Factor Growth and Equalized Factor Prices

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Abstract

This paper considers two simple questions relating to the Heckscher-Ohlin model: (i) How does factor growth affect the terms of trade between the North and the South? (ii) If factor prices are equalized by trade, at what level are they equalized? Regardless of where it occurs, labor growth improves the terms of trade of the capital-abundant region, whereas capital growth has the opposite effect. Equalized factor prices are "less" than a convex combination of autarky factor prices. A numerical example using Cobb-Douglas production and utility functions shows that world-wide free trade is likely to move the equalized wage rate closer to the autarky wages of developing countries and away from those of high-income countries of America and Europe.

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1. Introduction

The celebrated Heckscher-Ohlin model developed by Eli Heckscher (1919) and Bertil Ohlin contains four theorems. While attempts have been made to extend the model to many industries and factors,[1] the two-country, two-factor, two-commodity model can be considered complete.[2] Among the four cornerstones, the Stolper-Samuelson Theorem (1941) considers the effect of a tariff on factor prices within a single country, and its effect on the terms of trade in a two-country world has received thorough attention. The Rybczynski Theorem (1955) considers the effect of factor growth in a single country facing constant prices, but such factor growth in one country necessarily affects the terms of trade in a two-country framework. The Rybczynski Theorem in virtually all variations of the Heckscher-Ohlin model is derived for a small open economy facing constant prices. Samuelson (1948, 1949) shows that under certain conditions free trade equalizes factor prices between two countries, but the factor price equalization theorem does not address at what levels the factor prices will be equalized. Factor price equalization is also associated with income convergence.[3]

The present paper first analyzes the impact of factor growth on the terms of trade and demonstrates under conditions ensuring factor price equalization that factor growth, regardless of where it occurs, has the same effect on the terms of trade, and that terms of trade depend on the global capital-labor ratio. Second, it shows that the equalized factor prices are less than a convex combination of the autarky factor prices. Since the equalized output price lies somewhere between two autarky prices, one might expect that the equalized factor prices are also a weighted average of autarky factor prices. It is shown that this does not occur due to concavity of cost functions.

Section 2 presents the basic two-country, two-factor, two-commodity model. Section 3 examines the impact of factor growth on the terms of trade. Section 4 considers the equalized factor prices and Section 5 demonstrates two propositions in the Cobb-Douglas case.

2. The Two-Country Model

To investigate the impacts of free trade on the terms of trade and the equalized factor prices in an open economy, we employ the following assumptions:

(1) Consumers in the South and the North have identical and homothetic preferences.

(2) Two factors, capital and labor, are used to produce two goods, 1 and 2.

(3) The countries have identical production functions with constant returns to scale.

(4) Industry 2 is capital intensive and the North is capital-abundant.

(4) Factors are fully employed and mobile between sectors.

(5) Perfect competition prevails in product and factor markets.

(6) There are no transportation costs or trade barriers.

The Supply Side

Let Y_1 and Y_2 denote the domestic production of goods 1 and 2 of the South, respectively. An asterisk (*) is used to denote variables of the North. Let p_1 and p_2 denote the prices of goods 1 and 2. In the absence of transport costs and trade barriers, free trade equalizes output prices, i.e., $p_i = p_i^*$. Since full employment prevails in factor markets, input and output relations of the South may be written as

$$a_{L1}Y_1 + a_{L2}Y_2 = L, \text{ and } a_{K1}Y_1 + a_{K2}Y_2 = K, \qquad (1)$$

and full employment conditions in the North are given by

$$a_{L1}^{*}Y_1^{*} + a_{L2}^{*}Y_2^{*} = L^{*}, \text{ and } a_{K1}^{*}Y_1^{*} + a_{K2}^{*}Y_2^{*} = K^{*}, \tag{2}$$

where the cost-minimizing input-output coefficients are functions of regional factor prices,

$$a_{ij} = a_{ij}(w, r), \quad a_{ij}^{*} = a_{ij}^{*}(w^{*}, r^{*}).$$

Let $k_i \equiv K_i / L_i$ denote the capital intensity of industry i, i = 1, 2. Since industry 2 is capital intensive, $k_2 > k_1$. With the aid of Cramer's Rule, the outputs of the South are written as:

$$Y_1 = \frac{a_{K2}L - a_{L2}K}{\Delta}, \quad Y_2 = \frac{a_{L1}K - a_{K1}L}{\Delta}, \tag{3}$$

where $\Delta = a_{L1}a_{K2} - a_{K1}a_{L2} = a_{L1}a_{L2}(k_2 - k_1) > 0.$

Once factor prices are equalized, the identical technology assumption implies $a_{ij}(w, r) = a_{ij}^{*}(w, r)$ and $\Delta(w, r) = \Delta^{*}(w^{*}, r^{*})$. If factor endowments are within the cone of diversification, outputs in the North are given by

$$Y_1^{*} = \frac{a_{K2}L^{*} - a_{L2}K^{*}}{\Delta}, \quad Y_2^{*} = \frac{a_{L1}K^{*} - a_{K1}L^{*}}{\Delta}. \tag{4}$$

The world outputs of the two sectors are:

$$Y_1 + Y_1^{*} = \frac{a_{K2}(L + L^{*}) - a_{L2}(K + K^{*})}{\Delta}, \quad Y_2 + Y_2^{*} = \frac{a_{L1}(K + K^{*}) - a_{K1}(L + L^{*})}{\Delta}. \tag{5}$$

These show the total outputs in an integrated world equilibrium as described in Samuelson (1949), Dixit and Norman (1980) and Davis and Weinstein (2000). Once factor prices are equalized between countries, even though factors are not mobile between the two countries, *the world outputs are exactly equal to those when the world is a single country with* *aggregate resource endowments* $(L+L^*, K+K^*)$ *and factors are mobile throughout the world*.

