, c)$ which is derived from the optimal path $X = (k, c_1, c_2, \phi_1, \phi_2)$ where $c = c_1 + c_2$ holds. We denote it also the optimal path Y. We employ the following Lemma;

Lemma 1 (Arnold (1988))

Suppose that the right-hand side of the equation

$$\dot{x} = f(x), \quad x \in U \subset R^n \tag{27}$$

is differentiable, and let *K* be a subset of *U* which is the compact closure of a subdomain of the domain *U*. Then every solution of equation (27) with an initial condition in *K* can be extended to the right (left) either up to the boundary of *K* or infinity in time. The solution of a nonautonomous equation with an initial condition in a compact subset of the extended phase space $R \times U$ can be extended up to the boundary of the subset.

Recalling that the right-hand sides of equations (2), (5) and (12) are differentiable in k, c_1, c_2, ϕ_1 and ϕ_2 and employing the boundedness of ϕ_1 and ϕ_2 and Lemma 1, we obtain c and ϕ_i, $i = 1,2$ can be expressed as continuous functions of k globally with $c'(k) > 0$ for $c = c_1 + c_2 = c(k)$ holding globally. (See Appendix I.)

Theorem 1

For the open economy, the optimal path of world consumption *c* and world per capita capital *k* is globally stable so that

$c = c(k)$ with $k \to \bar{k}$ as $t \to \infty$ monotonically and $c'(k) > 0$ [4/].

(See Appendix I.)

To the best of our knowledge, this result is new, for it is obtained only by investigating the global properties of the world's per capita consumption. We utilize this result next to further investigate the global properties of each country's consumption.

III. 2 Characteristics of Consumption Path

Here we investigate the characteristics of the consumption path of both countries. The felicity function v_i, i=1,2 is identically equal to -1, i.e.,

$$v_1(c_1) = v_2(c_2) = -1.$$

The felicity function u_i reflects that country 1 is more impatient than country 2, i.e.,

$$u_1(c) > u_2(c) \text{ for any } c.$$

We specify

$$u_1(c_1) = c_1^{\beta} + \gamma_1 \text{ where } 0 < \beta < 1 \text{ and } \gamma_1 > 0$$

and

$$u_2(c_2) = c_2^{\beta} + \gamma_2 \text{ where } 0 < \gamma_2 < \gamma_1$$ [5/]

Here u_i satisfies the regularity conditions mentioned before. ($v = -1$ also satisfies their conditions.) Here $u_1(\bar{c}_1) = u_2(\bar{c}_2) = \bar{\rho} = f(\bar{k})$ implies $\bar{c}_1 < \bar{c}_2$.

Fig. 2

As for the local properties of the slopes of the c_i, $i = 1,2$ curves, we see that

$c'_1(k) < 0 < c'_2(k)$ at $k = \bar{k}$. (See Appendix II.)

Fig. 2 corresponds to our case. First we are concerned with the world's capital accumulation, i.e., $k < \bar{k}$. Here we want to show $c_1 < c_2$ holds always, i.e., the patient country 2's consumption c_2 is always larger for $k < \bar{k}$, generalizing the local result of Devereux and Shi(1991). Furthermore, as drawn in Fig.2, the patient country 2's consumption c_2 keeps increasing while the impatient country 1's consumption c_1 first increases if the initial world per capita capital stock k is low and then eventually starts decreasing.

Theorem 2

Let $v_i(c_i) = -1$, $i = 1,2$, $u_1(c_1) = c_1^{\beta} + \gamma_1$, and $u_2(c_2) = c_2^{\beta} + \gamma_2$ where $0 < \beta < 1$ and $0 < \gamma_2 < \gamma_1$. Let $k < \bar{k}$, i.e., the world's capital accumulates.

Then

(1) $c_1 < c_2$ holds, i.e., the more impatient country 1's consumption c_1 is always less than that of patient country 2.

(2)If $c'_1(k)$ and $c'_2(k)$ >0 hold initially, then these hold for a while, but $c'_1(k) < 0 < c'_2(k)$ holds as k approaches $\bar{k}$ That is, if the consumption of both countries increases as the world's capital stock increases, then it continuous to increase for awhile, but country 1's consumption starts decreasing while country 2's consumption keeps increasing.

(3) If $c'_1(k) < 0 < c'_2(k)$ holds initially, then it continues to hold as k approaches $\bar{k}$. That is, if country 1's consumption decreases, while country 2's consumption increases, then this consumption pattern remains unchanged.

Proof. (See Appendix III.)

Next we show the corresponding results for the world's capital decumulation, i.e., $\bar{k} < k$.

Theorem 3

Let the world's capital decumulate, i.e., $\bar{k} < k$.

Then

(1) either $c_1 < c_2$ holds always or if $c_1 > c_2$ holds initially, with $c_1 < c_2$ holding eventually as k decreases toward $\bar{k}$. That is, the more impatient country 1's consumption c_1 is always less than that of county 2, or if country 1's consumption is larger than that of country 2, then country 1's consumption becomes less than that of country 2 after a certain time.

(2)If $c'_1(k) > 0$ and $c'_2(k) > 0$ hold initially, then these hold for a while but $c'_1(k) < 0 < c'_2(k)$ holds as k decreases toward $\bar{k}$. That is, if the consumption of both countries decreases as the world capital stock decreases, then it continues to decrease after awhile but country 1's consumption starts increasing while country 2's consumption keeps decreasing.

(3))If $c'_1(k) < 0 < c'_2(k)$ holds initially, then it keeps holding. That is, if country 2's consumption decreases, while country 1's consumption increases, then this consumption pattern remains unchanged.

Proof (See Appendix IV)

As seen from Fig.2, the only difference from the world's capital accumulation case is the possibility of the change in the amounts of consumption between the two countries although this change occurs only once. That is, in the world's capital accumulation ($k < \bar{k}$), the impatient country 1's consumption c_1 is always smaller than that of the patient country 2's c_2 , but when the world's capital decumulates ($k > \bar{k}$), if the initial world per capita capital stock k is large enough, c_2 can be smaller than c_1, although this ranking is reversed once again as *k* decreases further. As shown later this possibility gives rises to the possibility of a change in trade pattern and asset-debt position.

Theorems 2 and 3 show the significance of global analysis in comparison with the local analysis. By restricting ourselves to the local analysis, we observe only the difference in the direction of the two countries' consumption path. However by generalizing to the global analysis we observe that when the starting points are not close to the stationary state, this direction is the same initially, and eventually the change in the direction of only country 1's consumption toward the stationary state occurs. As mentioned before, such a non monotonicity of one country's consumption and a monotonicity of the other country's consumption with respect to the world's capital accumulation can be observed only by global analysis.

Next we investigate the trade patterns and asset-debt positions.

III. 3 Trade Patterns and Asset-Debt Positions

Henceforth we specify the production functions to be of the Cobb-Douglas type, i.e.,

$$\left.\begin{array}{l} f_1(k_1) = \theta k_1^{\xi}, \theta \geq 1 \quad \text{and} \quad 0 < \xi < 1 \\ \text{and} \\ \qquad f_2(k_2) = k_2^{\xi} \end{array}\right\} \tag{28}$$

That is, country 1 is assumed to be technologically at least as good as country 2, $\theta \geq 1$.

