Fiscal Policy in An Open Economy with Home Production

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(Preliminary)

Abstract

Nonmarket home sector is large in both the input and output sides. Including home production into an otherwise standard open economy growth model, this paper investigates how the tax-free nonmaket sector will affect the resource allocations between different sectors. Change effects of income tax, consumption tax and government spending on factor allocation, capital formation and the current account are examined. We found that, introducing home production may alter the effects of fiscal shocks on key variables in the economy. We confirm this fact by examining various policy experiments both in the short run and long run.

JEL classification: H31, D13, O41

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1 Introduction

Production activities within households are substantial. Time and resources devoted to home production account for a considerable portion of the total resources devoted to production, even in advanced countries. According to Eisner (1988), in the United States an

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estimate of home produced output relative to measured gross national production is in the range of 20 to 50 percent. Wrase (2001) reports that a married couple in the US, on average, devotes 25 percent of discretionary time to unpaid home production and 33 percent of discretionary time to work in the marketplace for pay. Because of the nonmarket property of the home sector, an immediate question is whether the inclusion of this home sector affects the usual predictions of public policy.

The idea that home production may play a relevant role in macroeconomics has generated a large number of recent studies focusing on how households' production activities affect business cycles, macroeconomic policy performance and long-term economic growth.[1] Most of this literature has suggested that introducing a home production sector into otherwise standard macroeconomic models improves the models' ability to explain observed data. For example, Benhabib et al. (1991) and Greenwood and Hercowitz (1991) show that the introduction of home production into standard real business cycle theory significantly improves the performances of the calibrated models. The intuition behind this is that the incorporation of a home sector into the standard one-sector real business cycle model brings about the possibility of substitution between market and nonmarket production over time. Therefore, relative productivity differentials between the two sectors may enhance volatility in market activity. Furthermore, the substitution between home and market commodities at a given date, not just at different dates, affects the size of fluctuations induced by productivity shocks.[2] As for an explanation of the observed economic development facts, Parente et al. (2000) illustrate that, by adding a home production sector to the neoclassical growth model, international income differences can be well accounted for even when differences in policies are relatively small. This is because, in the presence of household production, fiscal policy affects not only capital accumulation, but also the shares between market and nonmarket activities.

[1] The idea that the home sector acts parallel to the market sector originates from Becker (1965).

[2] The empirical work of McGattan, Rogerson and Wright (1997) claims that the elasticity of substitution between home and market goods is very high.

Inspired by the work of Parente et al. (2000), the first objective of this paper is to explain economic development facts. As in Parente et al. (2000), we aim to analysis this problem from both theoretical and numerical side. Furthermore, this paper aims to explain diverse fiscal policy effects illustrated in existing empirical studies. We show that, depending on the factor intensity ranking between the market and home sectors, right opposite policy effects can be obtained.h

There is a huge body of public policy literature. In the framework of a small open economy, public policy, in particular fiscal policy and government spending, has been explored intensively.[3] We are particularly interested in resource allocation between the market and home sectors. Though there are existing studies that consider labor–leisure choice, rigorous public policy analysis that includes a nonmarket home sector is rare.[4] It has been shown in the field of macroeconomics that the nonmarket household production sector is a meaningful addition to the business sector in explaining observed economic fluctuations and development facts.[5] Although recently there have been studies in the field of international real business cycle models incorporating home production into the models (see, for example, Raffo, 2006), comprehensive studies of home production activity in an open economy framework are still rare. This paper contributes to the literature in this area.

The main findings of this paper are as follows. When income taxation does not change the factor intensity ranking between the market and home sectors, the model economy exhibits standard results. However, in the case that taxation on capital and labor incomes changes the factor intensity ranking between the market and home sectors, then different policy predictions from the standard models occur. In the steady-state analysis for given initial conditions, similar to analyses in the existing literature, there is one and only one steady state in the model economy that is locally saddle-point stable. However, as opposed to the existing small open economy literature, this paper finds that fiscal policy and the

[3] Among others, see Brock (1996) and Turnovsky (1997) and the references in these papers.

[4] Hu and Mino (2005) considers analytical policy investigation in an endogenous growth model with home production.

[5] Among others, see Benhabib, Rogerson and Wright (1991), and Parente, Rogerson and Wright (2000).

magnitude of the rate of substitution between market and home goods consumption affect the stability conditions of the steady state. Long-run policy effects are investigated analytically. We show, in general, that how endogenous variables change, following expansion in government spending or in the tax rate, depends on the ranking of factor intensity between the home and market sectors.

The analytical method that we adopt is in line with that of the dependent economy models[6] in international macroeconomics.[7] While "nontraded" goods are the focus of these studies, our interests lie in identifying the implication of the "nonmarket" sector in a small open economy. Because a nonmarket sector is isolated from taxation, the incorporation of a home production sector leads to asymmetry between sectors that play an important role in determining the policy effects.

The rest of the paper is arranged as follows. Section 2 lays out the model. The dynamic system and stability analysis of the model are reported in Section 3. Section 4 is devoted to long-run policy analysis, and Section 5 reports transitional dynamic results when shocks occur. Finally Section 6 concludes the paper.

2 The model

Consider a small open economy that faces an integrated capital market. There are three kinds of agents: firms, households and a government. Firms produce a consumable capital good using capital and labor. Households, as factor owners, supply capital and labor either to the factor markets to earn rent and wages, or to the home sector for producing nonmarket home goods, which are utility promoting. It is worth noting that it is the assumption of home production that distinguishes this study from most of the existing contributions in the literature. The central government levies a flat rate of income tax in order to finance its spending. To isolate the taxation effects, we assume the government repays the income

[6] According to Salter (1959), a dependent economy is an economy that is a price taker on world markets, but also produces nontraded goods for domestic use.

[7] In this respect, see Turnovsky (1997, ch. 4) for a detailed discussion.

after its spending to households in a lump sum form (tax or transfer depending on the relative size of government income and spending).

We assume that market goods and capital are tradable internationally, while home goods can be consumed only at home. Labor cannot move across borders; however, agents can choose to work in the marketplace or to stay at home engaging in nonmarket production. Both market and home goods need capital and labor as inputs. Both market and homemade goods are preference promoting, while only market goods can be reinvested in the domestic capital stock or in the world credit market. Furthermore, all markets are competitive.

2.1 Production

We specify the production functions in Cobb–Douglas form as follows:

$$Y_m = A_m K_m^{\alpha_m} L_m^{1-\alpha_m} and \quad Y_h = A_h K_h^{\alpha_h} L_h^{1-\alpha_h},$$

where variables with subscripts "m" and "h" represent the market sector and home sector, respectively. A_j represents total factor productivity, and Y_j, K_j and L_j are output, capital and labor in sector j ($j = m, h$), respectively. $x_j \equiv K_j/L_j$ represents the capital/labor ratio in sector j ($j = m, h$).

Market competition implies equalization between rental rates and marginal production in the market sector. That is:

$$R = \frac{\partial Y_m}{\partial K_m} = A_m \alpha_m x_m^{\alpha_m - 1} and \quad w = \frac{\partial Y_m}{\partial L_m} = A_m (1 - \alpha_m) x_m^{\alpha_m}, \tag{1}$$

where R and w are the (gross) rental rate and wage rate, respectively.

2.2 Households

Setting aside population growth and normalizing the number of households to unity, for given factor prices and world interest rate, the representative household maximizes its lifetime utility as follows:

$$\int_0^\infty u(c_m, c_h, n)e^{-\rho t}dt,$$

where c_m and c_h are consumption of the market and home goods, respectively, and n is pure leisure time. Market goods for consumption c_m could be domestically produced or imported. In order to concentrate on fiscal policy, we omit tariffs and assume that the domestically produced market good is the same as the imported good. We assume households own capital and labor. Suppose that each household owns one unit of labor at each moment of time, and denote aggregate capital as K. Then households allocate capital between the market and home sectors: $K_m + K_h = L_m x_m + L_h x_h = K$, and allocate time between market work, L_m, home work, L_h, and leisure $n = 1 - L_m - L_h$.

Following Benhabib, Rogerson and Wright (1991), we specify momentary utility as

$$u(c_m, c_h, n) = log[\mu c_m^\varepsilon + (1-\mu)c_h^\varepsilon]^{1/\varepsilon} + \gamma log n,$$

where $-\infty < \varepsilon < 1$ is the parameter expressing the rate of substitution between market and homemade goods, with $\mu > 0$ and $\gamma > 0$, and where c_h represents the proportion of home products for which close market substitutions exist. Because we do not consider trade policy in this paper, there is no need to distinguish domestic-made and imported market goods. Recognizing that some home activities (for example, sleep) have less market substitutes, we specify leisure and consumption in a log-additive form.

Facing a unified international capital market, the representative agent allocates his total income to goods consumption, physical capital investment and foreign assets investment. Denote B as the value of the economy's net claims on the rest of the world. Therefore, the flow budget constraint of the representative household is

$$\dot{B} = (1-\tau_k)RK_m + (1-\tau_l)wL_m + r^*B - (1+\tau_c)c_m - \phi(I,K) + T, \tag{2}$$

where τ_k, τ_l and τ_c are tax rates on market capital income, labor income and consumption, and T is a lump sum transfer (or tax) from the government. Because the model economy considered here is small and faces a perfect international credit market, then it takes the world interest rate r^* as exogenous.

To retain nondegenerate dynamics, we introduce a capital adjustment cost for capital accumulation. This is reflected in the function ϕ, which satisfies $\phi' > 0$ and $\phi'' > 0$. That is, to achieve a unit increase in physical capital stock, more than one unit of input is needed, and the larger the investment is, the more input per unit of investment is needed.