The Rybczynski Theorem deals with the impact of factor growth on the outputs of a single country facing constant prices. Each input-output coefficient $a_{ij}(w,r)$ depends on factor prices, which in turn depend on output prices. When prices are held constant, $a_{ij}(w,r)$ as well as $\Delta(w,r)$ also are fixed. Let $Y_{iL} \equiv \partial Y_i / \partial L$ and $Y_{iK} \equiv \partial Y_i / \partial K$. Differentiating (4) with respect to L and K, holding prices constant, we obtain the Rybczynski result:

$$Y_{1L} = \frac{a_{K2}}{\Delta} > 0, \quad Y_{2L} = -\frac{a_{K1}}{\Delta}, \quad Y_{1K} = -\frac{a_{L2}}{\Delta} > 0, \quad Y_{2K} = \frac{a_{L1}}{\Delta}. \tag{6}$$

From (3) and (6), we have $Y_i(L,K) = Y_{iL}L + Y_{iK}K,$ and is homogenous of degree 1 in factor endowments. The Reciprocity relation implies the Stolper-Samuelson Theorem,

$$\frac{\partial w}{\partial p_1} = \frac{a_{K2}}{\Delta} > 0, \quad \frac{\partial w}{\partial p_2} = -\frac{a_{K1}}{\Delta}, \quad \frac{\partial r}{\partial p_1} = -\frac{a_{L2}}{\Delta} > 0, \quad \frac{\partial r}{\partial p_2} = \frac{a_{L1}}{\Delta}. \tag{7}$$

Demand Side

Consider world demand for two goods. Consumers in the South and North are assumed to have identical and homothetic preferences. Their preferences are represented by monotone increasing and quasiconcave utility functions,

$$U = U(X_1, X_2), \text{ and } U^* = U(X_1^*, X_2^*),$$

where X_1 and X_2 denote the South's consumption of goods 1 and 2, and X_1^* and X_2^* are similarly defined for the North. Recall that the North is abundant in capital and the South in labor. Let good 1 be the numéraire so that $p_1 = 1$ and let $p = p_2 / p_1$ be the relative price of good 2 which the South imports. The budget constraints of consumers are:

$$X_1 + pX_2 = I, \text{ and } X_1^* + pX_2^* = I^*,$$

where I and I^* are incomes in terms of the numéraire good of the South and the North. Let $X_1 = X_1(p, I)$ and $X_2 = X_2(p, I)$ denote the demand functions for good 1 and 2 in the South, and $X_1^* = X_1^*(p, I^*)$ and $X_2^* = X_2^*(p, I^*)$ for the North. Indirect utility functions are written as:

$$V(p, I) \equiv U[X_1(p, I), X_2(p, I)], \text{ and } V^*(p, I^*) \equiv U^*[X_1^*(p, I), X_2^*(p, I^*)],$$

where national incomes are given by

$$I = Y_1 + pY_2, \text{ and } I^* = Y_1^* + pY_2^*.$$

The world market clearing price of good 2 is implicitly defined by

$$Y_2 + Y_2^* = X_2(p, I) + X_2^*(p, I^*). \tag{8}$$

Summing the budget constraints of the two countries, we get

$$p_1(X_1 + X_1^*) + p_2(X_2 + X_2^*) = p_1(Y_1 + Y_1^*) + p_2(Y_2 + Y_2^*).$$

If market 2 clears for a given p, market 1 does as well by Walras Law.

3. Factor Growth and Terms of Trade

How does labor growth affect the terms of trade? Producer revenue $R = Y_1 + pY_2$ is distributed to consumers as factor incomes. Partially differentiating consumer income $I = Y_1 + pY_2$ with respect to L and allowing endogenous price changes, we have

$$I_L = \left(Y_{1L} + pY_{2L}\right) + Y_2 p_L,$$

where $I_L \equiv \partial I / \partial L,\ p_L \equiv \partial p / \partial L,\ Y_{iL} \equiv \partial Y_i / \partial L$ and $Y_{iK} \equiv \partial Y_i / \partial K$. Labor growth in the South not only increases the regional income but also affects the North's income through a change in the terms of trade.

$$I_L^* = Y_2^* p_L.$$

Similarly,

$$I_{L^*}^* = \left(Y_{1L^*}^* + pY_{2L}^*\right) + Y_2^* p_{L^*}, \text{ and } I_{L^*} = Y_2 p_{L^*},$$

where $I_{L^*}^* \equiv \partial I^* / \partial L^*$ and $p_{L^*} = \partial p / \partial L^*$.

