International Trade Patterns under Quasi-Linear Preferences*

Elias Dinopoulos

Department of Economics, University of Florida

Kenji Fujiwara

Graduate School of Economics, Kobe University

Koji Shimomura†

RIEB, Kobe University

September 23, 2005

An electronic version of the paper is available at

http://bear.cba.ufl.edu/dinopoulos/Research.html

* An earlier version of this paper was presented at seminars held in Kobe University, the Spring 2005 Midwest International Economics Group Meetings at Vanderbilt University, and the Fall 2005 Meeting of the Japanese Economic Association at Chuo University. We would like to thank Jota Ishikawa, Yunfang Hu, Toru Kikuchi, Raymond Riezman, Nicolas Schmitt, Morihiro Yomogida and Laixun Zhao for useful comments and suggestions. The authors of the paper are solely responsible for any remaining errors.

† Corresponding author: RIEB, Kobe University, Rokkodai-cho, 2-1, Nada-ku, Kobe, 657-8501, Japan. Tel/Fax: +81-78-803-7006; Email: simomura@rieb.kobe-u.co.jp

ABSTRACT

The present paper analyzes the pattern of trade under quasi-linear preferences using a general-equilibrium, two-good, and two-country framework, in which country-size (measured by the number of consumers) can differ. Under perfect competition and constant returns to scale, a country exports the good in which it has comparative labor productivity advantage, or it exports the good that uses intensively its abundant factor of production. Under quasi-linear preferences, the volume of trade in factor services can be higher or lower compared to the one generated by homothetic ones. In the absence of factor price equalization, the factor-proportions theory holds only if the two countries are sufficiently different in their factor endowments. We apply our results to a variety of market structures including monopolistic competition, international duopoly and external scale economies.

Keywords: Heckscher-Ohlin theorem, comparative advantage, trade patterns, quasi-linear preferences.

JEL Classification: F10, F12.

1. INTRODUCTION

The logical link between the general-equilibrium theory of international trade and the partial-equilibrium analysis of commercial policy is paradoxical and incomplete when it comes to assumptions concerning the structure of consumer preferences. It is not an exaggeration to state that the analysis of trade patterns and income distribution has routinely utilized the assumption of identical and homothetic tastes, whereas the theory of commercial policy, especially its branch that deals with imperfect competition, has relied almost exclusively on quasi-linear tastes that allow the use of demand and supply curves.

Graduate textbooks of international trade theory invoke the properties of quasi-linear preferences in order to provide a "general-equilibrium" rationale for using demand and supply analysis. Consider, for instance, Feenstra (2004, Chapter 7), who sets up a partial-equilibrium framework based on the assumptions that every consumer in the global economy has a quasi-linear utility function, labor is the only factor of production, and that one unit of labor produces one unit of the numeraire good. According Feenstra (2004, p214), "It follows that wages in the economy are also unity, so that total income equals the fixed labor supply of L. These assumptions allow the economy to mimic a partial-equilibrium setting, where wages are fixed and trade is balanced through flows of the numeraire good".

Although the above-mentioned use of quasi-linear preferences is desirable, based on analytical feasibility and simplicity grounds, the above statement gives the strong impression that excessive and perhaps unrealistic general-equilibrium restrictions are required to justify the use of demand and supply curves. Is it true, for instance, that if both countries enjoy the same labor productivity in the numeraire good and the same wage, *only* then the partial-equilibrium analysis is valid? What kind of trade pattern holds in a Ricardian world economy in which the wage is equalized across the two countries and consumer tastes are quasi-linear? Is it possible that the pattern of trade implied by a general-equilibrium model with quasi-linear tastes differs from the trade pattern assumed in the partial-equilibrium analysis? More generally, under the assumption that labor is the only factor of production, does the Ricardian theory of comparative advantage hold for the case of quasi-linear (as opposed to homothetic) tastes?

Suppose further that one replaces the assumption of homothetic tastes with that of quasi-linear preferences in the standard two-country, two-factor, and two-goods trade model. One could then address the following questions: What are the determinants of the pattern of trade? Does the standard (or a modified version of the) Heckscher-Ohlin (HO, hereafter) theorem apply to a world of quasi-linear preferences? An affirmative answer to these questions would increase the robustness of partial-equilibrium analysis. And in doing so, it will enhance the comfort level of researchers- who assume implicitly that the determinants of the trade pattern under quasi-linear tastes are the same as those in the case of homothetic tastes- when analyzing the partial-equilibrium effects of commercial policy instruments. Finally, the above-mentioned questions and concerns can also be applied to imperfectly competitive product markets which have been analyzed almost exclusively under the assumption of quasi-linear preferences.

Recently, a small but growing literature has addressed some of the above-mentioned questions.[1] In a companion paper, Doi et al. [4] have addressed several questions concerning the pattern of trade under quasi-linear preferences and the assumption that each of the two countries has the same size measured by the number of consumers. These authors showed that, under quasi-linear preferences, the trade pattern follows approximately the HO theorem. It also established the robustness of the results in imperfectly competitive environments,

---

[1] The determinants of trade patterns under homothetic tastes and imperfectly competitive market structures have been analyzed extensively: Dixit and Norman [3] and Helpman and Krugman [7] have established the validity of HO theorem for the case of monopolistic competition; Lahiri and Ono [8], Shimomura [11], and Fujiwara and Shimomura [6] have done the same for the case of oligopoly and increasing returns.

increasing returns and Marshallian externalities. The present paper complements the analysis of Doi et al. [4] by considering the case of two countries with arbitrary labor endowments. This generalization allows us to isolate the role of quasi-linear preferences on the trade patterns from possible influences associated with the restrictive assumption of identical national labor endowments. Unlike Doi et al. [4], we analyze the role of quasi-linear preferences in the case of the standard Ricardian model of trade, and calculate the factor-content of trade in the context of the two-good, two-factor, two-country framework. We also establish conditions under which an exact (as opposed to a modified) version of the HO theorem holds for the case of quasi-linear preferences.

Our analysis generates several novel results. The pattern of trade under quasi-linear preferences depends on differences in per-capita output supplied of the non-numeraire good across the two countries. More specifically, Lemma 1 establishes that the country with a higher per-capita supply of the non-numeraire good exports that product. This novel result allows us to analyze the determinants of international trade patterns in a variety of contexts. For instance, under the assumption of constant returns to scale and perfect competition, the standard Ricardian comparative- advantage holds even in the case of quasi-linear preferences, and the factor proportions theory is also valid: A country exports the good with the higher comparative labor productivity or the good that uses intensively its abundant factor of production (Proposition 1). One could also use the concept of the integrated world equilibrium to derive the pattern and calculate the volume of trade in factor services. We establish that, as in the case of homothetic preferences, the Heckcher-Ohlin-Vanek (HOV) theorem holds in the presence of quasi-linear tastes, but the volume of factor-services trade differs between the two preference structures (Proposition 2). When factor price equalization is not feasible due to market structure considerations or external economies of scale, the factor proportions theory still holds in the case of quasi-linear tastes, under some restrictions on the distribution of world factor endowments across the two countries. We derive these restrictions and offer a modified version of the factor-proportions theory in this case (Proposition 3). Finally, the paper offers a variety of analytical tools and insights that can be used by other researchers analyzing similar issues.

The rest of the paper is organized as follows: Section 2 presents the basic properties of quasi-linear preferences and relates these properties to the pattern of trade. Section 3 establishes the robustness of Ricardian comparative advantage under quasi-linear preferences. Section 4 utilizes the two-good, two-factor, two-country model and analyzes the trade pattern for the case of factor price equalization (FPE). This section also establishes the validity of the HO theorem for the cases of perfect competition, monopolistic competition, and oligopoly. It also highlights the role of quasi-linear (as opposed to homothetic) preferences in calculating the factor content of trade. Section 5 analyzes the trade pattern under quasi-linear preferences in the absence of FPE. Section 6 offers concluding remarks. The algebra of proofs and several additional results related to sections 4 and 5 are relegated to appendices.

2 PREFERENCES

Consider now a two-good (good 1 and 2), two-country (Home and Foreign), world economy. Without loss of generality, good 2 is assumed to be the numeraire. However, in this paper we assume that, independently of the technology and market structure associated with good 1, good 2 is produced under perfect competition and constant returns to scale. Consumers are identical across the two countries and each consumer's preferences are described with the following quasi-linear utility function:

$$U = u(Q_1) + Q_2, \; u'(\cdot) > 0, \qquad u''(\cdot) < 0, \tag{1}$$

where U is the utility level of a typical consumer, Q_1 is the quantity consumed of good 1 and Q_2 is the quantity consumed of the numeraire good. Maximization of (1) subject to the standard budget constraint $I = pQ_1 + Q_2$, where I denotes each consumer's income and

p is the relative price of good 1 expressed in units of good 2, one obtains the per-capita demands for goods 1 and 2:

$$Q_1 = u'^{-1}(p) \equiv D(p), \qquad D'(\cdot) < 0 \tag{2}$$

$$Q_2 = I - pD(p). \tag{3}$$

Equation (2) is derived by inverting the inverse demand function for good 1, which is given by the first-order condition of the consumer-maximization problem $p = u'(Q_1)$. Equation (3) is derived by substituting (2) into the consumer's budget constraint and solving for Q_2. It is obvious from the above two equations that the demand for good 1 depends only on the relative price of good 1 and that any changes in consumer's income are reflected on the demand for the numeraire good. The demand for the latter decreases in its own price, $1/p$, and increases linearly in the consumer's income. In addition, unlike the case of homothetic preferences where the relative demand Q_1/Q_2 is independent of consumer income, under quasi-linear preferences the relative demand for good 1, Q_1/Q_2, declines in per-capita income I. Finally, observe that the aggregate demand for each good is directly proportional to the size of the market measured by number of consumers. Do these differences between quasi-linear and homothetic preferences make a difference in the determination of the trade pattern? We are now in the position to provide the first lemma that will shed some light at this important issue.

Denote with L and L* the number of consumers and with Y_1 and Y_1^* output of good 1 in Home and Foreign respectively, where an asterisk (*) will be used to identify Foreign's variables and functions. The market-clearing condition under free trade and no trade costs requires that the global demand for good 1 equals its supply:

$$LD(p) + L^*D(p) = Y_1 + Y_1^*. \tag{4}$$

The market-clearing condition (4) leads immediately to the following criterion that determines the pattern of trade:

LEMMA 1. *The country with the higher per-capita production of good 1 exports good 1 and imports the numeraire good 2 (i.e., Home exports good 1 if and only if* $Y_1 / L > Y_1^* / L^*$ *).*

Proof. (See Appendix A.)

