WILL TARIFF PROTECTION INDUCE PRODUCTIVITY GROWTH
AN OPTIMAL CONTROL MODEL

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4.1 Introduction

The last two decades have experienced a high tide of trade liberalization. Many developing countries have turned away from import-substituting oriented trade policies, and transferred towards freer trade patterns. The trade reform has provided valuable experience and thus opportunities to learn about the economic impact of trade liberalization. One of the current topics of research is the relationship between trade policy and economic growth in terms of growth in productivity. Though the magnitude of research in this area has been very impressive, a clear consensus has not yet been reached.

There are several studies that support the positive relationship between trade liberalization and productivity growth. In a cross-country analysis, Harrison (1995) found a generally positive association between growth and different measures of openness. In a firm-level analysis, Krishna and Mitra (1996) found evidence of productivity growth in India following trade liberalization. In a cross-industry study that related the increase in total factor productivity (TFP) to export expansion and import substitution, Nishimizu and Robinson (1984) found that in most cases, export expansion is positively associated, and import substitution negatively associated with TFP growth. However, there is room for doubt and further discussion. In a survey paper by Harrison and Revenga (1995), the relationship between trade liberalization and productivity growth in the long run is
ambiguous. Levine and Renelt (1992), employing different measures of trade policies, find no robust or even consistent positive relationship between trade liberalization and long run growth. In addition, Rodrik (1992) argues that it is extremely difficult to show that trade liberalization must have a positive impact on technical efficiency in general. His models demonstrate the ambiguous nature of the relationship. Tybout (1992) reviews a number of studies for developing countries, and concludes that the literature on the link between trade and productivity remains diverse and ambiguous.

Recently, the discussion on this issue has tended to be more specific. For example, Traca (1996) compares the substitution effect and the pro-competitive effect brought about by trade liberalization, and shows that if the initial productivity gap is small, the pro-competitive effect dominates. Consequently, a small liberalization stimulates productivity growth. However, if the initial productivity gap is big, he showed that the substitution effect dominates. As a result, firms choose not to fight and will simply exit the market eventually. Nevertheless, appropriate protection given to firms whose decisions were not to fight with foreign producers, might help these firms change their decisions to compete with foreign firms, and therefore enhance productivity growth. In other words, Traca has explored the direct link between the productivity gap and productivity growth. Trade policy reforms affect productivity growth by shrinking the productivity gap. Since the purpose of Traca’s study is to observe the firm’s behavior about decision making (to compete with imports or to concede the market) under the given trade policies, he left the relationship between trade policy and productivity growth open.

The purpose of this paper is, therefore, to investigate the direct link between productivity growth and trade policy. In particular, we intend to examine the cause of the ambiguity between trade policy and productivity growth, and the proper trade policy that can induce productivity growth and thus close the productivity gap.

The rest of the paper is arranged as follows. A static model is set up in the next section in which the pro-protection effect and pro-competitive effect are analyzed. In
Section 3 we set up a dynamic optimal control model and discuss how trade policy affects the path of productivity growth and the length of time needed to close the productivity gap. Finally, section 4 concludes the paper.

4.2 The Static Model

We assume that there is only one firm at home, and consider all of the foreign competitors as another firm. The domestic firm and the foreign firm produce differentiated manufacturing goods: \( m \) and \( m^f \) respectively.

The domestic firm produces \( m \) with linear technology \( m = \alpha L \), where \( \alpha \) represents productivity and \( L \) is the amount of labor used in producing \( m \) as the only input, the unit cost \( c \) is constant and given by \( c = w/\alpha \), where \( w \) is the wage rate. Obviously, the marginal cost is also a constant and is equals to \( c \). Unit cost can be reduced down to the minimum level \( \zeta \) by investing in innovation, where \( \zeta > 0 \).