On the other hand,

$$\dot{K} = I - \delta K. \tag{3}$$

The current value Hamiltonian of the representative household is

$$\begin{aligned} H &\equiv u(c_m, c_h, n) + p\dot{B} + q\dot{K} + \lambda(Y_h - c_h) \\ &= log[\mu c_m^{\varepsilon} + (1-\mu)c_h^{\varepsilon}]^{1/\varepsilon} + \gamma log(1 - L_m - L_h) + p\Big[(1-\tau_k)RK_m \\ &\quad +(1-\tau_l)wL_m + (1-\tau_b)rB - (1+\tau_c)c_m - (1-\tau_i)\phi(I) + T\Big] \\ &\quad +qI + \lambda[A_h(K-K_m)^{\alpha_h}L_h^{1-\alpha_h} - c_h]. \end{aligned}$$

The representative household maximizes its lifetime utility by choosing c_m, c_h, L_m, L_h,

K_m and I. At the interior solution, the first order conditions at each point of time are

$$\frac{\mu c_m^{\varepsilon-1}}{\mu c_m^{\varepsilon} + (1-\mu)c_h^{\varepsilon}} = p(1+\tau_c), \tag{4}$$

$$\frac{(1-\mu)c_h^{\varepsilon-1}}{\mu c_m^{\varepsilon} + (1-\mu)c_h^{\varepsilon}} = \lambda, \tag{5}$$

$$\frac{\gamma}{1-L_m-L_h} = p(1-\tau_l)w, \tag{6}$$

$$\frac{\gamma}{1-L_m-L_h} = \lambda(1-\alpha_h)A_h x_h^{\alpha_h}, \tag{7}$$

$$p(1-\tau_k)R = \lambda A_h \alpha_h x_h^{\alpha_h-1}, \tag{8}$$

$$p\phi'(I) = q, \tag{9}$$

and the intertemporal conditions are

$$\dot{p} = p(\rho - r^*), \tag{10}$$

$$\dot{q} = q(\rho+\delta) - p(1-\tau_k)R, \tag{11}$$

while the transversality conditions are

$$\lim_{t\to\infty} pBe^{-\rho t} = 0 = \lim_{t\to\infty} qKe^{-\rho t}. \tag{12}$$

Notice that (10) and (11) can be rearranged to express arbitrage conditions between the foreign asset and capital investments.

$$\frac{\dot{p}}{p} + r^* = \rho \text{ and } \frac{\dot{q}}{q} + \frac{p}{q}(1-\tau_k)R = \rho$$

2.3 The government

For the time being, we assume the government keeps its budget balanced at each point in time by transferring the gap between its income and expenditure to households in a lump-sum form. That is, for a given government spending G we have

$$T = \tau_k R K_m + \tau_l w L_m + \tau_c c_m - G.$$

2.4 Market equilibrium

In equilibrium, households consume all the homemade goods, that is:

$$Y_h = c_h. \tag{13}$$

In addition, complete employment in factor markets implies

$$n = 1 - L_m - L_h,\ L_m x_m + L_h x_h = K. \tag{14}$$

For the economy as a whole, the current account is

$$\dot{B} = R K_m + w L_m + r^* B - c_m - \phi(I) - G. \tag{15}$$

When the economy produces more output than domestic demand, it exports goods to gain ownership of foreign capital, which improves its current account. On the other hand, the trade deficit leads to a financial deficit and worsens its current account.

3 Equilibrium analysis

3.1 The dynamic system

Note firstly, it must be assumed that $\rho = r^*$ in order that the economy has a steady state. Thus (10) shows that p stays constant over time. That is, a small country, when facing a constant world interest rate, has this rate as its time preference rate as well.

>From (4) and (5), we have $c_m = c_m(\lambda, p; \tau_c)$, $c_h = c_h(\lambda, p; \tau_c)$ and

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} dc_m \\ dc_h \end{pmatrix} = \begin{pmatrix} d[p(1+\tau_c)] \\ d\lambda \end{pmatrix}$$

where

$$\begin{aligned} a_{11} &= \frac{-\mu c_m^{\varepsilon-2}\left[\mu c_m^{\varepsilon} + (1-\mu)(1-\varepsilon)c_h^{\varepsilon}\right]}{\left[\mu c_m^{\varepsilon} + (1-\mu)c_h^{\varepsilon}\right]^2} \\ a_{12} &= a_{21} = \frac{-\mu(1-\mu)\varepsilon c_m^{\varepsilon-1} c_h^{\varepsilon-1}}{\left[\mu c_m^{\varepsilon} + (1-\mu)c_h^{\varepsilon}\right]^2} \\ a_{22} &= \frac{-(1-\mu)c_h^{\varepsilon-2}\left[\mu(1-\varepsilon)c_m^{\varepsilon} + (1-\mu)c_h^{\varepsilon}\right]}{\left[\mu c_m^{\varepsilon} + (1-\mu)c_h^{\varepsilon}\right]^2} \end{aligned}$$

and the determinant of the coefficient matrix is

$$D_1 = a_{11}a_{22} - a_{12}a_{21} = \frac{\mu(1-\mu)(1-\varepsilon)c_m^{\varepsilon-2}c_h^{\varepsilon-2}}{\left[\mu c_m^{\varepsilon} + (1-\mu)c_h^{\varepsilon}\right]^2} > 0.$$

Then

$$\begin{aligned} dc_m &= \frac{a_{22}}{D_1} d[p(1+\tau_c)] - \frac{a_{12}}{D_1} d\lambda \\ dc_h &= \frac{a_{11}}{D_1} d\lambda - \frac{a_{12}}{D_1} d[p(1+\tau_c)] \end{aligned}$$

and

$$\begin{aligned} \frac{\partial c_h}{\partial \lambda} &= \frac{a_{11}}{D_1} = -\frac{\mu c_m^{\varepsilon} + (1-\mu)(1-\varepsilon)c_h^{\varepsilon}}{(1-\mu)(1-\varepsilon)c_h^{\varepsilon-2}} \\ \frac{\partial c_m}{\partial \lambda} &= -\frac{a_{12}}{D_1} = \frac{\varepsilon c_m c_h}{(1-\varepsilon)}. \end{aligned}$$

Lemma 1. (Market and home goods consumption)

$$\begin{aligned}
\text{sign}\left[\frac{\partial c_m}{\partial \lambda}\right] &= \text{sign}\left[\varepsilon\right], \ \frac{\partial c_m}{\partial p} < 0, \ \frac{\partial c_m}{\partial \tau_c} < 0 && (16)\\
\frac{\partial c_h}{\partial \lambda} &< 0, \ \text{sign}\left[\frac{\partial c_h}{\partial p}\right] = \text{sign}\left[\varepsilon\right], \ \frac{\partial c_h}{\partial \tau_c} > 0 && (17)
\end{aligned}$$

That is, an increase in one's consumption price lowers the consumption of these goods. The effect of this price change on the consumption of the other good depends on the rate of substitution of these two goods. For example, when $\varepsilon < 0$, that is, the market good consumption is complementary to the home good consumption, then the price increase in the home good will lower the consumption of the market good as well.

Similarly, from (6)-(8), we have $x_m = x_m(\lambda, p; \tau_k, \tau_l)$, $x_h = x_h(\lambda, p; \tau_k, \tau_l)$, which satisfies

$$\frac{x_m}{x_h} = \left(\frac{1-\alpha_h}{\alpha_h}\right)\left(\frac{\alpha_m}{1-\alpha_m}\right)\left(\frac{1-\tau_k}{1-\tau_l}\right) \tag{18}$$

and

$$\begin{aligned}
&\begin{pmatrix} p(1-\tau_l)A_m\alpha_m(1-\alpha_m)x_m^{\alpha_m-1} & -\lambda A_h\alpha_h(1-\alpha_h)x_h^{\alpha_h-1} \\ p(1-\tau_k)A_m\alpha_m(\alpha_m-1)x_m^{\alpha_m-2} & -\lambda A_h\alpha_h(\alpha_h-1)x_h^{\alpha_h-2} \end{pmatrix} \begin{pmatrix} dx_m \\ dx_h \end{pmatrix} \\
= &\begin{pmatrix} A_h(1-\alpha_h)x_h^{\alpha_h}d\lambda - A_m(1-\alpha_m)x_m^{\alpha_m}d[p(1-\tau_l)] \\ A_h\alpha_h x_h^{\alpha_h-1}d\lambda - A_m\alpha_m x_m^{\alpha_m-1}d[p(1-\tau_k)] \end{pmatrix}
\end{aligned}$$

where the determinant of the coefficient matrix is

$$\begin{aligned}
D_2 &\equiv pA_m\alpha_m(1-\alpha_m)x_m^{\alpha_m-2}\lambda A_h\alpha_h(1-\alpha_h)x_h^{\alpha_h-2}\begin{vmatrix} (1-\tau_l)x_m & -x_h \\ -(1-\tau_k) & 1 \end{vmatrix} \\
&= pA_m\alpha_m(1-\alpha_m)x_m^{\alpha_m-2}\lambda A_h\alpha_h(1-\alpha_h)x_h^{\alpha_h-2}\left[(1-\tau_l)x_m - (1-\tau_k)x_h\right].
\end{aligned}$$

That is, sign$[D_2]$ =sign$[(1-\tau_l)x_m-(1-\tau_k)x_h]$. Thus

$$dx_m = \frac{\lambda A_h \alpha_h (1-\alpha_h) x_h^{\alpha_h - 2}}{D_2} \left\{ \begin{array}{c} A_h x_h^{\alpha_h} d\lambda \\ -A_m x_m^{\alpha_m - 1}[(1-\alpha_m)(1-\tau_l)x_m + \alpha_m x_h (1-\tau_k)]dp \\ +pA_m(1-\alpha_m)x_m^{\alpha_m} d\tau_l + pA_m \alpha_m x_m^{\alpha_m - 1} x_h d\tau_k \end{array} \right\}$$

and

$$dx_h = \frac{pA_m \alpha_m (1-\alpha_m) x_m^{\alpha_m - 2}}{D_2} \left\{ \begin{array}{c} \left[(1-\tau_l)A_h \alpha_h x_m x_h^{\alpha_h - 1} + (1-\tau_k)A_h(1-\alpha_h)x_h^{\alpha_h}\right] d\lambda \\ -(1-\tau_l)(1-\tau_k)A_m x_m^{\alpha_m} dp + p(1-\tau_l)A_m \alpha_m x_m^{\alpha_m} d\tau_k \\ +p(1-\tau_k)A_m(1-\alpha_m)x_m^{\alpha_m} d\tau_l . \end{array} \right\}$$

