A Simple Model of Product Complexity

Kazunori Tanigaki*
Faculty of Economics
Ritsumeikan University

Abstract

This paper introduces the concept of product complex which is defined as necessary knowledge to produce goods, difficulty of learning task and achieving work. Using a simple labor model with product complexity and complex cost, I consider theoretically what is the source of economic development, how entry and exit to the market are determined, how comparative advantage is decided. I reveal that rich countries don't always export complex goods, and that firm size and wage are positively correlated. The paper provides what is the role of imitation and capability of learning tasks in our society. It is also shown that one direction of economic development is that complex work is simplified, that is, society is less complex.

Key word: Product complexity; Product complex cost; Capability of learning tasks; International trade.
JEL Codes: international trade theory, economic theory, economic development, labor economics

Address for correspondence:
Faculty of Economics, Ritsumeikan University
1-1-1 Nojihigashi Kusatsu, Shiga, 525-8577 Japan
E-mail :tanigaki@ec.ritsumei.ac.jp
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1. Introduction

This paper explicitly takes product complexity into consideration and reveals how product complexity works in our economic societies. Recently it is sometimes said that our society has been complex. It has been more difficult for us to understand the other tasks and sometimes ours. In some firms, learning by doing may be more necessary and the speed of the redeployment of labor within them is slow. These reasons would be due to the fact that tasks have been more complex and initial cost for learning new tasks has increased. Because of this, it is said that the demand for skilled labor with human capital has increased. It is noted that in this paper, the word complexity, is not that in so called complex literatures. Though society is made of complex process, the process itself is not directly relevant to complex task. Thus I don't consider complex systems and dynamics and process as in *Journal of Economics & Complex* or a recent paper, Durlauf(2005).

My paper is related to many literatures which include technology transfer, comparative advantage, O-ring theory, patent and the quality of labor. Berkowitz et al (2003) used product complexity as a title of his paper. They categorized goods to complex and simple. But his concept of complexity is different from my paper while it is similar, and his paper is relevant to law institutions. Product complexity is also used in Ono and Stango (2005). They say that product diversity make the complexity increase. Both papers are not theoretical but empirical. In literatures of international technological transfer, one of factors which influence the technology transfer is "the capacity to adopt technology". Also in my paper, this capacity is more necessary for complex and difficulty tasks. As an example of models which fit this description, there is the O-ring model (Kremer (1993)). Task mistakes are considered, and complexity cost is also partially that of mistake. We are not able to work perfectly, and the degree of task completeness depends on personality, nationality, experience and tradition. In the paper of Kremer, there is the expression that "one measure of product complexity is the number of different inputs“[1]. More different inputs would make production process complicated and raise the costs. Although I don’t consider learning by doing (for instance, Ohashi(2005)), its process would be that of the experience. Complex task needs more learning by doing, but complex cost is less for skilled labor with human capital.

While this paper reveals another aspect of work, that is, mistake, incompleteness and misunderstanding for tasks, this paper would contribute to economics of knowledge and economic development. Accumulation of knowledge in the whole society causes our tasks complex. To introduce new technology and more production process, employees are equipped to adjust to them. Employees with high human capital have the ability and their cost accompanied with the adjustment is small[2]. It is also shown that economic development is not only for tasks to be complex but also to be easier. Even average people have been able to achieve complex tasks which only skilled labor could do at one time[3]. In the following, I treat firm-level decisions and also consider production of aggregate level.

The next section 2 sets up the model. The notions of product complex, complex cost and the optimal complexity are introduced. It is explained what entry and exit from the market are. Some comparative statistic analyses are considered in section 3. We find that improvement in productivity and cost reductions don’t always increase in product complexity. International trade theory with product complex is analysed in section 4. I examine whether rich countries export complicated goods, and there are gains of international trade. The other applications are shown in section 5. First in this section, I analyse how new machines and patent make production process improve. Second, two types of labor, skilled and unskilled, are introduced. The paper suggests that unskilled labors engage in relatively complex work in developing countries compared with developed countries.

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[1] Kremer(1993, p.563)

[2] In the later, this cost is defined as complex cost.

[3] For example, software makes us release from complex programming tasks. Sewing machine is also similar.

Third, it is examined how firm size and wage are positively correlated. Fourth, complexity for consumer is analysed.

2. Setup of the model

2-1 Production with complexity

Let x denote a degree of complexity. A larger value of x corresponds to more complexity which means more process of production, more knowledge to produce and difficulty to learn tasks. When we use a new machine instead of human, the task would be easier and less complex. But the total knowledge which includes producing new machines and getting a patent would be increased. Thus, the variable, x, is not total knowledge.

Let y denote production. I assume that y depends on complexity x and labor L. A production function with complexity is given by:

$$y = \gamma \phi(x) Y(L) \text{ with } \phi' > 0, \quad \phi'' < 0, \; Y' > 0 \text{ and } Y'' < 0, \qquad (1)$$

where γ is productivity parameter, $\phi(x)$ is a degree of complexity which is related to production. An accumulation of knowledge would improve production process. However complexity is not always increased by the accumulation as this paper examine in the following. More careful and close works include complexity and it can increase production. We may think that high quality goods are produced by more complex production process. In this paper I don't analyse quality of goods and so I don't consider monopolistic competition among similar goods.

Figure 1 shows production function for $\varphi' > 0$ under given L[4]. In this figure, the production

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[4] Of course, in some cases, too large x would harm the production so that $\varphi(x)' < 0$. In this paper, I don't consider it.

function is shown for $\varphi'' > 0$ and small x. When complexity is low, the level of production may remain low and production may be increasing return to complexity. For sufficient production, enough complexity would be necessary. But, in the equilibrium, $\varphi''< 0$ is assumed in the following. Since $\gamma_1 > \gamma_0$, we find that $y_0 = \gamma_1\varphi(x_1)Y(L) = \gamma_0\varphi(x_0)Y(L)$, and $x_1 < x_0$. For a production y_0, we can produce the same y_0 with lower x_1 for high γ_1.

