Trade Openness, Real Exchange Rates, and the Exchange Rate Regime Choice

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Abstract

When financial markets are not complete, the advantage of flexible exchange rates lies in exploiting the ‘expenditure switching effect of depreciation’ for short run consumption stabilization in the face of adverse shocks for the households whose relative risk aversion of CRRA utility is greater than unity. Under flexible rates, policy makers are able to increase money supply and depreciate the exchange rate to worsen the terms of trade in favor of home goods’ price competitiveness, increasing exports, output, income and consumption. When an economy is too open in terms of trade in goods, however, depreciation itself weakens its ‘expenditure-switching effect’ by raising the imports prices and their citizens’ cost of living. In an extreme case where trading economies are completely integrated via trade [in the sense that households across trading economies have the same compositions of consumption baskets], depreciation increases their cost of living exactly by the amount that offsets the gains from the terms of trade worsening by itself completely. In this case, the performances of fixed and flexible rates in terms of short run consumption stabilization for the households with the degree of relative risk aversion of CRRA utility greater than unity would become indifferent.

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1. Introduction

Which exchange rate regime between flexible and fixed rates is optimal is an old
aged question in the international macroeconomics literature since the Gold Standard was
abandoned during the World War I. About 5 decades ago, Meade [1950], Friedman [1953],
and Scitovsky [1958] advocated for flexible exchange rates, telling that in the short run,
their adjustment can substitute for the inflexible relative price adjustment of home and
foreign produced goods to restore the external and internal equilibrium of an economy.
The effect through which, they pointed out, flexible rates help to stabilize one economy
facing adverse shocks from within and from abroad is the expenditure switching effect of
depreciation. For example, an increase in the relative price of home-produced goods
resulted from an adverse home output shock raises the cost of living of home households,
reducing home consumption. Under flexible exchange rates, for the purpose of short run
stabilization, policy makers are able to increase money supply and depreciate the exchange
rate in the foreign exchange market to induce agents across countries to switch to relatively
cheap home-produced goods, stimulating exports and home production, and increasing
home income and consumption. It was this effect that Obstfeld and Rogoff [2000, 2002]
emphasized greatly as the major advantageous feature of flexible exchange rates when they
objected against the creation of the European Monetary Union as of January 1, 1999 and
argued for adopting flexible exchange rates between major trading economies.\footnote{2}

\footnote{2 A monetary expansion has two effects of liquidity and expenditure switching in the short run in an open
economy. The liquidity effect is to boost consumption and investment via the lowered domestic interest rate.
In our model with the assumption of perfect capital mobility across countries, this liquidity effect is absent
because capital outflows immediately until the domestic interest rate is restored to their original level. In the
absence of this liquidity effect, our paper focuses solely on the effect of expenditure switching of a monetary
expansion in the short run.}

\footnote{5 Devereux and Engel [2001] argues that exchange rate changes may not have a great influence on relative
prices across countries if prices are fixed ex ante in consumers’ currencies, and therefore the expenditure-
switching effect may not be substantial to make the case for flexible exchange rates valid. In our research, we
assume that the pass-through of exchange rate changes onto consumer prices is perfect, leaving the empirical
study on the size of expenditure-switching effect in the setting of Local Currency Pricing to other researchers.}
Friedman [1953, pp180], however, himself acknowledges that if the rise in relative prices of foreign goods means a rise in the cost of living of home households, and this in turn gives rise to a demand for wage increases, the change in relative prices across trading countries that are supposed to worsen may actually remain unchanged and hence there are no market forces working toward the internal and external equilibrium. Mundell [1961] and McKinnon [1963] point out that as one of the conditions for a system of flexible exchange rates to work effectively, wages and profits should not be tied to a price index in which imported goods are heavily weighted. The theoretic and empirical works of Lane [1997], and Campillo and Miron [1997] respectively demonstrate that in smaller and more open economies, a monetary expansion causes exchange rate depreciation to reduce the benefits of the expansionary monetary policy by raising the amount of inflation associated with a given expansion of domestic output. As a result, Corsetti and Pesenti [2001] argue that a monetary expansion may turn out to be beggar-thy-self. Summarizing, if the economy is highly open, the expenditure switching effect of depreciation can be significantly weakened by the rise in the prices of imports and the relative cost of living of home households by depreciation itself." That is, for short run consumption stabilization, the expenditure switching effect of depreciation crucially depends on the size of the fall in the relative cost of living of home country (the extent of real depreciation), which, in turn, depends on the trade openness between trading economies.††


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For the empirical study on the size of expenditure-switching effect in the setting of Local Currency Pricing, see Engel [2002], Bhattacharya, Karayalcin, Thomakos [2003] and Dong [2005].

**" Trade openness is defined as the share of imports in the consumption index or in the consumption basket in our paper.

†† Consumption based real exchange rate is defined as the relative cost of living across countries.
of non-traded goods across countries, when country-specific shocks change the relative prices of goods produced in home and foreign countries. In the presence of non-traded goods, country-specific shocks make the cost of living of home and foreign agents different, resulting in the real exchange rate fluctuations. As the compositions of the consumption baskets become similar [or as the weight of non-traded goods in the consumption basket gets smaller], the size of real exchange rate fluctuations become smaller. In the dynamic general equilibrium models with non-traded goods, Obstfeld and Rogoff [2000], and Hau [2000] demonstrated that real exchange rate fluctuations are less volatile in more open economies.‡‡ To test their theoretical prediction, Hau [2002] treated the foreign country as a hypothetical ‘rest of the world country,’ and constructed trade-weighted effective real exchange rates for 21 OECD countries where relative shocks are of similar magnitude. The volatility of the real exchange rate is measured by the standard deviation of the percentage changes of the real exchange rate. Trade openness is defined as the share of imports in GDP. The following figure is the scatter plot of real exchange rate volatility against the degree of openness for 21 OECD countries.

