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A Supply Chain Model with Reverse Information Exchange

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We develop a model and analyze reverse information sharing, a growing business practice in supply chain management in which a manufacturer shares information about supply with a retailer. We model the manufacturer as a production queue with finished goods warehouse, the retailer as an inventory location, and other customers as an external demand stream. In our model, the manufacturer allows the retailer access to inventory status at the warehouse. To take advantage of this new information, the retailer changes from a single-level base-stock policy to a two-level, state-dependent base-stock policy. We provide an exact method for computing performance and develop a procedure for evaluating optimal policy. We demonstrate the impact of the new policy on the manufacturer and other customers. Numerical computations lead to insights about the value of information to the retailer, and to guidelines for the manufacturer on sharing information.

Key words: information sharing; stochastic production-inventory system; state-dependent base stock; Markov chains
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1. Introduction

In recent years, retailers and manufacturers have shown increasing interest in cooperating to improve the performance of the supply chain and increase their gains. Sharing information has emerged as one of the most critical practices in improving the performance of supply chains. Most of the work on information sharing to date has focused on cases in which the retailer shares information about current demand with the manufacturer (for example, see Lee and Whang 1999, Cheung and Lee 2002, Chen 1998, Cachon and Fisher 2000, Moinzadeh 2002). A different but parallel trend calls for the manufacturer to share information with the retailer. In these instances, information flows in *reverse* order; that is, instead of the retailer providing information about its demand and inventory status to the supplier, the supplier provides the retailer with information about its inventory availability status. While the practice appears to be growing, its consequences have not been fully analyzed. Specifically, how can the retailer use this information to its benefit, and what is the impact of the practice on the performance of the manufacturer?

In this paper, we consider a supply chain in which the manufacturer shares information about current inventory with a retailer that satisfies external demand through inventory. The manufacturer supplies goods first come, first served (FCFS) to the retailer and other "walk-in" customers, using its finished-goods inventory. Finished-goods inventory is replenished through production. Standard holding and backorder costs apply at inventory locations, but no ordering/setup cost is incurred. At the time of ordering, the retailer can ask about the availability of stock at the manufacturer's finished-goods inventory. To take advantage of this information, the retailer uses a state-dependent base-stock policy. Our main goals are to develop an exact evaluation of this system, and to develop insights for both the retailer and the manufacturer.

The new trend toward reverse information sharing is driven by two factors: (i) availability of technological capability for information sharing, and (ii) increased customer awareness of information sharing in supply chains (and, as a consequence, increased customer pressure on the manufacturer to share information). An example of new technological capability is the customer portal available in J. D. Edwards's software system. The portal allows a retailer to log in to a manufacturer's system. After logging in, the retailer not only has the ability to check the status of its outstanding orders and its financial account information, but can also choose "view inventory availability" option to get information on the manufacturer's stock availability. Here and in other examples, the manufacturer chooses to allow access to its system, and controls the amount of information available to customers. These "customer portals" or similar features (the terminology is not standard) are now available from most large business software application providers. They are the result of a confluence of three technologies developed in recent years: enterprise resource planning (ERP) systems, customer relationship management (CRM) systems, and business-to-business (B2B) exchanges.

Valspar Corporation, a leading paint manufacturer, offers an example of the second factor driving reverse information sharing, increased pressure from customers. Valspar Corporation administers a questionnaire to any business seeking to become a regular supplier. One of the qualifying questions determines whether the supplier provides product availability information. This kind of pressure from customers is reflected by the supplier's sales force. Hard data is difficult to come by, but in an informal U.S. InfoTel survey, sales representatives said that they could make more sales if they were able to answer availability questions in a customer's office. The desire to answer such questions may be part of the reason for growth in salesforce automation software that allows sales representatives to share inventory information with customers (Colombo 1994). Pressure for sharing information may also come from third-party trading hubs, such as Exostar, that promise customers "improved access to product availability information" (Plyler and Shaw 2001).

Driven by the availability of software applications and working under competitive pressures to keep customers satisfied, many manufacturers and distributors already employ customer portals in their supply chains. One such example (Smith 2002) is Osmonics Inc., a Minnesota-based manufacturer of water purification systems serving soft drink manufacturers, bottled water companies, and original equipment manufacturers. Osmonics uses the SAP AG R/3 ERP system for its internal data management, and HAHT Commerce's HAHT Commerce Suite as an interface to share information with its customers. Many customers tap into the company's ERP data and have varying degrees of access and control, depending in part on the sophistication of the customer's own software systems. Among other features, customers using the system can check the status of orders and inventory. Another example (PeopleSoft 2002) is Sager Electronics, one of the largest electronics distributors in the United States. The company upgraded its PeopleSoft ERP system to provide password-protected access to customers so that they can easily complete their supply chain management functions. In addition, the system is designed to tailor the portal for different users, and incorporates content changes for each user. In the business of spare parts, where base-stock policies are often used (Nahmias 1981), Pratt and Whitney Canada provides information related to inventory availability to its customers once they register on the Internet.

In our abstract model of these business situations, we consider a manufacturer who shares some private supply information with certain customers. The manufacturer uses a base-stock policy to manage its finished-goods inventory, and its production process is modeled as a single-server queue. The manufacturer shares information about product availability at its warehouse with the retailer. Other customers, called walk-ins, do not have access to this manufacturer information. To take advantage of the product availability information, the retailer employs a state-dependent base-stock ordering policy. Under this policy, the retailer uses two different base-stock levels, each corresponding to whether or not the product is available at the manufacturer's warehouse at the time the order is placed. We use our model to demonstrate that this policy lets the retailer use the product availability information opportunistically, sneaking in larger orders when the product supply is unavailable. We develop insights on the magnitude of the retailer's cost reduction under various settings. We show that the retailer's use of the information results in a bullwhip effect in the supply chain and investigate its impact on the number of orders in the manufacturer's production system, as well as on the on-hand inventory and backorder levels in the finished-goods warehouse. We then extend our model to consider the case of two-way information exchange in which the manufacturer may know the retailer's current inventory position. Finally, we use examples based on our model to develop insights into the reasons manufacturers are sharing their private supply information. We conclude with a discussion about the future of such information sharing.

Our contributions lie in two primary areas. First, we extend the existing literature on inventory systems with supply information. Among the literature that incorporates supply conditions in ordering policy, Song and Zipkin (1996) is noteworthy. They study this issue in a periodic setting by modeling the supply system as an evolving Markov chain in which the retailer knows the current state of the chain before ordering in each period. Their work builds on a stream of research initiated by Kaplan (1970) for modeling stochastic lead times. In Kaplan (1970), lead-time distribution is known at the time of ordering. By design, however, it does not provide any information about the lead-time distribution at the next ordering opportunity. Nahmias (1979) and Zipkin (1986) explain how to construct a supply system that produces dependent lead times (orders do not cross) and yet provides no information for the next lead time. In Song and Zipkin (1996), the retailer knows the current state of the Markov chain, which not only leads to the current lead-time distribution, but also provides information about the lead-time distribution at the next ordering opportunity. Thus, the dependency between consecutive lead times is explicitly modeled by Markov chain's transition probabilities. The future state of the Markov chain, however, is independent of the ordering activities at any given time. That is, the transition probabilities are independent of the order size. Chen and Yu (2002) provide a new algorithm for the Song and Zipkin (1996) model, and analyze a situation in which the retailer can use lead-time history data to predict the next state, even when the current state is unknown.