The economic intuition for Lemma 1 is the following: Although income per-capita might differ between Home and Foreign consumers, the per-capita demand for good 1 depends only on its relative price. Equation (2) implies that the country with the higher per-capita production of good 1 exhibits a lower relative price of good 1 in autarky and will export this good. Balanced trade implies that the country with a higher per-capita supply of good 1 imports the numeraire good. Lemma 1 implies that differences in per-capita income do not play a crucial role in the determination of trade patterns in the presence of quasi-linear preferences, although they affect the demand for the numeraire good and might influence the magnitude of the factor content of trade. In addition, Lemma 1 does not relate the fundamental elements of labor-productivity-or factor-endowment- differences across the two countries to the pattern of trade. The following sections relate these elements to per-capita output of good 1 in each country by introducing the production side of the global economy.

3 THE RICARDIAN MODEL

This section establishes that the basic proposition of the traditional Ricardian comparative advantage holds in the case of quasi-linear preferences: A country exports that good in which it has higher comparative labor productivity advantage. To establish this basic proposition, consider a two-country, two-good world economy in which labor is the only factor of production, and assume that perfect competition prevails in all markets. Denote with $\alpha_i (\alpha_i^*),\ i = 1, 2$ the unit labor requirement for the production of good i at Home (Foreign). Finally assume, without loss of generality, that $\alpha_1 / \alpha_2 < \alpha_1^* / \alpha_2^*$, which means that Home has a comparative advantage in the production of good 1. Independently of the preference structure, perfect competition implies that in the absence of trade the relative price of good 1 prevailing at Home, $p = \alpha_1 / \alpha_2$, would be lower than the autarkic relative price of good 1 in Foreign, $p^* = \alpha_1^* / \alpha_2^*$. Consequently, Home would specialize in the production of and export good 1, whereas Foreign would specialize in the production of and export the numeraire good 2 in accordance to the law of comparative advantage. In the case of complete specialization, the relative price of good 1 is determined by the market-clearing condition (4), where $Y_1^* = 0$ and $Y_1 = L / \alpha_1$:

$$p = D^{-1}\left(\frac{L}{\alpha_1(L+L^*)}\right). \tag{5}$$

If one of the two countries is sufficiently large, it could end up producing both goods after trade with its autarky price prevailing in both countries.

Figure 1 illustrates the determination of the terms of trade in the presence of quasi-linear preferences. The vertical axis measures the relative price p and the horizontal axis the world per-capita quantity of good 1, $(Y_1 + Y_1^*)/(L + L^*)$.

(Figure 1 around here)

The per-capita world supply of good 1 is illustrated by the step function 0ABCDE, and the per-capita demand for good 1 corresponds to the downward-sloping curve $D(p)$. The vertical distance 0A equals Home's autarky price α_1 / α_2, which is lower than distance 0G, which equals Foreign's autarky price α_1^* / α_2^*. Equation (5) corresponds to the intersection of the per-capita demand for good 1 with its supply at point F. It is obvious from Fig 1, that if the intersection of per-capita demand and supply occurs at a point located in the interior of the vertical segment BC, then Home specializes in the production of good 1 and exports this good;

Foreign specializes in the production of the numeraire good and exports it to Home. If the intersection between per capita demand and supply curves occurs at the horizontal segment AB (CD) then Home (Foreign) remains incompletely specialized and the international price equals the autarky price of Home (Foreign). The only difference between quasi-linear and homothetic tastes in the Ricardian model is the following: under quasi-linear preferences, the terms of trade depend only on *per-capita* supply and demand curves for good 1; whereas under homothetic tastes the determination of the terms of trade is based on the *relative* demand and supply curves defined as the ratio of the quantity of good 1 over the quantity of good 2.

The above results are novel and encouraging to researchers who use partial-equilibrium tools to analyze a variety of trade-related issues. However, strictly speaking, the Ricardian trade model generates horizontal supply curves and no producer surplus in the context of partial-equilibrium analysis. Consequently, the next section introduces quasi-linear preferences in the standard factor-proportions model of trade and analyses the resulting pattern and the volume of factor content of trade with the intention to establish the robustness of the aforementioned results.[2]

4 FACTOR-PROPORTIONS THEORY

In this section we assume that each of the two goods is produced with capital and labor under constant returns to scale and perfect competition. In addition, denote with K and K* the fixed endowments of capital at Home and Foreign respectively; and let w (w*) denote the Home (Foreign) wage of labor and let r (r*) be the Home (Foreign) rental of capital. As in the standard model, each good is produced with identical technology in both countries captured

[2] We do not analyze the pattern of trade in the sector-specific factor model because there is no a clear-cut prediction on the trade pattern in this model as in the Ricardian and factor-proportions theories of comparative advantage. See, for instance, Feenstra ([5], chapter 3) for a description of this model and its properties.

by the unit-cost functions, and assume that each of the two countries remains incompletely specialized after the introduction of free trade. The above standard assumptions imply that the production side of the world economy is described by following equations:

$$c_1(r,w) = p \tag{6}$$

$$c_2(r,w) = 1, \tag{7}$$

where $c_i(r,w), i = 1,2$ is a unit-cost function which is concave and homogeneous of degree one in both arguments. Equations (6) and (7) are the zero-profit conditions for industries 1 and 2 respectively. Under free trade, the relative price of good 1 is common in both countries, and therefore the above-two equations hold in both countries and imply factor-price equalization (i.e., $w = w^*$ and $r = r^*$).

Denote with $\alpha_{Ki}(r,w) = \partial c_i(r,w)/\partial r$ the per-unit capital requirement and with $\alpha_{Li}(r,w) = \partial c_i(r,w)/\partial w$ the per-unit labor requirement in the production of good i . The full-employment conditions of capital and labor at Home can be written as

$$\alpha_{K1}(r,w)Y_1 + \alpha_{K2}(r,w)Y_2 = K, \tag{8}$$

$$\alpha_{L1}(r,w)Y_1 + \alpha_{L2}(r,w)Y_2 = L. \tag{9}$$

Similarly, the full-employment conditions of capital and labor at Foreign are stated as:

$$\alpha_{K1}(r,w)Y_1^* + \alpha_{K2}(r,w)Y_2^* = K^*, \tag{10}$$

$$\alpha_{L1}(r,w)Y_1^* + \alpha_{L2}(r,w)Y_2^* = L^*. \tag{11}$$

The system of six equations (6) through (11) determines the values of six variables ($r, w, Y_1, Y_2, Y_1^*, Y_2^*$) for any given value of the relative price p . Equation (4), which is the market-clearing condition for good 1 provides the final equation that links the relative price to per-capita output of good 1.

In view of Lemma 1, it is straight forward to relate the per-capita output of good 1 to each country's factor endowments. The full-employment conditions at Home yield:

$$\frac{Y_1}{L} = \frac{\alpha_{L2}(r,w)\dfrac{K}{L} - \alpha_{K2}(r,w)}{\alpha_{K1}(r,w)\alpha_{L2}(r,w) - \alpha_{K2}(r,w)\alpha_{L1}(r,w)}\ , \tag{12}$$

and the full-employment conditions at Foreign generate:

$$\frac{Y_1^*}{L^*} = \frac{\alpha_{L2}(r,w)\dfrac{K^*}{L^*} - \alpha_{K2}(r,w)}{\alpha_{K1}(r,w)\alpha_{L2}(r,w) - \alpha_{K2}(r,w)\alpha_{L1}(r,w)}\,. \tag{13}$$

Comparing the two equations, it is easy to establish the desired result, namely,

$$sign\left\{\frac{Y_1}{L} - \frac{Y_1^*}{L^*}\right\} = sign\left\{\frac{K}{L} - \frac{K^*}{L^*}\right\}, \tag{14}$$

if and only if, $\alpha_{K1}(r,w)\alpha_{L2}(r,w) - \alpha_{K2}(r,w)\alpha_{L1}(r,w) > 0$ (i.e., good 1 is capital intensive). This result, together with Lemma 1, leads to

PROPOSITION 1. *Under quasi-linear preferences and incomplete production specialization, a country exports the good which uses intensively its abundant factor of production.*

Proposition 1 establishes the robustness of the HO theorem under perfect competition and factor-price equalization. The proof relies on the per-capita version of the Rybczynski theorem which states that an increase in factor abundance (measured by an economy's per capita capital) raises the per-capita production of capital intensive good and reduces the per-capita production of the labor intensive good. This result is sufficient to determine the pattern of trade even in the case of quasi-linear (non-homothetic) preferences.

At this point, we have established the robustness of the traditional theorems that determine the trade pattern in the Ricardian and the HO models under quasi-linear preferences. The reader might wonder though whether or not the assumption of quasi-linear preferences changes any of the traditional results. We now focus our analysis on the factor content of trade, which has received considerable attention recently by empirical researchers.[3] Following Dixit and Norman [3] define the world integrated equilibrium as the allocation of resources that would result in the global economy under free-trade and perfect international mobility of labor and capital. In other words, the integrated equilibrium is defined as the world closed-economy equilibrium.[4] The factor-price-equalization (FPE) set in the integrated world equilibrium is the set of all allocations of capital and labor between the two countries such that if each country employs fully its factor endowments and uses the techniques of production of the integrated equilibrium, free trade results in equalization of the wage rate and the return of capital across the two countries.

In the presence of quasi-linear tastes, each country's aggregate demand for good 1 becomes

$$LQ_1 = LD(p) \text{ and } L^* Q_1^* = L^* D(p) \tag{15}$$

Similarly, each country's aggregate demand for the numeraire good 2 becomes

$$LQ_2 = L\left[rk + w - pD(p)\right] \text{ and } L^* Q_2^* = L^*\left[rk^* + w - pD(p)\right], \tag{16}$$

where $k = K/L$ and $k^* = K^*/L^*$ denote each country's capital abundance. Figure 2 illustrates the case of good 1 being the capital intensive good and Home being the capital abundant country. In Figure 2, the diagonal correspond to the world's vector of capital and labor, each horizontal side of the box diagram measures the world endowment of labor and each vertical side measures the world endowment of capital. Point 0 corresponds to the

[3] See, for instance the seminal contributions on the empirics of factor content of trade by Trefler [12, 13] and Davis and Weinstein [1]. Davis and Weinstein [2] provide an excellent overview of the literature on factor content of trade.