Let consumer preferences be given by a utility function:
\[
U = \left( m^{1/2} + m^{f1/2} \right)^2
\]
with its budget constraint as:
\[
pm + p^f m^f = I
\]
where \( I \) is income and, \( p \) and \( p^f \) are the domestic prices of the home made product \( m \) and the imported good \( m^f \), respectively. With a tariff rate \( \tau \), the price of imports is
\[
p^f = p^b (1 + \tau )
\]
where \( p^b \) is the border price of import. The standard and simple utility maximization process yields the consumer’s demand for domestic good \( m^d \):
\[
m^d = \frac{p^{-2}}{p^{-1} + (p^F)^{-1}} I
\]
(4.2.1)
See appendix (IV.1) for the derivation.

Given \( p^F \) and the demand in (4.1.1), the firm maximizes its profit: \( \pi = (p - c)m^d \).
Through the regular profit maximization process we then obtain the profit function:

$$\pi(c, p^F; I) = \frac{z - 1}{z + 1} I,$$  \hspace{1cm} (4.2.2)

where

$$z = \left( \frac{p^F}{c} + 1 \right)^{1/2}$$

Appendix (IV.2) provides the detailed proof.

By applying the envelope theorem, the profit-maximizing output level, \(m(c, p^F; I)\), can be shown as

$$m = \frac{z - 1}{cz(z + 1)} I. \hspace{1cm} (4.2.3)$$

From here on, \(m\) refers to the profit-maximized output in the static model. Since the tariff increases the import price, from (4.1.2), it is easy to show:

$$\frac{\partial \pi}{\partial \tau} = \frac{I p^b}{(z + 1)^2 cz} > 0 \hspace{1cm} (4.2.4)$$

So we can see that the import tariff raises profits. However, the impact on the optimal output level \(m\) is ambiguous. In fact, this ambiguity arises from two counter effects: the pro-protection effect and the pro-competitive effect. We next turn to examine both effects.

4.2.1. The pro-protection effect and the pro-competitive effect

To solve the puzzle of the ambiguous impact of tariff on optimal output, we let \(R\) be the firm’s revenue so that \(m = R/p\). Differentiate \(m\) with respect to \(\tau\), using the chain rule, we have

$$\frac{\partial m}{\partial \tau} = \left( \frac{\partial R}{\partial \tau} \cdot \frac{1}{p} \right) \left( -\frac{\partial p}{\partial \tau} \cdot \frac{m}{p} \right), \hspace{1cm} (4.2.5)$$

where
\[
\frac{\partial R}{\partial \tau} = \frac{p^b I}{2cz^3} > 0, \quad \text{and} \quad \frac{\partial p}{\partial \tau} = \frac{p^b}{2z} > 0.
\]

The first parenthesis in the right-hand side of equation (4) shows that for a fixed domestic price, the rise of the tariff implies an increase in output. This is the pro-protection effect: if the price of the domestic good is kept constant, protection (i.e., an increase in the tariff) induces an expansion of output and the market share of the domestic firm. The second parenthesis is the pro-competitive effect, which has been introduced by Devarajan and Rodrik (1991): competition (a decrease in the tariff) encourages domestic output due to the fact that the decline in the price of imports leads to an increase in the price-elasticity of demand, which reduces the market power of the domestic firm and induces an expansion of output due to lower mark-up rates.

We next compare the magnitude of these two effects. Let \( J \) be the ratio of the two effects.

\[
J \equiv \frac{\frac{\partial R}{\partial \tau}}{\frac{\partial \tau}{\partial \tau}} = \frac{1}{p} = \frac{z+1}{z(z-1)}
\]

(4.2.6)

whether \( J \) is greater than 1 depends upon if

\[
\tau < \frac{2(1+\sqrt{2})c}{pb} - 1
\]

We set a critical value \( \tau' \) be

\[
\tau' = \frac{2(1+\sqrt{2})c}{pb} - 1
\]

(4.2.7)

It is easy to see that if the tariff rate is imposed below the critical value (\( \tau < \tau' \)), the pro-protection effect outweighs the pro-competitive effect (\( J > 1 \)). Thus the relationship between output and tariff (\( \frac{\partial m}{\partial \tau} \)) is positive. On the other hand, if the tariff is higher than the critical value (\( \tau > \tau' \)), the relationship is negative and thus \( \frac{\partial m}{\partial \tau} \) is negative.