Next we show when $c_1 < c_2$ holds

$$ex_1 = f_1(k_1) - c_1 - \dot{k}_1 > 0,$$

i.e., country 1's exports which equal its output $f_1(k_1)$ less its consumption c_1 and its investment $\dot{k}_1$, is always positive if its consumption c_1 is less than that of country 2. Since $f_1'(k_1) = f_2'(k_2) = r$ holds always, we obtain $k_1 / k_2 = \theta^{1/(1-\xi)} = \eta/(1-\eta)$ where $\eta = \theta^{1/(1-\xi)}/(1+ \theta^{1/(1-\xi)})$ with $\eta'(\theta) > 0$ and $\eta = 1/2$ for $\theta = 1$. Then $(1-\eta)k_1 = \eta k_2$ and hence $(1-\eta)\dot{k}_1 = \eta \dot{k}_2$ and $\dot{k}_1 = \eta \dot{k}$ from $k_1 = \eta k$ and $k_2 = (1-\eta)k$. Then it follows that

$$ex_1 = f_1(k_1) - c_1 - \dot{k}_1 = f_1(k_1) - c_1 - \eta\dot{k} = f_1(k_1) - c_1 - \eta(f_1(k_1) + f_2(k_2) - c_1 - c_2)$$

$$= (1-\eta) f_1(k_1) - \eta f_2(k_2) - (1-\eta)c_1 + \eta c_2 = -(1-\eta)c_1 + \eta c_2 \quad \text{in view of}$$

$f_1 / f_2 = \theta(k_1/k_2)^{\xi} = \theta^{1/(1-\xi)} = \eta/(1-\eta)$. Since $c_1 < c_2$ and $\eta/(1-\eta) \geq 1$ hold, $ex_1 > 0$ follows.

Next we observe

$$F_1(t) = -\int_t^{\infty} ex_1 n(t,\tau)d\tau \tag{29}$$

where $F_1 = F_1(t) = m_1 - k_1 > 0(< 0)$, i.e., is the net foreign asset (debt) holding by country 1's consumer and $n(t,\tau) = e^{-\int_t^{\infty} r(s)ds}$, is the time discount rate. (See Appendix V.)

Here we obtain the consumption demand function of country i from (12), (18) and (29);

$c_i(t) = h_i(t)N_i(t), \quad i = 1,2$

where $h_i(t)^{-1} = \int_t^{\infty} e^{\left[(1-\beta)^{-1}\int_t^{\infty}(r-\rho_i)ds - \int_0^t rds\right]} d\tau$

and $N_i(t) = F_i(t) + V_i(t),$ is the national wealth of country i and

$V_i(t) = \int (f_i(k_i) - \dot{k}_i)n(t,\tau)d\tau.$ (See Appendix VI.)

In our economy since the rates of time preference ρ_i differs, $h_1 \neq h_2$ follows. The national wealth effects N_i are composed of F_i, i =1,2 the distributional effects of the two countries and V_i, the capital stock accumulation effects. Since $f_1(k_1) = \eta f(k),$ $f_2(k_2) = (1-\eta)f(k),$ $k_1 = \eta k$ and $k_2 = (1-\eta)k$ hold in our specification, $V_1 > V_2$ follows. In short the capital accumulation effects of country 1 dominates. However since $F_1 < 0 < F_2(= -F_1)$, i.e., country 1 is a debtor, and $c_1 < c_2$ holds, we must conclude that in the consumption dynamics, the negative distributional effect of country 1 is seen to exceed its positive capital accumulation effects. This global analysis of distributional effects and capital accumulation effects on a country's consumption path corresponds to the local analysis done by Devereux and Shi(1991).

Lastly we show from $\overline{w}_1 \geq \overline{w}_2$ [6/], $\overline{c}_1 \leq \overline{c}_2$, and (25), $0 < \overline{m}_1 < \overline{m}_2$ and from $\overline{k}_1 = \eta(1-\mu)^{-1}\overline{k}_2 \geq \overline{k}_2$, $\overline{F_1} = \overline{m}_1 - \overline{k}_1 < 0$, i.e., the impatient country 1 is a debtor while the patient country 2 is a creditor at the stationary state. Based on the above arguments, we can generalize these results and obtain

Theorem 4

(1) When the world accumulates capital, $ex_1 > 0$ and $F_1 < 0$ hold always, i.e., the more impatient country 1 remains an exporter of the good as well as debtor throughout the transitional period.

(2) When the world decumulates capital, if $c_1 < c_2$ holds initially, then this and $ex_1 > 0$ and $F_1 < 0$ hold thereafter. If $c_1 > c_2$ holds initially, then eventually this relationship is reversed, but $ex_1 > 0$ and $F_1 < 0$ hold thereafter. That is, when the

world's capital decumulates, if the more impatient country 1's consumption is less than that of country 2 initially, then this difference remains thereafter and country 1 remains an exporter of the good as well as a debtor throughout the transitional period. If country 1's consumption is larger than that of country 2, then after a certain period, this difference is reversed, and country 1 becomes an exporter of the good as well as a debtor and remains so thereafter.

We note that Theorem 4 holds even if country 1 possesses no technological superiority, i.e., even if $\theta = 1$. In this case, trade patterns and asset-debt positions arise purely from the difference in the time preference rate of consumption. Furthermore with $\theta = 1$, $ex > 0$ if and only if $c_1 < c_2$ holds, i.e., country 1 is an exporter of the good if and only if its consumption is less than that of country 2. Furthermore the conclusion (1) of this theorem seems to be consistent with the rapid economic growth of some Asian countries (China, Korea and Thailand) whose exports are engines of growth. In these countries average propensities to consume are lower and they enjoy trade surpluses while remaining debtors.

IV. Concluding Remarks

We have derived the difference in per capita consumption based on the difference in the degree of impatience and employed this to explain the trade pattern and asset-debt positions of two countries in the simplest framework (one good, two countries, and two representative consumers) in a unified manner.

It is not difficult to introduce government expenditure into the model provided it does not affect consumption nor production. Also to generalize the framework to a multi-country model would not be so difficult provided the felicity function u_i is of the type assumed in the last section.

Perhaps one of the most crucial assumptions for the implications for trade patterns and asset-debt position is the Cobb-Douglas production function. In fact owing to this, country 1's capital k_1 is always proportional to country 2' capital k_2, i.e., $k_1 = \eta(1-\eta)^{-1} k_2$.

This does not hold even if we generalize the production function into a C.E.S. type. One of the merits of introducing a recursive type preference in the open model is that we can introduce capital accumulation into the model. In fact if we restrict our model one with different and exogenously fixed time preference rates ρ_i, $i = 1,2$, then we have to assume away capital accumulation to let the model work as done by Ikeda and Ono (1992), for with capital accumulation, $\rho_i = f'(\bar{k})$, $i = 1,2$ must hold at the stationary state.

We have tried to send two main findings in this paper. One is to show the characteristics of the optimal consumption path, trade patterns and asset-debt positions in the globally dynamic context. Second is that the trade surplus and foreign debt of the more impatient countries are the results of these countries' optimal choices. Consequently, it does not make sense to encourage such a country to realize a trade balance under the cause of "fair trade".

Appendix I

Here we show only a **brief sketch of the proof of Theorem 1.** A detailed proof is sent upon request.

I. First we show that the optimal path $Y = (k, c)$ converges globally to the unique stationary state $E(\overline{k}, \overline{c})$ irrespective of its initial point $Y_0 = (k_0, c_0)$ with $c'(k) > 0$.

I.① First we show Y to be unbounded so that starting from an arbitrary point near $E(\overline{k}, \overline{c})$ such as point E_1 along the optimal path Y in Fig.A.1, $Y \to (\infty, \infty)$ as $t \to -\infty$.

Fig.A.1

② If the path is bounded, then it must enter into the region below the $\dot{k} = 0$ curve where $k > 0$ holds as shown in Fig.A.1.

Fig.A.2

③ However then, instead of the optimal path starting from $S_2(k_1, c_2)$ in Figs.A.1 and A.2, we can construct the suboptimal path in Fig.A.2 starting from $S_3(k_1, c_3)$, arriving at $A(k_T, c_T)$ at the same time as the optimal path starting from $S_2(k_1, c_2)$ and following the optimal path after T.[7/] However this suboptimal path results in higher utility than the optimal path, for the former consumption is always higher than the latter for $t \leq T$.