‡‡ Hau [2002] calls this effect the ‘real exchange rate magnification effect of nontradables.’
Hau [2002] concludes that more open countries have the less volatile real exchange rate by stating that the OECD sample identifies trade openness as the most important determinant of real exchange rate volatility for developed countries.\(^5\)

In the 2 country dynamic stochastic general equilibrium model of intertemporal utility optimization featuring monopolistic competition, inflexible prices, and incomplete financial markets, we demonstrate that it is the real exchange rate rather than the terms of trade that determines the strength of the expenditure switching effect of depreciation in the short run consumption stabilization for households with the degree of relative risk aversion greater than 1 in CRRA utility function. In addition, we show that depreciation leads to real depreciation given price rigidity in the short run only if preferences across countries are home biased [or if there exist non-traded goods across trading economies]. As the degree of a country’s trade openness increases,*** the size of real depreciation becomes smaller given a certain extent of depreciation, and therefore the gain from the expenditure-switching effect of depreciation under flexible rates decreases. We also find that if countries are completely integrated via trade [or equivalently if countries have identical compositions of their consumption baskets], the depreciation increases home households’ relative cost of living by exactly to offset the terms of trade worsening by itself so that there would be no real depreciation making the expenditure switching effect of depreciation under flexible rates effective. Therefore, with complete goods’ market integration, flexible rates become indifferent to fixed rates in terms of short run consumption stabilization for households with the degree of relative risk aversion of CRRA utility greater than unity.

\(^5\) Hau [2002] finds that the basic regression (without controls) shows a surprisingly high adjusted R\(^2\) of 0.470, which means approximately half of the cross-sectional volatility is explained by trade openness. Inclusion of the log per capita GDP, an index of central bank independence, and a dummy for exchange rate commitment as control variables decreases the coefficient of the openness measure only slightly and remains highly significant at a 1 percent level.

\(^*\) As the degree of relative risk aversion of the CRRA utility rises from unity, or as the inverse of the elasticity of money demand to consumption increases from one, the size of real depreciation becomes smaller, too.
The following is the organization of our paper. Section II describes the structure of the 2 country dynamic stochastic general equilibrium model of intertemporal utility optimization featuring monopolistic competition, inflexible prices, incomplete financial markets, and non-traded goods. Section III solves for the real exchange rate as a function of the parameter of trade openness, and ex post country-specific output and monetary shocks. It shows how the real exchange rate is affected by the degree of trade openness given ex post country-specific output and monetary shocks. Section IV derives the relation between consumption and ex post country-specific output and monetary shocks. Sections V and VI respectively measure the degrees of consumption risk sharing under flexible exchange rates with Nash money rule and under fixed rates in the face of country-specific adverse output shocks. In Section VII, a utility based welfare comparison of flexible and fixed rate regimes is conducted via the numerical simulation. Section VIII concludes.

II. The Model

Preferences

In the world economy, there are two countries of the same economic size, Home and Foreign. In Home and Foreign, there are continuums of identical households, $0 \leq v \leq 1$ and $1 \leq v \leq 2$ respectively, each of who specializes in a single differentiated product indexed by $v$. The representative household $v$ in Home is assumed to maximize his lifetime utility given by

$$U_i = \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_{i,t}^{1-\rho}}{1-\rho} + \frac{\chi}{1-\varepsilon} \left( \frac{M_{i,t}}{P_t} \right)^{1-\varepsilon} - \eta Y_i(v) \right], 0 \leq \beta \leq 1, \quad 0 \leq \rho < \infty, \quad 0 < \varepsilon < \infty \text{[1]}$$

where $Y_i(v)$ is the amount of the representative product $v$ produced by the
representative household \( v \). \( \beta \) denotes the time discount rate, and \( \rho \) is the degree of relative risk aversion of CRRA utility function. \( C \) is the index of per capita consumption. Real money holding \( M/P \) provides a liquidity service via the reduction of transaction costs of goods and assets. The inverse of the elasticity of money demand with respect to consumption is \( \varepsilon \), and \( \chi \) is some constant. Technology shows constant returns to scale so that \( Y(v) = L(v) \), where \( L(v) \) denotes the amount of labor supplied by the representative household, \( v \). \( \eta \) is an expected adverse output shock arising in the home country that adversely affects home households’ utility.

Households’ preferences across countries are identically asymmetric since the weights on domestically produced goods and imports, \( \gamma \) and \( 1 - \gamma \), are the same. The indexes of per capita consumption of home and foreign countries are the following.

\[
C \equiv \frac{C_H^\gamma C_F^{1-\gamma}}{\gamma^\gamma (1 - \gamma)^{1-\gamma}};\quad C^* \equiv \frac{(C_H^*)^{1-\gamma} (C_F^*)^\gamma}{\gamma^\gamma (1 - \gamma)^{1-\gamma}}, \quad 0 \leq \gamma \leq 1
\]

[2]

where \( C_H \) and \( C_F \) are respectively the representative home household’s consumption of home and foreign produced goods, and \( C_H^* \), and \( C_F^* \) are the representative foreign household’s consumption of home and foreign produced goods respectively.

The sub-indexes of per capita consumption of home and foreign goods in home and foreign countries are respectively,

\[
C_H = \left[ \int_0^v C_H(v)^{\frac{\sigma-1}{\sigma}} dv \right]^{\frac{\sigma}{\sigma-1}};\quad C_F = \left[ \int_0^v C_F(v)^{\frac{\sigma-1}{\sigma}} dv \right]^{\frac{\sigma}{\sigma-1}}
\]

[3]

\[
C_H^* = \left[ \int_0^v C_H^*(v)^{\frac{\sigma-1}{\sigma}} dv \right]^{\frac{\sigma}{\sigma-1}};\quad C_F^* = \left[ \int_0^v C_F^*(v)^{\frac{\sigma-1}{\sigma}} dv \right]^{\frac{\sigma}{\sigma-1}}
\]

[4]

where \( C_H(v) \) and \( C_F(v) \) are respectively the representative home household’s
consumption of home and foreign produced goods, and $C_H(v)$, and $C_F(v)$ are the representative foreign household’s consumption of home and foreign produced goods respectively. The elasticity of substitution between goods produced within the same country is $\theta$ that is assumed to be greater than 1, while the elasticity of substitution between goods produced in Home and Foreign, $\sigma$ is assumed to be 1.