Notice that

$$\begin{aligned} \text{sign}\left[\left(\frac{1-\tau_l}{1-\tau_k}\right)x_m - x_h\right] &= \text{sign}\left[\frac{\alpha_m}{1-\alpha_m} - \frac{\alpha_h}{1-\alpha_h}\right] = \text{sign}\left[\alpha_m - \alpha_h\right] \\ \text{sign}\left[x_m - x_h\right] &= \text{sign}\left[\left(\frac{\alpha_m}{1-\alpha_m}\right)\left(\frac{1-\tau_k}{1-\tau_l}\right) - \frac{\alpha_h}{1-\alpha_h}\right]. \end{aligned}$$

That is, sign$[(1-\tau_l)x_m-(1-\tau_k)x_h]$ represents the pretax capital/labor ratio ranking between the market and home sector, while sign$[x_m - x_h]$ represents the market and home sector capital/labor ratio after the taxation. The above calculations provide the effect of the consumption price change on the capital/labor ratios in the market and home sectors.

Define ϕ_m and $\tilde{\phi}_m$ as the pretax and after-tax capital/labor ratios in the market sector, and ϕ_h as the capital/labor ratio in the home sector. That is,

$$\phi_m \equiv \frac{\alpha_m}{1-\alpha_m},\ \tilde{\phi}_m \equiv \left(\frac{\alpha_m}{1-\alpha_m}\right)\left(\frac{1-\tau_k}{1-\tau_l}\right),\ \phi_h \equiv \frac{\alpha_h}{1-\alpha_h}.$$

Lemma 2. (Capital/labor ratios) For $j = m, h$

$$\begin{aligned}\text{sign}\left[\frac{\partial x_j}{\partial \lambda}\right] &= -\text{sign}\left[\frac{\partial x_j}{\partial p}\right] = \text{sign}\left[\frac{\partial x_j}{\partial \tau_k}\right] = \text{sign}\left[\frac{\partial x_j}{\partial \tau_l}\right] = \text{sign}\left[\phi_m - \phi_h\right] \\ \frac{\partial x_m}{\partial p} &= \left(-\frac{x_m}{p}\right)\left(\frac{1}{\alpha_m - \alpha_h}\right).\end{aligned}$$

Proof See Appendix.

That is, how these ratios respond to a consumption price change depends on the capital/labor ratio ranking between the market and home sectors. For example, if the market good is relatively capital intensive, then a price increase in the home good raises the capital/labor ratios in the two sectors, while a price change in the market good sector will decrease these ratios.

Because $R = A_m \alpha_m x_m^{\alpha_m - 1}$ and $w = A_m(1-\alpha_m)x_m^{\alpha_m}$, we have sign$\left[\frac{\partial R}{\partial *}\right] = -$sign$\left[\frac{\partial x_m}{\partial *}\right]$, sign$\left[\frac{\partial w}{\partial *}\right]$ =sign$\left[\frac{\partial x_m}{\partial *}\right]$ *and* $* = \lambda, p, \tau_k, \tau_l$. Using Lemma 2, the following factor price results can be derived.

Lemma 3. (Rental rate, wage rate)

$$\begin{aligned}-\text{sign}\left[\frac{\partial R}{\partial \lambda}\right] &= \text{sign}\left[\frac{\partial R}{\partial p}\right] = \text{sign}\left[\phi_m - \phi_h\right] \\ \text{sign}\left[\frac{\partial w}{\partial \lambda}\right] &= -\text{sign}\left[\frac{\partial w}{\partial p}\right] = \text{sign}\left[\phi_m - \phi_h\right]\end{aligned}$$

and

$$R + P\frac{\partial R}{\partial p} = R\left(\frac{1-\alpha_h}{\alpha_m - \alpha_h}\right).$$

In words, when the market sector is relatively capital intensive ($\alpha_m > \alpha_h$), an increase in the price of the market good (p) raises the return rate to capital (R) and lowers the wage rate (w), while an increase in the implicit price of the home good (λ) has the opposite effect on them.

>From (6) and the factor market equilibrium conditions (14), we have $L_i = L_i(K, \lambda, p; \tau_k, \tau_l)$,

that is:

$$L_m = \frac{(1-n)x_h - K}{x_h - x_m}, \tag{19}$$

$$L_h = \frac{K - (1-n)x_m}{x_h - x_m} \tag{20}$$

and $n = n(\lambda, p; \tau_k, \tau_l)$

$$n = 1 - L_m - L_h = \frac{\gamma}{p(1-\tau_l)A_m(1-\alpha_m)x_m^{\alpha_m}}. \tag{21}$$

Lemma 4. (Labor and leisure time)

$$\begin{aligned} \text{sign}\left[\frac{\partial n}{\partial \lambda}\right] &= -\text{sign}\left[\phi_m - \phi_h\right], \ \text{sign}\left[\frac{\partial n}{\partial p}\right] = \text{sign}\left[\phi_m - \phi_h\right] \\ \text{sign}\left[\frac{\partial L_h}{\partial \lambda}\right] &= \text{sign}\left[\tilde{\phi}_m - \phi_h\right] \text{sign}\left[\phi_m - \phi_h\right] \\ -\text{sign}\left[\frac{\partial L_h}{\partial K}\right] &= \text{sign}\left[\tilde{\phi}_m - \phi_h\right] = \text{sign}\left[\frac{\partial L_m}{\partial K}\right] \end{aligned}$$

Proof. See Appendix.

It is worth noting that capital and labor income taxation can affect the factor intensity ranking between the market and home sectors. That is, sign$\left[\tilde{\phi}_m - \phi_h\right]$ may be different from sign$[\phi_m - \phi_h]$. While this factor-intensity reverse force does not disturb the households' choice of market and home good consumption and factor prices, it can affect the labor time allocation between sectors. For example, in the standard models without home production, sign$\left[\tilde{\phi}_m - \phi_h\right]$ =sign$[\phi_m - \phi_h]$ always. Therefore, an increase in the home goods price will lead to a corresponding increase in home work time. However, when income taxation is distorted sufficiently that sign$\left[\tilde{\phi}_m - \phi_h\right]$ = −sign$[\phi_m - \phi_h]$, then an increase in the home goods price will lead to less home work time.

Lemma 5. (Outputs)

$$\begin{aligned}
\text{sign}\left[\frac{\partial Y_h}{\partial \lambda}\right] &= \text{sign}\left[\tilde{\phi}_m - \phi_h\right] \text{sign}\left[\phi_m - \phi_h\right] = -\text{sign}\left[\frac{\partial Y_m}{\partial \lambda}\right] \\
\text{sign}\left[\frac{\partial Y_h}{\partial p}\right] &= -\text{sign}\left[\tilde{\phi}_m - \phi_h\right] \text{sign}\left[\phi_m - \phi_h\right] \\
\text{sign}\left[\frac{\partial Y_h}{\partial K}\right] &= -\text{sign}\left[\tilde{\phi}_m - \phi_h\right] = -\text{sign}\left[\frac{\partial Y_m}{\partial K}\right]
\end{aligned}$$

Proof. See Appendix.

Similarly to the previous result, it depends on the before-tax and after-tax factor intensity ranking as to whether an increase in the good price can raise the output of this good or not.

Lemma 6. (Implicit price of the home good) If sign$\left[\tilde{\phi}_m - \phi_h\right]$=sign$[\phi_m - \phi_h]$, then

$$\text{sign}\left[\frac{\partial \lambda}{\partial K}\right] = \text{sign}\left[\tilde{\phi}_m - \phi_h\right].$$

If $sign\left[\tilde{\phi}_m - \phi_h\right] = -sign\left[\phi_m - \phi_h\right]$ and $\tilde{\phi}_m - \phi_h$ is close to 0, then

$$\text{sign}\left[\frac{\partial \lambda}{\partial K}\right] = -\text{sign}\left[\tilde{\phi}_m - \phi_h\right].$$

Proof. See Appendix.

In summary, from (11), (9) and (3),

$$\dot{q} = q\rho - p(1-\tau_k)A_m\alpha_m x_m{}^{\alpha_m - 1}, \tag{22}$$

$$\dot{K} = (\phi'^{-1})(\frac{q}{p}), \tag{23}$$

together with the equilibrium condition in the home sector

$$0 = Y_h - c_h, \tag{24}$$

we obtain the dynamic system with respect to K, q, λ of the model economy.

3.2 The steady state

Notice that p is constant. From $\dot{K} = 0$ and $\dot{q} = 0$,

$$\bar{q}/p = \phi'(0), \tag{25}$$

$$\bar{q}\rho = p(1-\tau_k)A_m\alpha_m x_m(\lambda, p)^{\alpha_m - 1}. \tag{26}$$

>From the above relations and $Y_h(K, \lambda, p) = c_h(\lambda, p)$, we can derive the steady-state values of K, q and λ, which are denoted with barred notations.

Let $\dot{B} = 0$ in (15) and substituting $\bar{K}$ and $\bar{q}$ into it, we obtain the steady-state value of B and $\bar{B}$. Similarly, we can get the steady-state values of other variables.