Accordingly, we develop the output/tariff curve - \( m(\tau) \) curve in figure (4.1). We
name the range of tariff that is below the critical value $\tau'$ the low range. And we call the one that is above, the high range.

It is clearer now that the source of the debate about the impact of trade liberalization on output is generated from these conflicting effects. Those who argue that the competitive pressure arising from trade liberalization reduces the market share of domestic firms, assume that the pro-protective effect dominates. This is usually used by less developed economies to defend the out-coming economic invasions in order to protect their own national economies. While those who claim that the competitive pressure induces an expansion of output and economic efficiency, believe that the pro-competitive effect outweighs the pro-protective effect. The later, very often, is adopted by developed economies on the purpose to expand their market share in the global atmosphere.

Since increasing of tariff generates two effects. The combination effect of the two depends which effect dominates. It is impossible to say trade liberation would leads to an expansion of domestic production or reduction of the domestic production. To induce the positive effect, looking at (4.2.4), we can see there are several things we can do: First to increase the domestic income $I$; Secondly, to decrease the domestic consumption. Interestingly, from (4.2.2) it is clear that reducing domestic consumption can also increase the value of $z$, thus not only to enlarge the pro-protective effect, but reduce the pro-competitive effect as well. This conclusion meets with the traditional East Asian experiences in their past decades of economic development. In contrast to the western economy’s experience, Asian countries believe to save every penny from consumption and to turn it into investment would speed up economic growth. Based on the above analysis, we are now much more clear about the tariff effect towards domestic growth. If we can restrict domestic consumption, trade protection within a certain tariff range indeed promotes domestic growth.

4.2.2. Tariff protection and productivity growth
We now turn to the relationship between tariffs and the rate of productivity growth. There are two ways to see how tariffs affect the productivity growth. One is to find out how tariffs influence output, and then to see the relationship between output and productivity growth, and finally to see the impact of tariff on productivity growth. This approach is indirect and has been adopted by Rodrik (1992) and Traca (1996). An alternative way would be to directly link tariff and productivity growth. Here I use the latter.

To illustrate the main concepts, we assume that the decision to invest in the process of innovation takes place only once. Let $s$ be the cost of innovation and $r$ be the productivity growth rate. Equivalently, in the production defined here, $r$ is also the rate of decrease in production cost. We specify the relationship between them as $s = r^2$. This implies that the investment in the process of innovation has diminishing marginal returns. This may not fit for investment in human capital accumulation, but does fit for physical investment in general. The initial cost is $c_0$ and will decrease to $c_1$ if the firm invests in innovation. Thus, $c_1 = c_0 / (1+r)$. The firm chooses productivity growth rate $r$ to maximize $\pi(c, p^F) - s$. By applying the envelope theorem, the first order condition is

$$\frac{m(c_1, p^F)}{(1+r)^2}c_0 = 2r. \quad (4.2.8)$$

The optimal productivity growth rate can be obtained by solving (4.2.8).

We will use $r^*$ to denote this optimal rate of productivity growth in the static model. By graphing the right-hand side and left-hand side of equation (4.2.8), we can numerically show the existence of the optimal productivity growth rate $r^*$ and, the impact of tariff on $r$: if the tariff increases in the low range ($\tau < \tau'$), $r$ rises. At the same time, if the tariff is raised in the high range ($\tau > \tau'$), $r$ declines. Thus, the shape of the $r(\tau)$ curve (see figure (4.1)) is also a convex one, like that of the $m(\tau)$ curve. This is not surprising. The optimal output and the optimal rate of productivity growth are positively related, because we assume that the spending on innovation is independent of firm size.
Thus the larger the scale of output, the greater the benefits to the firm from a given reduction in costs. As a result, the tariff rate that promotes output growth also encourages productivity growth. Whereas the tariff rate that hinders output deters productivity from growth as well.

The function of optimal output and the function of optimal productivity growth throw light on an interesting point - the critical value $\tau'$. This critical tariff rate yields not only the highest profit-maximizing output level, but also the greatest optimal rate of productivity growth. I will call this critical value the “efficient tariff” throughout the paper. The policy implication here is that in order to lead the firm to catch up with the foreign technology in the most efficient way, the tariff should be set at the efficient tariff level. To see if this policy implication also applies to a dynamic setting, we next establish a dynamic model with continuous time.