[4] The world integrated equilibrium is formally defined by the market clearing condition for good 1 given by equation (4); the two zero profit conditions (6) and (7); and two global full-employment conditions, one for capital derived by adding equations (8) and (10), and one for labor derived by adding (9) and (11).

origin of Home and point 0^* corresponds to the origin of Foreign. Since the supply side of the economy is identical to that of the traditional factor proportions model of trade, the FPE set is given by the parallelogram which is shaped by the factor intensity of the two goods and the length of each size corresponds to the vector of global resources (labor and capital) allocated to the production of each of the two goods. Under the standard assumption that each country remains incompletely specialized after trade and assuming that Home is capital abundant, the point that determines the distribution of world factor endowments between the two countries must be located in the upper triangle of the FPE set, say point E. Once the endowment point E is given, the distribution of endowments that correspond to each country's national income which is consistent with point E is given by the negatively sloped straight line with slope w/r.

Quasi-linear preferences enter into the determination of each country's consumption point of factor services. Balanced trade and the assumption of perfect competition (absence of economic profits) imply that the consumption point must be located on the national income line. In the case of homothetic preferences, the consumption-endowment point is given by the intersection of the diagonal line 00^* and the income line, E F. However, in the case of quasi-linear preferences the location of the consumption point depends on the factor intensity ranking between the two goods.

Figure 2 considers the case in which good 1 is capital intensive. In this case, draw line LL^* which is vertical, passes through point E and intersects the diagonal at point E'. By drawing line $Q_1Q_1^*$ which has the same slope as the factor intensity of the labor-intensive good 2 and passes through point E', one can determine geometrically the factor content of consumption at Home and Foreign. In other words, $0Q_1$ is Home's aggregate consumption of factor services related to the production of good 1, and $0^*Q_1^*$ is the vector that corresponds to Foreign's aggregate consumption of factor services associated with the production of good 1.

(Figure 2 around here)

This is so because (15) implies that $0Q_1 / 0^* Q_1^* = L / L^*$, that is, each country's consumption is proportional only to its number of consumers and does not depend on the distribution of world capital endowment across the two countries as in the case of homothetic preferences. This proportionality is captured geometrically by the properties of similar triangles in Figure 2. These properties imply the following equalities: $0Q_1 / 0^* Q_1^* = 0E' / 0^* E' = 0L / 0^* L^*$. Therefore, under quasi-linear preferences, the point that determines the vector of factor services embodied in each country's consumption is given by the intersection of line $Q_1 Q_1^*$ and the national income line EFD, which is point D. Consequently, the factor content of trade under quasi-linear preferences is given by vector ED and implies that the capital abundant country will export the capital intensive good. In other words the Heckscher-Ohlin-Vanek (HOV) theorem holds for the case of quasi-linear preferences.

The validity of the HOV theorem for the case of quasi-linear preferences is not surprising and it is consistent with Proposition 1. Notice that, in general, points D and F (the consumption point under homogeneous preferences) will differ from each other. In Figure 2, ED is strictly greater than EF and therefore quasi-linear preferences generate more trade than homothetic tastes. However, this ranking depends on the intensity ranking between the two goods. Figure 3 illustrates the factor content of trade in the case where good 1 is the labor-intensive one. In this case, the HOV theorem holds but vector ED is strictly less than vector EF.

(Figure 3 around here)

In other words, in the presence of quasi-linear preferences the factor proportions model generates less (more) factor content trade than homothetic preferences if and only if the numeraire good is capital (labor) intensive. Trefler's [12] pioneering work has drawn attention to the mystery of "too much" factor content trade associated with the traditional factor proportions theory which is based on homothetic tastes.[5] Is the adoption of quasi-linear instead of homothetic tastes one step towards an explanation of the "missing trade" mystery? This novel question deserves more theoretical and empirical work which is outside the scope of the present paper.

The following proposition summarizes the above-mentioned results:

PROPOSITION 2. *Under quasi-linear preferences and incomplete specialization in production, each country exports indirectly its factor services related to its abundant factor (i.e., exports the good which uses intensively its abundant factor). The volume of trade in factor services predicted under quasi-linear preferences is smaller (larger) than that predicted under homothetic preferences if and only if the non-numeraire good is labor (capital) intensive.*

The first part of Proposition 2 is the mirror image of Proposition 1. The second part of Proposition 2 is somewhat surprising and requires an intuitive explanation. The latter depends on the non-homothetic nature of quasi-linear preferences and can be described with the help of Figure 2 and equations (15) and (16). Consider the case where the distribution of world factor endowments shown by point E is located in the diagonal, say point E coincides with point E' in Figure 2. In this degenerate case, there is no trade and therefore the amount of (zero) trade is identical for both homothetic and quasi-linear preferences. Assume now that the endowment point shifts above the diagonal on the vertical line LL^* from E' to E. This move is associated with an increase in Home's national income (and a reduction in the Foreign's national income) shown by a rightward shift in the income budget line EDF. In the

[5] Trefler ([13], p.1029) states that according to his analysis a country's "factor service trade is much smaller that its factor-endowment prediction".

case of homothetic preferences this increase in income raises Home’s demand for capital and labor embodied in both goods along the diagonal resulting in Home’s consumption vector OF. However, in the case of quasi-linear preferences the demand for capital and labor services embodied in the production of good 1 remains unchanged (see equation(15)) and the income increase affects only the demand for factor services embodied in the numeraire good. Thus, the consumption point moves along line $Q_1Q_1^*$ which is steeper than the diagonal if good 1 is labor intensive. This means that as Home becomes more capital abundant than Foreign, Home’s demand for capital services increases more under quasi-linear tastes than under homothetic tastes if good 1 is capital intensive. This, of course, implies that Home will trade less capital for labor with Foreign under quasi-linear (as opposed to homothetic) preferences. Balanced trade ensures that in this case Foreign will also trade less with Home.

Although the analysis focused on the case of perfect competition and constant returns to scale, it can be readily extended to the cases of monopolistic competition and increasing returns, oligopoly with free entry, and external economies. Appendix C, which is available by the authors upon request, demonstrates that Proposition 1 is valid in these cases as well.

5 A GENERALIZATION

This section considers the more general (and realistic) case of the factor proportions theory under quasi-linear preferences and in the absence of factor price equalization (FPE). Our main interest is to derive general conditions under which the factor proportion prediction regarding the pattern of trade holds. We also illustrate the validity of our results for the case of international duopoly and the case of country- specific (national) external economies.

We would like to maintain the tractability of the analysis and at the same time to be as general as possible. As a result, we will maintain our assumption that the numeraire good is produced under constant returns to scale and perfect competition. In the absence of FPE, the following equations summarize Home’s production side:

$$c_1(r,w) = \Gamma(Y_1, Y_1^*, L + L^*) \, , \tag{17}$$

$$c_2(r,w) = 1 \, , \tag{18}$$

$$\alpha_{K1}(r,w)\Psi(Y_1, Y_1^*) + \alpha_{K2}(r,w)Y_2 = K \, , \tag{19}$$

$$\alpha_{L1}(r,w)\Psi(Y_1, Y_1^*) + \alpha_{L2}(r,w)Y_2 = L \, , \tag{20}$$

where the right-hand-side of (17) is not necessarily equal to price but could be equal to the value of marginal revenue in the case of imperfect competition. Under the assumption of quasi-linear preferences, the market-clearing condition (4) implies that the demand for good 1 is a function of total output $Y_1 + Y_1^*$ and the total number of consumers $L + L^*$, and therefore total revenue and marginal revenue will be in general functions of Y_1, Y_1^* and $L + L^*$. This dependence is captured by function $\Gamma(\cdot)$ in (17). In addition, notice that in the presence of external economies the cost function is not directly proportional to national output but could depend on the output levels in both countries (especially in the case of global external economies). This possible dependence is captured by function $\Psi(Y_1, Y_1^*)$. For instance, in the case of perfect competition and constant returns, $\Psi(Y_1, Y_1^*) \equiv Y_1$, that is, the demand for factor services depends only on Home's output of good 1.

In general, both countries share the same functional forms regarding $\Gamma(\cdot)$ and $\Psi(\cdot)$, and therefore the production side in Foreign can be summarized with the following profit-maximization and full-employment conditions:

$$c_1(r^*, w^*) = \Gamma(Y_1^*, Y_1, L + L^*) \, , \tag{21}$$

$$c_2(r^*, w^*) = 1 \, , \tag{22}$$

$$\alpha_{K1}(r^*, w^*)\Psi(Y_1^*, Y_1) + \alpha_{K2}(r^*, w^*)Y_2^* = K^* \, , \tag{23}$$

$$\alpha_{L1}(r^*, w^*)\Psi(Y_1^*, Y_1) + \alpha_{L2}(r^*, w^*)Y_2^* = L^* . \tag{24}$$

We should mention at this point that the assumption that both countries share the same functional form of $\Gamma(\cdot)$ does not imply that free trade will establish factor price equalization because the arguments of $\Gamma(\cdot)$ and its value will be different across the two countries. In other words, in the case of an international duopoly and two asymmetric countries $\Gamma(\cdot)$

corresponds to the level of marginal revenue in each country, and under free trade $\Gamma(Y_1, Y_1^*, L+L^*) \neq \Gamma(Y_1^*, Y_1, L+L^*)$.

The present model consists of nine equations (4) and (17) - (24) which determine nine endogenous variables, $r, w, r^*, w^*, Y_1, Y_1^*, Y_2, Y_2^*$ and p. Fortunately, the above system of equations can be transformed into a system of two equations in two unknowns -each country's per-capita output of good 1, $y_1 = Y_1 / L$ and $y_1^* = Y_1^* / L$. In order to generate these two equations, consider Home's production side described by equations (17) through (20). Solving (17) and (18) for the rental of capital and the wage of labor yields:

$$r\left(\Gamma(Y_1, Y_1^*, L+L^*)\right), \quad w\left(\Gamma(Y_1, Y_1^*, L+L^*)\right).$$

Using these two expressions, Home's factor income can be written as

$$I \equiv rK + wL \equiv r\left(\Gamma(Y_1, Y_1^*, L+L^*)\right)K + w\left(\Gamma(Y_1, Y_1^*, L+L^*)\right)L .$$

Shimomura [11] has christened this function "factor-income function" and has explored its properties. Under constant returns to scale and perfect competition it becomes the familiar GDP function. The derivative of the factor-income function with respect to $\Gamma(\cdot)$ yields the "output" supply of good 1:

$$\Psi(Y_1, Y_1^*) = r'\left(\Gamma(Y_1, Y_1^*, L+L^*)\right)K + w'\left(\Gamma(Y_1, Y_1^*, L+L^*)\right)L , \qquad (25)$$

where $r'(\cdot) = dr(\cdot)/d\Gamma$ and $w'(\cdot) = dw(\cdot)/d\Gamma$. Equation (25) can be rewritten in terms of each country's per-capita output of good 1:

$$\Psi(Ly_1, L^*y_1^*) = r'\left(\Gamma(Ly_1, L^*y_1^*, L+L^*)\right)K + w'\left(\Gamma(Ly_1, L^*y_1^*, L+L^*)\right)L \qquad (26)$$

Similarly, Foreign's counterpart of (26) is given by

$$\Psi(L^*y_1^*, Ly_1) = r'\left(\Gamma(L^*y_1^*, Ly_1, L+L^*)\right)K^* + w'\left(\Gamma(L^*y_1^*, Ly_1, L+L^*)\right)L^* . \qquad (27)$$

Equations (26) and (27) determine the values of per-capita outputs of good 1, y_1 and y_1^*, as functions of each country's factor endowments.