Proposition 1 *For any given p, a unique steady state $(\bar{K}, \bar{B}, \bar{c_m}, \bar{c_h}, \bar{L_m}, \bar{L_h}, \bar{q})$ exists.*

Proof. From (25), $\bar{q} = p\phi'(0)$. Then, combining the above result with (26), we obtain

$$\bar{x}_m = \left[\frac{A_m\alpha_m(1-\tau_k)}{\rho\phi'(0)}\right]^{\frac{1}{1-\alpha_m}}. \tag{27}$$

Hence

$$\bar{n} = \frac{\gamma}{pA_m(1-\alpha_m)(1-\tau_l)}\left[\frac{\rho\phi'(0)}{A_m\alpha_m(1-\tau_k)}\right]^{\frac{\alpha_m}{1-\alpha_m}}.$$

Substituting $\bar{x}_m$ into (18) obtains

$$\bar{x}_h = \frac{\phi_h}{\tilde{\phi}_m}\left[\frac{A_m\alpha_m(1-\tau_k)}{\rho\phi'(0)}\right]^{\frac{1}{1-\alpha_m}}. \tag{28}$$

Therefore, from (8), the relative price of the home good is

$$\bar{\lambda}/p = \frac{[A_m\alpha_m]^{\frac{1-\alpha_h}{1-\alpha_m}}}{A_h\alpha_h[\rho\phi'(0)]^{\frac{\alpha_m-\alpha_h}{1-\alpha_m}}}\left(\frac{\phi_h}{\phi_m}\right)^{1-\alpha_h}(1-\tau_l)^{1-\alpha_h}(1-\tau_k)^{\frac{\alpha_m(1-\alpha_h)}{1-\alpha_m}}.$$

Substitute $\bar{\lambda}/p \equiv \bar{v}$ into (4) and (5); then

$$\bar{c}_m = \frac{1}{p}\Big[\frac{(\frac{1-\mu}{\mu})^{1/(\varepsilon-1)}(1+\tau_c)^{1/(\varepsilon-1)}\bar{v}^{\varepsilon/(1-\varepsilon)}}{(\frac{1-\mu}{\mu})^{1/(\varepsilon-1)}(1+\tau_c)^{\varepsilon/(\varepsilon-1)}\bar{v}^{\varepsilon/(1-\varepsilon)}+1}\Big] \tag{29}$$

$$\bar{c}_h = \frac{1}{p\bar{v}}\Big[\frac{1}{(\frac{1-\mu}{\mu})^{1/(\varepsilon-1)}(1+\tau_c)^{\varepsilon/(\varepsilon-1)}\bar{v}^{\varepsilon/(1-\varepsilon)}+1}\Big]. \tag{30}$$

Notice that for a standard model (with no home production considered), that is $\mu = 1$, we have $\bar{c}_h = 0$ and $\bar{c}_m = p^{-1}$.

From

$$A_h\left(\frac{K-(1-n)x_m}{x_h-x_m}\right)x_h^{\alpha_h} = c_h(\lambda, p)$$

and (15), we obtain

$$\bar{K} = \bar{x}_m\Big[1+\Big(\frac{x_h/x_m-1}{A_h}\Big)\bar{x}_h^{-\alpha_h}\bar{c}_h+\Big(\frac{\gamma}{A_m(1-\alpha_m)}\Big)\frac{\bar{x}_m^{-\alpha_m}}{p(1-\tau_l)}\Big], \tag{31}$$

$$\bar{B} = \frac{1}{r^*}\Big[\bar{c}_m+\phi(\psi(\bar{q}/p))-A_m\bar{L}_m\bar{x}_m^{\alpha_m}+G\Big] \tag{32}$$

where ψ is the inverse function of ϕ'.

3.3 Stability analysis

To investigate the local stability of the steady state, let us linearize the dynamic system (22)-(24) in the neighborhood of the steady state. This yields

$$\begin{pmatrix} \dot{K} \\ \dot{q} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & \partial\dot{K}/\partial q & 0 \\ 0 & \rho & -p(1-\tau_k)\partial R/\partial\lambda \\ \partial Y_h/\partial K & 0 & \partial Y_h/\partial\lambda - \partial c_h/\partial\lambda \end{pmatrix} \begin{pmatrix} K-\bar{K} \\ q-\bar{q} \\ \lambda-\bar{\lambda} \end{pmatrix} \tag{33}$$

where

$$\frac{\partial\dot{K}}{\partial q} = \frac{1}{p}\psi'(\bar{q}/p) > 0 \tag{34}$$

in view of $\psi' = 1/(\phi'') > 0$. The eigenequation of the coefficient matrix in the above linear system is

$$\chi^2 - \rho\chi + D = 0$$

where

$$D = -p(1-\tau_k)\frac{\partial R}{\partial\lambda}\frac{\partial Y_h}{\partial K}\frac{\partial\dot{K}}{\partial q}/\left(\frac{\partial Y_h}{\partial\lambda} - \frac{\partial c_h}{\partial\lambda}\right).$$

Proposition 2 (i) *If sign*$\left[\tilde{\phi}_m - \phi_h\right]$=*sign*$[\phi_m - \phi_h]$, *or (ii) sign*$\left[\tilde{\phi}_m - \phi_h\right]$=−*sign*$[\phi_m - \phi_h]$ *and* $\tilde{\phi}_m - \phi_h$ *is close to* 0, *then the steady state is locally saddle-point stable.*

Proof. Note first that $\partial\dot{K}/\partial q > 0$ always. From Lemmas 3 and 5, we know sign$[\partial R/\partial\lambda]$=−sign$[\phi_m - \phi_h]$ and sign$[\partial Y_h/\partial K]$=−sign$[\tilde{\phi}_m - \phi_h]$. On the other hand, $\partial c_h/\partial\lambda < 0$ and sign$[\partial Y_h/\partial\lambda]$ =sign$[\phi_m - \phi_h]$sign$[\tilde{\phi}_m - \phi_h]$ from Lemmas 1 and 5; then if sign$[\phi_m - \phi_h]$ =sign$[\tilde{\phi}_m - \phi_h]$, $\partial Y_h/\partial\lambda - \partial c_h/\partial\lambda > 0$, and $(\partial R/\partial\lambda)(\partial Y_h/\partial K) > 0$, therefore $D < 0$. If sign$[\phi_m - \phi_h]$ = −sign$[\tilde{\phi}_m - \phi_h]$, because $(\partial R/\partial\lambda)(\partial Y_h/\partial K) < 0$ now, in order for $D < 0$, $\partial Y_h/\partial\lambda - \partial c_h/\partial\lambda$ must be negative, which can be ensured when $\partial c_h/\partial\lambda$ is dominated by $\partial Y_h/\partial\lambda$.This is case when $\tilde{\phi}_m - \phi_h$ is sufficiently close to 0. □

It is worth noting that, unlike the standard model without home production, the stability of the steady state depends on the ranking of the capital/labor ratio between the market and home sector. In the case that taxation does not affect the ranking of the two sectors, the saddle-point stability of the steady state can be assured. However, when the post-tax capital/labor ratio ranking is reversed an additional condition is needed.

3.4 Current account and the determinant of p

Although p is constant over time, it is endogenously determined. In the following, through the dynamic analysis of the current account, the condition p must satisfy can be derived.

Recall the national budget constraint in (15) and take (12) into consideration; then we have

$$B_0 + \int_0^{\infty} [RK_m + wL_m - c_m - \phi(I) - G] e^{-rt} dt = 0.$$

This means that a net creditor country cannot run trade surpluses permanently; at some point it must run a trade deficit in order for the above relation to be satisfied. Under a given government spending G, this relation does not necessarily hold, because both production and consumption are determined by market forces. Therefore, in addition to the optimal conditions, the above relation adds an extra constraint to the economy. This additional constraint determines what value p should take.

To determine this endogenously determined constant, we rely on the linearized dynamic system (33). Having indicated the saddle-point stability of the steady state,[8] there must be a stable eigenvalue because K is predetermined. The stable eigenvalue of the system (33) is

$$\chi = \frac{1}{2}\left[\rho - \sqrt{\rho^2 - 4D}\right] < 0.$$

The stable saddle path on the $K - q$ plane can be expressed as

$$K - \bar{K} = a_1(K_0 - \bar{K})e^{\chi t},$$

$$q - \bar{q} = a_2(q_0 - \bar{q})e^{\chi t},$$

[8] Our discussion below is confined to this case only, which we think is economically meaningful.

where $(a_1, a_2)^T$ is an eigenvector of χ. Let $a_1 = 1$; we get $a_2 = \chi/\frac{1}{p}\psi'$. Because q_0 can be chosen freely, we can express the stable saddle path as

$$K - \bar{K} = (K_0 - \bar{K})e^{\chi t}, \tag{35}$$

$$q - \bar{q} = \frac{\chi}{\psi'/p}(K - \bar{K}). \tag{36}$$

From (15),

$$\dot{B} = Y_m(K, \lambda, p) + rB - c_m(\lambda, p) - \phi(\psi(\frac{q}{p})).$$

Thus, in the neighborhood of $(\bar{K}, \bar{q}, \bar{B})$, the above relation can be approximated by

$$\dot{B} = r(B - \bar{B}) + \Lambda(K - \bar{K}) \tag{37}$$

where

$$\Lambda \equiv \frac{\partial Y_m}{\partial K} + \left(\frac{\partial Y_m}{\partial \lambda} - \frac{\partial c_m}{\partial \lambda}\right)\frac{\partial \lambda}{\partial K} - \chi\phi'(\bar{q}/p).$$

Notice that, given the saddle-point property of the system, the first and third terms on the right-hand side of the above expression are positive, while the second term has the same sign as $[-v'(\bar{K})]$ because $c_m'(v(\bar{K})) > 0$. It appears that $\Lambda > 0$ is likely to be the case at least when sign$[x_m - x_h]$=sign$[\alpha_m - \alpha_h] < 0$.