The focus of these studies is on determining optimal policy in the periodic setting. In our work, we focus on developing a model that captures the impact of retailer ordering policy on the supply chain, and allows for exact analysis as well as the development of managerial insights. Most importantly, in our model we allow the orders placed by the retailer to influence the supply system. This relaxes the assumption made in the earlier studies that retailer orders do not influence the supply system. By explicitly capturing the retailer orders' impact on the state of the supply system, we find that the value of information for the retailer may be less than the value found in models in which the retailer's load is marginal to the system. We also identify situations in which the retailer would not benefit from using this information.

Our paper also makes a contribution to understanding the propagation of variability in supply chains. Lee et al. (1997) have suggested several reasons for the bullwhip effect in supply chains. One of these, the capacity-rationing game (see Cachon and Lariviere 1999), comes close to our situation. We demonstrate that reverse information sharing can actually create the bullwhip effect in the supply chain. While earlier work is driven by shortage of capacity in a single period, we show the effect in a dynamic model in which there is no shortage of capacity in the long term. In addition, Lee et al. suggest that in their model, the manufacturer sharing its inventory information with downstream members may prevent the bullwhip effect. In our model, however, it is the selective sharing of information that creates the bullwhip effect. We also present a rationale for the manufacturer to allow reverse information sharing.

In the next section, we describe our model and the notation. Section 3 is devoted to the development and the analysis of the continuous-time Markov chain (CTMC) describing the state of the system. In §4, we focus on the retailer's cost under the new policy, and provide a procedure to compute optimal policy parameters. We provide insights into the value of information for the retailer. Section 5 provides results on the manufacturer's performance measures in such systems. Finally, we summarize our main results and discuss the possible extensions to this research.

2. The Model

Consider a supply chain consisting of a manufacturer who supplies a finished good to a retailer and walk-in customers. Demand generated by the walk-in customers follows a Poisson process with rate λ_e (subscript e for external arrivals). Demand at the retailer is assumed to follow a Poisson process with rate λ_r. The retailer holds stock and satisfies its demand through its on-hand inventory, and we assume that excess demand at the retailer is backordered. To replenish its stock, the retailer places orders with the manufacturer. We assume that orders placed by either the retailer or walk-in customers are satisfied immediately if the manufacturer has stock; otherwise, they are delayed and served in an FCFS manner. The manufacturer manages its finished-goods inventory using a base-stock production policy. That is, for each order it receives from its customers, the manufacturer issues a production order. We model the manufacturer's production system as a single-server queue with exponentially distributed processing times with a mean of $1/\mu$. In addition, we assume that the order transit time from the manufacturer to the retailer is constant, T.

We assume that the manufacturer shares information about stock availability only with the retailer. At the time of order placement, the manufacturer informs the retailer whether the product is available in the manufacturer's finished-goods inventory. We acknowledge that manufacturers can share more detailed information. However, limiting ourselves to the simpler binary interpretation—stock is either available or unavailable—allows us to capture the main idea behind reverse information sharing while keeping the analysis tractable. Due to its limited scope, this interpretation of reverse information sharing may be more practical. For example, many online retailers, such as Amazon.com, share information on stock availability with their customers. Manufacturers may find it easier to overcome their natural reluctance to let customers see private information if the amount of that information is limited. Finally, this binary model of information sharing is equivalent to the manufacturer informing the retailer whether the product is backordered *before* the retailer places the order.

Because we observe differentiation between customers in practice, we classify the manufacturer's customers as retailers and walk-ins—those who have access to reverse information and those who do not. At Osmonics Inc. (Smith 2002), for example, "nearly 200 customers [are] actively accessing" its ERP data out of a total of "3,000 or so customers." Our modeling of walk-ins captures those customers who do not have access to information. Just like the retailer, walk-ins will carry inventory to satisfy their demand. In the absence of any supply information, however, the walk-ins will employ a standard one-for-one policy. Therefore, the walk-in order streams to the manufacturer will be exactly the same as their demand streams. It may be plausible that walk-ins will respond to any change in the supply chain by changing their base-stock level, but that would not affect their order stream. To keep matters simple, we do not explicitly model the inventory system for the walk-ins. Using the analysis presented here, incorporation of the inventory model at the walk-in level is straightforward. We believe, however, that it would not offer further insights. We should point out that if a walk-in customer learns by observing the dependence in the lead times and this results in a state-dependent ordering policy, it will be necessary to develop an explicit inventory model for such customers. We do not consider that case in the study.

The balance of the paper considers the impact of reverse information exchange from either the retailer's perspective or the manufacturer's perspective. In §4, we consider the retailer alone, and show that service received by walk-in customers may deteriorate as a result of the retailer's actions. Section 5 discusses the manufacturer's response to the retailer's policy. We consider only those manufacturer policies that leave the service received by walk-in customers unaffected, and so do not motivate a change in customer behavior.

2.1. Retailer's Policy

Let h and b denote the unit holding and backorder cost rates at the retailer. Assuming that the retailer's objective is to minimize the sum of average holding and backorder cost rates, we propose and discuss the form of the retailer's inventory policy. We present an exact formulation of the cost function in §4. Recall that the manufacturer allows the retailer access to information regarding stock availability at the manufacturer's finished-goods warehouse. So that the retailer can make use of this information, we propose a state-dependent base-stock retailer ordering policy as follows: When a demand occurs at the retailer, the retailer checks the availability of the stock at the manufacturer. If the manufacturer has stock available, it orders enough to bring its inventory position (on-hand + on-order − backorders) up to the base-stock level S_l. Otherwise, it orders enough to bring its inventory position up to the base-stock level S_u. Note that when the retailer's inventory position is equal to or greater than the target base-stock level, the retailer does not order. Under this scenario, the retailer's knowledge of the current state of the supply system is binary, and there is a base-stock level corresponding to each state. We will refer to this state-dependent base-stock policy as (S_l, S_u) policy.

Before we proceed, some discussion of the state-dependent base-stock policy is helpful. Previous research suggests that in settings such as ours, the choice of a state-dependent base-stock policy is a reasonable one: Our model can be formulated as a continuous-time Markov decision process, where the state transition probability is a function of the action taken (order size) by the retailer. In a periodic setting, with transition probabilities independent of the action, Song and Zipkin (1996) showed that the state-dependent base-stock policy is optimal. The dependence of transition probability on the form of the policy in our model makes it quite difficult to extend the standard optimality arguments. On an intuitive level, it is difficult to argue that our proposed policy may not perform well. We also follow the tradition of proposing simple-to-implement policies, especially in continuous inventory models. See, for example, Rubio and Wein (1996).

We assume that the retailer places an order only at its demand epochs, and accesses the availability information only at those times. Order placement at demand epochs, while not necessarily optimal (Moinzadeh 2001), is a common and reasonable assumption that has been widely used in many previous works (Axsäter 1990, Svoronos and Zipkin 1991). We also assume that the retailer knows the distribution of the manufacturer's processing time, its base-stock level and the total market demand. The retailer may have come to this knowledge based on historical data or by gathering business intelligence. This is akin to the retailer knowing the transition probabilities of the supply Markov chain in Song and Zipkin (1996) and in Chen and Yu (2002).