In the presence of quasi-linear preferences, the trade pattern is determined by the difference in per-capita supply of good 1 between the two countries (see Lemma 1), and these two equations can be used to analyze the dependence of the trade pattern on factor-endowments. Notice that in the absence of factor-price equalization, the concept of the world integrated equilibrium can not be used to analyze the pattern of trade. However, one can use equations (26) and (27) to characterize the combinations of Foreign's factor endowments in the $L^* - K^*$ space such that neither country has an incentive to trade. We call this curve the no-trade locus. In the case of homothetic preferences the no-trade locus is given by positively-sloped line, defined by equation $K^* = (K/L)L^*$. In the case of quasi-linear tastes, the no-trade locus can be derived is based on the system of equations (26) and (27). If none of the two countries has an incentive to trade, then Lemma 1 implies that $y_1 = y_1^* = \overline{y}_1$, that is the per-capita supply of good 1 must be the same in both countries. The following lemma summarizes the properties of the no-trade locus:

LEMMA 2. *Suppose that good 1 is capital intensive (labor intensive) and let* $\Omega \equiv r''(\cdot)K + w''(\cdot)L > 0$. *Then, the slope of the no-trade locus evaluated at* $y_1 = y_1^* = \overline{y}_1$ *is larger (smaller) than the factor intensity of the numeraire good* $k_2 = K_2 / L_2$, *if functions* $\Gamma(\cdot)$ *and* $\Psi(\cdot)$ *satisfy the following condition:*

$$\Psi_1 - \Psi_2 + \Omega(\Gamma_2 - \Gamma_1) > 0, \qquad (28)$$

where $\Psi_1 \equiv \partial\Psi(Y_1, Y_1^*)/\partial Y_1, \Psi_2 \equiv \partial\Psi(Y_1, Y_1^*)/\partial Y_1^*, \Gamma_1 \equiv \partial\Gamma(Y_1, Y_1^*, L + L^*)/\partial Y_1$ *and* $\Gamma_2 \equiv \partial\Gamma(Y_1, Y_1^*, L + L^*)/\partial Y_1^*$ *all of which are evaluated at* $y_1 = y_1^* = \overline{y}_1$.

Proof. (See Appendix A .)

Figure 4 illustrates the no-trade locus for the case in which good 1 is capital intensive. Line BB is a line with a slope equal to the capital intensity of the numeraire good. The no-trade locus in the presence of quasi-linear preferences and the absence of FPE is illustrated by the positively-sloped line $B'B'$, which has a higher slope than line BB, and crosses the latter at point E.[6] The coordinates of point E correspond to the case of two countries with identical endowments ($K^* = K$ and $L^* = L$). In the case of homothetic tastes, the no-trade locus corresponds to a line starting at the origin and having a slope equal to K/L.

(Figure 4 around here)

We are now in the position to utilize Lemmas 1 and 2 and characterize the trade pattern under quasi-linear preferences and without FPE. In order to do this, suppose that we consider the effects of a marginal increase in Foreign's endowment of capital K^* starting from $K^* = K$ and $L^* = L$, which corresponds to point E in Figure 4. This change will make Foreign the capital abundant country. In order to determine the effects of this marginal change on the pattern of trade, according to Lemma 1 it is sufficient to analyze the effects of an increase in K^* on each country's per capital supply of good 1. At point E, there is no trade because $y_1^* - y_1 = 0$, therefore if an increase in Foreign's endowment raises y_1^* more than y_1, then Foreign exports good 1 and imports good 2 in accordance with Lemma 1. Differentiating totally the system of equations (26) and (27), and using Crammer's rule yields:

$$\frac{d(y_1^* - y_1)}{dK^*} = \frac{r'(\Gamma)}{L\left[\Psi_1 - \Psi_2 + \Omega(\Gamma_1 - \Gamma_2)\right]}. \tag{29}$$

Lemma 2 is based on the assumption that the term in square brackets in the denominator of (29) is positive. In addition, if good 1 is capital intensive then $r'(\Gamma) > 0$ and the right-hand-side of (29) is positive. Therefore, the capital-abundant country exports the capital-intensive good even in the presence of non-homothetic quasi-linear preferences in accordance to the factor proportions theory. This result is stated in the following proposition:

[6] In the case of two countries which identical labor endowments, which was analyzed by Doi et al. [4], the no-trade locus coincides with line BB.

PROPOSITION 3. *If the factor abundance of both countries is such that the factor endowment point is located outside (above) the no-trade locus B'B' in Figure 4, then each country exports the good that uses intensively its abundant factor of production, under quasi-linear preferences and under no FPE.*

Of course, it is obvious from the above analysis that the trade pattern can be reversed when countries have similar (even identical) capital labor ratios. Therefore the robustness of the factor proportions theory depends on specific conditions and the differences between homogeneous and quasi-linear preferences are amplified with the introduction of market-structure considerations that prevent the realization of FPE. Appendix B applies the insights of proposition 3 and verifies its assumptions for the case of two popular market structures analyzed extensively in the literature: international duopoly and country-specific external economies coupled with perfect competition.[7]

6 CONCLUDING REMARKS

The present paper took the general equilibrium properties of quasi-linear preferences seriously. Our analysis was motivated by the existing gap in the logic between partial-equilibrium analysis of trade policy which is almost exclusively based on the assumption of quasi-linear preferences and the general equilibrium analysis of trade patterns and income distribution that uses homothetic preferences.

To our surprise, we discovered that quasi-linear preferences behave reasonably well in general-equilibrium settings: The Ricardian pattern of trade and the law of comparative advantage apply to the case of quasi-linear preferences; and the factor proportions theory of comparative advantage also remains robust if one replaces homothetic preferences with

[7] See Markusen [9], Fujiwara and Shimomura [6], and Doi et al. [4], among others.

quasi-linear ones under perfect competition and factor price equalization. This result enhances the analytical consistency of partial equilibrium analysis with the general-equilibrium analysis. In addition, even if we established the robustness of the traditional theory, we believe that the proofs basic propositions were novel.

We also discovered a number of important differences between the two structures of preferences: Even under perfect competition and factor price equalization, quasi-linear preferences generate too much or too little factor service trade compared to homothetic preferences. This result illustrates that quasi-linear preferences must be taken more seriously by empirical trade economists who are working on resolving the "missing trade" mystery, which states that the traditional factor proportions theory predicts too much trade in factor services compared to the observed volume of trade. Finally, in the absence of factor price equalization, the differences between quasi-linear and homothetic preferences are amplified. We established reasonable conditions under which the factor proportions theory holds even in this case and shown that these conditions hold for a couple of popular market structures such as the case of international duopoly and national external economies.

Although we focused exclusively on the general-equilibrium properties of quasi-linear preferences regarding trade patterns, we have abstracted from the analysis of the role of quasi-linear preferences in income distribution and trade. The non-homothetic nature of quasi-linear preferences generates additional interactions between the income distribution within and across countries and trade patterns. In addition our analysis assumed that the numerare good is produced under constant returns to scale and perfect competition. Relaxing this assumption might amplify the differences between homothetic and quasi-linear preferences. These important topics constitute a fruitful avenue for future research.

REFERENCES

1. D. R. Davis, D. Weinstein, An Account of Global Factor Trade, *Amer. Econ. Rev.* 91, (2001) , 1423-53.

2. D. R. Davis, D. Weinstein, The Factor Content of Trade, In: K. Choi, J. Harrigan, (Eds), *Handbook of International Trade*, Blackwell, Oxford, 2003.

3. A. K. Dixit, V. Norman, *Theory of International Trade*, Cambridge University Press, Cambridge, 1980.

4. J. Doi, K. Fujiwara, T. Kikuchi, K. Shimomura, A Modified Heckscher-Ohlin Theorem under Quasi-Linear Utility Functions, (2004), Mimeo, Kobe University.

5. R. C. Feenstra, *Advanced International Trade: Theory and Evidence*, Princeton University Press, Princeton, 2004.

6. K. Fujiwara, K. Shimomura, A Factor Endowment Theory of International Trade under Imperfect Competition and Increasing Returns, *Can. J. Econ.* 38, (2005), 273-89.

7. E. Helpman, P. R. Krugman, *Market Structure and Foreign Trade*, MIT Press, Cambridge, MA, 1985.

8. S. Lahiri, Y. Ono, The Role of Free Entry in an Oligopolistic Heckscher-Ohlin Model, *Int. Econ. Rev.* 36, (1995), 609-24.

9. J. R. Markusen, Trade and the Gains from Trade with Imperfect Competition, *J. Int. Econ* 11, (1981), 531-551.

10. J. R. Markusen, J. R. Melvin, Trade, Factor Prices, and the Gains from Trade with Increasing Returns to Scale, *Can. J. Econ.* 14, (1981), 450-469.

11. K. Shimomura, Factor Income Function and an Oligopolistic Heckscher-Ohlin Model of International Trade, *Econ. Letters* 61, (1998), 91-100.

