Patterns of Inflation Rates and Inflation Uncertainty in Quantile Regression

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Abstract

In contrast to conventional conditional mean approaches, this study uses quantile regression techniques to present some new statistical evidence on the link between inflation uncertainty and the level of inflation with cross sectional data in 161 countries for period from 1961-2002. We find new evidence that suggests positive inflation shocks have stronger impacts on inflation uncertainty across different quantiles of regression function. The findings suggest that inflation causes inflation uncertainty for sample countries, while inflation uncertainty causes inflation, too. Furthermore, empirical results indicate that increased uncertainty raises inflation, and increased inflation raises inflation uncertainty.

Keywords: Inflation, inflation uncertainty, Quantile regression, Parametric

JEL classification: C14; C21; O11; O15

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1 Introduction

Inflation is unpopular with the public. Price stability and inflation are wide consensus among macroeconomists and policymakers. It is rather surprising, therefore, that an issue about the relationship between these two variables is yet to emerge. While early studies by Friedman’s (1977) Nobel Lecture suggested an exploitable linkage between inflation uncertainty and the level of inflation, the large swings experience of the United States over 1960-88 belied this finding. Ball and Cecchetti (1990), Evans (1991) and Ball (1992) also provide evidence of such an effect. The most basic test of inflation-uncertainty consists of estimating a conditional mean approaches where the dependent variable is the average inflation rate and the explanatory variable is the standard deviation of inflation or a moving standard deviation of the variable under consideration. There are three main controversies in the empirical literature on the linkage effect of inflation on inflation uncertainty. First is that numerous recently developed models imply a positive association between the rate of inflation and inflation uncertainty. Second is whether models can exhibit a significant evidence of the higher inflation rate on the rate invoking the inflation variability incurring the higher cost. Third is the nature of the inflation-uncertainty effect that changes in the inflation regime are a major source of inflation uncertainty.

The contribution of this article is to estimate the unconditional inflation-uncertainty for broadly constituted samples using quantile regression. The estimated quantile regression process on the higher inflation rate exhibits a steeper upward at approximately 75th and 95th quantile. This finding suggests that there is evidence of unconditional inflation-uncertainty for countries in the upper tail of the conditional distribution of inflation rates but weak effect among countries in the lower tail. This result is in contrast with previous estimates obtained with conditional mean estimation methods such us generalized autoregressive conditional heteroskedasticity, henceforth GARCH. For instance, Ungar and Zilberfarb (1993) and Hwang (2001) show that a high rate of inflation does not necessarily imply a high variance of inflation.

The motivation to use quantile regression on the inflation-uncertainty is twofold. First, the
quantile regression estimator is robust to outlying observations on the dependent variable. This is an important point given that the unconditional inflation distribution is characterized by right tails, as can be seen, for instance, in Baillie et al. (1996). Second, the quantile regression estimator gives, potentially, one solution to each quantile. Therefore, we may assess how policy variables affect countries according to their position on the inflation distribution. By using quantile methods is an interesting way of capturing countries’ heterogeneity. In our case, the patterns of the inflation and inflation uncertainty imply that the coefficient on the inflation rates increases with the quantiles, suggesting that impact effect is stronger, in some sense, for countries in the upper quantiles. In other words, we assert that since the quantile estimates change so dramatically across the distribution, it is unlikely that mere data differences could be solely responsible.

Economists frequently study the relationship between inflation and inflation uncertainty because of its importance for policy analysis. Theoretically, Friedman (1977) first outlined an informal argument regarding the positive correlation between the level of inflation and inflation uncertainty, with higher inflation leading to greater uncertainty and lower output growth. On the other hand, reversing the causal effect link of the Friedman, Cukierman and Meltzer (1986) show that higher inflation uncertainty will lead to more inflation. These inferences, in turn, are easily recognized when we consider how uncertainty about inflation is likely to affect policy decision making. With these differences in mind, we shall use the cross sectional data with quantile regression model in subsequent sections to reexamine not only the variance of inflation that affects inflation rate, but also the inflation raises uncertainty into the future.