2-2 Cost function with complexity

We introduce a cost function which is corresponding to complexity. Complex costs represent those of learning, operation machines, managing complex organizations and defective products. If there are many defective products in production process, parts and components are wasted besides output reduction. Moreover recall and repair costs are included in this complex costs when production is inadequate. More complex products would be accompanied with theses costs. We may interpret that these costs are related to the fact that some labors can't understand their work and perform their work perfectly for complex task. It is represented as:

$$C = \beta\psi(x)c(L), \qquad (2)$$

where β is a cost parameter. It is assumed that $\psi' > 0$, $\psi'' > 0$, $c' > 0$ and $c'' > 0$. For too complex work, labor can't follow it or learning and operating costs are huge. Then assumptions of $\psi'> 0$, $\psi''> 0$ would be plausible. In Figure 2, the cost function, $\beta\psi(x)c(L)$, is depicted for given L. $\beta_1\psi(x)c(L)$ has more efficiency than $\beta_0\psi(x)c(L)$. When β is improved, the cost is same under higher x_1 or the cost is lowered under the same complexity x_0. This change would be caused by easier production process and labor with more human capital, and new knowledge. The reduction of β means that it is easier to learn tasks and work. For example, we have been released from making programs on computer by the development of the software package. By automatic machines, even average skilled persons have been able to realize complex work[5].

Then one view for a technological innovation is the reduction of β, that is, for complexity cost to decrease. The other view in our model is to an increase in γ. The general improvements of production which has been analyzed would be this type. For the same inputs to production, we can produce more by technological improvement. It is said that our society has been more complexity. But on the other hand, this society makes work easy and less complex so that even average persons can achieve complex task which professional only could do it in old times. However, it should be noted that total knowledge for production would be increased. That is, though someone can work easily, someone should work innovatively and knowledge is accumulated in professionals, and society on the whole would be more complex.

Total cost is defined as factor cost plus the complex cost. Then it is represented as:

$$c = wL + \beta\psi(x)c(L)$$
$$= C_f + C, \qquad (3)$$

where c is total cost, w is wage, C_f is factor cost which is given by wL.

2-3 Maximization Problems

For given price p and profit π, is given by:

$$\pi(x, L) = py - c$$
$$= p\gamma\varphi(x)Y(L) - wL - \beta\psi(x)c(L). \qquad (4)$$

Then profit maximization is shown by:

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5 To introduce new machines or new way of production and management, some costs are necessary. This is analyzed in later section 5-1.

$$\pi_x = \partial\pi/\partial x = \gamma p\varphi' Y - \beta\psi' c = 0, \quad (5)$$

$$\pi_L = \partial\pi/\partial L = \gamma p\varphi Y' - w - \beta\psi c' = 0. \quad (6)$$

Optimal complexity is depicted in Figure 3. When complexity function is C_1, optimal complexity x^* exists for given goods price and wage. When complexity function is C_2 and the profit is negative for any L, the firm is driven out from the market since its profit is also negative for any x. This is one of our characteristics in our model. If there is no complex cost, the firm can exist for higher wage by reducing labor employment since marginal product of labor is increased through the reduction of labor.

From $Y'' > 0$, $c'' > 0$, $\varphi'' < 0$ and $\psi'' > 0$, this profit function has the following properties:

$$\pi_{xx} = \gamma p\varphi'' Y - \beta\psi'' c < 0,\ \pi_{LL} = \gamma p\varphi Y'' - \beta\psi c'' < 0,\ \pi_{xL} = \pi_{Lx} = \gamma p\varphi' Y' - \beta\psi' c'.$$

For given L, we can consider production improvements using $\gamma p\varphi(x)Y(L)$ and $\beta\psi(x)c(L)$. The combinations of $\varphi(x)Y(L)$ and $\psi(x)c(L)$ are divided to 4 pairs. The case of figure 4-Ⅰ is a combination of low productivity and high complex cost and 4-Ⅳ is the reverse. In figure 4-Ⅰ and 4-Ⅱ, more complexity is not necessary for production and it even reduces production. In figure 4-Ⅰ, the complexity is a minimum in four cases since complex cost is large and complexity doesn't increase production. In figure 4-Ⅱ, though the complex cost is low, the low level of complexity would be adopted. In figure 4-Ⅲ, though complexity increases production, the degree of the complexity is small since marginal complex cost is large. In figure 4-Ⅳ, it is the reverse of 4-I and the complexity is a maximum in four cases. Economic development may be shown from Fig.4- I to 4-Ⅳ through 4-Ⅱ or 4-Ⅲ[6]. From Fig. 4-Ⅲ to 4-Ⅳ, the tasks are easier and From Fig. 4-Ⅱ to 4-Ⅳ,

[6] Though wage is constant in the figures, we can easily depict figures with higher wage

productivity is improved. The example of franchise stores in which unskilled labor are employed may be Figure 4-I, since more complexity is not necessary, and staffs are part-time and low-skilled without managers. Fig. 4-Ⅲ is the case in which unskilled labor produce using new expensive machine with complex process while there are many defects.