12. D. Trefler, International Factor Price Differences: Leontief was Right, *J. Polit. Econom*, 101, (1993), 961-87.

13. D. Trefler, The Case of Missing Trade and Other Mysteries, *Amer. Econ. Rev.*, 85, (1995), 1029-46.

Figures

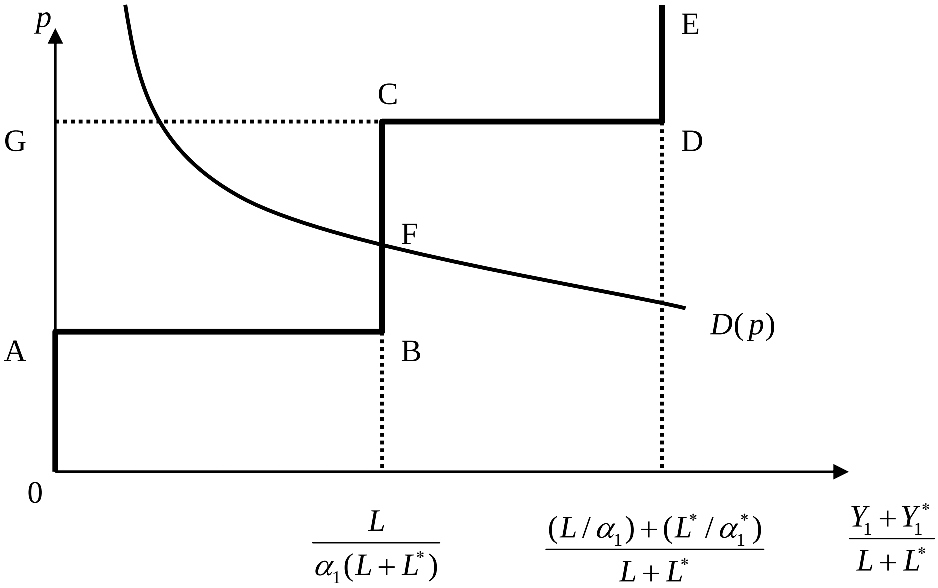

Figure 1: *The Ricardian Trade Model*

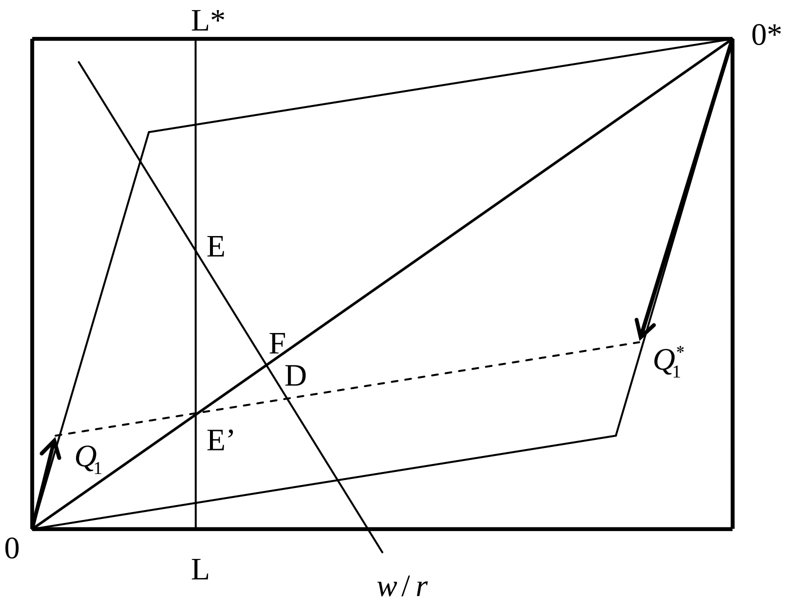

Figure 2: *Factor Content of Trade When Good 1 is Capital Intensive*

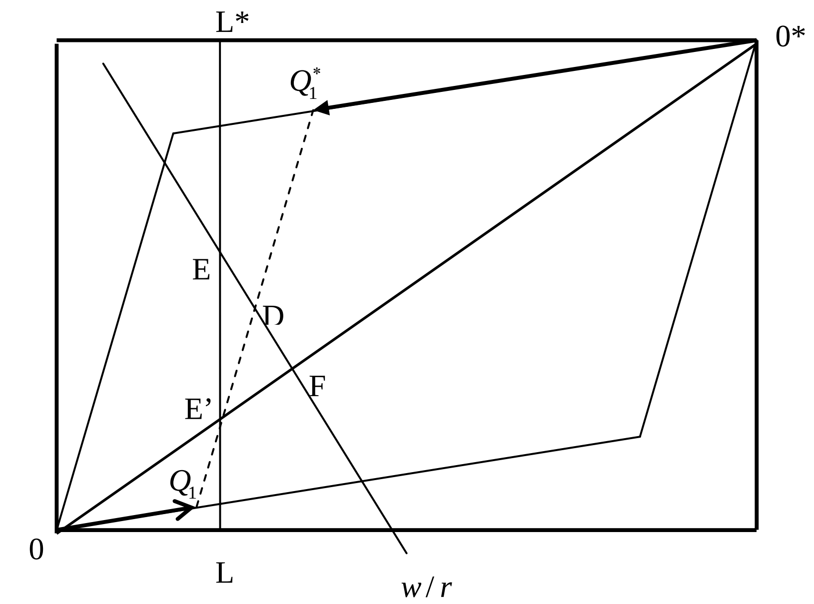

Figure 3: *Factor Content of Trade When Good 1 is Labor Intensive*

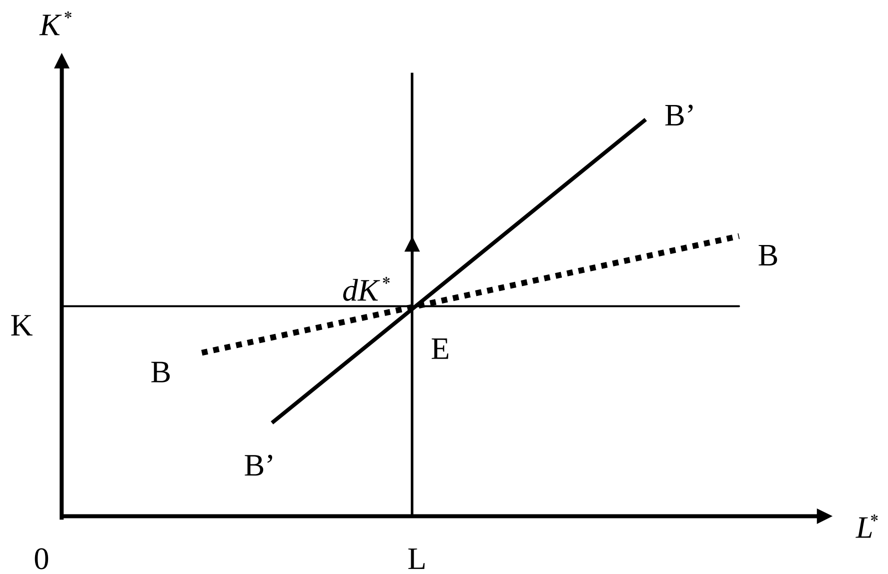

Figure 4: *Trade Patterns without Factor Price Equalization*

(in the Case That Good 1 is Capital-Intensive)

APPENDIX A

*Proof of Lemma 1***.** It suffices to consider the case where $Y_1 / L > Y_1^* / L^*$ because the same argument applies to the case where $Y_1 / L < Y_1^* / L^*$. The world market-clearing condition (4) can be rewritten as:

$$L\left[\frac{Y_1}{L} - D(p)\right] = L^*\left[D(p) - \frac{Y_1^*}{L^*}\right]. \quad \text{(A1)}$$

Equation (A1) implies that the signs of the terms in square brackets must be the same. Consequently, there are two possible cases:

$$\frac{Y_1}{L} - D(p) > 0, \frac{Y_1^*}{L^*} - D(p) < 0 \quad \text{and} \quad \frac{Y_1}{L} - D(p) < 0, \frac{Y_1^*}{L^*} - D(p) > 0 \quad \text{(A2)}$$

However, the latter sign pattern is excluded because it contradicts the assumption $Y_1 / L > Y_1^* / L^*$. Therefore, the former sign pattern is the only one consistent with our initial assumption and implies that Home has a positive export supply of good 1 and Foreign has a positive import demand for good 1. Therefore Home exports good 1 and imports the numeraire good to maintain a trade balance. Q. E. D.

Proof of Lemma 2. Our starting point is the system of (26) and (27). Making use of it, let us first find and characterize the locus on which both countries have no opportunity to trade in the $L^* - K^*$ space, which we call the no-trade locus. In the standard HO theorem, the no-trade locus is given by $K^* = (K/L)L^*$. In what follows, we shall characterize the no-trade locus based on the system of (26) and (27), which determines y_1 and y_1^* given K^* and L^* When no trade occurs between the countries, $y_1 = y_1^* = \bar{y}_1$. We will examine what relationship must hold between K^* and L^* which yields such a symmetry. To do this, substituting $\bar{y}_1$ into the system and totally differentiating it with respect to $\bar{y}_1, K^*$ and L^* yield

$$\begin{bmatrix} \Psi_1 L + \Psi_2 L^* - \Omega(\Gamma_1 L + \Gamma_2 L^*) & 0 \\ \Psi_1^* L^* + \Psi_2^* L - \Omega^*(\Gamma_1^* L^* + \Gamma_2^* L) & -r' \end{bmatrix} \begin{bmatrix} d\bar{y}_1 \\ dK^* \end{bmatrix} = \begin{bmatrix} \Omega(\Gamma_2 \bar{y}_1 + \Gamma_3) - \Psi_2 \bar{y}_1 \\ \Omega^*(\Gamma_1^* \bar{y}_1 + \Gamma_3^*) - \Psi_1^* \bar{y}_1 + w' \end{bmatrix} dL^*,$$

where $\Gamma_1 \equiv \partial \Gamma(Y_1, Y_1, L+L^*)/\partial Y_1$ and $\Gamma_1^* \equiv \partial \Gamma(Y_1^*, Y_1, L+L^*)/\partial Y_1^*$ and the other derivatives are similarly defined. Solving for dK^*/dL^*, yields

$$\begin{aligned}\frac{dK^*}{dL^*} &= -\frac{w'}{r'} + \frac{\Delta}{r'[\Psi_1 L + \Psi_2 L^* - \Omega(\Gamma_1 L + \Gamma_2 L^*)]} \\ &= k_2 + \frac{\Delta}{r'[\Psi_1 L + \Psi_2 L^* - \Omega(\Gamma_1 L + \Gamma_2 L^*)]},\end{aligned} \tag{A3}$$

where $$\begin{aligned}\Delta \equiv &\left[\Omega(\Gamma_2 \bar{y}_1 + \Gamma_3) - \Psi_2 \bar{y}_1\right]\left[\Psi_1^* L^* + \Psi_2^* L^* - \Omega^*(\Gamma_1^* L^* + \Gamma_2^* L)\right] \\ &- \left[\Omega^*(\Gamma_1^* \bar{y}_1 + \Gamma_3^*) - \Psi_1^* \bar{y}_1\right]\left[\Psi_1 L + \Psi_2 L^* - \Omega(\Gamma_1 L + \Gamma_2 L^*)\right],\end{aligned}$$

and $\Omega \equiv r''(\cdot)K + w''(\cdot)L > 0.$ Evaluating (A3) at $K^* = K$ and $L^* = L,$ it is simplified to

$$\left.\frac{dK^*}{dL^*}\right|_{K^*=K,\, L^*=L} = k_2 + \frac{y\left[\Psi_1 - \Psi_2 + \Omega(\Gamma_2 - \Gamma_1)\right]}{r'}. \tag{A4}$$

Thus, we have reached the result to be proved. Q. E. D.