**Cost of Living of the Representative Households in Home and Foreign**

The consumption-based price indexes of home and foreign countries are as follows.

$$P \equiv (P_H)^{(1/\gamma)}; \quad P^* \equiv (P_H^*)^{(1/\gamma)}$$

[5]

where $P_H$ and $P_F$ are home country’s price indexes for the goods produced in home and foreign countries, and $P_H^*$ and $P_F^*$ are foreign country’s price indexes for the goods produced in home and foreign countries.

The sub-price indexes for home and foreign goods are respectively,

$$P_H = \left[ \int_0^1 P_H(v)^{1-\theta} \, dv \right]^{1/\theta}; \quad P_F = \left[ \int_0^1 P_F(v)^{1-\theta} \, dv \right]^{1/\theta}$$

[6]

$$P_H^* = \left[ \int_0^1 P_H^*(v)^{1-\theta} \, dv \right]^{1/\theta}; \quad P_F^* = \left[ \int_0^1 P_F^*(v)^{1-\theta} \, dv \right]^{1/\theta}$$

[7]

where $P_H(v)$ and $P_F(v)$ are the prices of the representative goods produced in home and foreign countries in the home country, while $P_H^*(v)$ and $P_F^*(v)$ are the prices of the representative goods produced in home and foreign countries in the foreign country, respectively. The law of one price is assumed to hold for each individual good so that $P(v) = S P^*(v), \, \forall v \in [0,2]$, where $S$ is the spot exchange rate of home currency to
foreign currency. For the sub-price indexes such as \( P_H \) and \( P_F \), consumption-based purchasing power parity holds so that \( P_H = SP_H^* \), and \( P_F = SP_F^* \). Because home and foreign households do not have an identical preference on home and foreign-produced goods, consumption-based purchasing parity for overall consumer price indexes, \( P ≠ SP^* \), does not hold.

**Goods Market Equilibrium**

Under sub-demand functions [3] and [4], optimal intratemporal consumption choices for each differentiated goods are as follows.

\[
C_{H}(v) = \left[ \frac{P_{H}(v)}{P_H} \right]^{-\theta} C_{H} ; \quad C_{F}(v) = \left[ \frac{P_{F}(v)}{P_F} \right]^{-\theta} C_{F} \tag{8}
\]

\[
C_{H}^*(v) = \left[ \frac{P_{H}(v)}{P_H^*} \right]^{-\theta} C_{H}^* ; \quad C_{F}^*(v) = \left[ \frac{P_{F}(v)}{P_F^*} \right]^{-\theta} C_{F}^* \tag{9}
\]

where \( C_{H}(v) \) and \( C_{F}(v) \) are the demand for the representative home and foreign goods of the home representative household, while \( C_{H}^*(v) \) and \( C_{F}^*(v) \) are the demand for the representative home and foreign goods of the foreign representative household.

The Cobb-Douglas overall consumption indexes imply that the demands for home and foreign goods, \( C_H \), \( C_F \), \( C_H^* \), and \( C_F^* \) are given by

\[
C_H = \gamma \left( \frac{P_H}{P} \right)^{-1} C ; \quad C_F = (1 - \gamma) \left( \frac{P_F}{P} \right)^{-1} C \tag{10}
\]

\[
C_H^* = (1 - \gamma) \left( \frac{P_H^*}{P^*} \right)^{-1} C^* ; \quad C_F^* = \gamma \left( \frac{P_F^*}{P^*} \right)^{-1} C^* \tag{11}
\]

Combining [8] and [10], and [9] and [11] respectively gives
The world consumption for each individual good produced in home and foreign countries is defined as follows.

\[
C_{H_i}^w(v) = C_{H_i}^i(v) + C_{H_i}^H(v); \quad C_{F_i}^w(v) = C_{F_i}^i(v) + C_{F_i}^F(v) \tag{14}
\]

where \(C_{H_i}^w(v)\), and \(C_{F_i}^w(v)\) represent total world consumption for each individual good produced in Home and Foreign countries respectively. Plugging [12] and [13] into [14] gives

\[
C_{H_i}^w(v) = \gamma \left( \frac{P_{H_i}^*(v)}{P_{H_i}} \right)^{\theta} \left( \frac{P_{H_i}}{P} \right)^{-1} C + (1 - \gamma) \left( \frac{P_{H_i}^*(v)}{P_{H_i}} \right)^{\theta} \left( \frac{P_{H_i}}{P^*} \right)^{-1} C^* \tag{15}
\]

\[
C_{F_i}^w(v) = (1 - \gamma) \left( \frac{P_{F_i}^*(v)}{P_{F_i}} \right)^{\theta} \left( \frac{P_{F_i}}{P} \right)^{-1} C + \gamma \left( \frac{P_{F_i}^*(v)}{P_{F_i}} \right)^{\theta} \left( \frac{P_{F_i}}{P^*} \right)^{-1} C^* \tag{16}
\]

The goods market for each individual good produced in home and foreign countries clears when the demand equals the supply. Taking into account of the population of two countries and evaluating it at the symmetric equilibrium, where, \(P_{H_i}(v) = P_{H_i}\), and \(P_{F_i}(v) = P_{F_i}\), we obtain the world market clearing condition for each individual good produced in home and foreign countries as follows.

\[
C_{H_i}^w(v) = \gamma \left( \frac{P_{H_i}}{P} \right)^{-1} C + (1 - \gamma) \left( \frac{P_{H_i}^*}{P^*} \right)^{-1} C^* = Y(v) \tag{17}
\]
\[ C^*(v) = \left\{ \left(1 - \gamma \left( \frac{P_v}{P} \right)^{-1} \right) C + \gamma \left( \frac{P_v^*}{P^*} \right)^{-1} C^* \right\} = Y^*(v) \]  \[ [18] \]

**Asset Market Equilibrium**

Asset markets are assumed to be not complete either because Arrow-Debreu state-contingent futures contracts or real bonds ensuring one unit of composite consumption in the next period are not available, or because the number of risky assets to span idiosyncratic output shocks is limited. Because including risky assets that are not enough to span idiosyncratic shocks into the model would only complicate the analysis without any benefit, we exclude them. Only two nominal bonds denominated in home and foreign currencies are assumed to be available to the households across countries. Bonds denominated in home currency pay one unit of the home currency in the next period, while foreign currency-denominated bonds pay one unit of foreign currency. The current price of the futures contract in home currency is the discounted present value of one unit of home currency by the current domestic nominal interest rate, \( Q = 1/(1 + i) \). The current price of futures contract in foreign currency is the current spot exchange rate times the discounted present value of one unit of foreign currency by the current foreign nominal interest rate, \( S \cdot Q^* = S / (1 + i^*) \). The market clearing conditions in the world asset markets are respectively,

\[ B_{H,t} + B_{H,t}^* = 0, \quad B_{F,t} + B_{F,t}^* = 0 \quad \forall t \]  \[ [19] \]

where \( B_{H,t} \) and \( B_{F,t} \) are home and foreign currency denominated bonds.