On an empirical level, beginning with the early work of Evans’s (1991) discovers the link between inflation rates and inflation uncertainty that compares to linkage inflation-uncertainty effects found in Ball and Cecchetti (1990). They set out a model that puts Evans (1991) approach with in the time-varying parameter and ARCH specification good setting. From the linkage of inflation-uncertainty that includes time variation in the structure of inflation, the paper next covers a case of Brunner’s (1993), Markov switching model with inflationary dynamics
as inflation regimes, also proposed in Telatar and Telatar (2003); this model with temporal ordering added is used in Holland (1995). The paper then turns to cross-country models that compare to Davis and Kanago (1997). Then the paper sets out models with GARCH family approaches for estimating the relationship between inflation and inflation uncertainty, proposed in Grier and Perry (1996, 1998), Fountas (2001), Giordani and Söderlind (2003), Apergis (2004), Elder (2004), Kontonikas (2004), Berument and Dincer (2005), Daal et al. (2005) facilitate this analysis. Most existing evidences regarding the validity of the Friedman hypothesis are still far from incontrovertible. Finally, Cohrad and Karanasos (2005) are put forth that implies parametric models of long memory in both the conditional mean and the conditional variance of inflation to investigate the relationship between inflation and inflation uncertainty. Then, less robust evidence is found regarding the direction of the impact of an increased nominal uncertainty on inflation.

Yet, there are also empirical findings against the Friedman hypothesis. For instance, Davis and Kanago (1998) argue that the Friedman hypothesis works better for a cross section of countries at a point in time than for the evolution of inflation over time within countries. It turns out that the results do not support the existence of the Friedman (1977). Furthermore, Hwang (2001) use time series data with various ARFIMA-GARCH type models, but do not find evidence in favor of the Friedman’s view. In addition, Berument et al.’s (2005) evidence from using a time-varying parameter model with a GARCH specification, have flatly rejected that notion, contending that the inflation uncertainty does not necessarily signify the level of inflation rates.

However, many studies on the relationship between inflation and its uncertainty used GARCH type models are mainly focusing on estimating the conditional mean function while the mean effects obtained via the conditional mean regression offer intriguing summary statistics for measuring the impacts of covariates, they fail to characterize the full distributional impact. In contrast, this article applies the quantile regression introduced by Koenker and Bassett (1978), to examine the validity of the Friedman and the Cukierman and Meltzer hy-
potheses across different quantiles of the unconditional inflation distribution. As is well known, quantile regression has become an increasing important tool to estimate quantile-specific effects that describe the impact of variables not only on the center but also on the tails of the outcome distribution.

This article is divided as follows. Section 2 provides a brief review of the quantile regression estimation method and its properties. Section 3 introduces the estimates of the regression quantiles for the unconditional inflation-uncertainty equation. Section 4 describes the data sources, summarizes the empirical results. Finally, Section 6 concludes.

2 A brief introduction to quantile regression

Much of applied econometric may be viewed as an elaboration of the linear regression model and associated estimation methods of ordinary least squares (OLS) and least absolute deviation (LAD). The well known that the former method estimates by minimizing the sum of squared errors and results in an approximation to the mean function of the conditional distribution of the regressand. The later method minimizes the sum of absolute errors and fits medians to a linear function of covariates. Useful feature of the quantile regression is distinct from them as not bind that represent central tendency of a distribution. We could go further and compute several different regression curves corresponding to the various percentage points of the distributions and thus get a more complete picture of the set. As far as the entire conditional distribution is concerned, it is not satisfactory to characterize only the mean (or median) behavior. In other words, quantile regression is robust to the presence of outliers.

Now we briefly discuss the quantile regression estimation procedure and some properties of the quantile regression estimator. The quantile regression, first proposed by Koenker and Bassett (1978), has the appealing feature that it can estimate a family of conditional quantile functions that offer us a more complete picture of covariate effects. Given any real-valued random variable $X$ may be characterized by its distribution function as

$$F(x) = Pr(X \leq x)$$ (1)
The $\tau^{th}$ quantile, for $0 < \tau < 1$, is defined as

$$Q(\tau) = \inf \{ x : F(X) \geq \tau \}, \quad (2)$$

where $X$ is a random variable with distribution function given by eq.(1). The definition of quantile simply says that an observation in the $\tau^{th}$ percentile is greater than $\tau\%$ of the observations and smaller than $(1 - \tau)\%$ of the observations. We let $(y_i, x_i), i = 1, 2, 3, \ldots, n$, be a sample from some population, where $y_i$ is a real outcome variable of interest and $x_i$ is a vector of regressors include policy variables. The general quantile regression, described in Bunchinsky (1998), takes the linear form:

$$y_i = x_i' \beta + u_i \quad (3)$$

for $i = 1, 2, \cdots, n$, where $\beta$ is a $k \times 1$ vector of coefficients, $x_i$ is the column vector that is the transpose of the $i^{th}$ row of the $X_{n \times k}$ matrix of explanatory variables, $y_i$ is the $i^{th}$ observation of the dependent variable and $u_i$ is unknown error term. The $\tau^{th}$ conditional quantile of $y$ given $x$ can be rewritten as