Lemma 7. *(i) If* $\tilde{\phi}_m - \phi_h > 0$, $\phi_m - \phi_h > 0$ and $\varepsilon < 0$ or $\varepsilon \simeq 0$, $\Lambda > 0$*; (ii) if* $\tilde{\phi}_m < \phi_h$, $\phi_m < \phi_h$ *and* $-\varepsilon$ *is sufficiently large*, $\Lambda < 0$; *(iii) if* $\tilde{\phi}_m > \phi_h$, $\phi_m < \phi_h$, $\tilde{\phi}_m - \phi_h \simeq 0$ and $\varepsilon > 0$ or $\varepsilon \simeq 0$, $\Lambda > 0$. (iv) $\tilde{\phi}_m < \phi_h$, $\phi_m > \phi_h$ and $\tilde{\phi}_m - \phi_h \simeq 0$, $\Lambda < 0$.

Proof. Notice that $-\chi\phi'(\bar{q}/p) > 0$ always.

Case (i) $\tilde{\phi}_m > \phi_h$ and $\phi_m > \phi_h$; then $\partial Y_m/\partial K > 0$, $\partial Y_m/\partial \lambda > 0$, and $\partial \lambda/\partial K > 0$. Because sign$[\partial c_m/\partial \lambda]$=sign$[\varepsilon]$ $(-\infty < \varepsilon < 1)$, as long as $\varepsilon < 0$ or $\varepsilon \simeq 0$, we have $\Lambda > 0$.

Case (ii) $\tilde{\phi}_m < \phi_h$ and $\phi_m < \phi_h$; then $\partial Y_m/\partial K < 0$, $\partial Y_m/\partial \lambda > 0$, and $\partial \lambda/\partial K < 0$.For sufficiently large $-\varepsilon$, we have $\Lambda < 0$.

Case (iii) $\tilde{\phi}_m > \phi_h$ and $\phi_m < \phi_h$; then $\partial Y_m/\partial K > 0$, $\partial Y_m/\partial \lambda < 0$, and if in addition

$\tilde{\phi}_m - \phi_h \simeq 0$, $\partial\lambda/\partial K < 0$. As long as $\varepsilon > 0$ or $\varepsilon \simeq 0$, we have $\Lambda > 0$.

Case (iv) $\tilde{\phi}_m < \phi_h$ and $\phi_m > \phi_h$; then $\partial Y_m/\partial K < 0$, $\partial Y_m/\partial\lambda < 0$, and if in addition $\tilde{\phi}_m - \phi_h \simeq 0$, $\partial\lambda/\partial K > 0$. Hence, $\Lambda < 0$. $\square$

The solution of (37) is

$$(B_t - \bar{B})e^{-r^* t} = C + \Lambda(K_0 - \bar{K})\Big(\frac{1}{\chi - r^*}\Big)e^{(\chi - r^*)t}, \tag{38}$$

where C is a constant, given by

$$C = (B_0 - \bar{B}) - \Lambda(K_0 - \bar{K})\Big(\frac{1}{\chi - r^*}\Big).$$

On the other hand, the transversality condition in (12) means $C = 0$ in (38), so that

$$B_0 - \bar{B} = \Lambda(K_0 - \bar{K})\Big(\frac{1}{\chi - r^*}\Big). \tag{39}$$

$>$From Subsection 3.2, we know that $\bar{B} = \bar{B}(p)$ and $\bar{K} = \bar{K}(p)$. For given K_0 and B_0, substitute these expressions into the above relation, which is a unitary equation of p, from which the value of $p = p(K_0, B_0)$ can be determined.

We emphasize that the steady state to which the economy converges depends upon the initial conditions. The transition pattern is determined by the adjustment of the implicit price of the home good in terms of the market good p. As opposed to the corresponding closed economy, there is no room for a small economy to adjust its interest rate to achieve a unique steady state. Instead, the initial conditions affect the destination of the economy in the long run.

4 Long-run policy effect

To obtain the long-run effect of policy changes, by differentiating (15), (22)-(24) and (39) around the steady state $(\bar{K}, \bar{q}, \bar{\lambda}, \bar{B}, \bar{p})$, we have

$$\begin{bmatrix} 0 & 1/p\phi''(0) & 0 & 0 & -\phi'(0)/p\phi''(0) \\ 0 & \rho & -p(1-\tau_k)\partial R/\partial\lambda & 0 & -(1-\tau_k)\left(R + p\partial R/\partial p\right) \\ \partial Y_h/\partial K & 0 & \partial Y_h/\partial\lambda - \partial c_h/\partial\lambda & 0 & \partial Y_h/\partial p - \partial c_h/\partial p \\ \frac{\partial Y_m}{\partial K} & -\frac{\phi'(0)}{p\phi''(0)} & \frac{\partial Y_m}{\partial\lambda} - \frac{\partial c_m}{\partial\lambda} & r^* & \frac{\partial Y_m}{\partial p} - \frac{\partial c_m}{\partial p} + \frac{\phi'(0)^2}{p\phi''(0)} \\ \Lambda/(\chi - r^*) & 0 & 0 & -1 & 0 \end{bmatrix} \begin{pmatrix} d\bar{K} \\ d\bar{q} \\ d\bar{\lambda} \\ d\bar{B} \\ d\bar{p} \end{pmatrix}$$
$$= \begin{pmatrix} 0 \\ p\left[(1-\tau_k)\frac{\partial R}{\partial\tau_k} - R\right] d\tau_k + p(1-\tau_k)\frac{\partial R}{\partial\tau_l} d\tau_l \\ -\frac{\partial Y_h}{\partial\tau_k} d\tau_k - \frac{\partial Y_h}{\partial\tau_l} d\tau_l + \frac{\partial c_h}{\partial\tau_c} d\tau_c \\ dG - \frac{\partial Y_m}{\partial\tau_k} d\tau_k - \frac{\partial Y_m}{\partial\tau_l} d\tau_l + \frac{\partial c_m}{\partial\tau_c} d\tau_c \\ 0 \end{pmatrix} \tag{40}$$

from which

$$\frac{\partial\bar{p}}{\partial G} = \frac{1}{\Delta\rho\phi''(0)} \frac{\partial Y_h}{\partial K} \frac{\partial R}{\partial\lambda} p(1-\tau_k)$$

can be derived. Because $\text{sign}\left[\frac{\partial Y_h}{\partial K}\right] = -\text{sign}\left[\tilde{\phi}_m - \phi_h\right]$ and $\text{sign}\left[\frac{\partial R}{\partial\lambda}\right] = -\text{sign}[\phi_m - \phi_h]$, and the determinant of the coefficient matrix of the above system $\Delta < 0$ from the stable condition, then

$$\text{sign}\left[\frac{\partial\bar{p}}{\partial G}\right] = -\text{sign}\left[\tilde{\phi}_m - \phi_h\right] \text{sign}\left[\phi_m - \phi_h\right].$$

Here

$$\Delta = \frac{-1}{\rho\phi''(0)} \begin{vmatrix} 0 & -p(1-\tau_k)\frac{\partial R}{\partial\lambda} & \rho\phi'(0) - (1-\tau_k)\left(R + p\frac{\partial R}{\partial p}\right) \\ \frac{\partial Y_h}{\partial K} & \frac{\partial Y_h}{\partial\lambda} - \frac{\partial c_h}{\partial\lambda} & \frac{\partial Y_h}{\partial p} - \frac{\partial c_h}{\partial p} \\ \frac{\partial Y_m}{\partial K} + \frac{r^*\Lambda}{\chi - r^*} & \frac{\partial Y_m}{\partial\lambda} - \frac{\partial c_m}{\partial\lambda} & \frac{\partial Y_m}{\partial p} - \frac{\partial c_m}{\partial p}. \end{vmatrix}$$

4.1 Effects of government spending expansion

>From (40) and noticing that $\rho\phi'(0)/(1-\tau_k) = R$, we obtain

$$\begin{aligned}\frac{\partial\bar{q}}{\partial G} &= \phi'(0)\frac{\partial\bar{p}}{\partial G} \\ \frac{\partial\bar{\lambda}}{\partial G} &= \frac{\rho\phi'(0)-(1-\tau_k)\left(R+p\partial R/\partial p\right)}{p(1-\tau_k)\partial R/\partial\lambda}\frac{\partial\bar{p}}{\partial G} = -\frac{\partial R/\partial p}{\partial R/\partial\lambda}\frac{\partial\bar{p}}{\partial G}\end{aligned}$$

and

$$\begin{aligned}-\frac{\partial Y_h}{\partial K}\frac{\partial\bar{K}}{\partial G} &= \left[\left(\frac{\partial Y_h}{\partial p}-\frac{\partial c_h}{\partial p}\right)+\left(\frac{\partial Y_h}{\partial\lambda}-\frac{\partial c_h}{\partial\lambda}\right)\frac{\partial\bar{\lambda}}{\partial G}\right]\frac{\partial\bar{p}}{\partial G} \\ \frac{\partial\bar{B}}{\partial G} &= \left(\frac{\Lambda}{\chi-r^*}\right)\frac{\partial\bar{K}}{\partial G}.\end{aligned}$$

Capital level and net foreign assets. *If* sign$\left[\tilde{\phi}_m-\phi_h\right]$ =sign$[\phi_m-\phi_h]$, $\varepsilon>0$ *and* $\tilde{\phi}_m \approx \phi_h$, *then* $\frac{\partial\bar{K}}{\partial G}>0$. *Only in the case that* $\tilde{\phi}_m-\phi_h>0$, $\phi_m-\phi_h>0$ *and* $\varepsilon\simeq 0$, *is an unambiguous result,* $\frac{\partial\bar{B}}{\partial G}<0$, *derived.*

Proof. Notice that under the stable condition in Proposition 2

$$\text{sign}\left[\frac{\partial Y_h}{\partial\lambda}-\frac{\partial c_h}{\partial\lambda}\right] = \text{sign}\left[\tilde{\phi}_m-\phi_h\right]\text{sign}\left[\phi_m-\phi_h\right].$$