We assume that $S_l \leq S_u$. In studies of the periodic system, Song and Zipkin (1996) and Chen and Yu (2002) show that optimal state-dependent base-stock level is not necessarily nondecreasing in lead time. Based on their discussions and results, this counterintuitive phenomenon appears to occur when the lead time is not monotonically increasing in the state variable describing the supply system (see Theorem 4 of Song and Zipkin), and when lead time of zero is possible. These conditions do not hold in our setting. Thus, we have primarily focused on the case $S_l \leq S_u$. Our evaluation technique, however, also lends itself to the analysis of cases where $S_l > S_u$. (The appendix presents the case $S_l = S_u + 1$.) In all of the instances we evaluated (see §4.2), $S_l \leq S_u$ is always dominant.

The manufacturer receives an order stream from the retailer, and a second order stream, modeled as a Poisson process, from walk-in customers. If the retailer is using the information by following the proposed (S_l, S_u) policy, the retailer orders constitute a state-dependent bulk arrival process at the manufacturer. We assume that even though a retailer's order may consist of more than one unit, units in the same order do not wait for each other before being transported (i.e., partial shipment is allowed).

We assume that the manufacturer follows a base-stock production policy with base-stock level S_m to manage its finished-goods inventory. Thus, every unit demanded by a customer gives rise to a production order to replenish the inventory. Again, the production system is modeled as a single-server, FCFS queue with exponential service times with a mean rate of μ. Our assumption of exponential service time allows for tractability of analysis, and is commonly used in production and inventory literature.

The assumption of a base-stock policy for manufacturer finished-goods inventory is based on Sobel (1969), and Gavish and Graves (1980). In the absence of any fixed start-up or shutdown costs, but with standard holding and backorder cost rates, a base-stock policy is optimal when the order stream at the manufacturer is a renewal process. The assumption of renewal arrivals is not met when $S_l \neq S_u$. It can be argued that when $S_l \neq S_u$, the manufacturer may anticipate the retailer's ordering behavior and improve its current base-stock policy. We will explore such an improved policy for the manufacturer in §5, and show that the cost reduction for the manufacturer is miniscule. Here, we continue to assume a base-stock policy for the manufacturer.

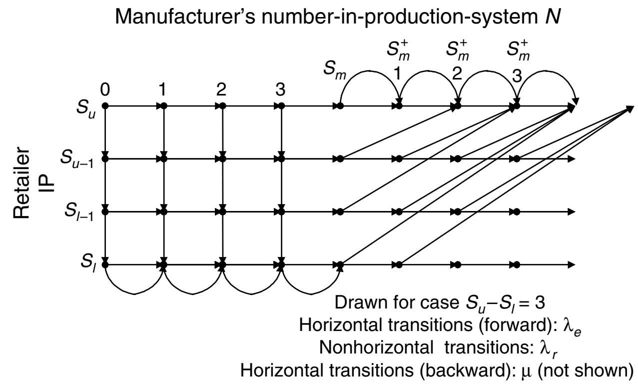
We close this section by defining all the relevant parameters in the system.

- λ_e Mean arrival rate of external walk-in customers
- λ_r Mean demand rate at the retailer
- μ Mean unit production rate of the manufacturer's production system
- ρ_e Load in the production system due to external arrivals $= \lambda_e/\mu$
- ρ_r Load in the production system due to retailer arrivals $= \lambda_r/\mu$
- ρ Overall load in the production system $= \rho_e + \rho_r$
- S_m Base-stock level for manufacturer's inventory in the finished-goods warehouse
- N Random variable representing the number of orders in the manufacturer's production system (also referred to as work in process)
- S_l Retailer's base-stock level when manufacturer has stock ($N < S_m$)
- S_u Retailer's base-stock level when manufacturer is out of stock ($N \geq S_m$)
- Δ $S_u - S_l$; $\Delta \geq 0$
- IP Random variable representing retailer's inventory position
- $\pi_{i,j}$ Steady-state probability of $\{IP = S_l + i,\ N = j\}$; $0 \leq i \leq \Delta$; $j \geq 0$
- h Unit holding cost/time
- b Retailer's unit backorder cost/time
- $C_{S_l,\Delta}$ Retailer's expected cost for policy parameters S_l and Δ
- $E[X]$ Expected value of random variable X

3. Analysis of the Reverse Information Sharing Model

In this section, we model the manufacturer's system as a CTMC and develop a solution for its steady-state probabilities. The state of the system is described by $\{IP, N\}$. The evolution of the system does not depend on the absolute values of the base-stock levels, S_l, S_u, but only on their difference Δ. Therefore, to formulate the Markov chain that describes the behavior of the system, we work with the equivalent state description $\{\omega, N\}$ where $\omega = IP - S_l$ $(0 \leq \omega \leq \Delta)$. This is the state of the system as seen by an external or retailer arrival (all assumed to be Poisson). The CTMC representing the system (see Figure 1) is as follows.

Figure 1 State-Transition-Rate Diagram of the Unrestricted Continuous-Time Markov Chain



We begin with describing the transitions between states in Figure 1. An external arrival (walk-in) will increase the number in the production system (N) by one, but will not influence the retailer's inventory position (ω). Therefore, all the horizontal forward transition arrows (in Figure 1) from state $\{i, j\} \rightarrow \{i, j+1\}$ represent a rate of λ_e. A service completion will also affect only the number in the production system, and not the retailer's inventory position. Horizontal backward transition arrows (not shown in the figures to avoid clutter) from state $\{i, j+1\} \rightarrow \{i, j\}$ represent a rate of μ. All the nonhorizontal transition arrows represent a demand arrival at the retailer with a rate of λ_r. We can classify the transitions initiated by retailer demand in the following three categories:

(a) The system is in state $\{i, j \mid i > 0,\ j < S_m\}$ and a retailer demand arrives; the retailer observes that the manufacturer's warehouse has stock and tries to achieve the base-stock level of S_l by not placing an order. This results in transition to the state $\{i-1, j\}$.

(b) The system is in state $\{i, j \mid i = 0,\ j < S_m\}$ and a retailer demand arrives; the retailer observes that the warehouse has stock and tries to achieve the base-stock level of S_l by placing an order of one unit. This results in transition to the state $\{i, j+1\}$.

(c) The system is in state $\{i, j \mid i \leq \Delta,\ j \geq S_m\}$ and a retailer demand arrives; the retailer observes that the warehouse has no stock and tries to achieve the base-stock level of S_u. Its current inventory position is $S_l + i - 1$, and therefore it places an order for

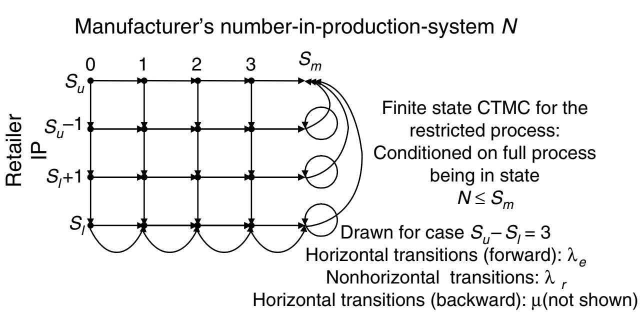
$\Delta-(i-1)$ units. This results in the transition to the state $\{\Delta, j+\Delta-(i-1)\}$.