$$Quant_{\tau}(y_i | x_i) = x_i' \beta_{\tau} \quad (4)$$

Its estimate is given by $x_i' \hat{\beta}_{\tau}$. As $\tau$ increases continuously, the conditional distribution of $y$ given $x$ is traced out. Then it is assumed that the conditional quantile of $y_i$, conditional on $x_i$, satisfies $Quant_{\tau}(y_i | x_i) = x_i' \beta_{\tau}$, for several different values of $\tau$, $\tau \in (0, 1)$, so that $Quant_{\tau}(y_i | x_i) = 0$. It is in this way that quantile regression allows for parameter heterogeneity across different types of regressors. Thus, the quantile regression estimator can be found as the solution to the following minimization problem:

$$\min_{\beta \in \mathbb{R}^k} \left[ \sum_{i \in \{i: y_i \geq x_i' \beta\}} \tau |y_i - x_i' \beta| + \sum_{i \in \{i: y_i < x_i' \beta\}} (1 - \tau) |y_i - x_i' \beta| \right] \quad (5)$$

The quantile function is a weighed sum of the absolute value of the residuals. Where the weights are symmetric for the median regression case in $\tau = 1/2$, the minimization problem above reduces to $\min_{\beta \in \mathbb{R}^k} \sum_{i=1}^n \{ |y_i - x_i' \beta| \}$, and asymmetric otherwise. By varying the value
of parameter $\tau$ from 0 to 1, we can generate the entire conditional distribution of $y$ given $x$. In practice, we consider the partial derivative of the conditional quantile of $y$ with respect to one of the regressors, coefficients of policy variable, can be interpreted as the marginal change in the dependent variable due to a marginal change in the policy variable. Note that since we have on $\beta$ for each $\tau$, the quantile regression approach allows us to identify the effects of the covariates on the regressand at different points on the distribution. In particular, as shown in Koenker and Hallock (2001), an attractive property of the quantile regression estimator is its robustness to the presence of outlying observations on the dependent variable. Interested readers are referred to Koenker (2004, 2005) for more details.

### 3 The unconditional inflation-uncertainty equation

Assuming that the ‘$\tau$’th quantile of the conditional distribution of dependent variable is linear in explanatory variable, following Koenker and Bassett (1978), the unconditional quantile regression model can be applied to the following two equations to examine the relationship between inflation and inflation uncertainty.

\[
\Pi_i = \alpha_\tau + \beta_\tau IU_i + u_\tau \tag{6}
\]

\[
IU_i = \gamma_\tau + \delta_\tau \Pi_i + v_\tau \tag{7}
\]

The terms $\Pi$ and $IU$ denote inflation and inflation uncertainty of equation (6) and (7), respectively. $\alpha_\tau$, $\beta_\tau$, $\gamma_\tau$ and $\delta_\tau$ are the unknown parameters to be estimated for different values of $\tau$, and $u_\tau$ and $v_\tau$ are the error terms. By varying the value of $\tau$ from 0 to 1, we can trace the entire distribution of dependent variable conditional on independent variable. Just as we can define the least squares estimators for obtaining the conditional mean function as the solution to the problem of minimizing a sum of squared residuals, the quantile estimators for $\beta_\tau$ and $\delta_\tau$ can be obtained by minimizing the following asymmetric linear penalty function as equation (5). For reasons discussed above, the quantile regression has the appealing feature that it can estimate a family of unconditional quantile functions that offer us a more complete picture of covariate effects.
4 Data descriptive and empirical results

4.1 Data sources

The data set used in this paper are collected primarily from “Global Development Finance & World Development Indicators, 2005” contains inflation rate and inflation uncertainty with cross sectional data in 161 countries for period from 1961 to 2002. Data for a number of developing countries, however, have a shorter span. Because of the uneven coverage, the empirical analysis is conducted using unbalanced panels. The inflation series is obtained by taking the logarithmic of the growth rate of the CPI index. The popular method for measuring inflation uncertainty is the standard deviation of inflation rate, as employed by Davis and Kanogo (1996). Table 1 shows the summary statistics of those variables.

4.2 The results of parametric quantile models

Table 2 provides the estimation results of equation (6) from the parametric mean and quantile regressions. In the simplest form, the conditional mean results in column (1) shows that the estimates of ‘IU’ is 4.0864, as significant at 1% level, and have the expected sign, thus, providing a preliminary support of the Cukierman-Meltzer’s hypothesis.