On the other hand,

$$\begin{aligned}\frac{\partial Y_h}{\partial p}-\frac{\partial c_h}{\partial p} &< 0,\ \text{if sign}\left[\tilde{\phi}_m-\phi_h\right]=\text{sign}\left[\phi_m-\phi_h\right]\ \text{and}\ \varepsilon>0 \\ \frac{\partial Y_h}{\partial p}-\frac{\partial c_h}{\partial p} &> 0,\ \text{if sign}\left[\tilde{\phi}_m-\phi_h\right]=-\text{sign}\left[\phi_m-\phi_h\right]\ \text{and}\ \varepsilon<0\end{aligned}$$

and

$$\text{sign}\left[-\frac{\partial Y_h}{\partial K}\right]=\text{sign}\left[\tilde{\phi}_m-\phi_h\right],\ \text{sign}\left[\frac{\partial\bar{p}}{\partial G}\right]=-\text{sign}\left[\tilde{\phi}_m-\phi_h\right]\text{sign}\left[\phi_m-\phi_h\right].$$

Hence,

(i) $\tilde{\phi}_m > \phi_h$ and $\phi_m > \phi_h$. Then $-\frac{\partial Y_h}{\partial K} > 0$ and $\frac{\partial \bar{p}}{\partial G} < 0$. If in addition $\varepsilon > 0$, we have $\frac{\partial Y_h}{\partial p} - \frac{\partial c_h}{\partial p} < 0$. From the sufficient stable condition in Proposition 2, we know $\frac{\partial Y_h}{\partial \lambda} - \frac{\partial c_h}{\partial \lambda} > 0$. Hence $\frac{\partial \bar{K}}{\partial G} > 0$.

(ii) $\tilde{\phi}_m > \phi_h$ and $\phi_m < \phi_h$. Then $-\frac{\partial Y_h}{\partial K} > 0$ and $\frac{\partial \bar{p}}{\partial G} > 0$. If in addition $\varepsilon < 0$, we have $\frac{\partial Y_h}{\partial p} - \frac{\partial c_h}{\partial p} > 0$. From the sufficient stable condition in Proposition 2, we know $\frac{\partial Y_h}{\partial \lambda} - \frac{\partial c_h}{\partial \lambda} < 0$. Hence sign$\left[\frac{\partial \bar{K}}{\partial G}\right]$ =?.

(iii) $\tilde{\phi}_m < \phi_h$ and $\phi_m < \phi_h$. Then $-\frac{\partial Y_h}{\partial K} < 0$ and $\frac{\partial \bar{p}}{\partial G} < 0$. If in addition $\varepsilon > 0$, we have $\frac{\partial Y_h}{\partial p} - \frac{\partial c_h}{\partial p} < 0$. From the sufficient stable condition in Proposition 2, we know $\frac{\partial Y_h}{\partial \lambda} - \frac{\partial c_h}{\partial \lambda} > 0$. Hence $\frac{\partial \bar{K}}{\partial G} > 0$.

(iv) $\tilde{\phi}_m < \phi_h$ and $\phi_m > \phi_h$. Then $-\frac{\partial Y_h}{\partial K} < 0$ and $\frac{\partial \bar{p}}{\partial G} > 0$. If in addition $\varepsilon < 0$, we have $\frac{\partial Y_h}{\partial p} - \frac{\partial c_h}{\partial p} > 0$. From the sufficient stable condition in Proposition 2, we know $\frac{\partial Y_h}{\partial \lambda} - \frac{\partial c_h}{\partial \lambda} < 0$. Hence sign$\left[\frac{\partial \bar{K}}{\partial G}\right]$ =?.

>From Lemma 7, we know $\Lambda > 0$ if $\tilde{\phi}_m - \phi_h > 0$, $\phi_m - \phi_h > 0$ and $\varepsilon \simeq 0$. That is, we have now $\frac{\partial \bar{B}}{\partial G} < 0$. This is the only case for which the sign of $\frac{\partial \bar{B}}{\partial G}$ can be determined. □

Market and home goods consumption. *(i) If* sign$\left[\tilde{\phi}_m - \phi_h\right]$*=sign*$[\phi_m - \phi_h]$ *and* $\varepsilon < 0$, *then an increase in government spending lowers households' market and home goods consumption. (ii) If* sign$\left[\tilde{\phi}_m - \phi_h\right]$*=-sign*$[\phi_m - \phi_h]$ *and* $\varepsilon < 0$, *then an increase in government spending lowers households' market and home goods consumption.*

The results here can be understood by inspecting the following calculations ($i = m, h$).

$$\begin{aligned} \frac{\partial \bar{c}_i}{\partial G} &= \frac{\partial c_i}{\partial \lambda}\frac{\partial \bar{\lambda}}{\partial G} + \frac{\partial c_i}{\partial p}\frac{\partial \bar{p}}{\partial G} \\ &= \left\{\frac{\partial c_i}{\partial \lambda}\left(-\frac{\partial R/\partial p}{\partial R/\partial \lambda}\right) + \frac{\partial c_i}{\partial p}\right\}\frac{\partial \bar{p}}{\partial G} \end{aligned}$$

The statement in case (i) describes the usual results, while those of case (ii) are new to the home production model. If income taxation distorts the economy sufficiently so that the factor intensity ranking between the market and home sectors is reversed, then when the substitution rate between the home good and market good is small ($\varepsilon < 0$), in the long run government spending increase causes the market good price to increase. Therefore, the demand on home consumption should increase. In the case that $\varepsilon < 0$, even facing a higher p, households' market good consumption still can increase.

Factor prices. *An expansion in government spending does not affect the factor intensity in the market sector, hence leaves the factor prices unaffected.*

$$
\begin{aligned}
\frac{\partial \bar{x}_m}{\partial G} &= \frac{\partial x_m}{\partial \lambda}\frac{\partial \bar{\lambda}}{\partial G} + \frac{\partial x_m}{\partial p}\frac{\partial \bar{p}}{\partial G} = \left[\frac{\partial x_m}{\partial \lambda}\left(-\frac{\partial R/\partial p}{\partial R/\partial \lambda}\right) + \frac{\partial x_m}{\partial p}\right]\frac{\partial \bar{p}}{\partial G} \\
&= \left[\frac{\partial x_m}{\partial \lambda}\left(-\frac{\partial x_m/\partial p}{\partial x_m/\partial \lambda}\right) + \frac{\partial x_m}{\partial p}\right]\frac{\partial \bar{p}}{\partial G}
\end{aligned}
$$

Labor time allocation. *An increase in government spending G causes leisure time n to increase if sign$\left[\tilde{\phi}_m - \phi_h\right]$ =sign$[\phi_m - \phi_h]$, and leisure time to decrease if sign$\left[\tilde{\phi}_m - \phi_h\right] \neq$sign$[\phi_m - \phi_h]$. The effects on the work time allocation between market and home work of this increase in G are generally ambiguous. In the case that sign$\left[\tilde{\phi}_m - \phi_h\right]$ =sign$[\phi_m - \phi_h]$, $\varepsilon > 0$ and $\tilde{\phi}_m \approx \phi_h$, sign$\left[\partial \bar{L}_m/\partial G\right]$=−sign$\left[\partial \bar{L}_m/\partial G\right]$=sign$[\phi_m - \phi_h]$.*

Notice that $\frac{\partial \bar{x}_m}{\partial G} = 0$ and $\frac{\partial n(x_m,p)}{\partial p} < 0$; then

$$
\text{sign}\left[\frac{\partial \bar{n}}{\partial G}\right] = -\text{sign}\left[\frac{\partial \bar{p}}{\partial G}\right] = \text{sign}\left[\tilde{\phi}_m - \phi_h\right]\text{sign}\left[\phi_m - \phi_h\right].
$$

$>$From the expressions of L_m, L_h in (20) and (19)

$$
\begin{aligned}
\frac{\partial \bar{L}_m}{\partial G} &= \frac{\partial x_h/\partial x_m}{\partial x_h/\partial x_m - 1}\left(-\frac{\partial \bar{n}}{\partial G}\right) - \frac{1/x_m}{\partial x_h/\partial x_m - 1}\frac{\partial \bar{K}}{\partial G} \\
\frac{\partial \bar{L}_h}{\partial G} &= \frac{1/x_m}{\partial x_h/\partial x_m - 1}\frac{\partial \bar{K}}{\partial G} - \frac{1}{\partial x_h/\partial x_m - 1}\left(-\frac{\partial \bar{n}}{\partial G}\right).
\end{aligned}
$$

4.2 Effects of taxation shocks

Similarly to the above analysis, we can investigate the effects of taxation shocks. Taking the change in the consumption rate as an example, from (40) we have

$$\frac{\partial \bar{p}}{\partial \tau_c} = \frac{p(1-\tau_k)}{\Delta \rho \phi''(0)} \frac{\partial R}{\partial \lambda} \left[\frac{\partial Y_h}{\partial K} \frac{\partial c_m}{\partial \tau_c} - \frac{\partial c_h}{\partial \tau_c} \left(\frac{\partial Y_m}{\partial K} + \frac{r^* \Lambda}{\chi - r^*} \right) \right]$$

$$\begin{aligned} \frac{\partial \bar{q}}{\partial \tau_c} &= \phi'(0) \frac{\partial \bar{p}}{\partial \tau_c} \\ \frac{\partial \bar{\lambda}}{\partial \tau_c} &= \left(-\frac{\partial R / \partial p}{\partial R / \partial \lambda} \right) \frac{\partial \bar{p}}{\partial \tau_c} \end{aligned}$$

and

$$\begin{aligned} \frac{\partial Y_h}{\partial K} \frac{\partial \bar{K}}{\partial \tau_c} &= \frac{\partial c_h}{\partial \tau_c} - \left[\left(\frac{\partial Y_h}{\partial p} - \frac{\partial c_h}{\partial p} \right) + \left(\frac{\partial Y_h}{\partial \lambda} - \frac{\partial c_h}{\partial \lambda} \right) \frac{\partial \bar{\lambda}}{\partial \tau_c} \right] \frac{\partial \bar{p}}{\partial \tau_c} \\ \frac{\partial \bar{B}}{\partial \tau_c} &= \left(\frac{\Lambda}{\chi - r^*} \right) \frac{\partial \bar{K}}{\partial \tau_c}. \end{aligned}$$

As long as the effect of τ_c's change on p is clear, we can derive the effect of τ_c's change on other variables. However, from Lemma 7, we know $\partial Y_m / \partial K$ and $r^* \Lambda / (\chi - r^*)$ always have opposite signs. Except for numerical calculations, it is generally impossible to determine the sign of $\partial \bar{p} / \partial \tau_c$. In the following, we will assume that $r^* \Lambda / (\chi - r^*)$ is dominated by $\partial Y_m / \partial K$ (this is at least the case when $x_m \approx x_h$); then we have the following results.