### 4.3 The dynamic model

We have so far investigated the link between productivity growth and trade policy in a static model. Here our study turns to a dynamic model when the process of innovation can be undertaken until the initial productivity gap is closed. It is a partial equilibrium set and the performance of the rest of the economy is taken as given.

Apparently, investment in innovation foregoes the current profits in order to increase the likelihood that production costs will fall, and thus increase future profits. The central question addressed here, therefore, is how much current profit at any point of time should be sacrificed to invest in the process of innovation in order to enhance future profit? Since investment in innovation results in the increased productivity, the central question can be transformed to: What is the rate of productivity growth that maximizes the present discounted value of future profits? The answer to this question takes the form of an optimal time path of the rate of productivity growth, which we try to seek in the following section.
4.3.1 The optimal control model

We first set up the model. The lower bound of the marginal cost $c$ is assumed to equal the border price of imports $p^b$. This implies that if the firm engages in innovative activities and hence decreases its unit cost down to the lower bound, it decreases the productivity gap between the domestic firm and the foreign sector and will eventually close it. Let “$T$” be the point of time at which the domestic firm fully catches up with the foreign technology, we hence have $c(T) = c = p^b$.

The unit cost declines at the rate $r$, which is also the productivity growth rate. The spending on innovation at time $t$ is $s(t)$, which yields the productivity growth $r^2(t)$. Or we can denote this as $s(t) = r^2(t)$. Therefore, the firm's objective functional is to maximize the present discounted value of profits while subtracting the present discounted value of spending on innovation before the firm completes the catching up process, plus the present discounted value of profits after the productivity gap is closed:

$$Max \int_0^T \left[ \pi(c(t), p^F) - s(t) \right] e^{-\delta t} dt + \int_T^\infty \pi(c, p^F) e^{-\delta t} dt$$

(4.3.1)

Subject to:

$$s(t) = r^2(t) ;$$

(4.3.2)

$$\dot{c}(t) = -r(t) ;$$

(4.3.3)

$$c(0) = c_0 ; \text{ and}$$

(4.3.4)

$$c(T) = c = p^b ,$$

(4.3.5)

where $c_0$, $c$, and $p^b$ are all fixed.

This is a free-terminal-time problem with an additive value function. We assume, the firm maximizes its stream of present discounted profits from time 0 to time $T$ by investing $s$ in innovation. The return to innovation is that the productivity grows at the rate $r$, which leads to the decrease in cost. Eventually, it contributes to the increments in profits. At time $T$, the firm fully catches up with the foreign technology. That is, the cost
level reaches its lower bound \( c \). From then on, the firm maximizes its stream of present discounted profits which is produced at the cost level \( c \). Since \( c \) is given, the additive value function can be simplified as

\[
\Phi \equiv \int_r^c \pi (c, p^F) e^{-\theta t} dt = \theta^{-1} \pi (c, p^F) e^{-\theta T}.
\]

(4.3.6)

The current value Hamiltonian \( (H_c) \) of the problem is defined as

\[
H_c \equiv He^{\theta t} = \pi (c, p^F) - r^2 + \mu(-r),
\]

where \( \mu \) is the multiplier.

Maximizing \( H_c \) with respect to the control variable \( r \), we find the first order condition is

\[-2r - \mu = 0. \]

(4.3.8)

The second derivative of (4.3.7) is negative. Hence the Hamiltonian does have a maximum solution.

Following the classical Keynes-Ramsey model (Oliver Jean Blanchard and Stanley Fischer, 1989), the maximum principle involves two equations of motion. They are

\[
\dot{\mu} = -\frac{\partial \pi(c, p^F)}{\partial c} + \mu \theta
\]

(4.3.9)

and

\[
\dot{c} = -r.
\]

(4.3.10)

Since the terminal time \( T \) is free, the firm has the freedom to choose \( T \) such that the stream of profits is maximized. We thus need the transversality condition to determine the optimal terminal time \( T^* \):

\[
H_c \left(c^*(T^*), r^*(T^*), \mu^*(T^*), T^* \right) e^{-\theta T} + \frac{\partial \Phi(T^*)}{\partial T} = 0.
\]

(4.3.11)

where \( \Phi(T) \) is defined in (4.3.6).