Factor growth affects consumption through changes in income and the terms of trade. Differentiating $X_2(p, I)$ and $X_2^*(p, I^*)$ with respect to p yields

$$\begin{aligned} \frac{\partial X_2}{\partial L} &= X_{2p} p_L + X_{2I} I_L = \left(X_{2p} + X_{2I} X_2 - Q X_{2I}\right) p_L + X_{2I}\left(Y_{1L} + pY_{2L}\right) \\ &= \left(X_{2p}^U - Q X_{2I}\right) p_L + X_{2I}\left(Y_{1L} + pY_{2L}\right). \end{aligned} \tag{9}$$

$$\frac{\partial X_2^*}{\partial L} = \left(X_{2p}^* + X_{2I*}^* Y_2^* \right) p_L = \left(X_{2p}^* + X_{2I*}^* X_2^* \right) - (X_2^* - Y_2^*) X_{2I*}^*$$
$$= X_{2p}^{*U} - Q^* X_{2I*}^* = \left(X_{2p}^{*U} + Q X_{2I*}^* \right) p_L. \tag{10}$$

where the South's excess demand is $Q = X_2 - Y_2$, the North's excess demand $Q^* = X_2^* - Y_2^*$, whereas the slope of the South's compensated demand curve is $X_{2p}^U = X_{2p} + X_2 X_{2I}$ is, and X_{2p}^{U*} is similarly defined for the North.

Producer revenue is distributed to consumers as labor and rental incomes,

$$R = Y_1 + pY_2 = wL + rK. \tag{11}$$

Note that labor growth can affect revenue by changing the product mix and through a change in the terms of trade. The effect of factor growth on producer revenue can be obtained by partially differentiating (11) with respect to *L*,[4]

$$\frac{\partial R}{\partial L} = Y_{1L} + pY_{2L} = w. \tag{12}$$

That is, an additional worker raises national income by his annual wage even when *p* is endogenous. Let $x_i(p) \equiv X_i(p,1)$ denote the South's demand for good i when income is unity. Given identical and homothetic preferences, income elasticity of demand for each good is unity. Thus, $x_i(p) = x_i^*(p)$, and

$$X_2(p,I) = x_2(p)I, \text{ and } X_2^*(p,I^*) = x_2(p)I^*,$$

which implies $X_{2I*}^* = x_2(p) = X_{2I}$. Combining (9) and (10) and using (12),

$$\frac{\partial(X_2+X_2^*)}{\partial L}=\left(X_{2p}^{U}+X_{2p}^{U*}\right)p_L+wX_{2I}. \qquad (13)$$

Differentiating the left side of (8), we get

$$\frac{\partial(Y_2+Y_2^*)}{\partial L}=Y_{2L}+\left(Y_{2p}+Y_{2p}^*\right)p_L. \qquad (14)$$

Equating (13) and (14), we have

$$p_L=\frac{Y_{2L}-wX_{2I}}{D_{2p}^{U}+D_{2p}^{U*}}>0, \qquad (15)$$

where $D_{2p}^{U}=X_{2p}^{U}-Y_{2p}$ and $D_{2p}^{U*}=X_{2p}^{U*}-Y_{2p}^*$ are the slopes of compensated excess demands for good 2. Similarly, for the North,

$$p_{L*}=\frac{Y_{2L}^*-wX_{2I}^*}{D_{2p}^{U}+D_{2p}^{U*}}>0. \qquad (16)$$

Note that $X_{1I}=x_1(p)>0$, $X_{2I}=x_2(p)>0$, and $Y_{2L}<0$ by the Rybczynski Theorem. The North exports good 2 and $p\equiv p_2/p_1$ represents the North's terms of trade. Thus, *whether it occurs in the North or the South, labor growth raises the price of the capital intensive good and improves the North's terms of trade*.

Let $\eta\equiv(\partial w/\partial p)(p/w)(<0)$ denote the price elasticity of the wage. By the Samuelson (1953) Reciprocity relation, $Y_{2L}=\partial w/\partial p=\eta w/p.$ Thus,

$$p_L\equiv\frac{\partial p}{\partial L}=\frac{w(\eta-x_2)}{D_{2p}^{U}+D_{2p}^{U*}}>0.$$

Given factor price equalization and identical technologies, $\partial w / \partial p = \partial w^* / \partial p$ and $\eta = \eta^* \equiv (\partial w^* / \partial p)(p / w)$. Thus,

$$\frac{\partial p}{\partial L^*} = \frac{w^*(\eta^* - x_2^*)}{D_{2p}^U + D_{2p}^{U*}} = \frac{w(\eta - x_2)}{D_{2p}^U + D_{2p}^{U*}} = \frac{\partial p}{\partial L}. \tag{17}$$

Thus, *labor growth has the same effect on the North's terms of trade whether it occurs in the North or South.*

Carrying out a similarly analysis on capital input, we get,

$$\frac{\partial p}{\partial K} = \frac{Y_{2K} - rX_{2I}}{D_{2p}^U + D_{2p}^{U*}}, \tag{18}$$

$$\frac{\partial p}{\partial K^*} = \frac{Y_{2K^*}^* - rX_{2I}^*}{D_{2p}^U + D_{2p}^{U*}}. \tag{19}$$

By the reciprocity relation, we have $Y_{2K} = \partial r / \partial p$. A change in *p* has a magnification effect on *r*. Let $\varepsilon \equiv (\partial r / \partial p)(p / r)(> 1)$ denote the price elasticity of rent. Thus,

$$Y_{2K} = \varepsilon r / p.$$

The budget constraint implies $x_1 + px_2 = 1$. Differentiating $X_2(p, I) \equiv x_2(p)I$ with respect to *I* gives $X_{2I} = x_2 = (1 - x_1) / p$, and (18) can be rewritten as

$$\frac{\partial p}{\partial K} = \frac{r(\varepsilon - 1 + x_1)}{p\left(D_{2p}^U + D_{2p}^{U*}\right)} < 0. \tag{20}$$

Given factor price equalization, $\partial r / \partial p = \partial r^* / \partial p$, $\varepsilon = \varepsilon^*$, and

$$\frac{\partial p}{\partial K*} = \frac{r(\varepsilon - 1 + x_1^*)}{p\left(D_{2p}^U + D_{2p}^{U*}\right)} = \frac{\partial p}{\partial K*} < 0, \qquad (21)$$

Note that due to the magnification effect, $\varepsilon > 1$. Thus, *whether it occurs in the North or South capital growth worsens the North's terms of trade*. *An increment of capital has the same effect on the terms of trade whether it occurs on the North or South*.