In contrast, five quantile estimates for the most basic specification are also obtained for \( \tau = 0.05, 0.25, 0.5, 0.75 \) and \( 0.95 \) and shown in columns (2) to (6). The all coefficients are significant at 1% level. The quantile process for inflation uncertainty exhibits a linear increasing trend. For countries in the bottom 5% of the conditional inflation distribution the estimated coefficient on inflation uncertainty is 0.8911, it increases to 4.0625 for countries in the conditional median, to increase again to 5.2733 in the top 5% of the distribution. This result suggests that the effect of inflation uncertainty has a stronger impact on countries in the upper tail of the inflation distribution. These findings are suggestive of the potential information gains associated with the estimation of the entire conditional inflation distribution, as opposed the conditional mean only. Moreover, a comparison of the estimates of the conditional median function with OLS estimates of the conditional mean function reveals that the traditional estimation techniques
are affected by the tails of the data distribution.

Table 3 shows the results of Wald test for equality of slope coefficients across the quantiles for the independent variables. These test results show that the slope coefficients indeed vary across the quantiles. The slopes are significantly different from each other between the 25th and 95th percentile for 5th quantile and between the 50th and 95th quantiles for 25th quantile. These test results confirm the argument that the relationship between the inflation and inflation uncertainty along with the inflation uncertainty affect the level of inflation rate differently across the quantiles. Figure 1 superimposes five estimated quantile regression curves on the scatter plot. The median regression line appears as a green line, the least-squares line as a yellow line and the 95th quantile line as a purple line. Once again, we find that the upper tails of the distribution, the larger effects of inflation uncertainty on inflation.

On the other hand, for the estimation results of equation (7) exhibit a similar pattern in Table 4. Inflation causes inflation uncertainty and would precede it at least slightly. Note that the OLS estimate of 0.1032, the estimated coefficient on the inflation is positive and significant at 1% level. The quantile regression process for the variable ‘II’ has an interesting pattern. It is positive for all the quantiles, as expected, however, it increases from $\tau = 0.05$ to $\tau = 0.50$, and then it increases again.

Table 5 presents the ANOVA test for the explanatory variable in equation (7) reject the null hypothesis that five percentiles are jointly not significant. It is also suggested that the inflation rates affect the inflation uncertainty differently across the inflation uncertainty distribution. These test results show that the slope of 95th quantile have significantly different from each other between the 5th and 75th quantiles. These evidences suggest positive inflationary shocks have stronger impacts on inflation uncertainty for upper tails of distribution.

Figure 2 shows the similar pattern with the parametric quantile regression. The 95th quantile still has steeper pattern with other quantiles. In this case, our results also support the Friedman hypothesis that inflation increases the inflation uncertainty across the different quantiles of regression function.
5 Concluding remarks

This paper presents a general linkage effect between inflation and inflation uncertainty using quantile regression methods. The estimates we gathered with the new set of specifications suggest that inflation causes inflation uncertainty each other. One particularly interesting result we find is the increasing linearity pattern of the regression quantile process on the inflation and inflation uncertainty coefficient. Each slope coefficient can be interpreted as a different impact of the inflation uncertainty to a change in an inflation variable, according to a country’s position on the inflation-uncertainty distribution. This is an interesting way of capturing parameter heterogeneity. This finding shows that the effect of inflation uncertainty (or inflation) on the inflation (inflation uncertainty) is stronger for countries in the upper quantiles than for countries in the lower quantiles.

Finally, our results can be subject to further investigation, and extended in several ways. Application of recent inferential methods in quantile regression such as semiparametrically and nonparametrically to avoid possible model mis-specifications is a natural extension of our framework. Moreover, the newest version of the IMF data set contains a number of important macroeconomic variables that we didn’t discuss here. Investigation on how these policy variables relate to inflation and inflation uncertainty as a robustness check also can be an interesting extension.
Reference


Table 1: Basic statistics

<table>
<thead>
<tr>
<th>variables</th>
<th>mean</th>
<th>median</th>
<th>std</th>
<th>min</th>
<th>max</th>
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</thead>
<tbody>
<tr>
<td>inflation rates(Π)</td>
<td>1.1353</td>
<td>0.9388</td>
<td>0.5858</td>
<td>0.4381</td>
<td>2.9640</td>
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<tr>
<td>inflation uncertainty (IU)</td>
<td>0.4648</td>
<td>0.4284</td>
<td>0.2100</td>
<td>0.1031</td>
<td>1.3306</td>
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</table>

† The logarithm of the raw (original) data.