Shadow price of wealth p. *An increase in the rate of consumption tax raises* p *if* *sign*$\left[\tilde{\phi}_m - \phi_h\right]$ =*sign*$[\phi_m - \phi_h]$, *and lowers* p *if sign*$\left[\tilde{\phi}_m - \phi_h\right] = -$*sign*$[\phi_m - \phi_h]$.

Notice that $\frac{\partial c_m}{\partial \tau_c} < 0$, $\frac{\partial c_h}{\partial \tau_c} < 0$, sign$\left[\frac{\partial Y_h}{\partial K}\right]$=$-$sign$\left[\frac{\partial Y_m}{\partial K}\right]$=-sign$\left[\tilde{\phi}_m - \phi_h\right]$ and sign$\left[\frac{\partial R}{\partial \lambda}\right] = -$sign$[\phi_m - \phi_h]$. When $r^* \Lambda / (\chi - r^*)$ is dominated by $\partial Y_m / \partial K$, we have

$$\text{sign}\left[\frac{\partial \bar{p}}{\partial \tau_c}\right] = \text{sign}\left[\tilde{\phi}_m - \phi_h\right] \text{sign}\left[\phi_m - \phi_h\right].$$

Market and home goods consumption. *(i) If sign*$\left[\tilde{\phi}_m - \phi_h\right]$=*sign*$[\phi_m - \phi_h]$ *and* $\varepsilon <$ 0, *then an increase in the rate of the consumption tax lowers market. (ii) If sign*$\left[\tilde{\phi}_m - \phi_h\right]$=-*sign*$[\phi_m - \phi_h]$ and $\varepsilon < 0$, *then an increase in the rate of the consumption tax raises home goods consumption.*

$>$From $(i = m, h)$

$$\begin{aligned}\frac{\partial \bar{c}_i}{\partial \tau_c} &= \frac{\partial c_i}{\partial \lambda}\frac{\partial \bar{\lambda}}{\partial \tau_c} + \frac{\partial c_i}{\partial p}\frac{\partial \bar{p}}{\partial \tau_c} + \frac{\partial c_i}{\partial \tau_c} \\ &= \left\{\frac{\partial c_i}{\partial \lambda}\left(-\frac{\partial R/\partial p}{\partial R/\partial \lambda}\right) + \frac{\partial c_i}{\partial p}\right\}\frac{\partial \bar{p}}{\partial G} + \frac{\partial c_i}{\partial \tau_c}\end{aligned}$$

the above results can be obtained.

Factor prices: *An increase in the rate of the consumption tax does not affect the factor intensity in the market sector, leaving the factor prices unaffected.*

$$\begin{aligned}\frac{\partial \bar{x}_m}{\partial \tau_c} &= \frac{\partial x_m}{\partial \lambda}\frac{\partial \bar{\lambda}}{\partial \tau_c} + \frac{\partial x_m}{\partial p}\frac{\partial \bar{p}}{\partial \tau_c} = \left[\frac{\partial x_m}{\partial \lambda}\left(-\frac{\partial R/\partial p}{\partial R/\partial \lambda}\right) + \frac{\partial x_m}{\partial p}\right]\frac{\partial \bar{p}}{\partial \tau_c} \\ &= \left[\frac{\partial x_m}{\partial \lambda}\left(-\frac{\partial x_m/\partial p}{\partial x_m/\partial \lambda}\right) + \frac{\partial x_m}{\partial p}\right]\frac{\partial \bar{p}}{\partial \tau_c}\end{aligned}$$

Capital level and net foreign assets. *If* sign$\left[\tilde{\phi}_m - \phi_h\right] = -$sign$[\phi_m - \phi_h]$, $\varepsilon < 0$, $\varepsilon \approx 0$ *and* $\tilde{\phi}_m \approx \phi_h$, *then* $\frac{\partial \bar{K}}{\partial \tau_c} < 0$. *In the case that* $\tilde{\phi}_m - \phi_h < 0$ *and* $\phi_m - \phi_h > 0$, *we have* $\frac{\partial \bar{B}}{\partial \tau_c} < 0$.

Notice that under the stable condition in Proposition 2

$$\text{sign}\left[\frac{\partial Y_h}{\partial \lambda} - \frac{\partial c_h}{\partial \lambda}\right] = \text{sign}\left[\tilde{\phi}_m - \phi_h\right]\text{sign}\left[\phi_m - \phi_h\right].$$

On the other hand,

$$\begin{aligned}\frac{\partial Y_h}{\partial p}-\frac{\partial c_h}{\partial p} &< 0, \text{ if sign}\left[\tilde{\phi}_m-\phi_h\right]=\text{sign}\left[\phi_m-\phi_h\right] \text{ and } \varepsilon>0\\ \frac{\partial Y_h}{\partial p}-\frac{\partial c_h}{\partial p} &> 0, \text{ if sign}\left[\tilde{\phi}_m-\phi_h\right]=-\text{sign}\left[\phi_m-\phi_h\right] \text{ and } \varepsilon<0\end{aligned}$$

and

$$\text{sign}\left[-\frac{\partial Y_h}{\partial K}\right]=\text{sign}\left[\tilde{\phi}_m-\phi_h\right],\ \text{sign}\left[\frac{\partial \bar{p}}{\partial \tau_c}\right]=\text{sign}\left[\tilde{\phi}_m-\phi_h\right]\text{sign}\left[\phi_m-\phi_h\right].$$

Hence,

(i) $\tilde{\phi}_m>\phi_h$ and $\phi_m>\phi_h$. Then $\frac{\partial Y_h}{\partial K}<0$ and $\frac{\partial \bar{p}}{\partial \tau_c}>0$. If in addition $\varepsilon>0$, we have $\frac{\partial Y_h}{\partial p}-\frac{\partial c_h}{\partial p}<0$. From the sufficient stable condition in Proposition 2, we know $\frac{\partial Y_h}{\partial \lambda}-\frac{\partial c_h}{\partial \lambda}>0$. Hence $\frac{\partial \bar{K}}{\partial \tau_c}$?.

(ii) $\tilde{\phi}_m>\phi_h$ and $\phi_m<\phi_h$. Then $\frac{\partial Y_h}{\partial K}<0$ and $\frac{\partial \bar{p}}{\partial \tau_c}<0$. If in addition $\varepsilon<0$, we have $\frac{\partial Y_h}{\partial p}-\frac{\partial c_h}{\partial p}>0$. From the sufficient stable condition in Proposition 2, we know $\frac{\partial Y_h}{\partial \lambda}-\frac{\partial c_h}{\partial \lambda}<0$. If additionally, $\varepsilon\approx 0$ so that $\frac{\partial c_h}{\partial \tau_c}$ is close to 0, then $\frac{\partial \bar{K}}{\partial \tau_c}<0$.

(iii) $\tilde{\phi}_m<\phi_h$ and $\phi_m<\phi_h$. Then $\frac{\partial Y_h}{\partial K}>0$ and $\frac{\partial \bar{p}}{\partial \tau_c}>0$. If in addition $\varepsilon>0$, we have $\frac{\partial Y_h}{\partial p}-\frac{\partial c_h}{\partial p}<0$. From the sufficient condition of a stable steady state in Proposition 2, we know $\frac{\partial Y_h}{\partial \lambda}-\frac{\partial c_h}{\partial \lambda}>0$. Hence $\frac{\partial \bar{K}}{\partial \tau_c}$?0.

(iv) $\tilde{\phi}_m<\phi_h$ and $\phi_m>\phi_h$. Then $\frac{\partial Y_h}{\partial K}>0$ and $\frac{\partial \bar{p}}{\partial \tau_c}<0$. If in addition $\varepsilon<0$, we have $\frac{\partial Y_h}{\partial p}-\frac{\partial c_h}{\partial p}>0$. From the sufficient stable condition in Proposition 2, we know $\frac{\partial Y_h}{\partial \lambda}-\frac{\partial c_h}{\partial \lambda}<0$. If additionally, $\varepsilon\approx 0$ so that $\frac{\partial c_h}{\partial \tau_c}$ is close to 0, then $\frac{\partial \bar{K}}{\partial \tau_c}<0$.

Examining the above cases for Λ, we find from Lemma 7 that only in case (iv) do we have a clear sign for $\Lambda:\Lambda<0$. Hence, $\frac{\partial \bar{B}}{\partial \tau_c}<0$. This is the only case where the sign of $\frac{\partial \bar{B}}{\partial G}$ can be determined. □

5 Transitional dynamics

For showing the transitional dynamics of the economy, we use an increase in government spending as an example. The analysis of other shocks can be accomplished in a similar way. Based on the linearized dynamic system (33), we know that, around the steady state, the stable equilibrium path takes a shape similar to SS' in Figure 1.

(CASE I): $\partial p/\partial G < 0$.