APPENDIX B: THE PETTERN OF TRADE WITHOUT FACTOR PRICE EQUALIZATION

B. 1: INTERNATIONL DUOPOLY

Consider now the case of international duopoly, which has been analyzed by Markusen [9] and Fujiwara and Shimomura [6]. The production function in each sector is given by:

$$\begin{aligned}&Y_1 = F(f^1(K_1, L_1)), \qquad F'(\cdot) > 0, \qquad F''(\cdot) > 0,\\ &Y_2 = f^2(K_2, L_2),\end{aligned}$$

where $f^1(\cdot)$ and $f^2(\cdot)$ are an increasing, strictly quasi-concave, and linearly homogeneous function. Hence, the cost functions associated with these production functions can be written as

$$\begin{aligned}&c_1(r,w)F^{-1}(Y_1) \equiv c_1(r,w)\phi(Y_1), \qquad \phi'(\cdot) > 0, \qquad \phi''(\cdot) < 0,\\ &c_2(r,w)Y_2,\end{aligned}$$

where the function $c_i(r,w), i = 1,2$ is an increasing, strictly concave, and linearly homogeneous function.

On the other hand, the inverse demand function $p((Y_1 + Y_1^*)/(L + L^*))$ is obtained as a solution to the following marker-clearing condition under free trade.

$$LD(p) + L^* D(p) = Y_1 + Y_1^* .$$

The home monopolist's profit is then defined as

$$p\left(\frac{Y_1 + Y_1^*}{L + L^*}\right) Y_1 - c_1(r, w)\phi(Y_1).$$

Then, the profit maximization condition for the home firm becomes

$$p'\left(\frac{Y_1 + Y_1^*}{L + L^*}\right)\frac{Y_1}{L + L^*} + p\left(\frac{Y_1 + Y_1^*}{L + L^*}\right) \equiv MR(Y_1, Y_1^*, L + L^*) = c_1(w, r)\phi'(Y_1),$$

from which the functions $\Gamma(\cdot)$ and $\Psi(\cdot)$ are specified as

$$\Gamma(Y_1, Y_1^*, L + L^*) \equiv \frac{MR(Y_1, Y_1^*, L + L^*)}{\phi'(Y_1)}, \quad \Psi(Y_1, Y_1^*) \equiv \phi(Y_1), \qquad \text{(B1)}$$

which yields

$$\Gamma_1 = \frac{MR_1 \cdot \phi' - MR \cdot \phi''}{(\phi')^2} = \frac{MR_1 - c_1\phi''}{\phi'}, \qquad \Gamma_2 = \frac{MR_2}{\phi'},$$

where $MR_1 \equiv \partial MR(\cdot)/\partial Y_1$ and $MR_2 \equiv \partial MR(\cdot)/\partial Y_1^*$. The numerator of Γ_1 is negative due to the second-order condition for profit maximization. On the other hand, (B1) gives $\Psi_1(\cdot) = \phi'(Y_1) > 0$ and $\Psi_2(\cdot) = 0.$ Based on the above equations, the sufficient condition for the establishment of the HO trade pattern reduces to

$$\Psi_1 - \Psi_2 + \Omega(\Gamma_2 - \Gamma_1) = \phi' + \frac{\Omega(MR_2 - MR_1 + c_1\phi'')}{\phi'} = \frac{\Omega\left[MR_2 - MR_1 + \frac{(\phi')^2}{\Omega} + c_1\phi''\right]}{\phi'} > 0. \text{(B2)}$$

To ensure that $MR_2 - MR_1 + (\phi')^2/\Omega + c_1\phi'' > 0$, let us consider the stability condition of the Cournot-Nash equilibrium. Following the existing literature, assume the following adjustment process:

$$\dot{Y}_1 = MR(Y_1, Y_1^*, L + L^*) - c_1$$

$$\dot{Y}_1^* = MR(Y_1^*, Y_1, L + L^*) - c_1^*.$$

Note that in the present general equilibrium system, c_1 is given by $\lambda(\phi(Y_1), K, L)$ which s a solution to[8]

$$\phi(Y_1) = r'(\lambda)K + w'(\lambda)L,$$

and hence $1/\Omega = \lambda_\phi \equiv \partial\lambda/\partial\phi.$ Then, the Jacobian matrix evaluated at $K^* = K$ and $L^* = L$ becomes

$$\begin{bmatrix} MR_1 - \lambda_\phi(\phi')^2 - \lambda\phi'' & MR_2 \\ MR_2 & MR_1 - \lambda_\phi(\phi')^2 - \lambda\phi'' \end{bmatrix}.$$

Stability of equilibrium requires that the trace of the above matrix is negative and that the determinant is positive. The former follows from the second-order condition for social welfare maximization.[9] On the other hand, the second requirement is

$$\det\begin{bmatrix} MR_1 - \lambda_\phi(\phi')^2 - \lambda\phi'' & MR_2 \\ MR_2 & MR_1 - \lambda_\phi(\phi')^2 - \lambda\phi'' \end{bmatrix}$$
$$= \left[MR_1 + MR_2 - \lambda_\phi(\phi')^2 - \lambda\phi''\right]\left[MR_1 - MR_2 - \lambda_\phi(\phi')^2 - \lambda\phi''\right] > 0.$$

In the case of $MR_2 > 0,$ the (social) second-order condition ensures that condition (B2) holds. In the case of $MR_2 < 0,$ the terms in the first brackets are negative and those in the second bracket must also be negative for stability to hold. Hence, condition (B2) holds whether $MR_2 > 0$, or $MR_2 < 0$ and the validity of the HO theorem holds.

So far in this section, we focused on local properties of the no-trade locus. However, if good 1 is produced under constant returns as in Markusen [9], the properties of the no-trade locus hold globally as summarized in the following proposition:

---

[8] The same is true of Foreign.

[9] Note that this is different from the second-order condition from the private viewpoint: $MR_2 - \lambda\phi'' < 0.$

PROPOSITION 4. *If constant returns to scale prevail in the production of the non-numeraire good and this good is capital-intensive (resp. labor-intensive), then the slope of the no-trade locus is globally larger (resp. smaller) than both Home's capital abundance* K/L *and the factor intensity of the numeraire good.*

Proof. We assume that all sectors are subject to constant returns to scale as in Markusen [9].[10] Then, the profit maximization condition for the home monopolist is given by

$$MR(Y_1, Y_1^*, L + L^*) = c_1(r, w).$$

Note that in the general equilibrium system the right-hand side is determined by $\lambda(Y_1, K, L)$, which is the solution to

$$Y_1 = r'(\lambda)K + w'(\lambda)L.$$

And making use of the definition of the marginal revenue, the above equilibrium condition is rewritten by

$$p'\left(\frac{Y_1 + Y_1^*}{L + L^*}\right)\frac{Y_1}{L + L^*} + p\left(\frac{Y_1 + Y_1^*}{L + L^*}\right) = \lambda(Y_1, K, L) = \lambda\left(\frac{Y_1}{L}, \frac{K}{L}, 1\right),$$

where the last equality follows from the fact that $q(\cdot)$ is homogeneous of degree zero. Furthermore, this condition is rewritten in terms of per-capita outputs as follows.

$$p'\left(\frac{Ly_1 + L^* y_1^*}{L + L^*}\right)\frac{Ly_1}{L + L^*} + p\left(\frac{Ly_1 + L^* y_1^*}{L + L^*}\right) = \lambda\left(y_1, \frac{K}{L}, 1\right).$$

[10] Figure 4 depicts the no-trade locus under international duopoly with constant returns in the case where good 1 is capital-intensive. Proposition 4 states that the non-trade locus is given like $B'B'$ *globally*. This proposition gives us the now-classical trade pattern proposition by Markusen [9]: the large country imports the monopolized good (good 1). To verify this, assume that the foreign endowment pair is given by F Then, the foreign country is defined as a labor-abundant country and imports good 1, which is a capital-intensive good. This is not the case in Doi et al. [4].

The foreign counterpart is similarly defined. Hence, these two equations determine y_1 and y_1^*.

The no-trade locus is defined when $y_1 = y_1^* = \bar{y}_1$ holds, which gives

$$p'(\bar{y}_1)\frac{L\bar{y}_1}{L+L^*} + p(\bar{y}_1) = \lambda\left(\bar{y}_1, \frac{K}{L}, 1\right)$$

$$p'(\bar{y}_1)\frac{L^*\bar{y}_1}{L+L^*} + p(\bar{y}_1) = \lambda\left(\bar{y}_1, \frac{K^*}{L^*}, 1\right).$$

We are now ready to prove Proposition 4. Subtracting these two equations yields

$$\frac{p'(\bar{y}_1)\bar{y}_1}{L+L^*}(L-L^*) = \lambda\left(\bar{y}_1, \frac{K}{L}, 1\right) - \lambda\left(\bar{y}_1, \frac{K^*}{L^*}, 1\right). \qquad \text{(B3)}$$

Consider the case of $L^* > L$ without loss of generality. Then, the left-hand side is positive, from which the right-hand side must also be positive. When good 1 is capital-intensive, $\lambda(\cdot)$ is decreasing in K/L Thus, in order for the right-hand side to be positive, we must have $K/L < K^*/L^*$. On the contrary, when good 1 is labor-intensive, $\lambda(\cdot)$ is increasing in K/L. Hence, the right-hand side is positive if $K/L > K^*/L^*$. Parallel arguments can be made for the case of $L^* < L$. In sum, when good 1 is capital-intensive, the absolute value of the slope of the borderline is larger than K/L while it is smaller than K/L if good 1 is labor-intensive. Q. E. D.