④ This contradiction shows the optimal path Y starting from E_1 near E never hits the $k = 0$ curve as $t \to -\infty$.

II. ⑤ Next we show by way of contradiction

$k \to +\infty$ as $t \to -\infty$ implies $c \to +\infty$, and

$c \to +\infty$ as $t \to -\infty$ implies $k \to +\infty$,

and hence $k \to +\infty$ and $c \to +\infty$ as $t \to -\infty$ follow.

Fig.A.3

III⑥ Next we consider the case of $k_0 < \overline{k}$ and show that the optimal path moves toward the origin as $t \to -\infty$ with $c'(k) > 0$. First the optimal path Y starting from E_2 in Fig.A.3 is shown never to enter into the region above the $k = 0$ curve where $k < 0$ holds.

⑦ If it ever enters into this region, then only such a path as that starting from F in Fig.A.3 is seen to be possible.

Fig.A.4

⑧ Next, corresponding to te optimal path starting from either F or K in Fig.A.4, we can construct the solution path $F''E''G''$. Since $\dot{c}=0$ holds at E'', E'' is another stationary point, contradicting its uniqueness. Hence we must conclude that the optimal path starting from E_2 never hits the $\dot{k}=0$ curve as $t \to -\infty$.

IV⑨ Next we show that the optimal path starting from E_2 never hits the horizontal axis as $t \to -\infty$ by way of contradiction, and hence that it must converge toward the origin as $t \to -\infty$.

⑩ Lastly employing Lemma 1 where the role of t is replaced by k, c_i and ϕ_i are seen to be expressed globally as functions of k, and from the phase diagram of the $\dot{c}=0$ curve and $\dot{k}=0$ curve in the (k, c) plane, $c=c(k)$ with $c'(k)>0$ is obtained.

Appendix II

I. Derivation of $c_1'(k)<0<c_2'(k)$ at $k=\bar{k}$

Although our specification of the felicity function u_i is slightly different from Devereux and Shi(1991)'s, since basically the same results hold, only a brief sketch of the proof is given. A detailed proof will be sent upon request.

Let $\dot{c}_i$, $\dot{\phi}_i$ and $\dot{k}$ be linearized around the stationary state E; then we obtain ,

$$\begin{pmatrix} \dot{c}_1 \\ \dot{c}_2 \\ \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{k} \\ \dot{m}_1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -a_1 & 0 & -b_1 & 0 \\ 0 & 0 & 0 & -a_2 & -b_2 & 0 \\ -e_1 & 0 & \bar{r} & 0 & 0 & 0 \\ 0 & -e_2 & 0 & \bar{r} & 0 & 0 \\ -1 & -1 & 0 & 0 & \bar{r} & 0 \\ -1 & 0 & 0 & 0 & \bar{F}_1 f'' & \bar{r} \end{pmatrix} \begin{pmatrix} c_1-\bar{c}_1 \\ c_2-\bar{c}_2 \\ \phi_1-\bar{\phi}_1 \\ \phi_2-\bar{\phi}_2 \\ k-\bar{k} \\ m_1-\bar{m}_1 \end{pmatrix} \tag{A-1}$$

where $a_i=-(v_i''-\phi_i u_i'')^{-1}\bar{r}u_i'>0$, $b_i=(v_i'-\phi_i u_i')(v_i''-\phi_i u_i'')^{-1}f''>0$, and $e_i=v_i'-\phi_i u_i'>0$.

Let A be the coefficient matrix of the above equation and B(5x5) be its upper left submatrix. Then we observe

$$f(\omega)=|A-\omega I|=(\bar{r}-\omega)(B-\omega I)=(\bar{r}-\omega)^2 g(\omega)$$

where

$$g(\omega)=\omega^2(\bar{r}-\omega)^2+(a_1e_1+a_2e_2+b_1+b_2)\omega(\bar{r}-\omega)+a_1a_2e_1e_2+ \tag{A-2}$$

$$e_2 a_2 b_1 + e_1 a_1 b_2 .$$

Now let ω_i , $i = 1, 2$ be the two negative solutions of $g(\omega) = 0$, i.e.,

$$\lambda_i = \omega_i (\bar{r} - \omega_i) , \quad i = 1, 2 . \tag{A-3}$$

Then we observe

$$\omega_i = \frac{\bar{r} - \sqrt{\bar{r}^2 - 4\lambda_i}}{2} , \quad i = 1, 2 \tag{A-4}$$

with $\omega_2 < \omega_1 < 0$. Now we can conclude the stationary state E is locally a saddle point with an optimal path that is a two dimensional manifold.

Then we obtain

$$\left.\begin{aligned}
c_1 - \bar{c}_1 &= -A_1 \frac{b_1(\bar{r} - \omega_1)}{\lambda_1 + a_1 e_1} e^{\omega_1 t} - A_2 \frac{b_1(\bar{r} - \omega_2)}{\lambda_2 + a_1 e_1} e^{\omega_2 t} , \\
c_2 - \bar{c}_2 &= -A_1 \frac{b_2(\bar{r} - \omega_1)}{\lambda_1 + a_2 e_2} e^{\omega_1 t} - A_2 \frac{b_2(\bar{r} - \omega_2)}{\lambda_2 + a_2 e_2} e^{\omega_2 t} , \\
\phi_1 - \bar{\phi}_1 &= -A_1 \frac{e_1 b_1}{\lambda_1 + a_1 e_1} e^{\omega_1 t} - A_2 \frac{e_1 b_1}{\lambda_2 + a_1 e_1} e^{\omega_2 t} , \\
\phi_2 - \bar{\phi}_2 &= -A_1 \frac{e_2 b_2}{\lambda_1 + a_2 e_2} e^{\omega_1 t} - A_2 \frac{e_2 b_2}{\lambda_2 + a_2 e_2} e^{\omega_2 t} , \\
k - \bar{k} &= A_1 e^{\omega_1 t} + A_2 e^{\omega_2 t} , \\
&\text{and} \\
m_1 - \bar{m}_1 &= A_1 \frac{\dfrac{-b_1(r - \omega_1)}{\lambda_1 + a_1 e_1} - \bar{F}_1 f''(\bar{k})}{\bar{r} - \omega_1} e^{\omega_1 t} + A_2 \frac{\dfrac{-b_1(r - \omega_2)}{\lambda_2 + a_1 e_1} - \bar{F}_1 f''(\bar{k})}{\bar{r} - \omega_2} e^{\omega_2 t} .
\end{aligned}\right\} \tag{A-5}$$

From the above we obtain $c_1^{'}(k) \to -b_1(\bar{r} - w_1)/(\lambda_1 + a_1 e_1) < 0$ and

$c_2^{'}(k) \to -b_2(\bar{r} - w_1)/(\lambda_1 + a_2 e_2) > 0.$

III. Determination of A_1 and A_2

Here from (A-5), by letting $t = 0$, we obtain

$$k_0 - \bar{k} = A_1 + A_2$$

where k_0 is the initial value of $k = k(t)$, i.e., $k_0 = k(0)$.

$$m_{10} - \bar{m}_1 = A_1 \left\{ \frac{-b_1}{\lambda_1 + a_1 e_1} - \frac{\bar{F}_1 f''}{\bar{r} - \omega_1} \right\} + A_2 \left\{ \frac{-b_1}{\lambda_2 + a_1 e_1} - \frac{\bar{F}_1 f''}{\bar{r} - \omega_2} \right\}$$

where m_{10} is the initial value of $m_1 = m_1(t)$, i.e., $m_{10} = m_1(0)$.