3. Comparative statistics

3-1 A firm and exogenous changes in *γ*, *β*, *w* and *p*

In this subsection 3-1, I analyze how the optimal *x* and *L* of a firm or a sector are changed by improvements of production and complex cost, wage rate and price. For exogenous changes of *γ*, *β*, *w* and *p,* we have from (5) and (6):

$$\begin{pmatrix} \pi_{xx} & \pi_{xL} \\ \pi_{Lx} & \pi_{LL} \end{pmatrix} \begin{pmatrix} dx \\ dL \end{pmatrix} = -\begin{pmatrix} \pi_{x\gamma} \\ \pi_{L\gamma} \end{pmatrix} d\gamma + \begin{pmatrix} \pi_{x\beta} \\ \pi_{L\beta} \end{pmatrix} d\beta + \begin{pmatrix} \pi_{xw} \\ \pi_{Lw} \end{pmatrix} dw + \begin{pmatrix} \pi_{xw} \\ \pi_{Lw} \end{pmatrix} dp. \tag{7}$$

$$\begin{pmatrix} dx \\ dL \end{pmatrix} = (-/\Delta) \begin{pmatrix} \pi_{LL} & -\pi_{xL} \\ -\pi_{Lx} & \pi_{xx} \end{pmatrix} \left(\begin{pmatrix} \pi_{x\gamma} \\ \pi_{L\gamma} \end{pmatrix} \mathrm{d}\gamma + \begin{pmatrix} \pi_{x\beta} \\ \pi_{L\beta} \end{pmatrix} d\beta + \begin{pmatrix} \pi_{xw} \\ \pi_{Lw} \end{pmatrix} dw \right). \tag{8}$$

where $\Delta = \pi_{xx}\pi_{LL} - \pi_{Lx}\pi_{xL}$ and it is positive from the stability condition.

From $\pi_{x\gamma} = p\varphi'Y > 0$ and $\pi_{L\gamma} = p\varphi Y' > 0$, $\pi_{x\beta} = p\psi'c > 0$, $\pi_{L\beta} = p\psi c' > 0$, $\pi_{xw} = 0$, $\pi_{Lw} = -1$, $\pi_{xp} = \gamma\varphi'Y > 0$ and $\pi_{Lp} = \gamma\varphi Y' > 0$, we find[7] :

$dL/dw < 0$,

and efficient production and low cost.

[7] From equation (8), we obtain:

$$(-\Delta)\begin{pmatrix} dx \\ dL \end{pmatrix} = \begin{pmatrix} \pi_{x\gamma}\pi_{LL} - \pi_{L\gamma}\pi_{xL} \\ -\pi_{x\gamma}\pi_{Lx} + \pi_{L\gamma}\pi_{xx} \end{pmatrix} d\gamma + \begin{pmatrix} \pi_{x\beta}\pi_{LL} - \pi_{L\beta}\pi_{xL} \\ -\pi_{x\beta}\pi_{Lx} + \pi_{L\beta}\pi_{xx} \end{pmatrix} \mathrm{d}\beta + \begin{pmatrix} \pi_{xw}\pi_{LL} - \pi_{Lw}\pi_{xL} \\ -\pi_{xw}\pi_{Lx} + \pi_{Lw}\pi_{xx} \end{pmatrix} \mathrm{dw}.$$

If $\pi_{Lx} = \pi_{xL} > 0$,

$dx/d\gamma > 0$ and $dL/d\gamma > 0$, $dx/d\beta < 0$ and $dL/d\beta < 0$,

$dx/dw < 0$, $dx/dp > 0$ and $dL/dp > 0$. (9)

The rise of wage makes the employment reduce. Usually we may think that the increase in productivity and cost reduction cause employment rise. But in our model it is found that there can be the reverse case. This could be occurred if $\pi_{Lx} = \pi_{xL} < 0$, that is, $\gamma p\varphi' Y' - \beta\psi' c' < 0$. Theses depend on the difference between $\gamma p\varphi' Y'$ and $\beta\psi' c'$. The negative sign of ($\gamma p\varphi' Y' - \beta\psi' c'$) means that an increase in the marginal product of complexity due to labor input is smaller than the increase in the marginal cost of complexity due to labor input.

Can this situation, $\pi_{Lx} = \pi_{xL} < 0$, continue for $d\gamma > 0$ and $d\beta < 0$? The answer would be no. From $\varphi'' < 0$, $Y'' < 0$, $\psi'' > 0$ and c" > 0, $\gamma p\varphi' Y'$ rises and $\beta\psi' c'$ reduces since $dx < 0$ and $dL < 0$ for $d\gamma > 0$ and $d\beta < 0$ when $\pi_{Lx} = \pi_{xL} < 0$. Then the negative sing of ($\gamma p\varphi' Y' - \beta\psi' c'$) would be the reverse eventually for $d\gamma > 0$ and $d\beta < 0$. So in the following, I analyze assuming that $dx/d\gamma > 0$ and $dL/dc < 0$ and $dL/d\beta < 0$ or $\pi_{Lx} = \pi_{xL} > 0$.

We may think $dx/dw < 0$ is strange because we expect that high wage countries produce complex goods. But in this section, dw is exogenous. In later subsection 3-3, endogenous dw, that is, the wage change due to parameters, γ and β, is analyzed. In this subsection 3-1, $dx/dw < 0$ refers to an increase in only wage change under constant γ and β. We find sing(dx/dw)= sing(π_{Lx}) since $\pi_{xx}dx + \pi_{xL}dL = 0$ from eq.(7) and $dL/dw<0$. Complexity is decreased because labor is reduced and marginal profit of x is reduced by the change of labor. Then we can say that complexity of a firm can be reduced if wage is increased under constant parameters of γ and β.

Then Proposition 1 is obtained.

PROPOSITION 1

In a firm, the improvement of productivity and cost raises the complexity and increases labor input of the firm. This result is obtained if $\gamma p\varphi(x)'Y(L)' - \beta\psi(x)'c(L)' > 0$. Complexity can be reduced by an exogenous wage rise and price reduction.

3-2 The changes of γ and β in all sectors

In this subsection 3-2, I assume that all firms or all sectors improve their parameters, γ and β for constant p. Then wage rate is endogenous variable. The whole labor supply, L^S, is fixed. Labor market equilibrium condition is shown as:

$L^D = L^D(\gamma, \beta, w) = L^S$

From $dL/dx < 0$ in equation (9), we have

$L^D{}_w < 0$ and $L^D{}_\gamma > 0$, $L^D{}_\beta < 0$ for the assumption of $\pi_{Lx} = \pi_{xL} > 0$.