B. 2: NATIONAL EXTERNAL RETURNS TO SCALE

The specification in the demand side does not deviate from the previous subsection so that we can skip to the supply side. Following most of the existing literature, we assume the following type of external returns to scale.

$$Y_1 = F(Y_1) f^1(K_1, L_1), \quad F'(\cdot) > 0, \quad F''(\cdot) < 0.$$

Then, the cost function becomes

$$c_1(r, w)\frac{Y_1}{F(Y_1)}.$$

Note that each firm in sector 1 seeks to maximize profits taking $F(\cdot)$ as given. Therefore, the profit-maximization condition becomes

$$p\left(\frac{Y_1 + Y_1^*}{L + L^*}\right) - \frac{c_1(r, w)}{F(Y_1)} = 0.$$

Allowing for this, $\Gamma(\cdot)$ and $\Psi(\cdot)$ are specified as

$$\Gamma(Y_1, Y_1^*) = p\left(\frac{Y_1 + Y_1^*}{L + L^*}\right)F(Y_1), \quad \Psi(Y_1, Y_1^*) = \frac{Y_1}{F(Y_1)},$$

which yield

$$\Gamma_1 = \frac{p'F}{L + L^*} + pF', \quad \Gamma_2 = \frac{p'F}{L + L^*} < 0, \quad \Psi_1 = \frac{F - F'Y_1}{F^2} > 0, \quad \Psi_2 = 0,$$

from which we have the sufficient condition for the HO trade pattern:

$$\Psi_1 - \Psi_2 + \Omega(\Gamma_2 - \Gamma_1) = \frac{F - F'Y_1}{F^2} - \Omega pF' > 0. \tag{B4}$$

The rest of our task is basically the same as the case of international duopoly. To show (B4), let us consider the following adjustment process.

$$\dot{Y}_1 = p\left(\frac{Y_1 + Y_1^*}{L + L^*}\right) - \frac{c_1}{F(Y_1)}$$

$$\dot{Y}_1^* = p\left(\frac{Y_1 + Y_1^*}{L + L^*}\right) - \frac{c_1^*}{F(Y_1^*)},$$

where c_1 and c_1^* satisfy

$$\frac{Y_1}{F(Y_1)} = r'(c_1)K + w'(c_1)L$$

$$\frac{Y_1^*}{F(Y_1^*)} = r'(c_1^*)K^* + w'(c_1^*)L^*$$

Then, stability of equilibrium requires that the trace of the associated Jacobian matrix is negative and that its determinant is positive. It is possible to show that this condition assures that condition

(B4) holds. The proof is omitted since it is exactly the same as that of international duopoly. Therefore, the HO theorem hold also in the case of national external returns to scale. And the following proposition establishes the global validity of the slope of the no-trade locus:

PROPOSITION 5. *Suppose that good 1 is capital-intensive (resp. labor-intensive). Then, the slope of the no-trade locus is larger (resp. smaller) than the factor intensity of good 2 but smaller (resp. larger) than* K/L.

Proof. The proof follows the same strategy as in the case of international duopoly. Then, the equilibrium condition such that $y_1 = y_1^* = \bar{y}_1$ reduces to

$$p(\bar{y}_1)F(L\bar{y}_1) = \lambda\left(\frac{\bar{y}_1}{F(L\bar{y}_1)}, \frac{K}{L}, 1\right)$$

$$p(\bar{y}_1)F(L^*\bar{y}_1) = \lambda\left(\frac{\bar{y}_1}{F(L^*\bar{y}_1)}, \frac{K^*}{L^*}, 1\right).$$

Subtracting the latter equation from the former yields

$$p(\bar{y}_1)\left[F(L\bar{y}_1) - F(L^*\bar{y}_1)\right] = \lambda\left(\frac{\bar{y}_1}{F(L\bar{y}_1)}, \frac{K}{L}, 1\right) - \lambda\left(\frac{\bar{y}_1}{F(L^*\bar{y}_1)}, \frac{K^*}{L^*}, 1\right). \qquad \text{(B5)}$$

Suppose $L^* > L$. Then, the left-hand side becomes negative and hence the right-hand side has to be negative as well. By the way, $L^* > L$ implies that $\bar{y}_1 / F(L\bar{y}_1) > \bar{y}_1 / F(L^*\bar{y}_1)$ which in turn implies

$$\lambda\left(\frac{\bar{y}_1}{F(L\bar{y}_1)}, \frac{K}{L}, 1\right) > \lambda\left(\frac{\bar{y}_1}{F(L^*\bar{y}_1)}, \frac{K}{L}, 1\right).$$

That is, $L^* > L$ s likely to make the right-hand side positive. Thus, in order to make the right-hand side negative, each country's factor endowment ratio must have the following relationship. When good is capital-intensive, $\lambda(\cdot)$ is decreasing in K/L. Accordingly, $K/L > K^*/L^*$ must hold. On the contrary, when good 1 is labor-intensive, $\lambda(\cdot)$ is increasing in K/L which establishes

$K/L < K^*/L^*$ to make the right-hand side negative. In sum, when good 1 is capital-intensive, the absolute value of the slope of the borderline must be smaller than K/L while the converse holds when good 1 is labor-intensive. Q. E. D.

This proposition asserts that the no-trade locus in Figure 4 is located between a curve with slope equal to K/L and a curve with slope equal to the factor intensity of good 2. One interesting by-product of Proposition 5 is that the larger country exports good 1, which is proved in Markusen and Melvin [9] under the assumption of a homothetic utility function.

B. 3: TRADE-PATTERN REVERSALS IN THE ABSENCE OF FPE

As shown in the main text, trade patterns follow the HO when FPE occurs, which implies that the no-trade locus is explicitly given by $K^* = kL^*$ in Figure 4. On the other hand, this is not the case in the market structures that are not associated with FPE, i.e., international duopoly and national economies of scale. Geometrically, the no-trade locus deviates from $K^* = kL^*$ in such market structures. The rest of this Appendix shows that this deviation raises the theoretical possibility that the capital-abundant country exports the labor-intensive good.

Recall that the no-trade locus under international duopoly is implicitly given by (B3). If the Foreign endowment pair satisfies it, the international equilibrium is characterized by $y_1 = y_1^* = \overline{y}_1$. Such a pair can be defined as $K^* = h\left(L^*; K, L\right)$. Under international duopoly, the no-trade locus is given by $B'B'$. According to Proposition 4, the Foreign exports the capital-intensive good if its endowment is above $B'B'$ and good 1 is labor-intensive. The same is true of the region between the diagonal of $K^* = kL^*$ and the no-trade locus $K^* = h\left(L^*; K, L\right)$. In this situation, the labor-abundant (resp. capital-abundant) Foreign exports the capital-intensive (resp. labor-intensive) good. The above result can be formally stated in:

PROPOSITION 6. *Under international oligopoly and constant returns to scale, each country **imports** the good that uses its abundant factor of production intensively, if the Foreign endowment pair satisfies the condition:*

$$\min\left\{kL^*, h\left(L^*; K, L\right)\right\} < K^* < \max\left\{kL^*, h\left(L^*; K, L\right)\right\}.$$

The conclusion derived under international duopoly applies to the other market structure which gives no FPE, i.e., national economies of scale. As Proposition 5 states, the no-trade locus is implicitly defined by (B5). Let us express the Foreign endowment pair which satisfies (B5) as $K^* = \tilde{h}\left(L^*, K, L\right)$. This function defines the no-trade locus in this case. By making an argument similar to that of international duopoly, we have:

PROPOSITION 7. *Under perfect competition and national external economies, each country **imports** the good that uses its abundant factor of production intensively, if the Foreign endowment pair satisfies the condition:*

$$\min\left\{kL^*, \tilde{h}\left(L^*; K, L\right)\right\} < K^* < \max\left\{kL^*, \tilde{h}\left(L^*; K, L\right)\right\}.$$

APPENDIX C: THE PETTERN OF TRADE UNDER QUASI-LINEAR PREFERENCES AND FACTOR PRICE EQUALIZATION

This Appendix analyzes the pattern of trade under quasi linear preferences and factor price equalization by applying the analysis of the main paper to three different market structures: Monopolistic competition and internal scale economies, free entry international oligopoly, and international external economies to scale. In all these cases, each country exports the good which uses its abundant factor intensively in its production, that is, the factor-proportions theory of comparative advantage holds.

C. 1: MONOPOLISTIC COMPETITION

Henceforth, sector 1 is assumed to be subject to economies of scale which are internal or external to firms. This section deals with a monopolistically competitive model developed by Dixit and Norman [1] and Helpman and Krugman [3]. Following their analysis, assume that the production function per-variety is given by a homothetic form:

$$x = F(f^1(k,l)), \quad F'(\cdot) > 0, \quad F''(\cdot) > 0,$$

where x is the per-variety output, k and l are the capital and labor inputs and $f^1(\cdot)$ is increasing, strictly quasi-concave and linearly homogeneous. This specification enables us to write the associated cost function as a multiplicatively separable form:

$$c_1(r,w)F^{-1}(x) \equiv c_1(r,w)\phi(x), \quad \phi'(\cdot) > 0, \quad \phi''(\cdot) < 0.$$

To derive the inverse demand function for each variety, consider the consumer behavior. For the time being, we focus on the home country. One consumer's utility function is assumed to take the form of

$$U = u(Q_1) + Q_2, \qquad u'(\cdot) > 0, \qquad u''(\cdot) < 0,$$

where Q_1 is the quantity index of the differentiated products:

$$Q_1 \equiv \left(\int_0^n q_i^\theta di + \int_0^{n^*} q_j^\theta di \right)^{\frac{1}{\theta}}, \quad \theta \in (0,1),$$

where q_i *(resp.* q_j) denotes the consumption of the home (resp. foreign) varieties and n *(resp.* n^*) is their range. The corresponding price index is defined as

$$P \equiv \left[\int_0^n p_i^{\frac{1}{\theta-1}} di + \int_0^{n^*} (p_j^*)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta-1}{\theta}},$$

where p_i *(resp.* p_j^*) is the price of the variety produced by firm i (resp. j) in the home (resp. foreign) country.

The consumer's utility maximization problem can be solved in two steps.[1] First, whatever the value of the quantity index, Q_1, q_i. and q_j need to be chosen so as to minimize the expenditure to attain Q_1. By solving the expenditure minimization problem, we obtain

$$q_i = Q_1 \left(\frac{p_i}{P} \right)^{\frac{1}{\theta-1}}, \quad i \in [0, n]$$

$$q_j = Q_1 \left(\frac{p_j^*}{P} \right)^{\frac{1}{\theta-1}}, \quad j \in [0, n^*],$$

from which we can obtain the minimum expenditure to attain Q_1 as

$$\begin{aligned} \int_0^n p_i q_i di + \int_0^{n^*} p_j^* q_j dj &= \int_0^n p_i Q_1 \left(\frac{p_i}{P} \right)^{\frac{1}{\theta-1}} di + \int_0^{n^*} p_j^* Q_1 \left(\frac{p_j^*}{P} \right)^{\frac{1}{\theta-1}} dj \\ &= Q_1 \left[\int_0^n p_i \left(\frac{p_i}{P} \right)^{\frac{1}{\theta-1}} di + \int_0^{n^*} p_j^* \left(\frac{p_j^*}{P} \right)^{\frac{1}{\theta-1}} dj \right] \\ &= \frac{Q_1}{P^{\frac{1}{\theta-1}}} \left[\int_0^n p_i (p_i)^{\frac{1}{\theta-1}} di + \int_0^{n^*} p_j^* (p_j^*)^{\frac{1}{\theta-1}} dj \right]. \end{aligned}$$

[1] See, for example, Helpman and Krugman (1985, Chap. 6).