Properties of the Autarky Price

Tadeus Rybczynski (1955, p. 340) was aware that factor growth lowers the autarky relative price of the good which intensively uses that factor. The effect of factor growth on the South's autarky price can be obtained from (15) and (18) by removing the North's production and consumption. Specifically,

$$\frac{\partial p}{\partial L} = \frac{Y_{2L} - wX_{2I}}{D_{2p}^U} > 0, \quad \frac{\partial p}{\partial K} = \frac{Y_{2K} - rX_{2I}}{D_{2p}^U} < 0.$$

We now show that a proportionate increase in factor endowments has no effect on the autarky price of good 2.[5] Let $k = K / L$ denote the South's capital-labor ratio, which is now held constant. Differentiating *p* with respect to *L* while allowing *K* to increase, we have

$$\frac{dp}{dL} = \frac{\partial p}{\partial L} + k\frac{\partial p}{\partial L} = \frac{LY_{2L} + KY_{2K} - (wL + rK)X_{2I}}{LD_{2p}^U} = 0.$$

To see this, first recall from (3) $Y_i(K, L)$ is homogeneous of degree 1 in factor endowments, i.e., $LY_{2L} + KY_{2K} = Y_2$. Next, $X_{2I} = x_2(p)$ and $(wL + rK)X_{2I} = X_2$.

Once factor prices are equalized, even though the factors are internationally immobile, the world market behaves as a single country with global resource endowments $(L+L^*, K+K^*)$. Equilibrium price depends on the aggregate capital-labor ratio,

$$p = h\left((K+K^*)/(L+L^*)\right). \tag{22}$$

A proportionate increase in the global capital and labor inputs has no effect on the terms of trade. Using (22) and the Stolper-Samuelson results in (7), we may write the *equalized factor prices* as functions of *p*,

$$w_o = w(p) = f\left(\frac{K+K^*}{L+L^*}\right), r_o = r(p) = g\left(\frac{K+K^*}{L+L^*}\right). \tag{23}$$

Thus,

$$\frac{\partial w_o}{\partial L} = \frac{\partial w}{\partial p_2} p_L < 0, \quad \frac{\partial r_o}{\partial L} = \frac{\partial r}{\partial p_2} p_L > 0, \quad \frac{\partial w_o}{\partial K} = \frac{\partial w}{\partial p_2} p_K > 0, \quad \frac{\partial r_o}{\partial L} = \frac{\partial r}{\partial p_2} p_K < 0. \tag{24}$$

The effects of factor growth on autarky prices can also be obtained by removing the foreign factor endowments in (23) and the signs remain the same,

$$\frac{\partial w_A}{\partial L} = \frac{\partial w}{\partial p_2} p_L < 0, \quad \frac{\partial r_A}{\partial L} = \frac{\partial r}{\partial p_2} p_L > 0, \quad \frac{\partial w_A}{\partial K} = \frac{\partial w}{\partial p_2} p_K > 0, \quad \frac{\partial r_A}{\partial L} = \frac{\partial r}{\partial p_2} p_K < 0. \tag{25}$$

Thus, *a growing factor necessarily brings about a decline in its autarky price*, while the Rybczynski Theorem holds in a small open economy facing constant prices. Moreover, labor growth necessarily lowers the equalized wage rate and capital growth the equalized rental rate, whether the growth occurs in the South or North.

The following propositions summarize these results.

Proposition 1: Whether it occurs in the North or the South, an increase in the labor endowment improves the North's terms of trade, while an increase in capital endowment worsens them.

Proposition 2: Factor growth has the same effect on the terms of trade, regardless of where it occurs, i.e.,

$$\frac{\partial p}{\partial L} = \frac{\partial p}{\partial L^*} > 0, \text{ and } \frac{\partial p}{\partial K} = \frac{\partial p}{\partial K} < 0.$$

Proposition 3: The terms of trade depends only on the global capital-labor ratio, i.e.,

$$p_o = h\big((K + K^*)/(L + L^*)\big).$$

Proposition 4: Labor growth lowers the equalized wage rate and raises the equalized rental rate, and capital growth has the opposite effects on factor prices, whether growth occurs in the North or South.

4. Levels of Equalized Factor Prices

If two countries establish a free trade area, will the equalized wage and rental rates be a weighted average of their autarky factor prices? Suppose p_1 is held constant, and let p_{2A} and p_{2B} now denote the *autarky* prices of good 2 in the South and the North, respectively.

Free trade ensures that equilibrium p_{2o} lies between the two autarky prices, p_{2A} and p_{2B}. That is,

$$p_{2o} = \mu p_{2A} + (1-\mu) p_{2B}, \tag{26}$$

where $0 < \mu < 1$. In a similar fashion, let (w_A, r_A) and (w_B, r_B) now denote the autarky factor prices of the South and the North, and let (w_o, r_o) denote the equalized factor prices. Since the equalized factor prices are trapped between their autarky levels, there exist θ_w and θ_r such that

$$w_o = \theta_w w_A + (1-\theta_w) w_B, \quad 0 < \theta_w < 1,$$

$$r_o = \theta_r r_A + (1-\theta_r) r_B, \quad 0 < \theta_r < 1,$$

but $\theta_w \neq \theta_r$.