We now develop a procedure for computing the steady-state joint probability mass function, π_{ij}, for this CTMC. The main idea is to exploit the distinct patterns in two different portions of the state-transition-rate diagram. We accomplish this by decomposing the Markov chain into two parts. The analysis proceeds in five steps. The first step defines how the process evolves over the reduced state space in one of the two parts—we call it the "restricted process." The complete Markov chain is called "unrestricted." The second step solves for the steady-state probabilities in the restricted process. The third and fourth steps take the analysis back to the full CTMC and solves for the steady-state probabilities for the rest of the states. The fifth step normalizes the state probabilities. We present the results of each step's analysis here. The details of the analysis are available in the appendix.

Step 1. Defining the Restricted Process. The restricted process captures the transitions between states with $N \le S_m$. We observe that whenever the restricted process is in states $\{i, S_m\}$, $0 \le i \le \Delta$, the next transition due to an external or retailer demand arrival can only result in the restricted process reverting either to the same state $\{i, S_m\}$, or to the state $\{\Delta, S_m\}$. This allows us to separate the states with $N \le S_m$ from those with $N > S_m$. Figure 2 presents this restricted process. The transition probabilities from states $\{i, S_m\} \to \{\Delta, S_m\}$ are developed in the appendix.

Step 2. Steady-State Probabilities of the Restricted Process. The next step is to develop the steady-state probabilities of the restricted process's Markov chain.

Figure 2 **State-Transition-Rate Diagram of the Restricted Continuous-Time Markov Chain**



Let z_1, z_2 $(z_1 > z_2)$ be the two roots of the characteristic equation $z^2-(1+\rho)z+\rho_e=0$. The following expressions summarize the steady-state probabilities of the unrestricted process (π_{ij}) in terms of $\pi_{\Delta 0}$:

$$\pi_{\Delta.j} = \left(\frac{\rho - z_2}{z_1 - z_2} z_1^j + \frac{z_1-\rho}{z_1-z_2} z_2^j\right)\pi_{\Delta 0} \quad \text{for } 0 \le j \le S_m. \quad (1)$$

$$\pi_{i.j} = z_2\pi_{i.j-1} + \rho_r \sum_{k=j}^{S_m-1} \frac{1}{z_1^{k-j+1}} \pi_{i-1.k} \quad \text{for } 0 < i < \Delta \text{ and } 1 \le j \le S_m \quad (2)$$

$$\pi_{i.0} = \frac{\rho_r}{\rho - z_2} \sum_{k=0}^{S_m-1} \frac{1}{z_1^k} \pi_{i-1.k} \quad \text{for } 0 < i < \Delta \quad (3)$$

$$\pi_{0.S_m} = \frac{\rho_r}{(z_1-1)} \sum_{k=0}^{S_m-1} \pi_{1.k} \quad (4)$$

$$\pi_{0.j} = \frac{1}{\rho}\pi_{0.j+1} + \frac{\rho_r}{\rho}\sum_{k=0}^{j}\pi_{1.k} \quad \text{for } 0 \le j \le S_m - 1. \quad (5)$$

With the sequential application of (1) to (5), all steady-state probabilities $\pi_{i.j}$, $0 \le i \le \Delta$, $0 \le j \le S_m$ can be directly expressed as multiples of $\pi_{\Delta.0}$. We do not present the complete expressions here, as they are cumbersome and add nothing to our understanding.

Step 3. Steady-State Probabilities for States with $\omega < \Delta$, $N > S_m$. We now turn back to the complete CTMC. For states with $\omega < \Delta$, balance equations for any state involve only the neighboring states. This allows us to find a simple solution to the balance equations as following:

$$\pi_{i.j} = \pi_{i.S_m} z_2^{j-S_m} \quad \text{for } 0 \le i < \Delta;\ j \ge S_m. \quad (6)$$

Step 4. Steady-State Probabilities for States with $\omega = \Delta$; $N > S_m$. Next, we present equations that allow us to express the steady-state probabilities of the states with $\omega = \Delta$; $N > S_m$.

$$\pi_{\Delta.j} = \rho\pi_{\Delta.j-1} + (\rho - z_2)\sum_{k=0}^{\Delta-1}\pi_{k.S_m} z_2^{[j-S_m-1-\Delta+k]^+} \quad \text{for } j \ge S_m+1. \quad (7)$$

At this point, we present the confirmation that our reduction of the unrestricted chain into a restricted chain is indeed valid, and yields relationships that provide solutions for the unrestricted chain.

Theorem 1. *Equations* (1)–(7) *provide a solution to the unrestricted CTMC.*

Proof. See the appendix.

Step 5. Determining $\pi_{\Delta.0}$. Equations (1) to (7) allow us to write all probabilities $\pi_{i.j}$ in terms of $\pi_{\Delta.0}$. We can now take the traditional normalization step of equating the sum of probabilities ($\sum_{i=0}^{\Delta}\sum_{j=0}^{\infty}\pi_{i.j}$) to 1 and determining $\pi_{\Delta.0}$. The following theorem provides a simplification of this step by showing that the sum of probability masses at states with $N=0$ equals $1-\rho$. Even though our model represents a queuing system that is controlled by the state-dependent actions of the retailer, the following relationship is similar to the one observed in a simple uncontrolled single-server queuing system, such as an M/M/1 queue.

Theorem 2. $\sum_{i=0}^{\Delta}\pi_{i.0}=1-\rho$.

Proof. See the appendix.

4. Retailer's Performance Measures

In this section, we determine the optimal policy and cost for the retailer under reverse information sharing. We also present computational results that provide managerial insights into the value of this information for the retailer.

4.1. Retailer's Optimal Policy and Cost

To develop retailer's cost, we follow the unit-costing convention where each unit ordered by the retailer is assigned a cost incurred for satisfying the corresponding demand. Let $C[i,j,k]$ denote the expected cost assigned to the kth unit ordered when $\omega=i$ and $N=j$. Let $C[i,j]$ be the expected total cost assigned to an ordering epoch when the state is $\omega=i$ and $N=j$.

$$C[i,j]=\sum_{k=1}^{\Delta-i+1}C[i,j,k] \quad \text{for } j\ge S_m$$

$$C[i,j]=C[i,j,1] \quad \text{for } i=0;\ j<S_m$$

$$C[i,j]=0 \quad \text{for } i>0;\ j<S_m$$

We can write the average total cost per time unit for the retailer as $C_{S_l.\Delta}=\lambda_r\sum_{i=0}^{\Delta}\sum_{j=0}^{\infty}\pi_{i.j}C[i,j]$.

The average cost assigned to each unit ordered, $C[i,j,k]$, depends on the time difference between the arrival of the unit at the retailer and the occurrence of the corresponding demand that will be satisfied by this unit. For $j<S_m$, the ordered unit will arrive after the transportation time T. When $j\ge S_m$, in addition to the fixed transportation time T, the ordered units will each experience a delay at the manufacturer as the manufacturer is out of stock at the time the retailer placed the order. In such cases, the delay experienced by the kth unit in the order will be equal to the time until $j-S_m+k$ units have been processed in the production system. In either case, the corresponding demand, which will be satisfied by the kth unit in the order, will arrive after $S_l+i+k-1$ subsequent demands.