Using the definition of P, we have

$$\int_0^n p_i q_i di + \int_0^{n^*} p_j^* q_j dj = PQ_1.$$

Therefore, the upper-level step of the consumer's problem can be formulated as

$$\max_{Q_1, Q_2} \quad u(Q_1) + Q_2$$

subject to $PQ_1 + Q_2 =$ (Per-capita income).

Solving this, we obtain the "demand function" for Q_1 as

$$Q_1 = D(P),$$

where $D(\cdot) \equiv u'^{-1}(\cdot)$.

Similarly, the demand function for a typical variety in Foreign is given by

$$q_i^* = Q_1^* \left(\frac{p_i}{P} \right)^{\frac{1}{\theta - 1}}, \quad i \in [0, n]$$

$$q_j^* = Q_1^* \left(\frac{p_j^*}{P} \right)^{\frac{1}{\theta - 1}}, \quad j \in [0, n^*],$$

and therefore the sector wide demand is given by:

$$Q_1^* = D(P).$$

Putting these results together yields the inverse demand function for the representative variety in Home and Foreign

$$p_i = A(P)(Lq_i + L^* q_i^*)^{\theta - 1} = A(P) x_i^{\theta - 1}, \quad i \in [0, n] \tag{C1}$$

$$p_j^* = A(P)(Lq_j + L^* q_j^*)^{\theta - 1} = A(P) x_j^{\theta - 1}, \quad j \in [0, n^*], \tag{C2}$$

where $A(P) \equiv P \cdot [(L + L^*) D(P)]^{1-\theta}$ and the last equalities in (C1) and (C2) follow from the market-clearing condition for each variety: $Lq_i + L^* q_i^* = x_i$ and $Lq_j + L^* q_j^* = x_j$.

Making use of the inverse demand functions obtained in (C1) and (C2), we can write a representative home firm's profit as[2]

[2] For notational convenience, the subscript i is omitted.

$$\pi = A(P)x^{\theta} - c_1(r,w)\,\phi(x).$$

Each firm chooses its output to maximize profits by setting marginal revenue equal to marginal cost.[3]

$$\theta A(P)x^{\theta-1} = c_1(r,w)\,\phi'(x). \tag{C3}$$

Free entry generates zero profits at equilibrium, implying that a price equals the average cost:

$$A(P)x^{\theta} = c_1(r,w)\,\phi(x). \tag{C4}$$

Equations (C3) and (C4) determine simultaneously the equilibrium common output level of each variety $\bar{x}$, such that

$$\theta = \frac{\phi'(\bar{x})\bar{x}}{\phi(\bar{x})}. \tag{C5}$$

The same condition can be obtained for the foreign varieties, which leads to $x = x^* = \bar{x}$. Then, the market-clearing condition for each differentiated product is: $q + q^* = \bar{x}$.

Under quasi-linear preferences, each country consumes the same amount of good 1 under free trade and the world demand for good 1 is

$$Q_1 + Q_1^* = (L + L^*)D(P).$$

Using the definitions of Q_1 and Q_1^*, (C4), $Lq + L^*q^* = \bar{x}$ and this condition, we obtain

$$(n+n^*)^{\frac{1}{\theta}}\bar{x} = (L+L^*)D\left((n+n^*)^{\frac{\theta-1}{\theta}}\frac{\phi(\bar{x})c_1(r,w)}{\bar{x}}\right).$$

Solving this equation for $c_1(\cdot)$ yields

$$\frac{D^{-1}\left(\dfrac{(n+n^*)^{\frac{1}{\theta}}\bar{x}}{L+L^*}\right)}{(n+n^*)^{\frac{\theta-1}{\theta}}\dfrac{\phi(\bar{x})}{\bar{x}}} = c_1(r,w). \tag{C6}$$

[3] Note that P, therefore $A(P)$, is taken as given by each firm.

Finally, let us relate the above model to the argument made so far. Denoting the sum of outputs of all varieties in the home (resp. foreign) country by Y_1 (resp. Y_1^*), we have

$$Y_1 = n\bar{x} \text{ and } Y_1^* = n^*\bar{x}.$$

Hence, in the present model, $\Gamma(\cdot)$ and $\Psi(\cdot)$ take the forms of

$$\Gamma(Y_1, Y_1^*, L+L^*) = \frac{D^{-1}\left(\frac{(Y_1+Y_1^*)^{\frac{1}{\theta}}\bar{x}^{1-\frac{1}{\theta}}}{L+L^*}\right)}{(Y_1+Y_1^*)^{\frac{\theta-1}{\theta}}\bar{x}^{\frac{1-2\theta}{\theta}}\phi(\bar{x})}$$

$$\Psi(Y_1, Y_1^*) = \frac{Y_1\phi(\bar{x})}{\bar{x}}.$$

Accordingly, the sufficient conditions in Proposition 3 in the main text are established and so each country is a net exporter of the good that uses intensively its abundant factor.

C. 2: FREE ENTRY OLIGOPOLY

In this section, we continue to assume that the production function of good 1 is given by the homothetic function as in the case of monopolistic competition. On the other hand, the per-capita utility function is given by

$$U = u(Q_1) + Q_2, \qquad u'(\cdot) > 0, \qquad u''(\cdot) < 0,$$

where $Q_i, i = 1,2$ is the consumption of each good. The derived demand function of good 1 is then

$$Q_1 = D(p) \equiv u'^{-1}(p), \qquad D'(\cdot) < 0.$$

The market structure is specified by Cournot-Nash oligopoly with free entry dealt with by Lahiri and Ono [4] and Shimomura [5]. Then, the market-clearing condition under free trade becomes

$$(L+L^*)D(p) = nx + n^*x^*,$$

where n (resp. n^*) and x (resp. x^*) respectively denote the number of oligopolistic firms and the output per firm in the home (resp. foreign) country. Hence, the inverse demand function becomes

$p((nx+n^*x^*)/(L+L^*))$ and the profit maximization and zero profit conditions for the home country's representative firm are

$$p\left(\frac{nx+n^*x^*}{L+L^*}\right)\left[1-\frac{x}{(nx+n^*x^*)\eta\left(p\left(\frac{nx+n^*x^*}{L+L^*}\right)\right)}\right]=c_1(r,w)\phi'(x)$$

$$p\left(\frac{nx+n^*x^*}{L+L^*}\right)x=c_1(r,w)\phi(x),$$

where $\eta(\cdot)$ is the price elasticity of good 1 which is defined by

$$\eta\left(p\left(\frac{nx+n^*x^*}{L+L^*}\right)\right)^{-1}\equiv-\frac{d\ln\left[p\left(\frac{nx+n^*x^*}{L+L^*}\right)\right]}{d\ln\left[\frac{nx+n^*x^*}{L+L^*}\right]}.$$

Combining the above two conditions yields

$$1-\frac{x}{(nx+n^*x^*)\eta\left(p\left(\frac{nx+n^*x^*}{L+L^*}\right)\right)}=\frac{\phi'(x)x}{\phi(x)}, \tag{C7}$$

for a typical Home firm and similarly

$$1-\frac{x^*}{(nx+n^*x^*)\eta\left(p\left(\frac{nx+n^*x^*}{2}\right)\right)}=\frac{\phi'(x^*)x^*}{\phi(x^*)}, \tag{C8}$$

for a typical Foreign firm. We can now establish $x=x^*=\bar{x}$ if $(1-\phi'x/\phi)/x$ is a monotonic function of x.[4] Therefore, invoking that $Y_1=nx$ and $Y_1^*=n^*x^*$, we see from (C7) and (C8) that $\bar{x}$ is determined as a function of $Y_1+Y_1^*$, $\bar{x}(Y_1+Y_1^*)$. Hence, the functions $\Gamma(\cdot)$ and $\Psi(\cdot)$ are specified as

$$\Gamma(Y_1,Y_1^*,L+L^*)=\frac{p\left(\frac{Y_1+Y_1^*}{L+L^*}\right)\bar{x}(Y_1+Y_1^*)}{\phi(\bar{x}(Y_1+Y_1^*))}$$

[4] Shimomura [5] provides a more detailed proof.

$$\Psi(Y_1, Y_1^*) = \frac{Y_1\phi(\bar{x}(Y_1 + Y_1^*))}{\bar{x}(Y_1 + Y_1^*)},$$

from which we see that the conditions for Proposition 3 in the main text are valid.

C. 3: INTERNATIONAL EXTERNAL RETURNS TO SCALE

In the previous two subsections we assume that economies of scale are internal to each firm. We now turn to international external economies of scale as in Ethier [2] and Wong [6]. Assume then that the production function of good 1 is specified by

$$Y_1 = F(Y_1 + Y_1^*) f^1(K_1, L_1), \quad F'(\cdot) > 0, \quad F''(\cdot) < 0,$$

where K_1 and L_1 are the capital and labor inputs and $f^1(\cdot)$ is increasing, strictly quasi-concave and linearly homogeneous. The corresponding cost function is

$$c_1(r, w)\frac{Y_1}{F(Y_1 + Y_1^*)}.$$

Each firm seeks to maximize profits by taking the term $1/F(\cdot)$ as given. Thus, the profit-maximization condition in each country becomes

$$p\left(\frac{Y_1 + Y_1^*}{L + L^*}\right) - \frac{c_1(r, w)}{F(Y_1 + Y_1^*)} = 0$$

$$p\left(\frac{Y_1 + Y_1^*}{L + L^*}\right) - \frac{c_1(r^*, w^*)}{F(Y_1 + Y_1^*)} = 0,$$

which imply that functions $\Gamma(\cdot)$ and $\Psi(\cdot)$ take the following form:

$$\Gamma(Y_1, Y_1^*, L + L^*) = p\left(\frac{Y_1 + Y_1^*}{L + L^*}\right)F(Y_1 + Y_1^*)$$

$$\Psi(Y_1, Y_1^*) = \frac{Y_1}{F(Y_1 + Y_1^*)}.$$

Hence, the sufficient conditions for Proposition 3 in the main text hold in this case as well.

REFERENCES

1. A. K. Dixit, V. Norman, *Theory of International Trade*, Cambridge University Press, Cambridge, 1980.

2. W. J. Ethier, Internationally Decreasing Costs and World Trade, *J. Int. Econ.* 9, (1979), 1-24.

3. E. Helpman, P. R. Krugman, *Market Structure and Foreign Trade*, MIT Press, Cambridge, MA, 1985.

4. S. Lahiri, Y. Ono, The Role of Free Entry in an Oligopolistic Heckscher-Ohlin Model, *Int. Econ.Rev.* 36, (1995), 609-624.

5. K. Shimomura, Factor Income Function and an Oligopolistic Heckscher-Ohlin Model of International Trade, *Econ. Letters*, 61, (1998), 91-100.

6. K. Wong , *International Trade in Goods and Factor Mobility*, MIT Press, Cambridge, MA, 1995.