**The Budget Constraint**
Given intra-temporal consumption choices, the budget constraint of the representative household in the home country is as follows.

\[ P_j C_j + M_j + Q_j B_{H_j} + S_j Q_j B_{F_j} = M_{t-1} + P_{H_j} (v) Y_j (v) + B_{H_j-1} + S_j B_{F_j-1} + T_j \]  \[ 20 \]

where \( Q_j \) and \( Q_j^* \) are the prices of home and foreign currency denominated bonds. \( T_j \) is the monetary transfer from the government to each citizen. Only domestic currency is assumed to be held by the household in each country.

The government budget constraint is given as follows. The change in the money supply by the government is transferred directly to the each household. There are no government expenditures over time.

\[ M_j = M_{j-1} + T_j \]  \[ 21 \]

**First Order Conditions for the Representative Households in Home and Foreign**

The problem of the representative household in each country is to choose rules for holding nominal money balances, \( M \), home-currency denominated bonds, \( B_{H_i} \), and foreign-currency denominated bonds, \( B_{F_i} \), by maximizing his lifetime expected utility \[ 1 \] given his life-time budget constraint that are the present discounted sum of \[ 20 \]. The initial values of \( M \), \( B_{H_i} \), and \( B_{F_i} \) are given.

First order conditions for the representative home household are as follows.

\[ Q_j = \beta E_j \left( \frac{P_{t+1}^{-1} C_{t+1}^{-\rho}}{P_j^{-\rho} C_j^{-\rho}} \right) \]  \[ 22 \]

\[ Q_j^* = \beta E_j \left( \frac{S_{t+1}^{-1} P_{t+1}^{-1} C_{t+1}^{-\rho}}{S_j^{-\rho} P_j^{-\rho} C_j^{-\rho}} \right) \]  \[ 23 \]
First order conditions for the representative foreign household are as follows.

\[
Q_t = \beta \frac{E_t(S_{t+1}^{-1}P_{t+1}^{-1}C_{t+1}^{-\rho})}{S_t^{-1}P_t^{-1}C_t^{-\rho}}
\]

\[
\dot{Q}_t = \beta \frac{E_t(P_{t+1}^{-1}C_{t+1}^{-\rho})}{P_t^{-1}C_t^{-\rho}}
\]

\[
\left(\frac{M_t}{P_t}\right)^\varepsilon = \frac{\chi C_t^\rho}{1 - Q_t}
\]

**Short Run Inflexible Prices in Goods Markets**

The monopoly prices of the representative consumer-producers in home and foreign countries, \(P_{H,t-1}(v)\) and \(P_{F,t-1}(v)\) are determined by maximizing their lifetime expected utility, [1], given the information at time \(t-1\) and their life-time budget constraint at the symmetric equilibrium where \(P_{H,t-1}(v) = P_{H,t-1}^*\) and \(P_{F,t-1}(v) = P_{F,t-1}^*\).

\[
P_{H,t-1}(v) = P_{H,t-1} = \left(\frac{\theta}{\theta - 1}\right) \frac{E_t[\eta Y_t(v)]}{E_t\left\{\frac{Y_t(v)}{P_t C_t^\rho}\right\}}
\]

\[
P_{F,t-1}^*(v) = P_{F,t-1}^* = \left(\frac{\theta}{\theta - 1}\right) \frac{E_t[\eta^* Y_t^*(v)]}{E_t\left\{\frac{Y_t^*(v)}{P_t^* C_t^*\rho}\right\}}
\]

**Consumption Risk Sharing Condition**

From the first order conditions for bond holdings, equating [22] and [25], and [23] and [26] respectively gives the following two equations.
\[ Q_t = \beta \frac{E_t(P_t^{-1}C_t^{-\rho})}{P_t^{-1}C_t^{-\rho}} = \beta \frac{E_t(S_t^{-1}P_t^{-1}C_t^{-\rho})}{S_t^{-1}P_t^{-1}C_t^{-\rho}}; \quad [30] \]

\[ Q_t^* = \beta \frac{E_t(S_t^{-1}P_t^{-1}C_t^{-\rho})}{S_tP_t^{-1}C_t^{-\rho}} = \beta \frac{E_t(P_t^{-1}C_t^{-\rho})}{P_t^{-1}C_t^{-\rho}} \quad [31] \]

From [30] or [31], assuming that both countries are initially in the symmetric equilibrium where \( P_t^{-1}C_t^{-\rho} = (P_t^*)^{-1}(C_t^*)^{-\rho} \) gives the following expression.

\[ \Rightarrow \left( C_t/C_t^* \right)^\rho = \frac{S_tP_t^*}{P_t} \quad \forall t \quad [32] \]

The equation [32] says that the presence of asset markets for nominal one-year bonds in different currency denomination ensures that the ratio of the marginal utility of consumption in home and foreign countries equals the real exchange rate. It implies that relative consumption between countries varies with respect to the changes in the real exchange rate. This condition is called ‘consumption risk sharing condition.’


\[ \frac{C_t}{C_t^*} = \left( \frac{S_tP_t^*}{P_t} \right)^{\frac{\rho}{1-\rho}} \quad \forall t \quad [33] \]

From the equation [33], we see that relative consumption relies not only on the change in the terms of trade but also on the parameters of the CRRA utility function of the share of the home-produced goods in the consumption basket, the degree of relative risk aversion.