From these, A_1 and A_2 are determined as

$$A_1 = \frac{m_{10} - \overline{m}_1 + \left\{ b_1/(\lambda_2 + a_1 e_1) + \overline{F}_1 f''/(\overline{r} - \omega_2) \right\}(k_0 - \overline{k})}{-b_1/(\lambda_1 + a_1 e_1) - \overline{F}_1 f'' \left\{ (\overline{r} - \omega_1)^{-1} - (\overline{r} - \omega_2)^{-1} \right\} + b_1/(\lambda_2 + a_1 e_1)}$$

$$A_2 = \frac{-\left\{ b_1/(\lambda_1 + a_1 e_1) + \overline{F}_1 f''/(\overline{r} - \omega_1) \right\}(k_0 - \overline{k}) - m_{10} + \overline{m}_{11}}{-b_1/(\lambda_1 + a_1 e_1) - \overline{F}_1 f'' \left\{ (\overline{r} - \omega_1)^{-1} - (\overline{r} - \omega_2)^{-1} \right\} + b_1/(\lambda_2 + a_1 e_1)}$$

Appendix III

Proof of Theorem 2.

First we show that for $k < \overline{k}$

I. $\operatorname{sgn} c_1'(k)$ **changes only once while** $\operatorname{sgn} c_2'(k) > 0$ **holds always.**

Since $c'(k) > 0$ holds globally, $c(k) \to 0$ as $k \to 0$ and $c_1 \le c$ and must hold always, we observe both c_1 and c_2 must decrease to zero as $k \to 0$.

Fig. A.5a **Fig.A.5b**

From the conditions of competitive equilibrium, we obtain

$$\dot{c}_i \ge 0 \ , \ i = 1,2 \quad \Leftrightarrow \ f'(k) \ge \dot{k}_i \cdot u_i'(c_i) + u_i(c_i) \ , \ i = 1,2 . \qquad \text{(A-6)}$$

(a detailed proof of (A-6) is sent upon request.)

Here recalling $\dot{k} \ge 0 \Leftrightarrow \dot{k}_i \ge 0 \Leftrightarrow k \le \overline{k}$ [8/] along the optimal path of $Y = (k, c)$ (See Theorem 1.), we obtain the phase diagram of (k, c_i) , $i = 1,2$ as shown in Figs. A.5a and A.5b. First we note that in both figures, $f'(k) \le (\text{resp.} \ge) u_i(c_i)$, $i = 1,2 \Leftrightarrow (k, c_i)$ lies in the upper-right region (resp. lower-left region) of the $f'(k) = u_i(c_i)$ curve and $\dot{k}_i \ge 0$ (resp. ≤ 0) $\Leftrightarrow (k, c_i)$ lies in the left (resp. right) side of the $\dot{k}_i = 0$ vertical line, and hence the $\dot{c}_i = 0$ curve must lie between these two curves as a dotted line passing through $E = (\overline{k}, \overline{c}_i)$. By construction $\dot{c}_i \ge 0$, $i = 1,2 \Leftrightarrow (k, c_i)$, $i = 1,2$ lies in the lower-left region of the $\dot{c}_i = 0$ curve, $i = 1,2$.

The optimal path $Y_i = (k, c_i)$, $i = 1,2$ is shown by the arrowed lines for c_1 and c_2 corresponding to Figs.A.5a and A.5b respectively.

As seen from the above phase diagrams, it is easy to see that the $c_1 = c_1(k)$ curve changes sign only once, but the $c_2 = c_2(k)$ curve never change sign $c_2'(k)$.[9/]

Fig. A.6

II. c_1 **curve and** c_2 **curve never intersect.**

We here first show c_1 curve and c_2 curve never intersect. First we note that since

$$\dot{c}_i = -(u_i'/u_i'')(f'(k) - \rho_i) \ , \ i = 1,2 \qquad \text{(A-7)}$$

holds from (12) and $\rho_i = -1/\phi_i$ from (13)

$$\rho_i'(k) > 0 \Leftrightarrow \phi_i'(k) > 0 \text{ , } i = 1,2 \tag{A-8}$$

where $\rho_i = \rho_i(k)$, $i = 1,2$, we obtain

$$c_i'(k) > 0 \Leftrightarrow f'(k) > \rho_i \text{ , } i = 1,2$$

from (A.7) and $\dot{k} > 0$ for $k < \bar{k}$. Suppose, for a contradiction, the c_1 and c_2 curves intersect at least twice at E_1 and E_2 as shown in Fig.A.7. In Fig.A.6, the curve starting from E passing through E_2 and E_1, corresponds to the movement of the c_1 and c_2 curves starting from $k = \bar{k}$ in Fig. A.7. As seen in Fig.A.6, the curve EE_2E_1 is below the curve $u_1(c_1) = c_1^{\alpha} + \beta_1 = c_2^{\beta} + \beta_2 = u_2(c_2)$, implying $u_1(c_1) > u_2(c_2)$ for $t > t^1$ where $k^1 = k(t^1)$ and $(c_1^1,\ c_2^1) = E_1$ with $c_i^1 = c_i\,(k^1)$, $i = 1,2$. In short at E_1 and thereafter (i.e., $t > t^1$) $u_1(c_1) > u_2(c_2)$ holds always and hence $\phi_1(t^1) > \phi_2(t^1)$ by construction (from (14) with $v_i = -1$, $i = 1,2$). However as seen from Fig.A.7, at E_1 $dc_1/dk \geq dc_2/dk$ holds and hence from (A-7) with $u'_1/u_1'' = u_2'/u_2''$ at E_1 and $\rho_i = -1/\phi_i$, $i = 1,2$, $\rho_1 \leq \rho_2$ or $\phi_1(t^1) \leq \phi_2(t^1)$ must hold, a contradiction. Here we note that the above arguments hold especially when E_1, happens to be the origin.

Hence the c_1 and c_2 curves never intersect for $0 < k \leq \bar{k}$.

Fig. A.7

III. c_1 curve and c_2 curve never touch.

Fig.A.8

Next we show c_1 curve and c_2 curve never touch. We can employ similar arguments as those used in I, with $t^1 = t^2$. Suppose, for a contradiction $c_1 = c_2$ at $E_1 = E_2$ where $t = t^1 = t^2$ and $c_1 < c_2$ but $u_1(c_1) > u_2(c_2)$ for t with $0 < t^1 \leq t < +\infty$ as can be seen in Fig. A.6 with $E_1 = E_2$. Then by construction

$$\phi_1(t^1) > \phi_2(t^1)$$

must hold. However at $E_1 = E_2$ as seen from Fig.A.8 $dc_1/dk = dc_2/dk$ holds, and hence (at $E_1 = E_2$, $c_1 = c_2$ and hence $u_1'/u_1'' = u_2'/u_2''$ follows)

$$\phi_1(t^1) = \phi_2(t^1)$$

from (A-7) and $\rho_i = -1/\phi_i$, $i = 1,2$, a contradiction. This shows the c_1 and c_2 curves never touch as shown in Fig.A.8.

Here we investigate the phase diagram of $\phi_i(k)$, $i = 1,2$. From (6) we observe

$\operatorname{sgn} d\phi_i / dk\Big|_{\dot{\phi}_i=0} = \operatorname{sgn} c_i'(k)$, recalling $\operatorname{sgn} c_i'(k)$ changes at most once, we obtain the phase diagrams of (k, ϕ_i) as shown in Figs. A.9a and A.9b. Here the dotted lines show the optimal path of (k, ϕ_i), $i = 1,2$.

Fig. A.9a **Fig. A.9b**

Fig. A.10

Then we obtain the two curves $\phi_1(k)$ and $\phi_2(k)$ for case $A_1 / A_2 > 0$ as drawn in Figs. A.9a and A.9b where A_1 .and A_2 are shown in Appendix II.