Does there exist a common weight θ $(0 < \theta < 1)$ such that

$$(w_o, r_o) = \theta(w_A, r_A) + (1-\theta)(w_B, r_B)?$$

The unit cost functions, $p_1(w,r)$ and $p_2(w,r)$, are concave in factor prices.[6] By Jensen's inequality, we have

$$p_2\left(\mu w_A + (1-\mu) w_B, \mu r_A + (1-\mu) r_B\right) > \mu p_2(w_A, r_A) + (1-\mu) p_2(w_B, r_B) = p_{2o}.$$

That is, if the autarky factor prices (w_A, r_A) and (w_B, r_B) had the same weights, μ and $(1-\mu)$, as the autarky output prices, the resulting p_2 will be "too high" in the sense that it is greater than the equalized output price. However, the equalized factor prices (w_o, r_o) are a positive linear combination of the autarky factor prices, i.e., there exists λ_A and λ_B such that[7]

$$(w_o, r_o) = \lambda_A \cdot (w_A, r_A) + \lambda_B \cdot (w_B, r_B). \tag{27}$$

If the equalized factor prices were a convex combination of the two autarky factor prices, the sum of the two weights ($\lambda_A + \lambda_B$) would be unity. We now show that the equalized factor prices are “less” than a convex combination, i.e., the sum of the weights is less than one, $\lambda_A + \lambda_B < 1.$

Note that the price of the numéraire is fixed and the equalized factor prices must satisfy the constraint $p_1(w, r) = 1.$[8] Since $p_1(w, r)$ is a concave function, the factor price frontier or the contour of (w, r) along the curve $p_1(w, r) = 1$ is convex to the origin and lies below the line connecting two autarky factor prices, (w_A, r_A) and (w_B, r_B). Any point along the latter line in Figure 1 is a convex combination of the autarky factor prices, and hence $\lambda_A + \lambda_B = 1.$ However, since the price of the numéraire is fixed, the equalized factor prices must move along curve $p_1(w, r) = 1$.

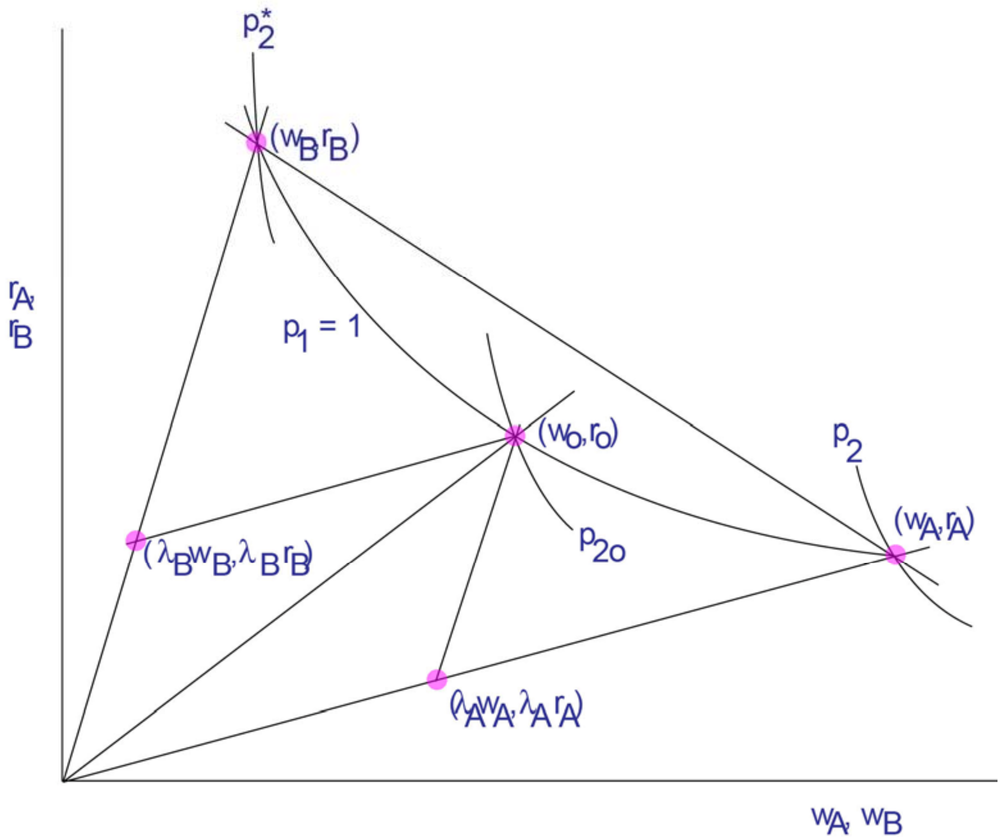

Figure 1. Equalized factor prices and autarky factor prices

These results are summarized below:

Proposition 5: Let the equalized price $p_{2o} = \mu p_{2A} + (1-\mu) p_{2B}$ and equalized rent $r_o = \rho r_A + (1-\rho) r_B$ be weighted averages of autarky prices and rents of the North and the South. Then

(i) $p_2\left(\mu w_A + (1-\mu) w_B, \mu r_A + (1-\mu) r_B\right) > p_{2o}$, and

(ii) the equalized factor prices are less than a convex combination of the autarky factor prices, i.e., $(w_o, r_o) = \lambda_A \cdot (w_A, r_A) + \lambda_B \cdot (w_B, r_B)$, $\lambda_A + \lambda_B < 1$.