III. Real Exchange Rates and ex post Country-specific Output and Monetary
Shocks

Suppose that the growth of output and money supply in both home and foreign countries follow a random walk.

\[
\begin{align*}
\log \eta_t &= \log \eta_{t-1} + \xi_t; \quad \log \eta_t^* = \log \eta_{t-1}^* + \xi_t^* \\
\log M_t &= \log M_{t-1} + \mu_t; \quad \log M_t^* = \log M_{t-1}^* + \mu_t^*
\end{align*}
\]

where \( \xi_t^* \sim N(0, \sigma_{\xi}^2) \), and \( \mu_t^* \sim N(0, \sigma_{\mu}^2) \) for every date \( t \) are defined as output and monetary shocks. Distributions from which output and monetary shocks are redrawn are assumed to be time-invariant and lognormal.

Log-linearizing the money demand equations of the home and foreign countries, [24] and [27] at a non-stochastic steady state where \( Q = Q^* = \overline{Q} \) gives

\[
\begin{align*}
\epsilon \{ m_t - p_t \} &= \log \chi - \log(1 - Q) + \rho \epsilon_t \\
\epsilon \{ m_t^* - p_t^* \} &= \log \chi^* - \log(1 - Q^*) + \rho \epsilon_t^*
\end{align*}
\]

Adding two equations [36] and [37] under the assumption that at the initial equilibrium, \( \log M_{t-1} = \log M_{t-1}^* \), \( \chi = \chi^* \), and \( Q = Q^* \) gives the following expression.

\[
\rho \{ \epsilon_t + \epsilon_t^* \} = \epsilon \{ \mu_t + \mu_t^* \} - \epsilon \{ p_t + p_t^* \}
\]

where \( \rho(\epsilon_t - \epsilon_t^*) = (2 \gamma - 1)(s_t + p_{f,i-1}^* - p_{H,i-1}) \) [39]

\[
\begin{align*}
p_t &= \gamma p_{H,i-1} + (1 - \gamma)s_t + (1 - \gamma)p_{f,i-1}^* \\
p_t^* &= (1 - \gamma)p_{H,i-1} - (1 - \gamma)s_t + \gamma p_{f,i-1}
\end{align*}
\]

Plugging [40] and [41] into [38] and [39], and solving for \( \epsilon_t \) and \( \epsilon_t^* \) gives

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* Equivalently, \( i = i' = i \).
\[ \varepsilon_i = \frac{(2\gamma-1)}{2\rho} \left[ \varepsilon_i - (p_{H,i-1} - p_{F,2,i-1}) \right] + \left( \frac{\varepsilon}{2\rho} \right) \left[ \mu_i + \mu_i' \right] - \left( \frac{\varepsilon}{2\rho} \right) \left[ p_{H,i-1} + p_{F,2,i-1} \right] \]  \hspace{1cm} [42]

\[ \varepsilon_i' = -\frac{(2\gamma-1)}{2\rho} \left[ \varepsilon_i' - (p_{H,i-1} - p_{F,2,i-1}) \right] + \left( \frac{\varepsilon}{2\rho} \right) \left( \mu_i + \mu_i' \right) - \left( \frac{\varepsilon}{2\rho} \right) \left[ p_{H,i-1} + p_{F,2,i-1} \right] \]  \hspace{1cm} [43]

Subtracting the equation [37] from [36] under the assumption that at the initial equilibrium,

\[ \log M_{t-1} = \log M_{t-1}^* = 0, \quad \chi = \chi^*, \quad \text{and} \quad Q = Q^* \] gives the following expression.

\[ \rho \{ \varepsilon_i - \varepsilon_i' \} = \varepsilon \{ \mu_i - \mu_i' \} - \varepsilon \{ p_i - p_i' \} \]  \hspace{1cm} [44]

Plugging [40] and [41] into [44] and [39] gives the following expression of the ex post terms of trade as a function of ex post monetary shock and price revisions in the current period.

\[ \varepsilon_i + p_{F,2,i-1} - p_{H,i-1} = \left( \frac{\varepsilon}{(2\gamma-1) + 2\varepsilon(1-\gamma)} \right) \left( \mu_i - \mu_i' \right) - \left( \frac{\varepsilon}{(2\gamma-1) + 2\varepsilon(1-\gamma)} \right) \left( p_{H,i-1} - p_{F,2,i-1} \right) \]  \hspace{1cm} [45]

Taking a log of [28] and [29] in the current period gives

\[ p_{H,i-1} = \log \left( \frac{\theta}{\theta - 1} \right) + E \log \eta_i + E p_i + \rho E \varepsilon_i \]  \hspace{1cm} [46]

\[ p_{F,2,i-1} = \log \left( \frac{\theta}{\theta - 1} \right) + E \log \eta_i^* + E p_i^* + \rho E \varepsilon_i^* \]  \hspace{1cm} [47]

Subtracting [47] from [46] under [34], [35], and the assumption that \( \log \eta_{i-1} = \log \eta_{i-1}^* \), and combining it with the equations [39], [40] and [41] gives the ex ante terms of trade.

\[ p_{H,i-1} - p_{F,2,i-1} - E \varepsilon_i = (\xi_i - \xi_i^*) \]  \hspace{1cm} [48]

where \( E \varepsilon_i = \left( \frac{(\varepsilon - 1)(2\gamma-1)}{(2\gamma-1) + 2\varepsilon(1-\gamma)} \right) \left( p_{H,i-1} - p_{F,2,i-1} \right) \)  \hspace{1cm} [49]
The expected exchange rate in [49] is derived by taking the expectation of the equation [45] under the assumption that future period monetary surprises are not expected by the households. Combining [48] and [49] gives

\[ p_{H,t-1} - p_{F,t-1}^* = \left( \frac{(2\gamma - 1) + 2\epsilon(1-\gamma)}{\epsilon} \right) (\xi_t - \xi_t^*) \]  \[ 50 \]

Plugging [50] into [45] gives the ex post terms of trade as a function of ex post relative money supply and relative output shocks.