By applying (4.3.11) to the problem (4.3.6) and (4.3.7), we have

\[
\pi(c^*(T^*), p^F) - r^*(T^*) + \mu^*(-r^*(T^*)) + (-\theta) \theta^{-1} \pi(c^*(T^*), p) = 0
\]

From this we can solve for the optimal rate of productivity growth at time \( T \):

\[
r^*(T) = 0.
\]

(4.3.12)
Equation (4.3.12) states that the optimal growth rate of productivity at the terminal time $T$ should be zero. That is because all of the benefits from innovation have been exhausted before $T$. Further sacrifice of current profit and consumption to spend on innovation makes no more contribution to the future profits. (We suppose that there is no further technological improvement) From the above explanation, we can see that the real dynamic path comes from the first part of the equation (4.3.1) before time $T$. The second part of (4.3.1) is a constant once the time $T$ is set. This concludes the model. We next turn to the phase diagram analysis.

4.3.2 Phase diagram analysis

We begin by differentiating (4.3.8) with respect to time. That yields:

$$\dot{r} = -\frac{1}{2} \mu .$$  \hspace{1cm} (4.3.13)

Using this equation, together with equations (4.3.8) and (4.3.9) and recall that

$$\pi = (p - c)m^d$$

we can use envelope theory and apply it into (4.3.13), it yields

$$\dot{r} = \theta r - \frac{1}{2} m(c, p^r).$$  \hspace{1cm} (4.3.14)

Equation (4.3.10) and (4.3.14) forms a two dimensional system of differential equations in the two variables $c$ and $r$. Accordingly, two locus $\dot{c} = 0$ and $\dot{r} = 0$, are constructed in figure (4.2) in $c$ and $r$ dimensions.

The $\dot{c} = 0$ curve shows up as an L-shaped curve, as illustrated in figure (4.2).

As we can see from (4.3.10), if $r$ is zero, $\dot{c} = 0$. Therefore $c$ can be any positive number between $c_0$ and $\zeta$. This forms the horizontal part of the curve. The second situation is, $c$ stops declining when it reaches $\zeta$, meaning $\dot{c}$ equals zero when $c$ is at $\zeta$. In such a case $r$ can take any positive value. Thus this part of the curve must be a vertical straight line. To the right and above the locus $\dot{c} = 0$, the unit cost $c$ is higher than its lower bound $\zeta$, and
the productivity growth rate is positive \((r > 0)\). In that case, \(c\) is decreasing, as showed by the arrowhead in figure (4.2).

The equation for \(\dot{r} = 0\) curve, from (4.3.14), is

\[
\dot{r} = \frac{1}{2\theta} m(c(t), p^r).
\] (4.3.15)

Since \(\frac{\partial m}{\partial c} < 0\) and \(\frac{\partial^2 m}{\partial c^2} > 0\), the curve is downward sloping and convex, as showed in figure (4.2). When the curve reaches \(c\), the transversality condition requires \(r(T)\) to be zero. This forms the vertical part of the curve. Interestingly, as shown in Figure (4.2), at the point \(c = c\), the dynamic path \(\dot{c} = 0\) and \(\dot{r} = 0\) share the same curve. Above the locus \(\dot{r} = 0\), according to (4.3.14), \(r\) is increasing \((\dot{r} > 0)\). Below the locus, on the other hand, \(r\) is decreasing \((\dot{r} < 0)\).

Although only one initial condition \((c_0)\) is given to these variables, the transversality condition (4.3.7) determines a terminal condition. Thus there are enough boundary conditions to determine a unique solution to the dynamic system. This unique solution is the time path that converges to the stationary state \((c, 0)\).

The phase diagram in figure (4.3) illustrates the dynamic system. The unique solution of the system -- the optimal path of productivity growth -- converges to the point \((c, 0)\). This steady state is a corner solution. Once \(c\) reaches \(c\), it stays at this point since \(r\) cannot take negative values to raise \(c\). On the other hand, the transversality condition requires \(r(T)\), the optimal productivity growth rate at the time \(T\), when \(c\) reaches \(c\), to be zero. These conditions indicate the stability of the steady state.