When a sudden increase in G occurs, p jumps up immediately. Because $\bar{q}$ must move in the same way as p in view of (25), then $\bar{q}$ decreases. On the other hand, Proposition 3 shows $\bar{K}$ will move to a higher level. Then this shock results in a rightwards shift of the stable arm on the $K-q$ plane, to S_1S_1' for example. Because the $K-q$ dynamic system is complete, moving patterns of other variables can be derived, except those of B, which we will examine below.

In view of (38), the relation between B and K around the steady state can be illustrated as XX' in Figure 2, which is negatively sloped on the $K-B$ plane. An increase in G lowers $\bar{B}$ and raises $\bar{K}$ in the long run. This corresponds to a right downward movement along locus XX'.

(CASE II): $\partial p/\partial G > 0$.

As for the above case, a similar analysis can be made.

6 Concluding remarks

Including the home sector into a standard small open neoclassical growth model, in this paper, we explored the equilibrium dynamics and policy effects of the model economy analytically. We found that the nonmarket home sector and fiscal policy asymmetry together play an important role in determining the equilibrium property and policy effects of the model economy. We also show that the rate of substitution between home and market goods consumption can affect the above analysis.

We found also that a solely theoretical study has limitations in understanding the complete effects of policy. For some parameter ranges, we have to rely on numerical calculations to determine the exact effects. This numerical study is left for the future.

Appendix

A.1 Proof of Lemma 2

>From the expression of dx_m in Section 3.1, the definition of D_2 and the relation

$$x_m(1-\tau_l)(1-\alpha_m) = \left(\frac{1-\alpha_h}{\alpha_h}\right)\alpha_m(1-\tau_k)x_h,$$

we have

$$\begin{aligned}\frac{\partial x_m}{\partial p} &= \frac{-1}{D_2}A_m x_m^{\alpha_m-1}\lambda A_h\alpha_h(1-\alpha_h)x_h^{\alpha_h-2}[(1-\alpha_m)(1-\tau_l)x_m+\alpha_m x_h(1-\tau_k)] \\ &= \left(-\frac{x_m}{p}\right)\left(\frac{1}{\alpha_m-\alpha_h}\right).\end{aligned}$$

A.2 Proof of Lemma 4

>From Lemma 2

$$\frac{1}{\alpha_m}+\frac{p}{x_m}\frac{\partial x_m}{\partial p} = \frac{-\alpha_h}{\alpha_m(\alpha_m-\alpha_h)}.$$

On the other hand, from $n = n(\lambda, p; \tau_k, \tau_l)$ in (21)

$$\begin{aligned}\frac{\partial n}{\partial p} &= \frac{-\gamma\alpha_m}{A_m(1-\alpha_m)(1-\tau_l)}\frac{(1/\alpha_m + p/x_m\cdot\partial x_m/\partial p)}{p^2 x_m^{\alpha_m}} \\ &= \frac{\gamma\alpha_h/(\alpha_m-\alpha_h)}{A_m(1-\alpha_m)(1-\tau_l)p^2x_m^{\alpha_m}}.\end{aligned}$$

A.3 Proof of Lemma 5

$>$From $Y_j = A_j L_j(K,\lambda,p) x_j^{\alpha_j}(\lambda,p), j = m,h,$

$$
\begin{aligned}
Y_h &= \frac{A_h}{1-x_m/x_h}\left[\frac{K}{x_h}-(1-n)\frac{x_m}{x_h}\right]x_h^{\alpha_h} \\
Y_m &= \frac{A_m}{x_h/x_m-1}\left[(1-n)\frac{x_h}{x_m}-\frac{K}{x_m}\right]x_m^{\alpha_m},
\end{aligned}
$$

then

$$
\begin{aligned}
\frac{\partial Y_h}{\partial \lambda} &= -\left(\frac{A_h}{1-x_m/x_h}\right)\left[\begin{array}{c} \frac{{x_h}^{\alpha_h}x_m}{x_h}\left(-\frac{\partial n}{\partial \lambda}\right) \\ +(1-\alpha_h)\frac{K{x_h}^{\alpha_h-1}}{x_h}\frac{\partial x_h}{\partial \lambda}+\frac{(1-n)\alpha_h {x_h}^{\alpha_h-1}x_m}{x_h}\frac{\partial x_h}{\partial \lambda} \end{array}\right] \\
\frac{\partial Y_h}{\partial p} &= \left(\frac{A_h}{1-x_m/x_h}\right)\left\{x_m x_h^{\alpha_h-1}\frac{\partial n}{\partial p}-\left[(1-\alpha_h)K+\alpha_h(1-n)x_m\right]x_h^{\alpha_h-2}\frac{\partial x_h}{\partial p}\right\}
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial Y_m}{\partial \lambda} &= \left(\frac{A_m}{x_h/x_m-1}\right)\left[\begin{array}{c} \frac{x_h {x_m}^{\alpha_m}}{x_m}\left(-\frac{\partial n}{\partial \lambda}\right) \\ +(1-\alpha_m)\frac{K{x_m}^{\alpha_m-1}}{x_m}\frac{\partial x_m}{\partial \lambda}+\frac{(1-n)\alpha_m x_h {x_m}^{\alpha_m-1}}{x_m}\frac{\partial x_m}{\partial \lambda} \end{array}\right] \\
\frac{\partial Y_h}{\partial K} &= \frac{A_h {x_h}^{\alpha_h}}{x_h-x_m},\ \frac{\partial Y_m}{\partial K} = -\frac{A_m {x_m}^{\alpha_m}}{x_h-x_m}.
\end{aligned}
$$

A.4 Proof of Lemma 6

Totally differentiating the two sides of $Y_h(K,\lambda,p) = c_h(\lambda,p)$, we obtain

$$
\frac{\partial \lambda}{\partial K} = -\frac{\partial Y_h}{\partial K}/\left(\frac{\partial Y_h}{\partial \lambda}-\frac{\partial c_h}{\partial \lambda}\right).
$$

If sign$\left[\tilde{\phi}_m-\phi_h\right]$ =sign$[\phi_m-\phi_h]$, then $\partial Y_h/\partial\lambda - \partial c_h/\partial\lambda > 0$; hence sign$[\partial\lambda/\partial K]$ = $-$sign$[\partial Y_h/\partial K]$ =sign$\left[\tilde{\phi}_m-\phi_h\right]$. If sign$\left[\tilde{\phi}_m-\phi_h\right] = -$sign$[\phi_m-\phi_h]$, as long as $\tilde{\phi}_m-\phi_h$ is sufficiently close to 0, $\partial c_h/\partial\lambda$ will be dominated by $\partial Y_h/\partial\lambda$, that is $\partial Y_h/\partial\lambda - \partial c_h/\partial\lambda < 0$. Then in this case, sign$[\partial\lambda/\partial K]$ =sign$[\partial Y_h/\partial K] = -$sign$\left[\tilde{\phi}_m-\phi_h\right]$.

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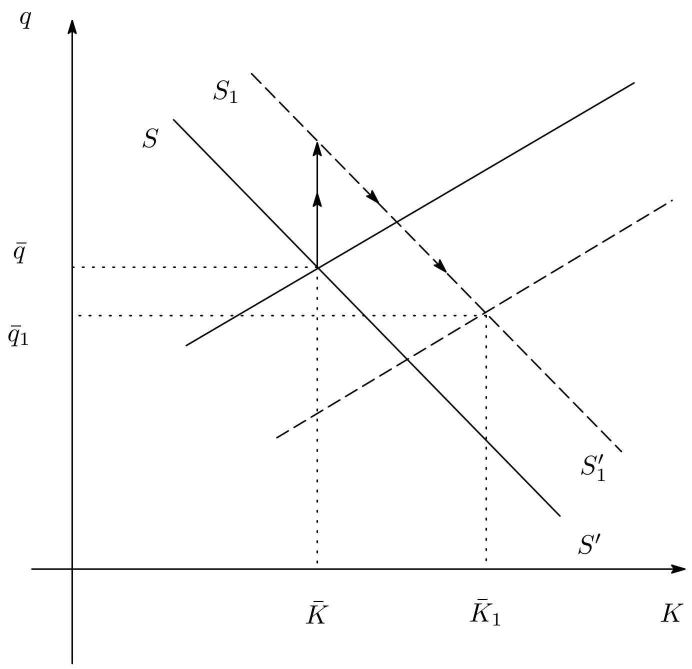

Figure 1

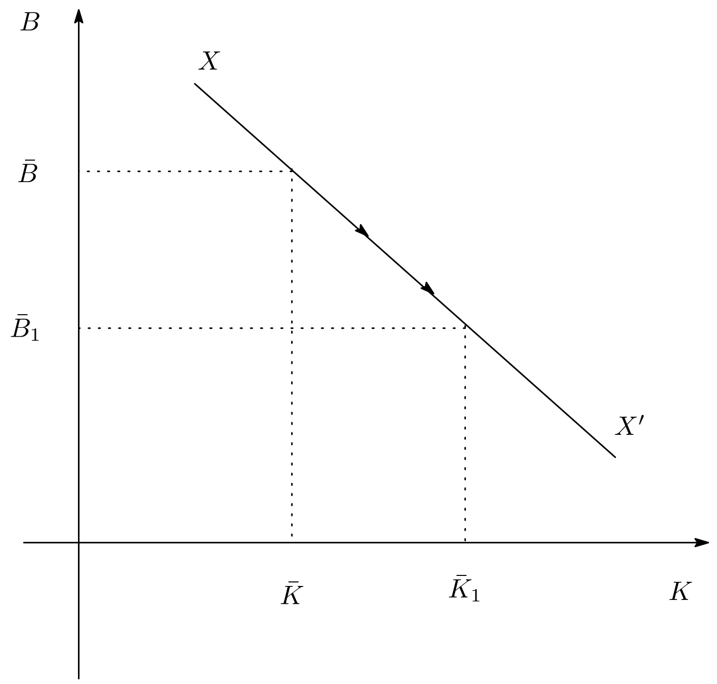

Figure 2