\[ s_t + p_{F,t-1}^* - p_{H,t-1}^* = \left( \frac{\epsilon}{(2\gamma - 1) + 2\epsilon(1-\gamma)} \right) \left[ \mu_t - \mu_t^* \right] - (\xi_t - \xi_t^*) \]  \[ 51 \]

The real exchange rate is defined as the relative cost of living across countries. Combining it with the home and foreign price indices and log-linearizing it gives

\[ s_t + p_t^* - p_t = (2\gamma - 1) (s_t + p_{F,t-1}^* - p_{H,t-1}^*) \]  \[ 52 \]

Combining [51] and [52] gives the following expression for the real exchange rate as ex post relative money and relative output shocks.

\[ s_t + p_t^* - p_t = (2\gamma - 1) \left[ \left( \frac{\epsilon}{(2\gamma - 1) + 2\epsilon(1-\gamma)} \right) \left[ \mu_t - \mu_t^* \right] - (\xi_t - \xi_t^*) \right] \]  \[ 53 \]

From [53], we notice that the change in the real exchange rate is influenced by the degree of trade openness given ex post relative money and output shocks. When \( \gamma > \frac{1}{2} \) (there are non-traded goods across countries), the adverse home relative output shock appreciates the real exchange rate. The more open the economies are, the less are the fluctuations of real
exchange rates as in Hau [2000, 2002].

IV. Consumption and ex post Country-specific Output and Monetary Shocks

Plugging [40], [41], [46], and [47] into [38] at the initial symmetric equilibrium under [34], [35], and the assumption that \( \log \eta_{j,t-1} = \log \eta_{j,t-1}^* \) gives

\[
p_{H,t-1} + p_{F,t-1} = (\mu_t + \mu_t^*) + \frac{1}{\epsilon} (\xi_t + \xi_t^*) \tag{54}
\]

Plugging [51] and [54] into [41] and [42] respectively brings home and foreign consumption as functions of ex post country-specific output and monetary shocks.

\[
\epsilon_t = \frac{(2\gamma - 1)}{2\rho} \left( \frac{\epsilon}{(2\gamma - 1) + 2\epsilon(1-\gamma)} \right) \{\mu_t - \mu_t^*\} - \frac{\gamma}{\rho} \{\xi_t\} - \frac{(1-\gamma)}{\rho} \{\xi_t^*\} \tag{55}
\]

\[
\epsilon_t^* = -\frac{(2\gamma - 1)}{2\rho} \left( \frac{\epsilon}{(2\gamma - 1) + 2\epsilon(1-\gamma)} \right) \{\mu_t - \mu_t^*\} - \frac{(1-\gamma)}{\rho} \{\xi_t\} - \frac{\gamma}{\rho} \{\xi_t^*\} \tag{56}
\]

With respect to monetary shocks, home consumption rises by a half of real depreciation multiplied by relative monetary expansion when Home and Foreign are symmetric (that is, they have equal economic sizes). Domestic adverse output shock reduces home consumption by the share of home goods in the consumption basket discounted by the degree of relative risk aversion, while foreign adverse output shock by the share of imports discounted by the degree of relative risk aversion.

V. Consumption Risk Sharing By Nash Money Rule under Flexible Rates

Since no output persistence is assumed, the monetary authority only needs to maximize the period utility for their citizens’ intertemporal utility maximization. Since all
shocks in our model are ex post, taking the expectation of the period utility to evaluate it is not necessary. From the lifetime utility of the representative home household, [1], the money utility component can be ignored since the utility from money holdings is trivial as compared to the utility from consumption and labor.

\[
U \equiv \frac{C^{1-\rho}}{1-\rho} - \eta Y(\rho)
\]  

[54]

Combining the goods market equilibrium condition [17] and the monopoly pricing equation [28], and evaluating it at the symmetric equilibrium where \( C = C^* \) gives

\[
\{\eta Y\} = \left(\frac{\theta - 1}{\theta}\right)\{\rho C^{1-\rho} + (1-\gamma)C^{1-\rho}\} = \left(\frac{\theta - 1}{\theta}\right)\{C^{1-\rho}\}
\]  

[55]

Combining [54] and [55] gives the following expression for the period utility.

\[
U \equiv \frac{C^{1-\rho}}{1-\rho} - \left(\frac{\theta - 1}{\theta}\right)C^{1-\rho} = \left(\frac{\theta - (1-\rho)(\theta - 1)}{\theta}\right)\frac{C^{1-\rho}}{1-\rho}
\]  

[56]

Log-linearizing [56] at the symmetric equilibrium where \( C = C^* \) gives

\[
\nu \equiv \log\left(\frac{\theta - (1-\rho)(\theta - 1)}{(1-\rho)\theta}\right) - (\rho - 1)c
\]  

[57]

If \( \rho = 1 \), home households’ utility would not be affected by the change in consumption. If \( \rho > 1 \), minimizing the change in consumption is equivalent to maximizing the change in the utility.

We assume that there are neither velocity innovations in the money demand nor expected autonomous money supply shocks. We use the same notations of money supply.
shocks $\mu$ and $\mu^*$ for accommodative money rules of the home and foreign countries.

Two countries, Home and Foreign, have two objectives of maximizing $U$ and $U^*$, and two monetary instruments of $\mu$ and $\mu^*$. Nash rule is such that a country facing an adverse shock manipulates the terms of trade in favor of their products and thereby improve domestic welfare at the expense of foreign welfare. The optimal Nash money rules for home and foreign countries are obtained by optimizing each country’s consumption changes [54] and [55] with respect to home and foreign monetary instruments simultaneously under the assumption that the other country’s monetary policy is given.

$$
\mu_i = \left( \frac{2\gamma}{2\gamma-1} \right) \left( \frac{2\gamma-1 + 2\varepsilon(1-\gamma)}{\varepsilon} \right) \{\xi, \gamma\} + \left( \frac{2(1-\gamma)}{2\gamma-1} \right) \left( \frac{2\gamma-1 + 2\varepsilon(1-\gamma)}{\varepsilon} \right) \{\xi^*, \gamma\} \quad [58]
$$

$$
\mu_i^* = \left( \frac{2(1-\gamma)}{2\gamma-1} \right) \left( \frac{2\gamma-1 + 2\varepsilon(1-\gamma)}{\varepsilon} \right) \{\xi, \gamma\} + \left( \frac{2\gamma}{2\gamma-1} \right) \left( \frac{2\gamma-1 + 2\varepsilon(1-\gamma)}{\varepsilon} \right) \{\xi^*, \gamma\} \quad [59]
$$