Then we obtain:

$$dw/d\gamma = L^D{}_\gamma/(-L^D{}_w) > 0 \text{ and } dw/d\beta = L^D{}_\beta/(-L^D{}_w) < 0. \tag{10}$$

We can say that countries with high labor productivity and low complex cost have high wage. However is the complexity of rich country high? If also $\pi_{Lx} = \pi_{xL} > 0$, it is obtained that $dx/d\gamma > 0$ and $dx/d\beta > 0$. But $dx/dw < 0$ for $\pi_{Lx} = \pi_{xL} > 0$.

From equation (9) and (10), we can write x-function as:

$$x = x(\gamma, \beta, w(\gamma, \beta)).$$

Now we omit β in the following since the results are symmetric to γ.

We have:

$dx/d\gamma = x_\gamma + x_w dw/d\gamma$. (11)

In equation (11), we call x_γ direct effect ($x_\gamma > 0$) and $x_w dw/d\gamma$ indirect effect ($x_w dw/d\gamma < 0$) through wage rise.

Although, the sign is ambiguous, we obtain a sufficient condition for $dx/d\gamma>0$:

$p\varphi Y' > L^D_\gamma/(-L^D_w)$ $(= dw/d\gamma)$ for $dx/d\gamma > 0$[8].

Then we have Proposition 2.

PROPOSITION 2

Complexity is increased by the improvement of productivity and cost in all sectors if a direct positive effect is larger than a negative indirect effect by wage rise.

4. International trade theory

4-0 Real marginal product of labor

Before applying to international theory, I introduce a concept of real marginal product of labor, *RML*.

Equation (6) is rewritten as:

$w = p\gamma\varphi Y' - \beta\psi c' = p(\gamma\varphi Y' - \beta\psi c'/p) = pRML$, (12)

where *RML* is real marginal product of labor, $RML = (\gamma\varphi Y' - \beta\psi c'/p)$. $\beta\psi c'/p$ is real complex cost in

[8] From the definition of equation (9), $x_w = (-1/\Delta)\pi_{xL}$, $x_\gamma = (-1/\Delta)(\pi_{x\gamma}\pi_{LL} - \pi_{L\gamma}\pi_{xL})$. Then $(-\Delta)dx/d\gamma = x_\gamma + x_w dw/d\gamma = p\varphi' Y\pi_{LL} + \pi_{xL}(dw/d\gamma - p\varphi Y')$.

terms of this goods. *RML* is marginal products of labor considering complex costs. In other words, mistake or failure costs of labor are included in *RML*. From equations (6) and (12), and the above comparative statistics, 3-1, we find:

$d(RML)/d\gamma = d(RML)/d\beta = 0$ for constant w and p, (13-1)

$d(RML)/dp < 0$ for constant w, γ and β, (13-2)

$d(RML)/dw > 0$ for constant p, γ and β. (13-3)

RML and equations (13) are used for later analyses.

4-1 Do rich countries export in complicated goods?

To consider this problem, we assume a simple international trade model in which home country is small, and completely same as the rest of the world, and there are only two goods. Let * denote the rest of the world. Moreover it is assumed that preferences are also same, homothetic and international trade is nothing initially, and that there is no profit for simplicity[9]. Using equation (12), we have for i-th goods:

$$p_i RML_i = w = w^* = p_i^* RML_i^*, \tag{14-1}$$

where p_i*and w^* are exogenous, and $i = 1, 2$.

Labor market equilibrium condition is rewritten as:

$$L^D = L^D{}_1(\gamma_1, \beta_1, w) + L^D{}_2(\gamma_2, \beta_2, w) = L^S.$$

Now, suppose that the sector 1 in domestic country improves their parameters, γ_1, and suppose that

[9] From (4), profit π is not always zero for given p. When the profit is positive, another model would be necessary.

the other parameters and the rest of the world are not changed. Then from the analysis of subsection 3-2 with equation (14-1), constant p and the above labor equilibrium condition, we have:

$dw/d\gamma_1 > 0.$

Then (14-1) is rewritten to:

$$p_i{*}RML_i = w > w{*} = p_i{*}RML_i{*} \qquad (14\text{-}2)$$

Let L_i, x_i and y_i denote the employment, complexity and production of sector i (i = 1, 2) in home country respectively. From equation (9), we have $\partial L_2/\partial w < 0$ and $\partial x_2/\partial w < 0$. We introduce a net production which is defined as:

$$z_i = z_i(L_i, x_i, \gamma_i) = \gamma_i\varphi_i Y_i - \beta_i\psi_i c_i/p_i. \qquad (i = 1, 2) \qquad (15)$$

Since $\partial z_i/\partial x_i = \pi_x/p_i = 0$,

$dz_i = (\partial z_i/\partial L_i)dL_i + (\partial z_i/\partial x_i)dx_i + (\partial z_i/\partial \gamma_i)d\gamma_i = (\partial z_i/\partial L_i)dL_i + (\partial z_i/\partial \gamma_i)d\gamma_i$

From $\pi_{L\,i} = 0,$

$\partial z_i/\partial L_i = w/p_i > 0$[10].

Since $dL_2/d\gamma_1 = (\partial L_2/\partial w)(dw/d\gamma_1) < 0$ and $L_1 + L_2 = L^S$, we have :

$dL_1/d\gamma_1 = -\, dL_2/d\gamma_1 > 0.$ Thus, we obtain:

$dz_1/d\gamma_1 = (\partial z_1/\partial L_1)(dL_1/d\gamma_1) + \partial z_1/\partial \gamma_1 > 0$ and $dz_2/d\gamma_1 = (\partial z_2/\partial L_2)(dL_2/d\gamma_1) < 0.$

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[10] $\partial z_i/\partial L_i = (p_i\gamma_i\varphi_i Y_i' - w - \beta_i\psi_i c_i')(1/p_i) + w/p_i = \pi_{L\,i}(1/p_i) + w/p_i.$

Thus the net production of sector 1 is increased and that of sector 2 is decreased.

From homothetic preferences, we find home country with high wage exports goods 1 and imports goods 2.