First, consider the case $j\ge S_m$. Let $u=S_l+i+k-1$ and $v=j-S_m+k$. The random variable representing the time until the corresponding demand occurs is Erlang(λ_r,u), and time until supply is available is Erlang(μ,v). Note that Erlang(α,w) represents a random variable that is equal to the sum of w exponentially distributed random variables, each with rate α. Now

$$\begin{aligned}C[i,j,k]&=hE[\text{Erlang}(\lambda_r,u)-T-\text{Erlang}(\mu,v)]^+\\&\quad+bE[T+\text{Erlang}(\mu,v)-\text{Erlang}(\lambda_r,u)]^+\\&=(h+b)E[\text{Erlang}(\lambda_r,u)-T-\text{Erlang}(\mu,v)]^+\\&\quad-b\left[\frac{u}{\lambda_r}-T-\frac{v}{\mu}\right],\end{aligned}$$

where $[x]^+=\max(0,x)$. For the sake of completeness, when $u\le 0$, the cost function is given by

$$C[i,j,k]=b\left[T+\frac{v}{\mu}+\frac{u}{\lambda_r}\right].$$

Next, consider the case $j<S_m$. Clearly, for $1\le i\le\Delta$, the retailer does not place an order, and therefore the cost assigned to the order epoch is zero. For $i=0$, the retailer always orders one unit. Because the material is available in the warehouse, the lead time for this order is always T, and therefore its corresponding average cost is always the same. Thus, $C[i,j,1]$ for $i=0$; $j<S_m$ has the same value; let us call it $C[0,0,1]$, which can be easily computed as

$$C[0,0,1]=(h+b)E[\text{Erlang}(\lambda_r,S_l)-T]^+-b\left[\frac{S_l}{\lambda_r}-T\right] \quad \text{for } 0\le j<S_m.$$

The computational burden mainly consists of computing $C[i,j,k]$ values. The following observation helps us ease this burden: $C[i,j,k]=C[i+k-1,j+k-1,1]$. We only need to evaluate $C[i,j,1]$ once for $j<S_m$,

and once for each state $\{i, j\}$, $j \geq S_m$. For the probabilities $\pi_{i.j}$ required for computing $C_{S_l.\Delta}$, we only need to know the values of $\pi_{i.S_m}$, $0 \leq i \leq \Delta$; because $\sum_{j=0}^{S_m-1} \pi_{0.j}$, as well as all $\pi_{i.j}$, $0 \leq i \leq \Delta$, $j \geq S_m + 1$, can be directly expressed in terms of $\pi_{i.S_m}$, $0 \leq i \leq \Delta$. See proof of Theorem 2 for the expression of $\sum_{j=0}^{S_m-1} \pi_{0.j}$.

For a given manufacturer base-stock S_m, and retailer's ordering policy (S_l, S_u), $\Delta = S_u - S_l$, the retailer cost rate is denoted by $C_{S_l.\Delta}$. We now prove a property of $C_{S_l.\Delta}$.

Theorem 3. *For a given Δ and S_m, $C_{S_l.\Delta}$ is convex in S_l.*

Proof. See the appendix.

This result allows us to efficiently search for the optimal $S_l^*(\Delta)$ for a given Δ. An observation based on extensive computations further simplifies the search for optimal policy: $S_l^*(\Delta)$ is nonincreasing in Δ. The algorithm we present below arrived at the optimal policy parameters (confirmed by an exhaustive search) in all the cases in extensive experimentation.

Step 1. Set $\Delta = 0$, compute $S_l^*(0)$ and $C_{S_l^*(0),0}$.

Step 2. $\Delta = \Delta + 1$. Find $S_l^*(\Delta)$ as follows:

2(a) Set $S_l^*(\Delta) = S_l^*(\Delta - 1)$, compute $C_{S_l^*(\Delta),\Delta}$

2(b) $S_l^*(\Delta) = S_l^*(\Delta) - 1$, If $C_{S_l^*(\Delta),\Delta} < C_{S_l^*(\Delta)+1,\Delta}$, go to Step 2(b) else, $S_l^*(\Delta) = S_l^*(\Delta) + 1$, go to Step 3.

Step 3. If $C_{S_l^*(\Delta),\Delta} < C_{S_l^*(\Delta-1),\Delta-1}$, go to Step 2, else $\Delta^* = \Delta - 1$, stop.

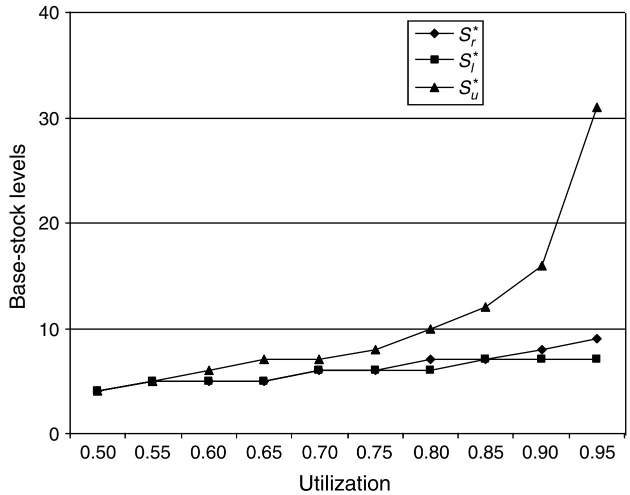
Finally, it is evident intuitively that the retailer can be no worse off using the information through a state-dependent base-stock policy. If current information suggests a congested supply system, the retailer has an opportunity to order extra before the external arrivals generated by walk-ins create further congestion. The retailer uses this information to opportunistically sneak in orders for extra units, and thus avoids possibly longer lead times. As we discuss in the next section, however, we have identified instances when the retailer continues to use the single base-stock policy after it obtains access to current information.

4.2. Computational Results for the Retailer

In this section, we study the impact of reverse information sharing on the retailer, using a numerical experiment. We set the mean service rate, μ, and holding cost rate, h, to unity. Then, we vary the overall system utilization, ρ; the fraction of retailer load in the total utilization, ρ_r/ρ, the transportation time, T; and the backorder cost rate at the retailer, b. The values of the manufacturer's base-stock level, S_m, are set to provide the service level, α_m, under the case $S_l = S_u$. The values of T and b are chosen relative to μ and h, respectively. We report our observations based on computing the optimal policy and corresponding performance measures for a set of 216 problem instances consisting of the following parameter values: $\rho \in \{0.7, 0.8, 0.9\}$, $\rho_r/\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $\alpha_m \in \{0.7, 0.9, 0.95\}$, $T \in \{1, 5\}$, $b \in \{7/3, 9, 19\}$. We chose these values to reflect the range found in practice. For utilization ρ, the historical data in the Federal Reserve Statistical Release (2004) suggests that our chosen range represents a wide variety of industries. For relative retailer size ρ_r/ρ, the chosen values represent a wide range of possibilities. In the context of our model, what matters is the *relative* size ρ_r/ρ because the load imposed by a large retailer may constitute either a large or a small portion of the manufacturer's total load. For example, a large retailer such as Wal-Mart is a major customer to Procter and Gamble, but it makes up only about 10% of Procter and Gamble's dollar revenue. For a smaller manufacturer, on the other hand, Wal-Mart may be responsible for a larger portion of total load. To be complete, we focus on a wide range of ρ_r/ρ values in our numerical experiment. For manufacturer's service level, α_m, we also consider a wide range of values, as previous work has reported and used service levels as low as 0.5 (Ernst and Cohen 1992) and as high as 0.99 (Cachon and Fisher 1997). The values of b have been chosen to represent the same range in the newsboy service levels for the retailer.