To see the Nash outcome, subtracting [59] from [58] gives

$$
\{\mu_i - \mu_i^*\} = 2 \left( \frac{2\gamma-1 + 2\varepsilon(1-\gamma)}{\varepsilon} \right) \{\xi, \gamma - \xi^*\} \quad [60]
$$

Nash rule is to increase money supply by twice as much as to offset the real appreciation by the shock. Substituting [60] into [52] and [53] gives the consumption of home and foreign agents after the Nash rule implementation.

$$
\varepsilon_i = -\frac{1-\gamma}{\rho} \{\xi, \gamma\} - \frac{\gamma}{\rho} \{\xi^*\} \quad [61]
$$

$$
\varepsilon_i^* = -\frac{\gamma}{\rho} \{\xi, \gamma\} - \frac{1-\gamma}{\rho} \{\xi^*\} \quad [62]
$$

Under Nash rule, consumption of home and foreign households are reduced by the share
of imports in their consumption baskets divided by the degree of relative risk aversion. The
higher the degree of relative risk aversion, the smaller is the impact of the domestic adverse
output shock on domestic consumption. The levels of home and foreign consumption are
switched after the Nash rule implementation because optimal Nash money rule is to raise
money supply by twice as much as to offset the original real appreciation by the adverse
output shock.

Subtracting [52] from [61] under the assumption of no autonomous money shocks
gives the degree of consumption risk sharing by the Nash rule under flexible rates.

\[
\frac{\epsilon_{1, \text{after Nash}} - \epsilon_{1, \text{before Nash}}}{\text{flexible rates}} = \frac{2 \gamma - 1}{\rho} (\xi - \xi^*)
\]

[63]

Nash rule is to increase home consumption by the extent of real depreciation via the
expenditure-switching effect of depreciation.

**VI. Consumption Risk Sharing under Fixed Rates**

Under fixed exchange rates, home and foreign monetary authorities adjust their money
supplies to restore their fixed bilateral exchange rate with respect to relative output shocks.
Plugging the equation [50] into [51] gives the spot exchange rate as a function of ex post
relative output and monetary shocks.

\[
s_t = \frac{\epsilon}{(2 \gamma - 1) + 2 \epsilon (1 - \gamma)} (\mu_t - \mu_t^*) - \frac{(\epsilon - 1)(2 \gamma - 1)}{\epsilon} (\xi_t - \xi_t^*)
\]

[64]

We again assume that there are neither velocity innovations in the money demand nor
autonomous money supply shocks. We use the same notations of money supply shocks \( \mu \)
and \( \mu^* \) for accommodative money rules of the home and foreign countries. If \( \gamma = \frac{1}{2} \), a
relative output shock does not change the spot exchange rate because it doesn’t have an influence neither on relative cost of living nor on relative consumption. See the equation [33]. If \( \varepsilon = 1 \), a relative output shock alters the relative money demand via the change in relative consumption on one hand, and the relative money supply via the change in relative cost of living on the other hand by the same degree so that the spot exchange rate does not change. Therefore, either if \( \gamma = \frac{1}{2} \) or if \( \varepsilon = 1 \), with respect to a relative output shock, relative money supply restoring the exchange rate to the original level is 0.

If \( \gamma > \frac{1}{2} \) and \( \varepsilon > 1 \), a relative output shock reduces relative real money supply via the rise in the relative cost of living more than relative money demand via the drop in relative consumption, appreciating the spot exchange rate. Relative money supply fixing the exchange rate can be solved for by equating the equation [64] to 0.

\[
\mu_s - \mu_s^* = \left( \frac{(2\gamma - 1) + 2\varepsilon(1 - \gamma)(\varepsilon - 1)(2\gamma - 1)}{\varepsilon^2} \right) (\xi_s - \xi_s^*)
\]

Since home and foreign countries are symmetric, their monetary policies are symmetric, too.

\[
\mu_s = \left( \frac{(2\gamma - 1) + 2\varepsilon(1 - \gamma)(\varepsilon - 1)(2\gamma - 1)}{2\varepsilon^2} \right) (\xi_s - \xi_s^*)
\]

\[
\mu_s^* = \left( \frac{(2\gamma - 1) + 2\varepsilon(1 - \gamma)(\varepsilon - 1)(2\gamma - 1)}{2\varepsilon^2} \right) (\xi_s - \xi_s^*)
\]

Plugging [65] into [55] and [56] gives the ex post consumption of home and foreign households after the monetary accommodation under fixed exchange rates.

\[
\xi_s = \left\{ \frac{(2\gamma - 1)}{2\rho} \left\{ \frac{(\varepsilon(1)(2\gamma - 1)}{\varepsilon} - \frac{\gamma}{\rho} \right\} \right\} (\xi_s) - \left\{ \frac{(2\gamma - 1)}{2\rho} \left\{ \frac{(\varepsilon(1)(2\gamma - 1)}{\varepsilon} - \frac{1 - \gamma}{\rho} \right\} \right\} (\xi_s^*)
\]

\[
\xi_s^* = \left\{ \frac{(2\gamma - 1)}{2\rho} \left\{ \frac{(\varepsilon(1)(2\gamma - 1)}{\varepsilon} + \frac{1 - \gamma}{\rho} \right\} \right\} (\xi_s) + \left\{ \frac{(2\gamma - 1)}{2\rho} \left\{ \frac{(\varepsilon(1)(2\gamma - 1)}{\varepsilon} - \frac{\gamma}{\rho} \right\} \right\} (\xi_s^*)
\]
Under fixed rates, the consumption of the country facing an exchange rate appreciation by an adverse output shock is raised by the accommodative expansionary monetary policy while the consumption of its trading counterpart is decreased by the monetary contraction to offset the change in the spot exchange rate.