Then how about $dx_1/d\gamma_1$? From equation (11), it is shown as:

$$dx_1/d\gamma_1 = x_{1\gamma} + x_{1w} dw/d\gamma_1.$$

where $x_{1\gamma} = \partial x_1/\partial \gamma_1 > 0$, $x_{1w} = \partial x_1/\partial w < 0$. Thus the sign of $dx_1/d\gamma_1$ is ambiguous. We call the first term direct effect and the second term wage effect. Then, we can say that if wage effect is overwhelmed by direct effect, complexity is increased. In other words, rich countries export complex goods compared with foreign if $dx_1/d\gamma_1 > 0$. Moreover, home country imports less complexity goods since the degree of complexity in the other sector is reduced because of wage rise. We may think that developed countries have comparative advantage in relative complicated goods to the other domestic sectors and developing countries. But this paper shows that this conjecture doesn't always hold[11].

Then we have Proposition 3.

PROPOSITION 3

A high wage country does not always export more complex goods compared with foreign country. Developed countries tend to import less complexity goods.

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[11] We may think that the result is obtained from $p_i = w/RML_i < w^*/RML_i^* = p_i^*$, or $1 < w/w^* < RML_i/RML_i^*$. But I can't obtain clear conditions for $d(RML)_i/d\gamma_i = \partial(RML)_i/\partial\gamma_i + (\partial(RML)_i/\partial L_i)(dL_i/d\gamma_i) + (\partial(RML)_i/\partial x_i)(dx_i/d\gamma_i) > 0$ and $d(RML)_i/d(-\beta)_i > 0$, since $(\partial(RML)_i/\partial L_i)(dL_i/d\gamma_i)$ is negative .

4-2 The gains from International Trade

From the above subsection 4-1, we show there is international trade gain in this subsection. From (13) we obtain:

$$(R\hat{M}L_1) > (R\hat{M}L_2)^{12}. \qquad (16)$$

Since $RML_1/RML_2 = RML_1^*/RML_2^*$ before improving product parameter, we obtain:

$$RML_1/RML_2 < RML_1^*/RML_2^* \qquad (17)$$

This is basically same as a simple Ricardo model since relative real labor marginal products or opportunity costs are different between two countries[13]. Home country and the rest of the world have trade gains when labors in the 2-th sector move to the 1-th sector in home country, and the opposite conditions occur in the rest of the world. Then we have Proposition 4.

PROPOSITION 4

If $dx_1/d\gamma_1 > 0$, a high wage country can obtain trade gain by exporting complicated goods and importing less complicated goods in home country.

5 Other Applications

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[12] $(p^*_1\hat{R}ML_1) = \hat{p}^*_1 + (R\hat{M}L_1) = (R\hat{M}L_2)$, and $\hat{p}^*_1 = 0$.

[13] Of course, in Ricardo model, marginal product of labor is fixed. So this is applied to a small change.

5-1 Improvement productivity and cost by introducing machines and patents

In this subsection, we take endogenous parameter changes into consideration. We assume that parameter, γ and β, are function of machines and patents. Let M denote machines and patents and its price p_M. Then profit function of a firm is given by:

$$\pi(x, L, M) = \gamma(M)p\varphi Y - wL - \beta(M)\psi c - p_M M. \quad (18)$$

The optimizing of M is given by:

$$\pi_M = \gamma'(M)p\varphi Y - \beta'(M)\psi c - p_M = 0,$$

where $\gamma'(M) > 0$ and $\beta'(M) < 0$. Figure 5 represents it. In the figure, for simplicity, we consider only cost function. When new production or management process by machine and patents is introduced, sometimes we have to take careful of total cost which includes complicate costs. Complicate machine make tasks difficult and raise the complicate cost. It is depicted as C^2 in Figure 5. In the Figure, $p_M M$ expresses change in direct cost according to introducing machines or patents. C^0 is an original cost and it is assumed that M is zero initially. Lines C^0 and C^1 are parallel and the difference is $p_M M$ only. Suppose that complex cost parameter is changed by M. $C^3 = \beta_3\psi(x)c(L)$ is the case in which complex cost is decreased and the situation is the opposite of C^2. If initial complexity, x_0, is not changed after introducing M and it is larger than x^*, total cost is decreased. The fact that developing countries have been able to produce more complex goods may be explained by C^3. This would be one of technological progress phenomenon and technological transfer. If complex cost function is changed to C^2 in developing countries, the economic development would not be able to expect. In C^1 and C^2, firms don't introduce M. In the case of C^3, firms would purchase M when complexity is larger than x^*. Then, we find that for small complexity, firms would not tend to purchase expensive machines and patents.

The analysis in this subsection represents also technological transfer through trade (for instance, Dilavoro(2005)). If M denotes import, this reducing of complex cost can be interpreted as technological transfer. For the cost to reduce, imitation by using M, and human capital or capability for the imitation would be necessary.

5-2 Skilled and unskilled labor

In this subsection, I take the difference of labor quality into consideration[14]. Now it is assumed there are two types of labor, skilled labors L_s and unskilled labors L_u. Let w_s denote wage of skilled labor and w_u unskilled with $w_s > w_u$, and x^b a boundary complexity for complexity cost. x^b satisfies the following:

$$w_s L_s + \beta_s \psi_s (x^b) c(L_s) = w_u L_u + \beta_u \psi_s (x^b) c(L_u), \qquad (19)$$

where $\beta_s < \beta_u$, $\psi_s(x^b) < \psi_u(x^b)$. Thus x^b is the value which both complexity costs are equal. Now we ignore production sides for simplicity and assume each labor input is fixed. Then we can know the way how firms assign the complexity to each labor according to the difference of complex costs.

Now suppose that unskilled labor is reduced. Then w_u is raised because of labor scarcity in unskilled labor market. Then from equation (19), the change in x^b is given by

$$(\beta_s \psi_s'(x^b) c(L_s) - \beta_u \psi_s'(x^b) c(L_u)) dx^b = L_u dw_u.$$

[14] As a similar paper to this subsection, Yeaple (2005) treats endogenous choosing technology and hire different type of woks.