From figure (4.3), the optimal path of the productivity growth rate \(r\) increases before the stream line reaches the locus \(\dot{r} = 0\), then decreases after that until it reaches zero at time \(T\). This indicates that \(r\), the productivity growth rate that the firm chooses optimally, is not constant through time.

4.3.3 The impact of trade policy on the path of productivity growth
To find the effect of trade policy on the optimal time path of \( r \), we differentiate the equations \( \dot{c} = 0 \) and \( \dot{r} = 0 \) with respect to \( \tau \). Only the \( \dot{r} = 0 \) curve is affected by the change of tariff.

\[
\frac{\partial r(t)}{\partial \tau} = \frac{1}{2\theta} \frac{\partial m(t)}{\partial \tau}.
\] (4.3.16)

Equation (4.3.16) tells that at each point of time, the tariff influences the productivity growth through the firm's output level. How tariff affects output level has been discussed in section 4.2. Through the comparison of two conflicting effects, it has been concluded that if the pro-protection effect prevails, a small tariff raises optimal output \((\partial m(t)/\partial \tau > 0)\) and hence optimal productivity growth \((\partial r(t)/\partial \tau > 0)\). In this case the \( \dot{r} = 0 \) curve shifts to the northeast (see figure (4.4)). As a result, the optimal path of productivity growth shifts upward. The \( \dot{r} = 0 \) curve shifts to the highest position when the path of the tariff is at the efficient tariff level. This means the optimal path of productivity growth is maximized as well. This path is showed in figure 4 as \( r(\tau') \).

If the tariff is raised above \( \tau' \), the \( \dot{r} = 0 \) curve would shift to the southwest, the case in which that pro-competitive effect dominates. Consequently, the firm chooses a lower optimal path of productivity growth as a response to the overprotection. Therefore, the policy implication is that the tariff rate should be targeted at the time path of efficient tariff rates \( \tau'(t) \), because it would encourage the firm to choose the highest optimal rate of productivity growth at each point in time.

Figure (4.4) also shows that the steady state \((c, 0)\) is not affected by the trade policy. The firm's target, regardless of a high or low tariff rate, remains at the lower bound of marginal cost.

Though this is the case, trade policy does change the targeted terminal time \( T \). At a higher productivity growth rate less time is needed to catch up with the technology level of the competitors. It is easy to see that the targeted terminal time \( T \) is the shortest when the path of tariff rates is at the efficient level.
There are two policy implications generated from this model. First, the best environment in which a firm with a productivity gap to grow is one which gives proper protection on one hand, and which expose the firm to appropriate competition on the other. Without any protection, the firm may fail to grow due to the severe competition. And thus it might have to leave the market eventually, as pointed out by Traca (1996). But this is not to say that the firm should be protected under high trade barriers as many developing countries have done in the past several decades. To expose the firm to a proper degree of competition leads the firm to learn and develop. Second, The fastest way to catch up with the competitors is to impose the tariff at the efficient level. If the initial tariff level is too high, trade reform is necessary. Traca (1996) shows that a gradual trade reform should be undertaken instead of a radical trade reform because the latter might cause the firm to concede the market. It has been shown in this model. More precisely, that the trade reform should target at setting the tariff to the efficient level.

To sum up, trade policy or trade reform for an industry with a productivity gap requires the policy maker to avoid extreme positions: such as an overprotection of the market or complete liberalization of market. Instead, there is a time path of efficient tariff rate under which the market should be protected. That is the path of efficient tariff rate that can lead the firm to choose the highest productivity growth rate so as to emulate the foreign competitor in the shortest time.

4.4. Conclusion

This paper investigates the cause of the ambiguity of the relationship between trade policy and productivity growth in an import-competing sector in which there is initially a productivity gap between home and foreign countries.