Subtracting \([52]\) from \([68]\) under the assumption of no autonomous money shocks gives the degree of consumption risk sharing under fixed exchange rates.

\[
(\epsilon_{t, \text{after accommodative}} - \epsilon_{t, \text{before accommodative}})_{\text{fixed rates}} = \frac{(2\gamma - 1)}{2\rho}\left(\frac{(\varepsilon - 1)(2\gamma - 1)}{\varepsilon}\right)(\xi_t - \xi^*_t) \tag{70}
\]

By the accommodation rule under fixed rates, home consumption rises by the extent of the real depreciation caused by the depreciation offsetting the appreciation due to the adverse home output shock.

VII. Welfare Comparison of Flexible Rates with Nash Money Rule and Fixed Rates

To compare the degrees of consumption risk sharing of flexible rates with Nash money rule and fixed rates responding to a relative output shock when \(\rho > 1\), subtracting \([70]\) from \([63]\) gives the following expression.

\[
(\epsilon_{t, \text{after Nash}} - \epsilon_{t, \text{before Nash}})_{\text{flexible rates}} - (\epsilon_{t, \text{after accommodative}} - \epsilon_{t, \text{before accommodative}})_{\text{fixed rates}} = \frac{2\gamma - 1}{\rho}\left(1 - \frac{(\varepsilon - 1)(2\gamma - 1)}{2\varepsilon}\right)(\xi_t - \xi^*_t) \tag{71}
\]

The difference in the degrees of consumption risk sharing under flexible rates with Nash money rule and under fixed rates is a function of 3 parameters of the share of home-produced goods in the consumption basket, \(\gamma\), the degree of relative risk aversion of...
CRRA utility function, $\rho$, and the inverse of money demand elasticity of consumption, $\varepsilon$. Since it is possible to analyze it only in 3 dimensions, first, we fix $\varepsilon$ and later $\rho$.

We focus mainly on the effect of $\gamma$ on the difference in the degrees of consumption risk sharing under two regimes.

The 3D diagram below shows the change in the difference in the degrees of consumption risk sharing under flexible and fixed exchange rate regimes with respect to an ex post relative home output shock, as $\gamma$ changes from 0 to 1 and $\rho$ from 1 through 10 under the assumption that the inverse of the money demand elasticity is at 5, following McCallum [1989].

![Diagram 1](image)

The diagram 2 below is the left hand side of the diagram 1 above. It shows that as $\gamma$ approaches $\frac{1}{2}$ (complete goods market integration in our model), the difference in the degrees of consumption risk sharing under flexible and fixed exchange rate regimes with respect to an ex post relative home output shock diminishes because the increase in the cost of living by depreciation offsets the relative gain from expenditure-switching effect of

---

$\text{‡‡‡ In addition, it relies on other parameters of intratemporal elasticity of substitution, } \sigma, \text{ and the economic size, } n. \text{ In our model, } \sigma \text{ is assumed to be } 1 \text{ and } n \text{ to be } 0.5.$
Nash rule under flexible rates to the accommodation under fixed rates. If $\gamma = \frac{1}{2}$, the welfare effects under flexible and fixed rates become indifferent because any shocks including monetary accommodations would not change the real exchange rate.

The diagram 3 below is the right hand side of the diagram 1 above. It shows how the difference in the degrees of consumption risk sharing under flexible and fixed exchange rate regimes with respect to an ex post relative home output shock diminishes as $\rho$ rises from 1. As $\rho$ becomes larger from 1, it decreases because more risk-averse households self-insure themselves so that consumption risk to be shared by the expenditure-switching effect of depreciation is not much left.

The 3D diagram below shows the change in the difference in the degrees of consumption risk sharing of flexible and fixed exchange rate regimes with respect to a relative output shock as $\gamma$ changes from 0 to 1 and $\epsilon$ from 1 through 10 under the assumption the degree of relative risk aversion in CRRA utility function, $\rho$, is 2, following Krugman [1981].
The diagram 5 below is the right hand side of the diagram 4 above. It shows the change in the difference in the degrees of consumption risk sharing of flexible and fixed exchange rate regimes with respect to a relative output shock decreases as $\epsilon$ rises from 1. It diminishes because as $\epsilon$ becomes larger, the size of the exchange rate appreciation by the adverse home output shock grows so that the extent of monetary expansion to restore the exchange rate gets larger.

**VIII. Conclusion**

When an economy suffers from an adverse shock from within or abroad, flexible exchange rates compared to fixed rates have an advantage in that the expenditure-switching effect of depreciation can be exploited by the policy-makers to stabilize the intertemporal
consumption of their risk averse citizens whose relative risk aversion is greater than unity in the short run. Depreciation under flexible rates can be used to manipulate the terms of trade in favor of home goods’ competitiveness in the world goods market to increase home exports, output, income and consumption. In our paper, we demonstrated that the effectiveness of this strategy depends not on the extent of the terms of trade worsening but on the size of real depreciation, which, in turn, depends on the elasticity of money demand with respect to consumption, $\varepsilon$, and the parameters of households’ CRRA utility function such as the degree of relative risk aversion $\rho$, and especially the share of home produced goods in the consumption basket [trade openness] $\gamma$.

When an economy is too open or small with great dependency on imported goods, the increase in the cost of living by depreciation itself weakens the expenditure-switching effect of depreciation on consumption. In an extreme case where trading economies are completely integrated in terms of trade so that agents across trading economies have the same compositions of their consumption baskets, the depreciation under flexible exchange rates raises the cost of living exactly by the amount that completely offsets the increase in real consumption so that depreciation would not have any real effect on consumption. Therefore, with complete trade integration, the performances of fixed and flexible rates in short run consumption stabilization for the risk-averse households with relative risk aversion greater unity would become indifferent. In-between, the more open a country, the less the gains that are generated from the flexible exchange rate regime. Policy makers in more open economies find the incentive to exploit the expenditure-switching effect of depreciation for short run consumption stabilization less attractive and may be willing to easily give it up.
References


[13] Lane, Philip [1997], Exchange Rate Regimes and Monetary Policy in Small Open Economies