In short, we obtain Theorem 2.

Appendix IV

Proof of Theorem 3

As already shown in Figs.A.5a and A.5b of Appendix III, the optimal path of the $c_1 = c_1(k)$ curve changes its sign at most once while the optimal path of $c_2 = c_2(k)$ curve is always positively sloped from both $k_0 < \bar{k}$ and $k_0 > \bar{k}$. (Here as noted in footnote (9), if the $c_1 = c_1(k)$ curve does not change its sign for $k_0 > \bar{k}$, then the curve is always negatively sloped.)

Now we show that the c_1 and c_2 curves, should they ever intersect, do so just once as shown in Fig.2. Since the c_2 curve is positively sloped always for $k > \bar{k}$, these two curves never meet when the c_1 curve is negatively sloped for $k > \bar{k}$. We can show these two curves meet at most just once at E_1, for $k > \bar{k}$ by way of contradiction, supposing the two curves meet at E_2 north east of E_1, as shown in Fig. A.11.

Fig. A.11

Here we note first

$dc_1 / dk \geq dc_2 / dk \Leftrightarrow$ (from $\dot{k} < 0$) $\dot{c}_1 \leq \dot{c}_2 \Leftrightarrow$ (from (A-7) $\rho_1 > \rho_2 \Leftrightarrow$ (from $\rho_i = -1/\phi_i$, $i = 1,2$) $\phi_1 \geq \phi_2$.

Since $dc_1 / dk \geq$ (resp. $\leq$)dc_2 / dk holds at E_1 (resp. E_2), we obtain

$$\phi_1(t^1) = -\int_{t^1}^{\infty} e^{-\int_{t^1}^{\tau} u_1 ds} d\tau \geq -\int_{t^1}^{\infty} e^{-\int_{t^1}^{\tau} u_2 ds} d\tau = \phi_2(t^1) . \tag{A-9}$$

from (A-7) and $\rho_i = -1/\phi_i$ where $t = t^1$ at E_1, and

$$\phi_1(t^2) = -\int_{t^2}^{\infty} e^{-\int_{t^2}^{\tau} u_1 ds} d\tau \le -\int_{t^2}^{\infty} e^{-\int_{t^2}^{\tau} u_2 ds} d\tau = \phi_2(t^2)\,, \tag{A-10}$$

similarly since $dc_1/dk \le dc_2/dk$ holds at E_2 where $t = t^2$ at E_2 with $t^2 < t^1$. Furthermore since $c_1 > c_2$ for t with $t^2 < t < t^1$, we observe $u_1(c_1) > u_2(c_2)$ for $t^2 < t^1$ and hence

$$-\int_{t^2}^{t^1} e^{-\int_{t^2}^{\tau} u_1 ds} d\tau > -\int_{t^2}^{t^1} e^{-\int_{t^2}^{\tau} u_2 ds} d\tau\,. \tag{A-11}$$

Here recalling $e^{-\int_{t^2}^{t^1} u_1 ds} < e^{-\int_{t^2}^{t^1} u_2 ds}$, we obtain from (A- 9).

$$-\int_{t^1}^{\infty} e^{-\int_{t^2}^{t^1} u_1 ds - \int_{t^1}^{\tau} u_1 ds} d\tau > -\int_{t^1}^{\infty} e^{-\int_{t^2}^{t^1} u_2 ds - \int_{t^1}^{\tau} u_2 ds} d\tau$$

or

$$-\int_{t^1}^{\infty} e^{-\int_{t^2}^{\tau} u_1 ds} d\tau > -\int_{t^1}^{\infty} e^{-\int_{t^2}^{\tau} u_2 ds} d\tau \tag{A-12}$$

By rearranging (A-10) as

$$-\int_{t^2}^{t^1} e^{-\int_{t^2}^{\tau} u_1 ds} d\tau - \int_{t^1}^{\infty} e^{-\int_{t^2}^{\tau} u_1 ds} d\tau \le -\int_{t^2}^{t^1} e^{-\int_{t^2}^{\tau} u_2 ds} d\tau - \int_{t^1}^{\infty} e^{-\int_{t^2}^{\tau} u_2 ds} d\tau\,,$$

and from (A-12), we observe

$$-\int_{t^2}^{t^1} e^{-\int_{t^2}^{\tau} u_1 ds} d\tau < -\int_{t^2}^{t^1} e^{-\int_{t^2}^{\tau} u_2 ds} d\tau\,,$$

contradicting (A-11). This contradiction shows that the c_1 curve and the c_2 curve intersect only once at E, for $k > \bar{k}$.

Next we show that the c_1 curve and the c_2 curve never touch.

Fig. A. 12 **Fig. A.13**

Suppose not. Then the c_1 curve and the c_2 curve touch at E_1 as shown in Fig. A.12. At E_1, $\phi_1 = \phi_2$ holds from $dc_1/dk = dc_2/dk$, (A-7) and $\rho_i = -1/\phi_i(k)$, $i = 1, 2$. $c_1 = c_2$ implies $u_1(c_1) > u_2(c_2)$ and hence

$0 < d\phi_2/dk < d\phi_1/dk$ at E_1,

from (6), $v_i = -1$, $i = 1, 2$ and $\dot{k} < 0$. (Since the $\phi_2 = \phi_2(k)$ curve is positively sloped as seen from Fig. A.9b — the $\dot{\phi}_2 = 0$ curve is positively sloped from (6) and $c_2'(k) > 0$ (i.e., $\operatorname{sgn} d\phi_i/dk\big|_{\dot{\phi}_i=0} = \operatorname{sgn} c_i'(k)$), hence from the phase diagram $\phi_2'(k) > 0$ follows.—) Let k' be slightly smaller than k^1 such that $k' = k(t')$. Then as seen from Fig. A.13 which is obtained from Figs. A.9a and A.9b, $\phi_1(t') < \phi_2(t') \Leftrightarrow \rho_1 < \rho_2$ holds from $\phi_i = -1/\rho_i$. From Fig. A.12, we obtain

$0 < dc_2/dk < dc_1/dk$ at $k = k'$, implying

$$0 > c_2(f' - \rho_2) > c_1(f' - \rho_1)$$

from (A-7) and $\dot{k} < 0$. Hence from $c_1 < c_2$ it follows that

$$0 > f' - \rho_2 > f' - \rho_1 \Leftrightarrow \rho_2 < \rho_1,$$

a contradiction. This shows the c_1 curve and the c_2 curve never touch. Hence we obtain the results of Theorem 3.

Appendix V

Derivation of (29)

In the competitive equilibrium, the utility maximization problem of coutry i's consumer is solved by forming the Hamiltonian;

$$H_i = \alpha_i v_i(c_i) + \tilde{\lambda}_i(rm_i + w_i - c_i) - \tilde{\phi}_i \alpha_i u_i(c_i) \text{ , } i = 1,2 \tag{A-13}$$

and by obtaining the first order conditions;

$$\alpha_i v_i'(c_i) - \tilde{\lambda}_i - \tilde{\phi}_i \alpha_i u_i'(c_i) = 0 \text{ , } i = 1,2 \tag{A-14}$$

$$\dot{\tilde{\lambda}}_i = -\tilde{\lambda}_i r \text{ , } i = 1,2 \tag{A-15}$$

$$\dot{\tilde{\phi}}_i = -v_i + \tilde{\phi}_i u_i \text{ , } i = 1,2 \tag{A-16}$$

and the transversality conditions

$$\tilde{\lambda}_i m_i \to 0 \text{ as } t \to \infty \tag{A-17}$$

and

$$\tilde{\phi}_i \alpha_i \to 0 \text{ as } t \to \infty \text{ , } i = 1,2. \tag{A-18}$$