Assuming $\beta_s\psi_s'(x^b)c(L_s) < \beta_u\psi_u'(x^b)c(L_u)$, that is, the marginal complex cost of skilled labor is lower, we obtain $dx^b/dw_u < 0$. Similarly we have $dx^b/dw_s > 0$. We have asymmetric results for each wage rise. If skilled labor endowment is relatively much, it would happen that $dw_u > 0$ and $dw_s < 0$, and $dx^b < 0$. This suggests that unskilled labor would engage in less complex task, and skilled labor engage in relatively easy task for such society, in which skilled labor is much and unskilled is small. Such economy is usually interpreted as developed countries.

Then we have Proposition 5.

PROPOSITION 5

Relatively less (more) complex tasks are conducted by unskilled labor in developed (developing) countries.

The above analysis is shown in Figure 6-I. The figure 6- I is depicted for exogenous wage rise. In the figure, c_s denotes complex cost of skilled labor and c_u denotes that of unskilled. These cost functions are depicted for given labor and the changes are corresponding to wage rise in unskilled labor. When such wage rise is due to cost reduction, that is, $d\beta < 0$, the curve of c_{u1} may be shifted under while $dw_u > 0$ (Figure 6-Ⅱ). Then new boundary complexity $x^b{}_1$ is increased in some cases and dx^b is not always negative.

5-3 Firms are larger in rich country or firm size and wage are positively correlated.

In this subsection, the relationship between firm size and per income is considered using our complex model[15]. Let define rich countries as high wage and assume that the source of wage rise is due to $d\gamma > 0$ and $d\beta < 0$. Suppose that the size of firm is expressed as labor employment. From the

[15] Shaffer (2002) examines relationship between growth and firm size. He shows that the size is negatively with growth rates. But his sample is US domestic firms.

above analysis, we find that for $d\gamma > 0$ and $d\beta < 0$, labor demand is increased if $\pi_{Lx} = \pi_{xL} > 0$. Then we may think our model predicts it. But the answer is not always right. If parameters are improved in all firms, the size of each firm may be not changed and wage rates rise. Instead if parameters are improved unequally, we find that from $dL/dw < 0$ and $dx/dw < 0$, less inefficient firms with no improvement on technology reduce employment. Then the small firms may be not necessary to exit from the market since marginal products of labor is increased by the reduction of employment. Then average firm size would not change. But in our model, profit can be negative since there is a term of $\beta\psi(x)c(L)$. Some these firms would have negative profit and exit from the market and the number of firms would be reduced. Then, it can be predicted from our model that the relation between wage and firm size trend to exist[16].

5-4 Consumer preferences to Complexity

In this subsection, complexity is defined not for production but for consumers. Although we may think this is not product complexity, firms decide complexity for users. In this sense, it is also a kind of product complexity[17]. Using complexity, I show that firms decide the degree of consumption complexity according to consumer preferences. Consumers always don't prefer simple goods since complexity goods make consumers expand the opportunity of consumption. For instance, complex cameras are preferred by professional or amateur cameramen who are intensively interested in camera. On the contrary, simple cameras are preferred by people who are not interested and feel with difficulty in operating cameras.

Let consider goods which has diversity. Suppose that price, p, is increasing function of complexity, x, and utility is also increasing function of complexity while there is complex cost which operates

[16] On the other hand there can be new entry-firms. In our model these firms are not considered. If there are new entries and these firms are small, the average firm size may not be large.
[17] Users include not only consumers but also producers. I have already analyzed in 5-1 for the case in which users are producers.

the goods. Consumer choose optimal x among many goods to maximize utility. Then maximization problem is shown as:

$$\underset{x}{Max} \ [U(x) - U(C(x)) - U(p(x))],$$

where $-U(C(x))$ is disutility accompanied with complexity, and $-U(p(x))$ is disutility accompanied with expenditure to this goods, $C(x)$ is operation cost. Figure 7 depicts it for two prices, p^E and p^C, where p^E is that of expensive and complex, and p^C cheap and simple. If people prefer expensive and complex goods, each utility function is shown as U^E. Instead, if people prefer cheap, each utility function is U^C. Optimal complexity is x^{E*} for people who like expensive goods, and x^{C*} for cheap goods. Although goods which have many functions with complexity are convenient, it is expensive if they are too complex for consumers to use full functions and consumers confuse to use. Such persons choose cheap goods.

Concluding and Remarks

Using the concept of complexity, this paper extends many analyses which include labor, international trade and strategic interaction within firms. Our applications which are shown in this paper are partial. As another example, there is an analysis of outsourcing. Ono and Stango (2005) consider outsourcing with product diversities which can be interpreted as product complexity. They find that product diversities are associated with outsourcing. If firms produce many goods with diversity, some goods may not have comparative advantage to other firms and foreign countries. Then the outsourced goods of these firms may have low or high complexity. Sometimes firms may outsource the goods which they can not produce at low cost because of too complexity[18]. In this

[18] Deardorff (2005) examine outsourcing in a North and South model with skilled and

case these goods have high complexity. There is also the reverse case in which easy tasks don't make skilled labor fit in. As another instance, there is the analysis of patent. If firms consider that imitation is impossible, they don't apply patent since they can receive monopolistic profit for eternal[19]. In our model, this can be interpreted as huge complex cost since it is too difficult for rival firms to imitate.

The complex cost may be hidden cost and not accounted in firms. Empirical analysis for complex cost would be not easy. But as in Berkowitz et al (2003), we can analysis it indirectly by interpret complexity as the number of production process. In some cases, firms don't employ labors of whom ability is too low to achieve their tasks, and firms remove such employments, and firms don't invest directly to foreign countries which have high complex costs. Thus, the complex costs may be seemingly low.

The complex cost may also occur in learning by doing. The period of learning by doing is longer or the learning is more important if complexity of tasks increases. Then the speed of redeployment of labor is slow and professional workers with skill for specific tasks increase.