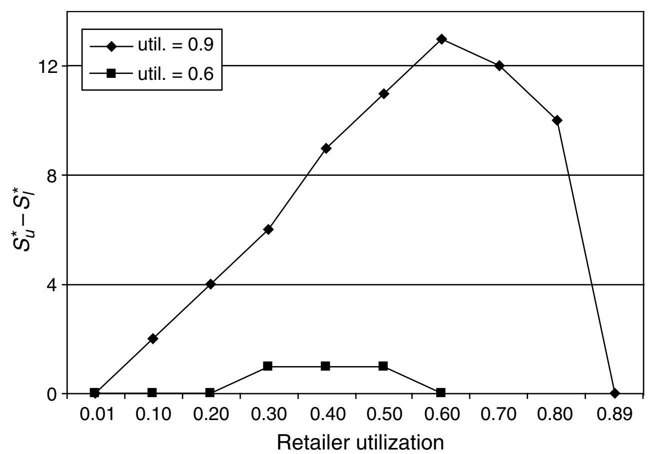
4.2.1. Retailer's Optimal Policy Parameters. We report our observations by presenting results for a specific combination of parameters. However, these observations hold for all problem instances we have described previously. Figure 3 depicts the optimal policy parameters (S_l^*, S_u^*) as well as S_r^*, the optimal base-stock level if the retailer does not use the information, as the overall utilization, ρ, is varied, while the ratio ρ_r/ρ is fixed. Note that while S_l^* tracks S_r^* quite closely, S_u^* increases as ρ increases. We can intuitively explain this behavior by considering the roles of each of the two base-stock levels S_l^* and S_u^*. We suggest that S_l^* is mainly determined by considering the

Figure 3 **Optimal Policy Parameters vs. Utilization ρ**



Note. $\rho_r/\rho = 0.5$; $T = 10$; $\alpha_m = 0.9$; $b = 9$.

Figure 4 **Optimal Policy Spread vs. Retailer Utilization ρ_r**



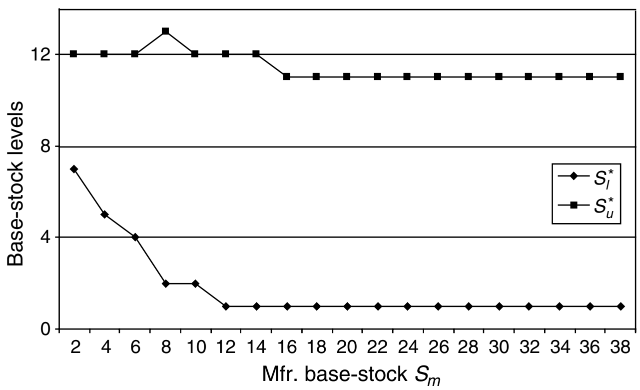
Note. Total utilization (util.) is kept constant at ρ; ρ_r changes. $T = 1$; $\alpha_m = 0.9$; $b = 9$; $\rho_e = \rho - \rho_r$.

holding and backorder cost trade-off at the retailer. Therefore, S_l^* behaves in a manner similar to S_r^*; it increases as the congestion grows, which results in a deterioration of the manufacturer's service. On the other hand, S_u^* is the base-stock level the retailer wants to achieve after it sees a stockout. It depends on the retailer's estimation of future lead times. Once a stockout has occurred, all retailer orders join the queue and future lead times depend on the utilization of the queueing system. As the congestion at the manufacturer grows, lead times increase, resulting in an increase in S_u^*.

Figure 3 shows that there are cases in which all three optimal base-stock levels are the same. That is, the retailer continues to operate a single-level base-stock policy ($\Delta^* = 0$) even when reverse information is available. Clearly, when $\Delta^* = 0$, the retailer does not benefit from the available information. To help us further investigate the circumstances that lead to $\Delta^* = 0$, Figure 4 presents Δ^* as the retailer utilization, ρ_r, changes for a given overall utilization ρ ($\rho = 0.6, 0.9$). For $\rho = 0.9$, $\Delta^* = 0$ occurs only at the extremely low and high values of ρ_r. For $\rho = 0.6$, the retailer seldom adopts the two-level base-stock policy at all. This leads us to two observations. First, for a given ρ, if the retailer makes up either a very small or a very large portion of the overall load, reverse information sharing is not beneficial to the retailer. Second, the range of relative retailer load ρ_r/ρ over which it benefits from reverse information is smaller for a smaller ρ.

The retailer benefits from information about stock availability when it signals that a stockout is likely at the next ordering epoch and ordering a large batch enables the retailer to preempt orders from other customers. When the retailer is a very small part of the system, intuition suggests that current stock availability will not be a strong predictor of a stockout at the next ordering opportunity. This is because during the relatively long period between the retailer's orders, activity at the manufacturer such as walk-in arrivals and production completions is likely to be significant. Information about stock availability, then, is not a reliable predictor of stockout at the next ordering opportunity, and opportunistic ordering is not likely to help the retailer. When the retailer is a very large part of the system, its knowledge of its own backorders closely mimics the state of the manufacturer's systems. Therefore, information about stock availability adds little to the retailer's knowledge of the system. In addition, ordering a large batch does not have much preemptive value due to low external traffic. This argues that opportunistic ordering will not be beneficial. Indeed, if there are no external arrivals, the retailer will not see any benefit at all. Finally, when overall utilization is low, knowing that the system may have a stockout at the next ordering opportunity is not of much use because low utilization implies that

Figure 5 Retailer Optimal Policy vs. Manufacturer Base-Stock S_m



Note. $\rho = 0.9$; $\rho_r/\rho = 0.5$; $T = 1$; $b = 9$.

the manufacturer will most likely resolve its backorders shortly. These arguments suggest that before a retailer pressures a manufacturer to share reverse information, it should first make an assessment, based on its relative load, of whether it will see any benefit.

To understand the retailer's policy with respect to changes in S_m, Figure 5 provides an example of the retailer optimal policy parameters for different values of S_m for a specific set of parameter values. Two observations can be made. First, at low values of S_m, a marginal increase in S_m often results in decreasing S_l^*. As pointed out earlier, S_l^* is primarily concerned with the marginal cost trade-off at the retailer, and its behavior is similar to a single base-stock level S_r^*. As S_m increases, the manufacturer's stockout probability decreases, which in turn results in lower optimal retailer base-stock levels. As the manufacturer's service level increases, further reduction in the retailer's base-stock level is only marginally better for the retailer, and therefore S_l^* does not change significantly. Our second observation about Figure 5 is that at moderate or high values of S_m, the optimal policy parameter S_u^* is relatively insensitive to S_m. As pointed out earlier, S_u^* depends on lead times after a stockout, which in turn depend on the utilization of the queuing system. While S_m may have an initial impact by influencing the number of orders in queue at the time of stockout, it does not influence the system utilization and has limited impact on future lead times and S_u^*. We note that the range of S_m over which the retailer optimal policy remains undisturbed corresponds to a range of reasonable service levels α_m. For example, $S_m = 21 \Leftrightarrow \alpha_m = 0.9$ and $S_m = 11 \Leftrightarrow \alpha_m = 0.7$.