From the transversality condition $\tilde{\lambda}_i m_i \to 0$ as $t \to \infty$, $i = 1,2$ (A-17) and $\lambda k \to 0$ as $t \to \infty$ ($\tilde{\lambda}_1 / \beta = \tilde{\lambda}_2 = \lambda$ and hence $\lambda(m_1 + m_2) \to 0$ as $t \to \infty \Leftrightarrow \lambda k \to 0$ as $t \to \infty \Leftrightarrow$ $\lambda k_i \to 0$ as $t \to \infty$, $i = 1,2$) imply $\lambda F_i \to 0$ as $t \to \infty$, $i = 1,2$. Then from (A-15), we obtain

$$\tilde{\lambda}_i(t) = \tilde{\lambda}_i(0) e^{-\int_0^t r(t) d\tau} = \tilde{\lambda}_i(0) n(0, t) \text{ , } i = 1,2.$$

Substituting this into the transversality condition $\tilde{\lambda}_i F_i \to 0$ as $t \to \infty$, $i = 1,2$, we obtain:

$$\text{NPG(No-Ponzi-Game) Condition: } \lim_{t \to \infty} F_i n(0, t) = 0. \tag{A-19}$$

Next by integrating the flow budget constraint (17), we obtain

$$F_1(t) = F_1(t_1)n(t, t_1) + \int_{t_1}^{t} ex_1 n(t_1, \tau) d\tau \, .$$

By letting $t_1 \to \infty$, and from NPG, we obtain (29).

Appendix VI

Derivation of the Consumption Demand Function

From (18) and (29)、we obtain the national wealth of country i, N_i to be

$$N_i(t) = F_i(t) + V_i(t) = \int_t^{\infty} c_i(\tau) n(t, \tau) d\tau \qquad \text{(A-20)}$$

where

$$V_i(t) = \int_t^{\infty} (f_i(k_i) - \dot{k}_i) n(t,\tau) d\tau = \int_t^{\infty} (f_i(k_i) - \dot{k}_i - w_i) n(t,\tau) d\tau + \int_t^{\infty} w_i n(t,\tau) d\tau = W_i + H_i ,$$

$W_i(t) = \int_t^{\infty} (f_i(k_i) - \dot{k}_i - w_i) n(t,\tau) d\tau$, is the value of the firm of country *i*, and

$H_i(t) = \int^{\infty} w_i n(t,\tau) d\tau$ is the value of human wealth of country *i* and

$V_i(t) = W_i(t) + H_i(t)$ is the sum of value of firm and the value of human wealth of country *i*, *i* =1,2.

Next from (12) with $\nu_1 = \nu_2 = -1$ and $u_i = c_i^{\beta} + \gamma_i , i = 1,2$ we obtain

$$c_i(\tau) = c_i(t) e^{(1-\beta)^{-1} \int_t^{\tau} (r - \rho_i) d\tau} , i = 1,2$$

and by substituting this into (A-20) and rearranging we obtain the consumption demand function of country *i*,

$$c_i(t) = h_i(t) N_i(t)$$

where $h_i(t)^{-1} = \int_t^{\infty} e^{\left[(1-\beta)^{-1} \int_t^{\infty} (r - \rho_i) ds - \int_0^{t} r ds \right]} d\tau$.