By introducing two conflicting effects -- the pro-protection effect and the pro-competitive effect, we realize that a profit-maximizing firm responds differently depending on the level of the import tariffs. In a lower range of tariff, the pro-protection
effect dominates. Thus the domestic firm expands its output due to the protection coming from reduction of import pressures. While in a high-tariff range, the relatively low intensity of import pressures gives the firm more monopoly power. As a result, price rises but output drops, because consumers demand less at the higher price. Therefore, if the initial tariff were imposed in the high range, where pro-competitive effect prevails, a trade reform would raise output level. An output/tariff curve is thus developed: the firm's optimal output increase when tariffs are raised in the low range and then decreases when tariffs are levied in the high range. We thus find an interesting point: in between the range of low and high tariff, there exists a protection level - the optimal efficient tariff rate, which yields the highest optimal output.

To examine the relationship between trade policy and productivity growth, we assume that the firm can invest in process innovation, which result in productivity growth. Optimal output is positively related to the rate of productivity growth due to the fact that a higher output level brings a greater return from the investment in innovation. As a result, tariff protection has a similar effect on productivity growth to output level. The relationship between productivity growth and tariff, therefore, is positive when pro-protection effect dominates and negative when the pro-competitive effect outweighs the other. As expected, the efficient tariff induces the greatest productivity growth as well. Therefore, a policy maker should set the protection level at the efficient tariff level since it yields the greatest investment in innovation.

To see if an efficient tariff ensures the fastest productivity growth, we set up an optimal control model. This dynamic model shows that trade policy, depending on how high the trade barrier is, can help (hurt) the catching up process in a way that it shortens (delays) the length of time that the process takes. Relatively speaking, a path of small tariff, where the pro-protection effect prevails, helps the firm to choose an optimal time path of a higher rate of productivity growth, and to catch up with the rest of the world in a shorter time. On the other hand, if the firm is overprotected by a path of high tariff, productivity growth occurs at a slower pace. In this case, it takes a longer period of time
for the firm to close the productivity gap. As a result, between the path of low tariff and the path of high tariff, there is a path of efficient tariff, which yields the path of highest productivity growth rate. We thus conclude that if the policy maker adopts the path of efficient tariff, the productivity gap between the domestic firm and the foreign competitor would be closed at a shortest time.
Figure (4.1)

Low range

High range

$M, r$

$\tau'$

$\tau$

$m(\tau)$

$r(\tau)$
Figure (4.2)
Figure (4.3)
\[ C_0 = C_r(t_0) = \tau_r(t_0) = \tau_r' \]

Figure (4.4)
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APPENDIX IV:

APPENDIX AND SOLUTIONS TO CHAPTER (4)

Appendix (IV.1) The utility maximization and solution in a static model:

To $\text{Max} \ (m^{1/2} + (m^f)^{1/2})^2$

s.t. $pm + p^f m^f = I$

The Lagrange is:

$L = (m^{1/2} + (m^f)^{1/2})^2 + \lambda (I - pm - p^f m^f)$

The first order condition implies the solution of demand in domestic and foreign goods $m^d$ and $m^f$ as:

$m^d = \frac{p^f}{p^n p^f + p^n I}, \quad m^f = \frac{p}{p^{f2} + pp^f I}$

Multiply $m^d$ with $p^{-2} p^{f-1}$ to both numerator and denominator, we have equation (4.2.1).
Appendix (IV.2) The profit maximization and solution in a static model:

Using (4.1.2) the profit function can be expressed as:

$$\pi = (p - c)m^d = \frac{p^f I(1-cp^{-1})}{p^f + p}$$

The first order condition implies that

$$\frac{p^f I(cp^{-2}(p^f + p) - (1 - cp^{-1}))}{(p^f + p)^2} = 0$$

From this equation we can solve for price:

$$p = c(1 + (1 + p^f / c)^{1/2})$$

Let $z = (1 + p^f / c)^{1/2}$, we have $p = c(z+1)$. Substitute $p$ into the profit function, we have

$$\pi = \frac{p^f I\left(\frac{z}{z+1}\right)}{p^f + c(z+1)}$$

Since $z = (1 + p^f / c)^{1/2}$, $p^f = (z^2 - 1)c$

then we can rewrite $\pi = \frac{z - 1}{z + 1}I$. This is (4.1.2).