4.2.2. Value of Information for the Retailer. In this section, we look at the magnitude of the benefits and costs of reverse information sharing for the retailer. We report the percentage reduction in the retailer's optimal cost as a result of employing reverse information. When the retailer has no information, it uses a single base-stock level, that is, $S_l = S_u$. The percentage reduction in retailer cost (value of information), averaged across all cases with a fixed parameter value, is presented in the second column of Table 1. As a result of reverse information sharing, we observe a reduction in inventory costs at the retailer. The average percent cost reduction across all cases is 9.51%. In our numerical experiment, the minimum cost reduction is none (e.g., $\rho = 0.7$, $\rho_r/\rho = 0.2$, $\alpha_m = 0.9$, $T = 1$, $b = 9$), and the maximum cost reduction is 46.82% (e.g., $\rho = 0.9$, $\rho_r/\rho = 0.4$, $\alpha_m = 0.9$, $T = 1$, $b = 19$).

In Table 1, average percent cost reduction increases as the backorder cost rate for the retailer increases. Considering individual cases, all but 3 out of 72 comparisons between $b = 7/3$ and $b = 9$, and all but 4 out of 72 comparisons between $b = 9$ and $b = 19$, show an increase in percentage cost reduction. Our explanation for the few anomalies lies in the discrete nature of the units of goods. Discreteness constraints on optimal decisions may result in higher cost than the unconstrained optimal. The cost increase may not be the same with or without the reverse information. The difference in cost increases may result in the observed anomalies. The dominant effect is that the value of information increases with backorder cost. This follows intuition, because the main benefit of opportunistic ordering for retailers is reducing the possibility of backorders and backorder costs.

The value of information for the retailer is also greatly influenced by the ratio ρ_r/ρ. For averages across cases in Table 1, as well as for each individual case, the percent cost reduction first increases and then decreases with the increase in ρ_r/ρ. This suggests that, similar to the behavior of Δ^* in Figure 4, the value of information increases with ρ_r/ρ at lower values of ρ_r/ρ, and the trend is reversed as ρ_r/ρ gets larger. The implication here is that information is more valuable to the retailer when its demand is neither a very small nor a very large part of the overall system demand. Furthermore, we observe that the average percent cost reduction at the retailer increases

Table 1 Benefits and Costs of Reverse Information Sharing

Fixed parameter and value	Average % decrease in retailer cost	Average % increase in $E[N]$	Average % decrease in $E[S_m - N]^+$	Average % increase in $E[N - S_m]^+$	Average % increase in walk-in backorders	Avg. Δ^*
$b = 19$	14.63	21.50	4.32	125.76	46.80	6.00
$b = 9$	9.95	15.06	3.07	83.21	32.17	4.36
$b = 7/3$	3.97	5.27	0.99	29.49	11.78	1.83
$\rho = 0.7$	3.52	8.93	1.42	51.17	11.49	1.40
$\rho = 0.8$	8.05	13.82	2.74	76.74	24.94	2.96
$\rho = 0.9$	16.98	19.08	4.23	110.56	54.32	7.83
$\rho_r/\rho = 0.2$	11.90	5.47	1.15	28.78	11.54	1.93
$\rho_r/\rho = 0.4$	12.97	13.30	2.79	71.72	27.81	3.89
$\rho_r/\rho = 0.6$	9.55	20.70	4.26	113.55	43.80	5.50
$\rho_r/\rho = 0.8$	3.65	16.30	2.98	103.89	37.86	4.94
$T = 1$	11.89	15.23	3.06	86.68	32.76	4.29
$T = 5$	7.15	12.65	2.53	72.30	27.73	3.83
$\alpha_m = 0.7$	9.45	22.47	5.71	59.48	21.25	3.51
$\alpha_m = 0.9$	9.67	12.56	1.90	86.84	32.23	4.26
$\alpha_m = 0.95$	9.43	6.78	0.78	92.15	37.29	4.42

as the congestion at the manufacturer increases. On a case-by-case basis, only 4 out of 72 comparisons between $\rho = 0.7$ and $\rho = 0.8$, and none of the comparisons between $\rho = 0.8$ and $\rho = 0.9$, show a decrease in percentage cost reduction. We attribute these few anomalous cases to the discreteness constraint on the optimal policy decision.

Finally, we discern from Table 1 that in cases in which the transportation time, T, is long compared to the production times, information sharing has less value for the retailer. This observation holds for all cases we consider. Because the transportation time is known and deterministic, the value of knowing about supply uncertainty and order delay at the manufacturer is less when this knowledge benefits a smaller fraction of the total lead time (order delay + transportation). Similar to Figure 5, the value of the manufacturer service level does not appear to have a great influence on the average percent cost reduction for the retailer.

5. Manufacturer's Performance Measures

In this section, we take the manufacturer's perspective. We begin by discussing the impact of the retailer's state-dependent policy on the supply chain. Next, we propose and evaluate an improved production policy for the manufacturer. Finally, we discuss why such information sharing may make economic sense for the manufacturer.

5.1. Variability Propagation in the Supply Chain

We focus first on understanding the impact on the manufacturer's performance measures of a retailer who follows a state-dependent base-stock ordering policy. Under a state-dependent ordering policy, the retailer's ordering does not necessarily follow a Poisson process. Because the retailer orders in batches at certain demand epochs while at other times orders nothing at all, the order stream experienced by the manufacturer is more variable than the demand experienced by the retailer. Thus, sharing supply information with the retailer can result in a bullwhip effect in the supply chain. This bullwhip effect is of no concern to the retailer because the opportunistic ordering that creates the effect also ensures that the retailer benefits from it. The question is how this affects the manufacturer and walk-in customers.

To understand the impact of the bullwhip effect, we compare the manufacturer's performance measures under two cases: (i) the retailer uses the information and employs the two-level policy (superscript *RI* for reverse information), and (ii) the retailer uses no information and employs a single-level policy (superscript *NI* for no-information). Rather than focus on a single cost measure, we examine three performance measures at the manufacturer. These are: (i) the average work in process in the production system, $E[N]$, (ii) the average finished-goods on-hand inventory, $E[S_m - N]^+$, and (iii) the average backorders at the finished-goods warehouse, $E[N - S_m]^+$. Here we assume that the manufacturer does not react to the retailer's new policy, and continues to follow the base-stock policy with the same base-stock S_m to manage its finished-goods stock. In §5.3, we assume the manufacturer modifies its policy in response to the retailer's state-dependent policy. We find that the manufacturer gains only marginally from changing its policy, which implies that the analysis in this section is valid.

The following shows that as a direct consequence of the retailer following the state-dependent ordering policy, the number in the production system is stochastically larger in the reverse information sharing system.

Theorem 4. $N^{RI} \geq_{st} N^{NI}$.

Proof. See the appendix.

When the retailer observes that no stock is available at the manufacturer's warehouse, that is, the production system is congested, it takes the opportunity to place a larger order, and thus further increases the congestion. This suggests that the larger the maximum order size, as represented by Δ, the larger the disruption in the supply system created by the retailer's new policy. The immediate implication of Theorem 4 is that for a given base-stock level at the manufacturer, both the average work in process $E[N]$ and average backorders $E[N-S_m]^+$ will increase when the retailer employs the state-dependent base-stock policy.