Figures

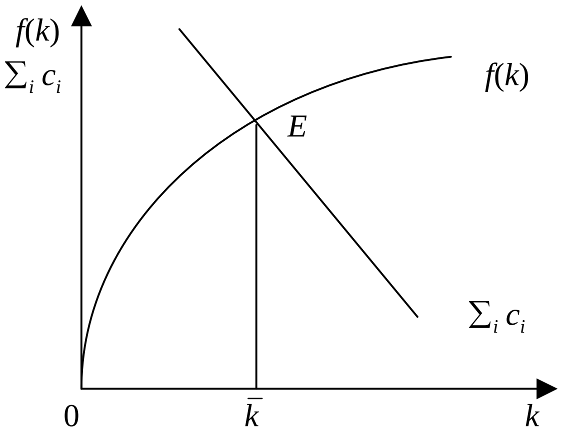


Fig. 1 Existence and Uniqueness of the Stationary State

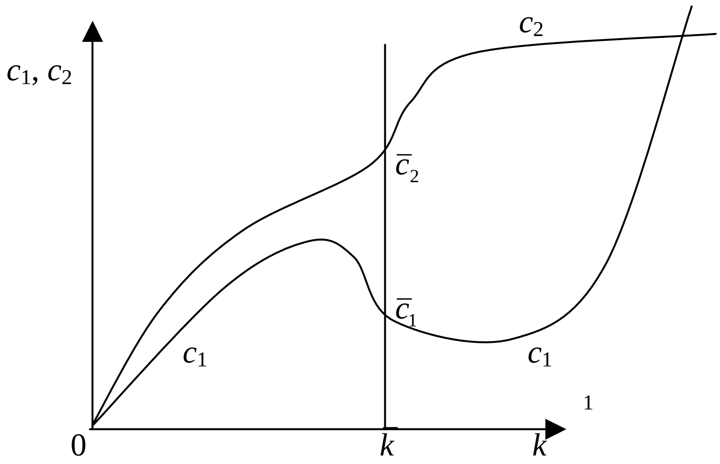


Fig. 2

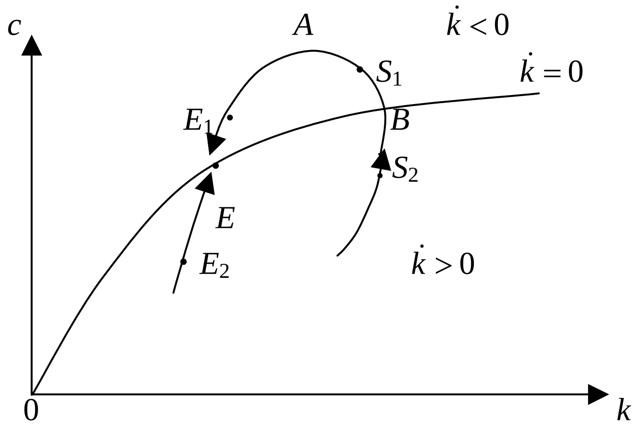


Fig.A.1

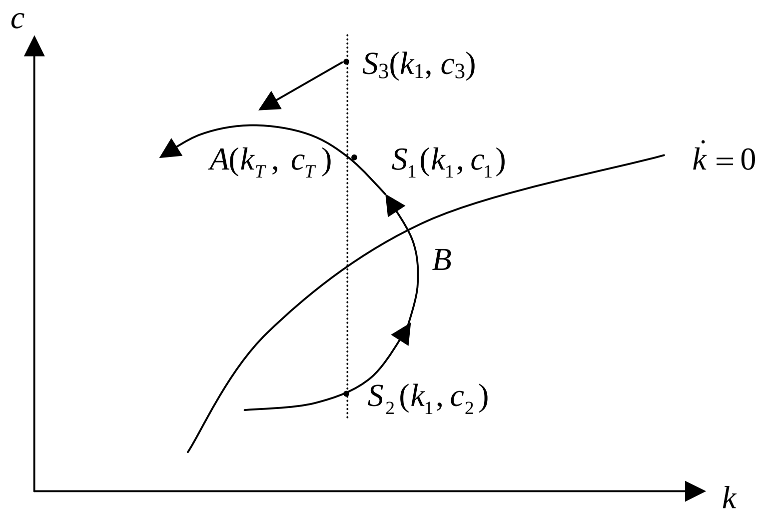


Fig. A.2

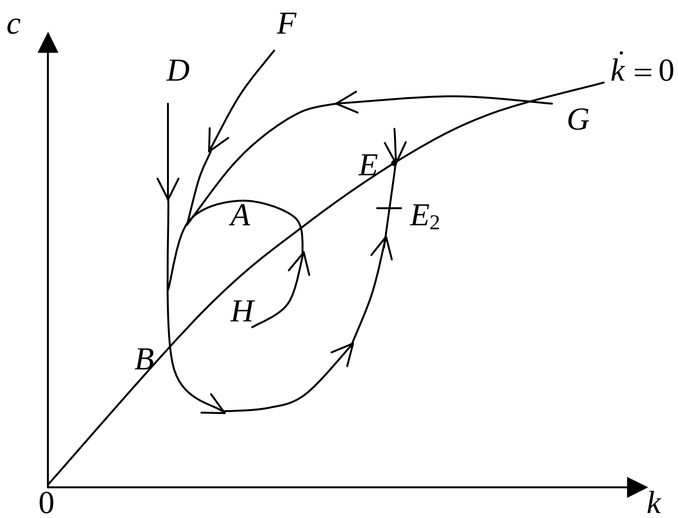


Fig. A.3

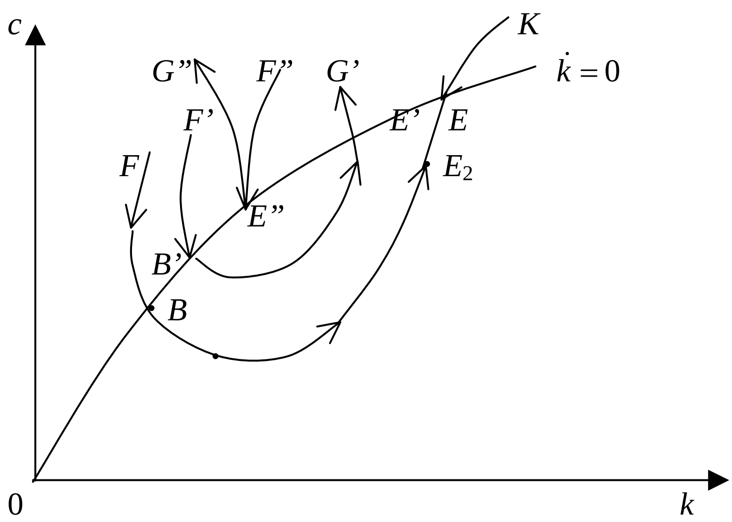


Fig. A.4

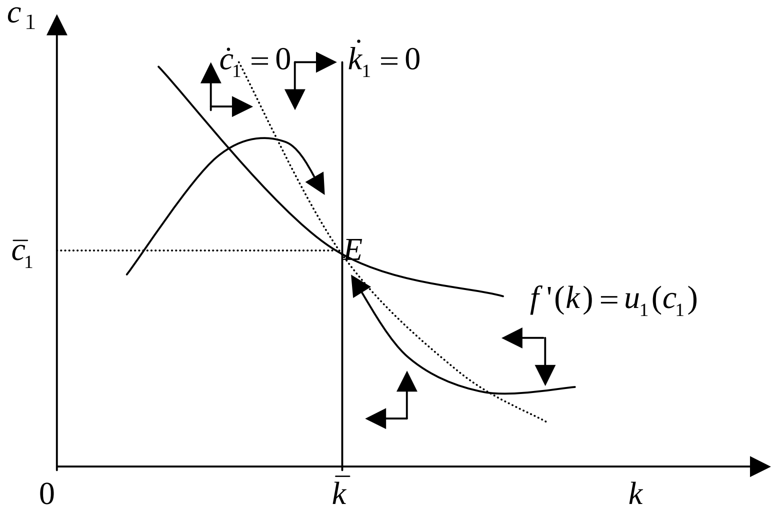


Fig. A.5a

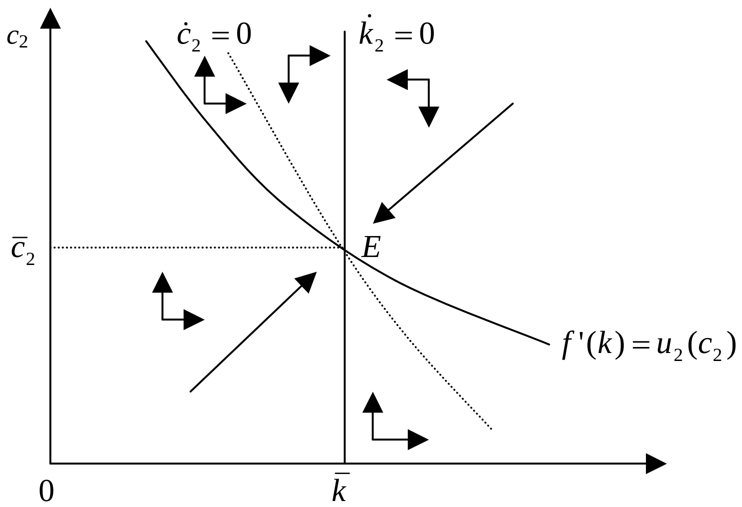


Fig. A.5b

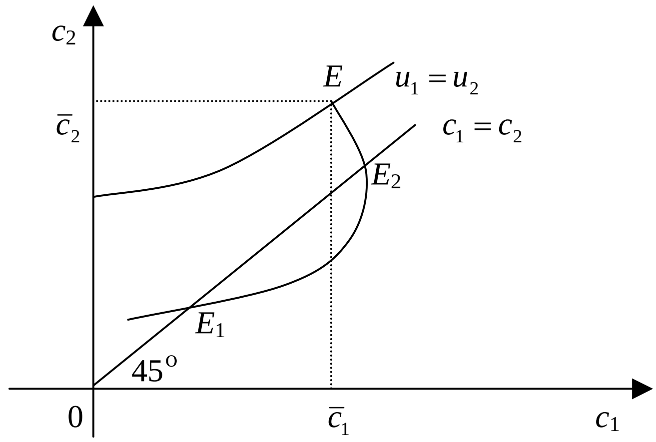


Fig. A.6

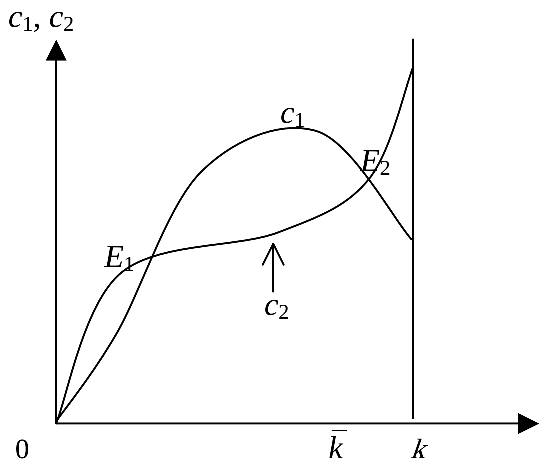
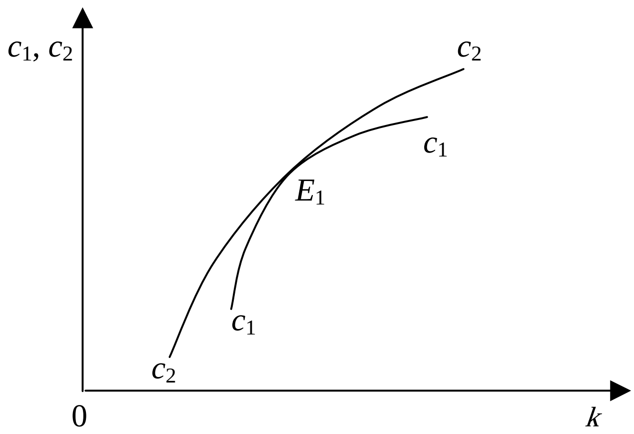