Proposition 5 has an important implication on the equalized factor prices. If developed economies have free trade with developing regions such as Asia and Africa, the resulting equalized factor prices will be "less" than a convex combination of autarky prices of the developed and developing economies.

Consider a numerical example. Suppose factor endowments of China are $(L_B, K_B) = (600, 2000)$ and those of America are $(L_A, K_A) = (150, 20000)$. Workers are measured in millions and capital stock is measured in terms of the numéraire. In this situation, China's capital stock is one-tenth of the U.S. capital stock, whereas China's labor force is four times the U.S. counterpart. Assume that the two countries have identical utility functions, $U = X_1 X_2$, and identical Cobb-Douglas production functions: $y_i = K_i^{\alpha_i} L_i^{\beta_i}$ with $(\alpha_1, \beta_1) = (1/4, 3/4)$ and $(\alpha_2, \beta_2) = (3/4, 1/4)$. In this case, $k_2 = 9k_1$, i.e., industry 2 is nine times more capital intensive than industry 1. As shown in Table 1, when $p_1 = 1$, the autarky price p of the capital-intensive good is \$0.5477 in China and \$0.0866 in America. The detailed derivation used in Table 1 is in the Appendix. The equilibrium price under free trade is \$0.1846, and China's weight in determining the world equilibrium price is μ_B = 21.26%.

China's autarky factor prices are $(w_B, r_B) = (0.7700, 0.2310)$, and America's are $(w_A, r_A) = (1.9365, 0.0145)$. The equalized factor prices are:

$$(w_o, r_o) = (1.3262, 0.0452) = 0.1566(w_B, r_B) + 0.6226(w_A, r_A).$$

The sum of the weights is $\lambda_B + \lambda_A = 0.7792 < 1$ as predicted by Proposition 5. The equalized factor prices are "less" than a convex combination of autarky factor prices.

5. Concluding Remarks

In the present Doha Round of trade negotiations developing countries have asked developed nations to liberalize agricultural trade so that they would have access to the markets of developed economies. Increased trade between populous regions of Asia and Africa and capital abundant regions of Europe and America will exert its pressure to reduce wage and rent gaps between the two regions. True, the equalized wage rate will be a convex combination of pre-trade wage rates, and the equalized rent is another weighted average of autarky rental rates, but the weights for rental and wage will be different. When a single weight is chosen for both autarky factor prices of each country, the equalized factor prices will be “less” than a weighted average of autarky factor prices, i.e., the sum of the weights for autarky factor prices of the two countries is less than unity.

The Cobb-Douglas case in Appendix is particularly interesting in that the factor price outcomes are independent of factor shares. The weights of the equalized wage-rent ratio are simply the population ratios. For instance, if the South is five times as populous as the North, the weight of the South’s autarky wage-rent ratio is 5/6 and that of the North 1/6. The HO model assumes capital is mobile only between industries but increased capital mobility will further expedite rental equalization. This example paints a grim picture that in the absence of technological improvements, workers in the North should expect their wage to decline significantly when trading with populous regions, such as China and India.

This paper also points out that it is not possible for the capital-abundant North to grow without spilling the benefits of capital accumulation on the labor-abundant South. Capital growth necessarily worsens the North’s terms of trade, and labor growth the South’s.

Appendix

This appendix illustrates Proposition 2 using Cobb-Douglas production and utility functions. Assume that the two countries have identical Cobb-Douglas production functions: $y_i = K_i^{\alpha_i} L_i^{\beta_i}$ with $(\alpha_1, \beta_1) = (1/4, 3/4)$ and $(\alpha_2, \beta_2) = (3/4, 1/4)$. Consumers have identical Cobb-Douglas utility functions: $U = X_1 X_2$ and $U^* = X_1^* X_2^*$. From the cost minimization problems, we have

$$wL_1 = \beta_1 p_1 y_1, \quad wL_2 = \beta_2 p_2 y_2,$$

$$rK_1 = \alpha_1 p_1 y_1, \quad rK_2 = \alpha_2 p_2 y_2.$$

Thus,

$$\frac{w}{r} = \frac{\beta_1 p_1 y_1 / L_1}{\alpha_1 p_1 y_1 / K_1} = \frac{\beta_1 k_1}{\alpha_1} = \frac{\beta_2 k_2}{\alpha_2}. \qquad (28)$$

It follows that

$$k_2 = \phi k_1, \qquad (29)$$

where $\phi = \beta_1 \alpha_2 / \beta_2 \alpha_1$. From the resource constraints, we have

$$\beta_1 p_1 y_1 + \beta_2 p_2 y_2 = wL, \qquad (30)$$

$$\alpha_1 p_1 y_1 + \alpha_2 p_2 y_2 = rK. \qquad (31)$$

Let $\delta \equiv \beta_1 \alpha_2 - \alpha_1 \beta_2 = \alpha_2 - \alpha_1$. Then

$$p_1 y_1 = \frac{\alpha_2 wL - \beta_2 rK}{\Delta}, \quad p_2 y_2 = \frac{\beta_1 rK - \alpha_1 wL}{\Delta},$$

which shows the Rybczynski Theorem. In autarky, $y_1 = x_1$, or

$$\frac{wL + rK}{2 p_1} = \frac{\alpha_2 wL - \beta_2 rK}{\delta}.$$

Thus, the wage-rent ratio is:

$$\frac{w}{r} = \gamma \frac{K}{L}, \tag{32}$$

where $\gamma = (2 - \alpha_1 - \alpha_2)/(\alpha_1 + \alpha_2) > 0$. Also, from (28) and (29), we have

$$k_1 = \frac{\sigma_1 K}{L}, \quad k_2 = \frac{\sigma_2 K}{L}, \tag{33}$$

where $\sigma_1 \equiv \gamma\alpha_1 / \beta_1$ and $\sigma_2 \equiv \gamma\alpha_2 / \beta_2$.