Table 2: Regression results of coefficients across quantiles

<table>
<thead>
<tr>
<th>Variables</th>
<th>OLS</th>
<th>5th Quant</th>
<th>25th Quant</th>
<th>50th Quant</th>
<th>75th Quant</th>
<th>95th Quant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Πi = α + β IU + u</td>
<td></td>
<td>0.7065 ***</td>
<td>0.8605 ***</td>
<td>0.9538 ***</td>
<td>0.6189 ***</td>
<td>0.8038 **</td>
</tr>
<tr>
<td>Constant</td>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.011)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>IU</td>
<td>4.0864 ***</td>
<td>0.8911 ***</td>
<td>1.9126 ***</td>
<td>4.0625 ***</td>
<td>4.9815 ***</td>
<td>5.2733 ***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.069)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>

† ***, **, and * denote significant at 1%, 5% and 10% level, respectively.
Table 3: Wald test for equality of coefficients across quantiles

<table>
<thead>
<tr>
<th>Quantiles</th>
<th>5th Quant</th>
<th>25th Quant</th>
<th>50th Quant</th>
<th>75th Quant</th>
</tr>
</thead>
<tbody>
<tr>
<td>5th Quant</td>
<td>3.6502 *</td>
<td>(0.057)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25th Quant</td>
<td>23.162 ***</td>
<td>19.086 ***</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>50th Quant</td>
<td>25.89 ***</td>
<td>18.857 ***</td>
<td>2.2282</td>
<td>(0.000)</td>
</tr>
<tr>
<td>75th Quant</td>
<td>25.89 ***</td>
<td>18.857 ***</td>
<td>2.2282</td>
<td>(0.000)</td>
</tr>
<tr>
<td>95th Quant</td>
<td>8.1252 ***</td>
<td>5.0433 **</td>
<td>0.677</td>
<td>0.0457</td>
</tr>
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</table>

† The numbers present F-statistic of equality of the slope coefficients at \( \Pi_i = \alpha_r + \beta_r IU_i + u_r \) across quantiles with \((1, N - k)\) degrees of freedom.
‡ The associated p-values are reported in parentheses.
* **, and * denote significant at 1%, 5% and 10% level, respectively.

Table 4: Regression results of coefficients across quantiles

<table>
<thead>
<tr>
<th>Variables</th>
<th>OLS</th>
<th>5th Quant</th>
<th>25th Quant</th>
<th>50th Quant</th>
<th>75th Quant</th>
<th>95th Quant</th>
</tr>
</thead>
<tbody>
<tr>
<td>( IU_i = \gamma_r + \delta_r \Pi_i + v_r )</td>
<td>0.1980 ***</td>
<td>0.0790</td>
<td>0.1531 ***</td>
<td>0.2003 ***</td>
<td>0.2817 ***</td>
<td>0.3154 ***</td>
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<tr>
<td>Constant</td>
<td>(0.000)</td>
<td>(0.227)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>( \Pi )</td>
<td>0.1032 ***</td>
<td>0.0552 ***</td>
<td>0.0825 ***</td>
<td>0.0997 ***</td>
<td>0.1070 ***</td>
<td>0.1685 ***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.004)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>

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Table 5: Wald test for equality of coefficients across quantiles

<table>
<thead>
<tr>
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<th>5th Quant</th>
<th>25th Quant</th>
<th>50th Quant</th>
<th>75th Quant</th>
</tr>
</thead>
<tbody>
<tr>
<td>5th Quant</td>
<td>1.8858</td>
<td>4.6968 **</td>
<td>5.7309 **</td>
<td>12.653 ***</td>
</tr>
<tr>
<td></td>
<td>(0.171)</td>
<td>(0.031)</td>
<td>(0.017)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>25th Quant</td>
<td></td>
<td>1.3637</td>
<td>1.9958</td>
<td>6.5386 **</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.216)</td>
<td>(0.159)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>50th Quant</td>
<td></td>
<td></td>
<td>1.9958</td>
<td>6.2611 ***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.548)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>75th Quant</td>
<td></td>
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<td></td>
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<td>95th Quant</td>
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‡ The associated p-values are reported in parentheses.
* ** ** *, and * denote significant at 1%, 5% and 10% level, respectively.
Figure 1: Effect of an increase in inflation uncertainty under quantiles

Figure 2: Effect of an increase in inflation under quantiles