Section 4 considers international trade theory. This is a kind of Ricardo model and reveals the role of the technology factor from the view point of complexity. This paper is complement to empirical analysis in which international technology difference is important for explaining international trade[20].

This paper sheds lights on another side of technology and development, that is, making tasks easy. In production process, the standardizing of technology and reducing parts causes the complex cost decrease. The concept of product complexity recognizes us the relationship between development and imitation. From this paper, for developed countries and firms, we can understand

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skilled labors.

19 Generally speaking, the period when patent is valid is limited.

20 See for instance, Trefeler (1995).

that it is important for developing countries and rival firms too complex to imitate[21].

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Figure 1 Production function

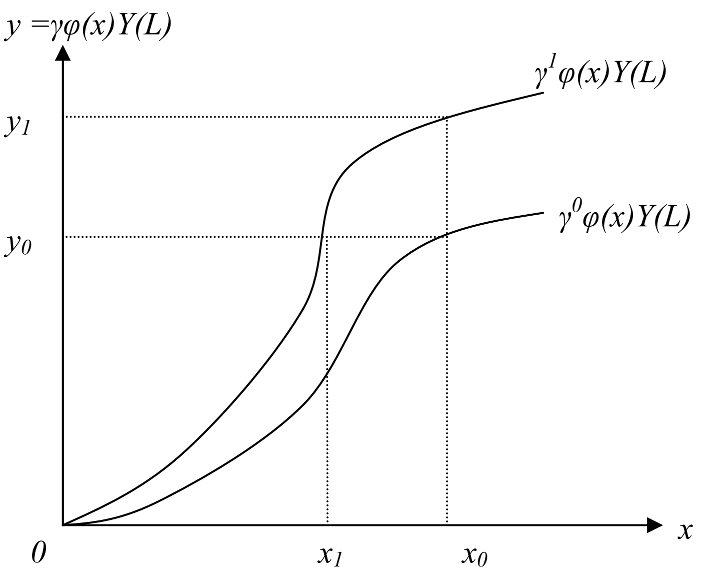


Figure 2 Improvement on complexity cost

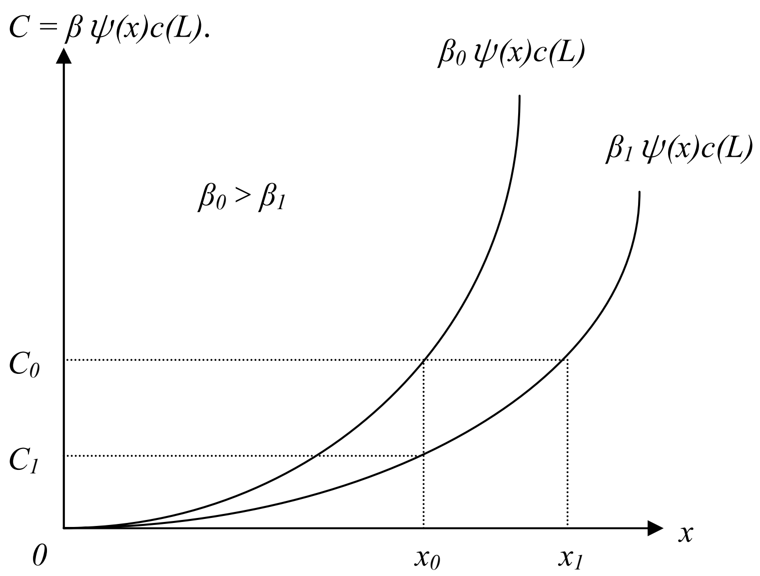


Figure 3 Optimal complexity for given L and w

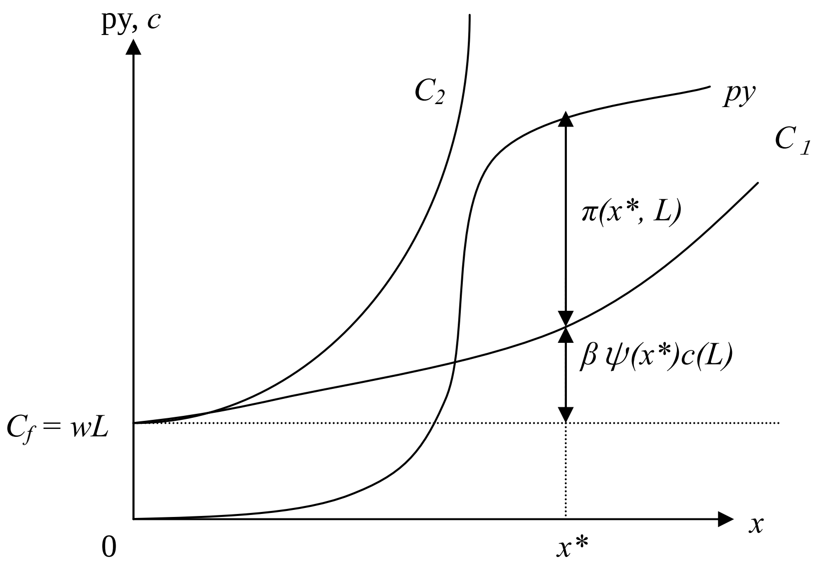


Table 4 Optimal *x** for given *L*, and economic development

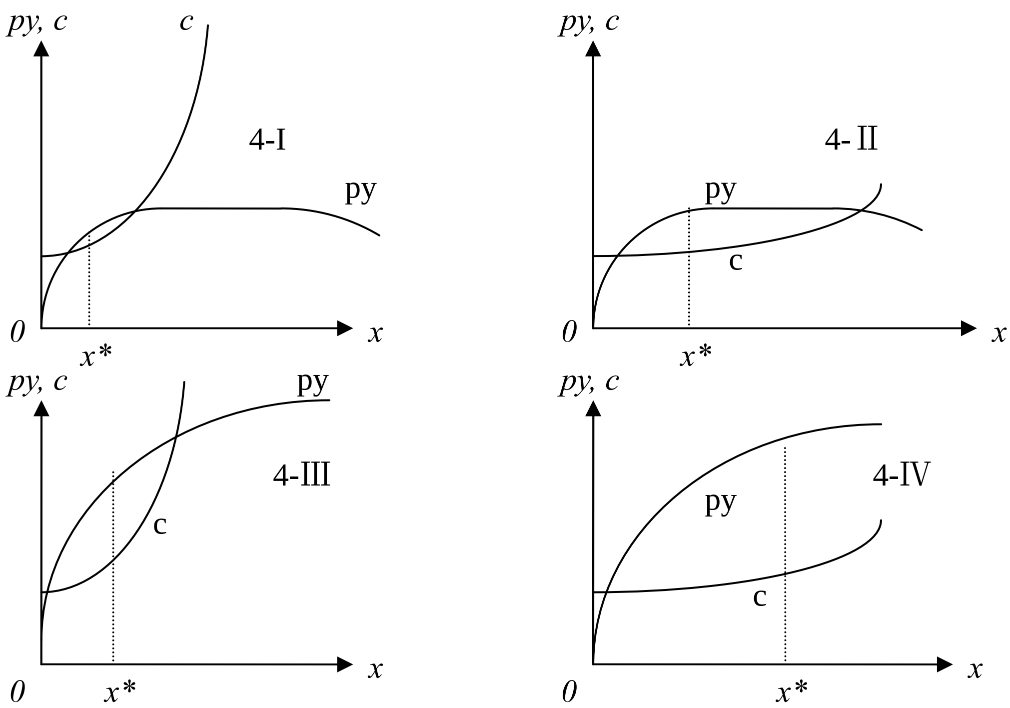


Figure 5 Total cost and an increase in machines and patents for given w and L

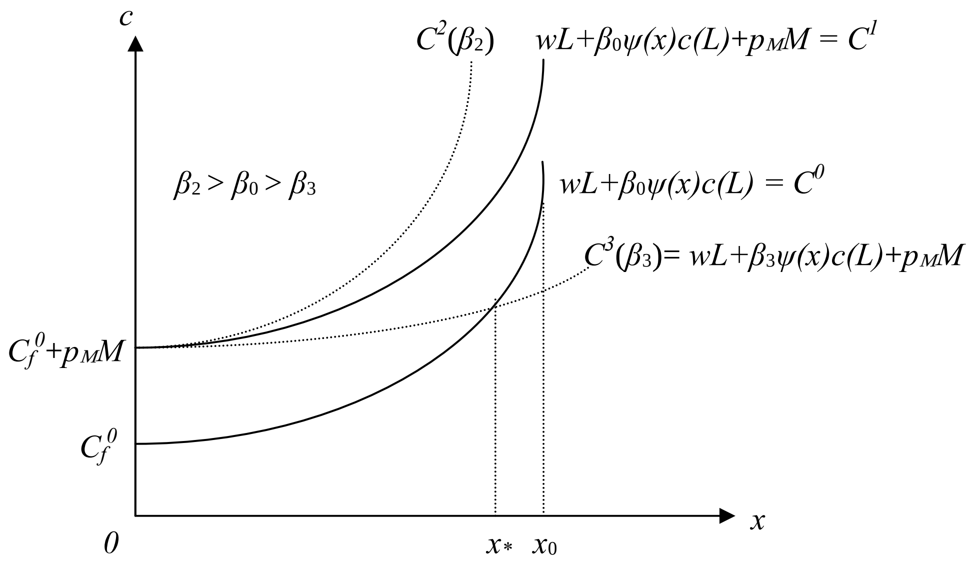
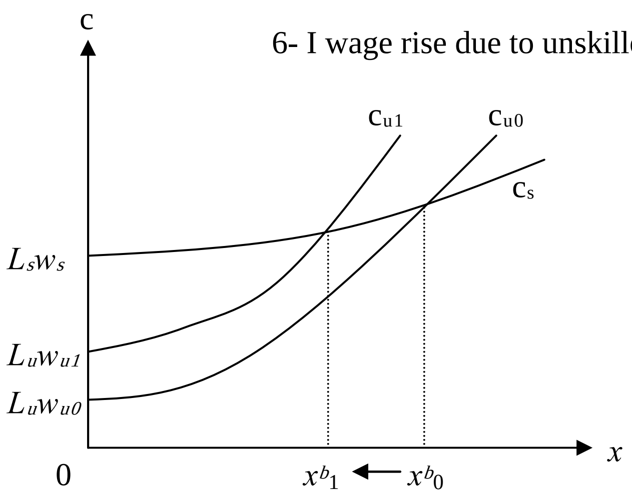


Figure 6 Skilled and Unskilled labor, and boundary of complexity



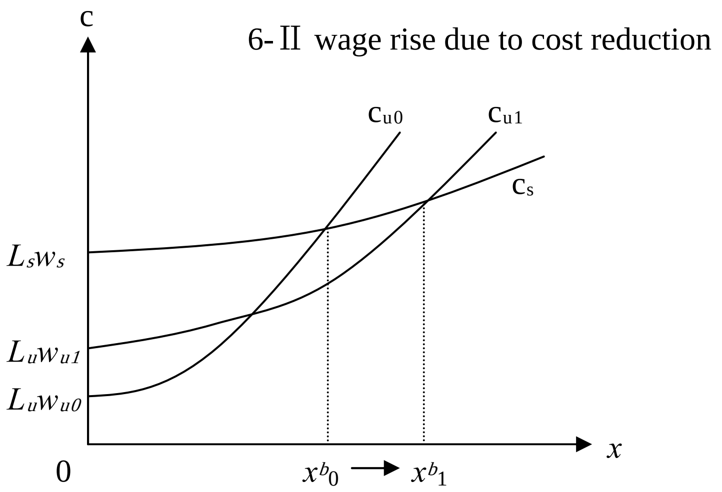


Figure 7 Optimal complexities for consumers