Industry outputs are written as:

$$y_1 = L_1 (k_1)^{\alpha_1}, \quad y_2 = L_2 (k_2)^{\alpha_2}. \tag{34}$$

Thus,

$$\frac{y_1}{y_2} = \frac{L_1}{L_2} \frac{\beta_2}{\beta_1} p = \frac{K_1}{K_2} \frac{\alpha_2}{\alpha_1} p. \tag{35}$$

Using (34) and the first equality in (35), we get

$$\frac{\beta_2}{\beta_1} p = k_1^{\alpha_1} k_2^{-\alpha_2}.$$

Using (29) and (33), we have

$$p = (\beta_1 / \beta_2) k_1^{\alpha_1 - \alpha_2} \left(\frac{\beta_1 \alpha_2}{\alpha_1 \beta_2} \right)^{-\alpha_2} .$$

$$p = k_1^{\alpha_1-\alpha_2} (\beta_1 / \beta_2)^{\beta_2} (\alpha_1 / \alpha_2)^{\alpha_2} = (\sigma_1 K / L)^{\alpha_1-\alpha_2} (\beta_1 / \beta_2)^{\beta_2} (\alpha_1 / \alpha_2)^{\alpha_2} = \omega(\sigma_1 K / L)^{\alpha_1-\alpha_2} . \quad (36)$$

where $\omega \equiv (\beta_1 / \beta_2)^{\beta_2} (\alpha_1 \alpha_2)^{\alpha_2} > 0.$ Note that *p* is a concave function of *L*, but a convex function of *K*. That is, an increase in *L* increases *p* at a decreasing rate, while an increase in *K* lowers *p* at a decreasing rate. Note that since $\alpha_2 > \alpha_1,$ in (36) autarky price p is a decreasing function of (K / L) as shown in Proposition 2. Also, a proportionate increase in capital and labor inputs has no effect on the terms of trade.

While the calculation is tedious, expressions for the wage and rent can be obtained from (36). First, note that $k_1^{\alpha_1-\alpha_2} = p(\beta_1 / \beta_2)^{-\beta_2} (\alpha_1 / \alpha_2)^{-\alpha_2}$. Thus,

$$k_1 = p^{-1/(\alpha_2-\alpha_1)} (\beta_1 / \beta_2)^{\beta_2/(\alpha_2-\alpha_1)} (\alpha_1 / \alpha_2)^{\alpha_2/(\alpha_2-\alpha_1)}. \quad (37)$$

From (34), $w = \beta_1 p_1 k_1^{\alpha_1}$, or

$$w = p^{-\alpha_1/(\alpha_2-\alpha_1)} \phi_w, \quad (38)$$

where $\phi_w \equiv \beta_1 (\beta_1 / \beta_2)^{\alpha_1\beta_2/(\alpha_2-\alpha_1)} (\alpha_1 / \alpha_2)^{\alpha_1\alpha_2/(\alpha_2-\alpha_1)} > 0.$ This illustrates the Stolper-Samuelson theorem. If industry 2 is capital intensive ($\alpha_2 > \alpha_1$), an increase in the price of the capital-intensive good lowers *w*. Differentiating (38) with respect to *p* twice yields:

$$\frac{\partial w}{\partial p} = -\frac{\alpha_1 \phi_w}{\alpha_2 - \alpha_1} p^{-\alpha_2/(\alpha_2-\alpha_1)} < 0,$$
$$\frac{\partial^2 w}{\partial p^2} = \frac{\alpha_1 \alpha_2 \phi_w}{(\alpha_2 - \alpha_1)^2} p^{\alpha_1 - 2\alpha_2} > 0.$$

This implies that the wage function $w = f(p)$ is convex in p. Jensen's inequality implies

$$w(p_o) = w\left(\mu p_A + (1-\mu) p_B\right) < \mu w(p_A) + (1-\mu) w(p_B). \tag{39}$$

Thus, in the Cobb-Douglas case, the equalized wage rate at the free trade equilibrium is less than the weighted average of autarky wage rates, using the same weights for the equalized price. That is, the equalized wage rate is closer to the wage of the low-wage or labor abundant country than the weights of the equalized output price suggest.