Fig. A.7



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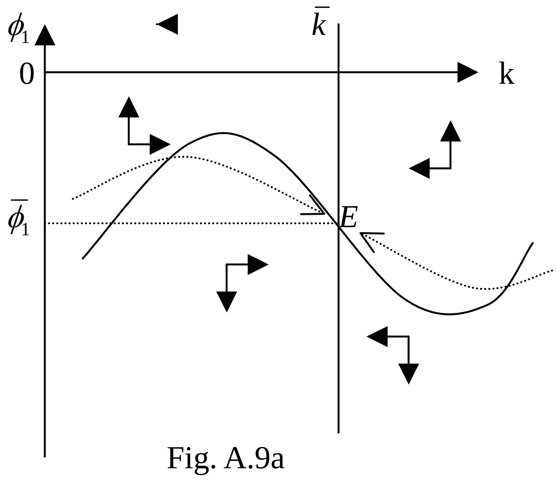

Fig. A.9a

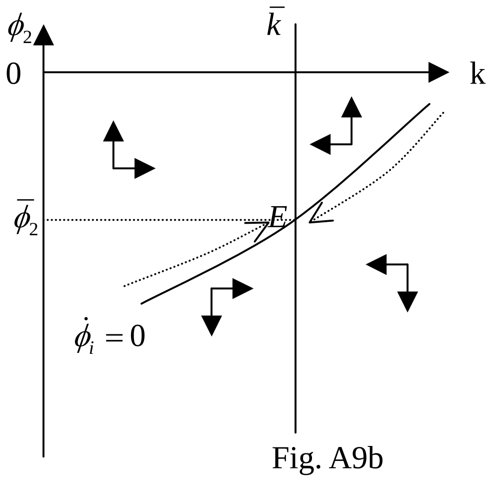

Fig. A9b

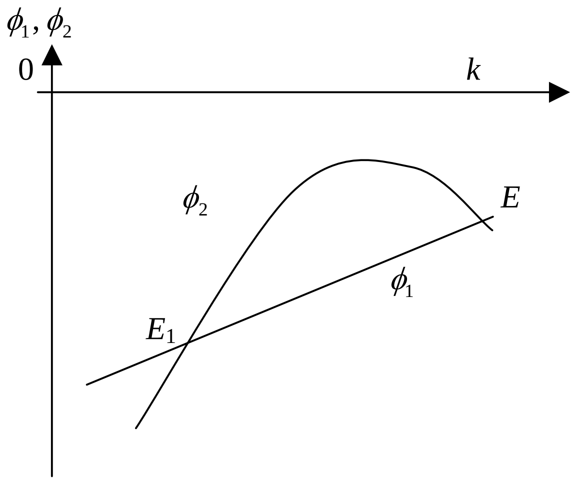

Fig.A.10

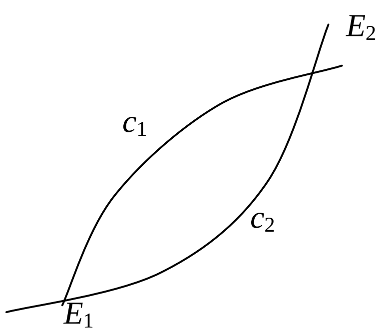

Fig. A.11

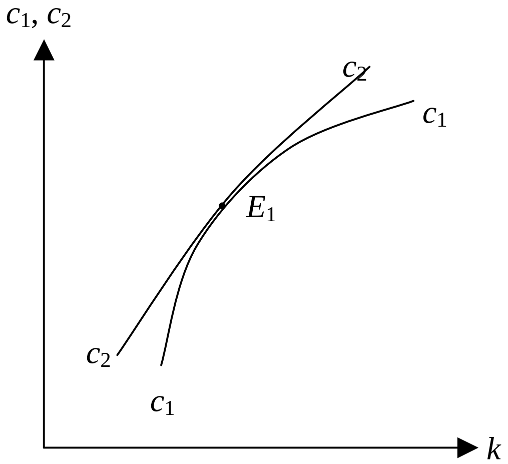

Fig. A.12

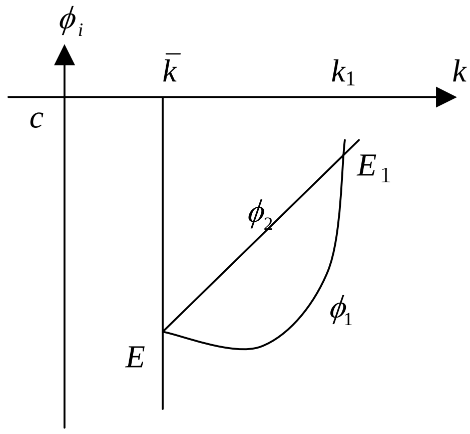

Fig. A.13

Notes

1. We thank an anonymous referee for pointing this out.
2. Again we owe an anonymous referee for pointing this out.
3. As for the recent data of these countries' high growth rates, trade surplus and low average propensities to consume, see, the tables of GDP Deflators, Balance of Payments and Final Consumption Expenditure as Percentage of GDP of *International Financial Statistics, Yearbook*(2004). Regarding the foreign dept position, see the table of Total external and public/publicly guaranteed long term debt of developing countries in *Statistical Yearbook*(2003), pp.797-803.
4. As seen in Theorem 1, the global stability of the optimal path *Y* is obtained with fewer sufficient conditions than Epstein(1987b). His sufficient conditions include (1) the desire for a smooth consumption profile, as measured by $-u_i''/u_i'$ is small, (2) the rate of time preference ρ_i increases quickly with the level of consumption in a constant program and (3) the rate of return in production is small. These are not needed in our proof.
5. Debereux and Shi (1991) specified $v_i = -1$ and $u_i(c_i) = \delta_i + \log(c_i + \alpha)$, $i = 1,2$ with $\delta_1 > \delta_2 > 0$.

6. From $w_i = f_i - k_i f_i'(k_i)$ and $f_i(k_i) = \theta_i k_i^{\xi}$ where $\theta_i = \theta \geq 1$ and $\theta_2 = 1$, we observe $w_i = \theta_i (1-\xi) k_i^{\xi}$, $i = 1,2$. Since $f_i'(k_i) = r$ holds always, we obtain $k_1 / k_2 = \theta^{1/(1-\xi)}$ or $k_1 \geq k_2$, and hence $w_1 \geq w_2$ holds always.
7. This suboptimal path can be constructed easily. Let $c_T = c_3 e^{-\beta T}$ which defines the constant decreasing rate of c to be $\beta = \log(c_3 / c_T)^{1/T}$. Next let γ be the constant decreasing rate of capital k so that $k_T = k_1 e^{-\gamma T}$ or $\gamma = \log(k_1 / k_T)^{1/T}$. Then from (2), $\dot{k} = f(k) - c = f(k_1 e^{-\gamma t}) - c_3 e^{-\beta t}$, c_3 is defined implicitly so that $c_3 - c_T \equiv \log(c_3 / c_T)^{1/T} \left[\int_0^T f(k_1 e^{-\gamma t}) dt + k_1 - k_T \right]$.
8. From $f'(k) = f_i'(k_i)$, $i = 1,2$, $k_i = k_i(k)$ with $k_i' > 0$ and hence $\dot{k}_i = k_i' \cdot \dot{k}$, $i = 1,2$ follow.
9. Although the slopes of the optimal path of Y_1 are drawn to change their signs for $k_0 > \bar{k}$ in Fig. A.5a and A.5b, this is not definite. Since the $\dot{c}_1 = 0$ curve never touches the horizontal axis, the optimal path $c_1 = c_1(k)$ can always lie below the $\dot{c}_1 = 0$ curve.

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