The expression for the autarky rent is given by $r = \alpha_1 p_1 k_1^{-\beta_1}$, or

$$r = p^{\beta_1/(\alpha_2-\alpha_1)} \phi_r, \tag{40}$$

where $\phi_r \equiv \alpha_1 (\alpha_1/\alpha_2)^{-\beta_1 \alpha_2/(\alpha_2-\alpha_1)} (\beta_1/\beta_2)^{-\beta_1 \beta_2/(\alpha_2-\alpha_1)} > 0.$ Differentiating (40) with respect to p twice, we have

$$\frac{\partial r}{\partial p} = \frac{\beta_1 \phi_r}{\alpha_2 - \alpha_1} p^{-\alpha_1/(\alpha_2-\alpha_1)} > 0,$$
$$\frac{\partial^2 r}{\partial p^2} = -\frac{\alpha_1 \beta_1}{\alpha_2 - \alpha_1} p^{-\alpha_2/(\alpha_2-\alpha_1)} < 0.$$

Thus, $r(p)$ is a concave function of p. By Jensen's inequality,

$$r(p_o) = r\left(\mu p_A + (1-\mu) p_B\right) > \mu r(p_A) + (1-\mu) r(p_B). \tag{41}$$

Thus, the equalized rent is greater than the weighted average of the autarky rents using the weights of the equalized output price for the Cobb-Douglas case. That is, the equalized rent is closer to the autarky rent of the labor-abundant country than the weights of the equalized output price suggest.

From (27), we obtain the weights for the equalized factor prices,

$$\lambda_A = \frac{r_B w_C - w_B r_C}{w_A r_B - r_A w_B}, \quad \lambda_B = \frac{w_A r_C - r_A w_C}{w_A r_B - r_A w_B}.$$

In the above example, $\lambda_B \cong 0.1566$ and $\lambda_A \cong 0.6226.$ The weights of the equalized output price are, $\mu_B \cong 0.2126$ and $\mu_A \cong 0.7814,$ which add up to unity. Proposition 5 only predicts that $\lambda_A + \lambda_B < 1,$ and the Cobb-Douglas case confirms it.

Wage-Rent Ratio

From (32), the autarky wage-rent ratio of the South is given by $w_A / r_A = \gamma K_A / L_A.$ For the North, $w_B / r_B = \gamma K_B / L_B.$ Under free trade, the equalized wage-rent ratio is

$$\frac{w_o}{r_o} = \gamma \frac{K_A + K_B}{L_A + L_B}.$$

Then it can be shown that in the Cobb-Douglas case, the free trade wage-rent ratio is a weighted average of the autarky wage-rent ratios,

$$\frac{w_C}{r_C} = \frac{L_A}{L_A + L_B}\frac{w_A}{r_A} + \frac{L_B}{L_A + L_B}\frac{w_B}{r_B}. \tag{42}$$

where $L_A/(L_A + L_B)$ and $L_B/(L_A + L_B)$ are the labor shares of the South and the North, respectively. For example, when $(L_B, K_B) = (600, 2000)$ and $(L_A, K_A) = (150, 20000)$, China's population is four times the U.S. population but its capital stock is one-tenth of the U.S. capital stock. The autarky wage-rent ratio is 3.33 in China and 133.33 in the United States. The latter is 40 times that of the former. The equalized wage-rent ratio is 29.33. In determining the equilibrium output price, China's weight is only about 21%, but for the equalized wage-rent ratio, China's weight is 80%. That is, the equalized wage-rent ratio is closer to China's autarky level while the equalized output price is closer to America's autarky level.

Table 1. Equalized Factor Prices in the Cobb-Douglas Case

	US	China	World	weight
L	150	600	750	
K	20,000	2,000	22000	
α1	0.25	0.25	0.25	
α2	0.75	0.75	0.75	
β1	0.75	0.75	0.75	
β2	0.25	0.25	0.25	
ε	1	1	1	
A1	0.438691338	0.438691338	0.438691338	
B1	1.316074013	1.316074013	1.316074013	
θ	0.577350269	0.577350269	0.577350269	
q	0.577350269	0.577350269	0.577350269	
w/r	133.3333333	3.333333333	29.33333333	
s1	0.333333333	0.333333333	0.333333333	
σ2	3	3	3	
k1	44.44444444	1.111111111	9.777777778	
k2	400	10	88	
s1K/L	44.44444444	1.111111111	9.777777778	
p eq	0.08660254	0.547722558	0.184637236	0.212601259
p confirm	0.08660254	0.547722558	0.184637236	
p1	1	1	1	
w b1p1k1a1	1.936491673	0.770017572	1.326237501	0.523161355
r a1p1k1-b1	0.014523688	0.231005272	0.045212642	0.141762426
Λ			0.43615629	
λ			0.156577005	

*λ**			0.622605546
			0.779182552
y1A	290.473751	462.0105432	994.678126
y2A	3354.101966	843.5119878	5387.202197
p aut	0.08660254	0.547722558	0.184637236

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Endnotes

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[1] For extensions to higher dimensions, see for instance, Ethier (1984), Neary (1985), and Choi (2003).

[2] Due to inability to predict the commodity trade pattern in a world with more than two goods, some developed a fifth proposition, the Heckscher-Ohlin-Vanek theorem, which predicts that a country exports the services of its abundant facto through trade.

[3] For income convergence, see Rassekh and Thompson (1997) and Slaughter (1998).

[4] Even when factor prices respond to a change in *L*, as Henry Thompson pointes out, cost minimization implies $dR = wdL + (L_1 + L_2)dw + (K_1 + K_2)dr = wdL.$

[5] This result on autarky price was shown in Jones (1965). See also Batra (1973).

[6] It is well known that the cost function is concave in all factor prices, not just in *w* and *r*. Thus, the argument holds for many factor prices. A diagram is used only for illustrative purposes.

[7] These weights are in the Appendix.

[8] It does not matter which good is used as a numéraire because it is the convexity of factor price frontiers that drives the result.