This has direct implications for the performance measures of walk-in customers. A randomly arriving walk-in customer will have to wait for an average of $E[N-S_m]^+$ service completions before being serviced. Therefore, average manufacturer lead time for walk-in customers is $(E[N-S_m]^+ + 1)/\mu$ and average number of walk-in backorders is $(\lambda_e(E[N - S_m]^+ + 1))/\mu$. As $E[N - S_m]^+$ increases, the average walk-in backorders also increase. Any benefit realized by the retailer will come at the expense of walk-in customers.

This result does not explicitly provide an understanding of the reverse information's effect on the manufacturer's average on-hand finished-goods inventory. In the next proposition, we show that the manufacturer's average on-hand inventory decreases when the retailer employs the state-dependent policy.

Proposition 5. $E[S_m - N^{RI}]^+ \leq E[S_m - N^{NI}]^+$.

Proof. See the appendix.

At first glance, the decrease in the manufacturer's on-hand inventory may not appear intuitive. Using reverse information, the retailer places orders earlier than it would if it had no information. This suggests that under reverse information, whenever a unit is produced, a retailer order is more likely to be waiting for it. The length of time on-hand inventory is zero and the length of time a unit of inventory resides in the manufacturer's stock may be shorter using reverse information. Thus, the manufacturer will see its finished-goods on-hand inventory levels decrease while its work in process and backorders increase. Depending on the manufacturer's cost structure, it is conceivable that cost may decrease. For example, in cases in which the manufacturer may not be charged an *explicit* backorder cost, the manufacturer's cost may decrease. This is because the finished-goods holding cost is likely to exceed the work-in-process holding cost, and a decrease in the former cost may trump an increase in the latter. Note, however, that even in such cases, the increase in the manufacturer's average backorder levels will result in an increase in average lead time for walk-in customers. In the next section, we look at performance measures for all parties in the supply chain.

5.2. Computational Results for the Manufacturer's Performance Measures

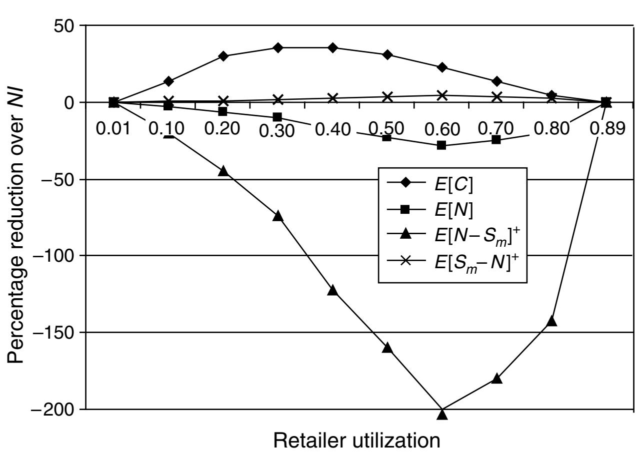
Using a numerical experiment, we now study the behavior of average walk-in backorders, and the three performance measures described above as we move from no information sharing to reverse information sharing. In addition, we report changes in Δ^* as a measure of variability caused by the retailer's two-level policy. Table 1 reports the average % increase/decrease in the performance measures discussed across all system parameters in our numerical experiment in §4.2 when one of the parameters is fixed.

Looking at Table 1, our first observation concerns the magnitude of the effects we described in the last section. The retailer benefits by reverse information sharing while walk-in customers see degradations in its performance measure. The manufacturer sees a percentage increase in its work in process, which is numerically larger than the reduction it sees in its finished-goods on-hand inventory. From the manufacturer's perspective, the retailer's push to share information results in degradation in the service walk-in customers receive. The outcome, whether information is shared or not, reflects a balance between these two observations. Clearly, our model does not capture all of the relevant aspects of the business situation that may influence the outcome, such as the power of the retailer versus the power of the walk-in customer, and the profit function of the manufacturer. In the context of our model, however, we can discuss how various parameters may affect the outcome.

Our second observation is that, as ρ_r/ρ increases, the percent increase in $E[N]$, percent decrease in $E[S_m - N]^+$, and percent increase in $E[N - S_m]^+$ all get larger at first, and then decrease. Thus, the improvement in the retailer's performance measure and the degradation in walk-in customers' performance measure follow the same broad pattern.

To further analyze this observation, we study the percentage changes in the performance measures described above as ρ_r changes for a specific set of parameter values (see Figure 6). In Figure 6, we observe that, while the retailer's maximum cost savings occurs at lower values of ρ_r/ρ ($\rho_r = 0.2$), the maximum disruption imposed on the rest of the system occurs at higher values of ρ_r/ρ ($\rho_r = 0.6$). This suggests that, in the long run, one is more likely to see an information-sharing outcome when the retailer's demand is a smaller share of the total system (say, ρ_r of 0.2 versus 0.6 in the Figure). This is because at $\rho_r = 0.2$, the retailer and the walk-in customers are both better off than at $\rho_r = 0.6$. Such observations suggest a testable hypothesis: It is more likely that only a small share of the manufacturer's total load is privy to the reverse information. Of course, this hypothesis will only be borne out to the extent that the dynamics analyzed in this model matter in practice versus other factors.

Figure 6 % Change over *NI* Scenario vs. Retailer Utilization ρ_r



Note. $\rho = 0.9$; $T = 1$; $\alpha_m = 0.9$; $b = 9$; $\rho_e = \rho - \rho_r$.

5.3. An Improved Manufacturer's Policy

Thus far we have considered a manufacturer following a base-stock policy. In cases where the retailer does not use the information (i.e., $S_l = S_u$), this policy is indeed optimal (Sobel 1969). More recently, Ha (1997) pointed out that for a single-product production-inventory model with independent renewal demand, a base-stock policy is optimal when the cost of starting production is negligible. However, when the retailer is taking advantage of the information (i.e., $S_l < S_u$), the demand faced by the manufacturer is neither independent of the current state, nor is it a renewal process. This raises the question of how the manufacturer should respond to the retailer's two-level policy. Here, we assume that the manufacturer's objective is to provide a given service level, α_m, while minimizing the finished-goods on-hand inventory holding cost. We also provide a model of two-way information exchange in which the retailer knows the manufacturer's inventory availability and the manufacturer knows the retailer's inventory position.

Before proposing a policy, we make two observations. First, any manufacturer policy in our setting can be described as a rule to stop and start production. Note that one can view any stationary production policy at a manufacturer as a binary decision rule, start production or stop production, applied at the epochs of changes in net inventory. In the absence of any starting cost, one can assume starting inventory level is just one less than the stopping inventory level (Gavish and Graves 1980). Second, the manufacturer knows the retailer's inventory position, and thus can anticipate the period when the retailer will not be ordering in an effort to bring down its inventory position from S_u to S_l. The manufacturer can take advantage of this anticipated low-demand period by carrying lower on-hand inventory than it does during periods when the retailer is ordering. The proposed policy can be stated this way: If the retailer inventory position is less than or equal to S_{ip} ($S_l \leq S_{ip} < S_u$), stop production when the manufacturer's on-hand inventory reaches S_{mh}; otherwise, stop production at S_{ml} (where $S_{ml} < S_{mh}$). Start production whenever on-hand inventory drops below the stopping level. We note that it is possible to implement a version of this policy with $S_{ip} = S_l$, even if the manufacturer has no knowledge of the retailer's inventory position. Our methodology for Markov chain analysis, developed earlier, is easily applicable here. (See the appendix for details.)

Unfortunately, from the manufacturer's perspective, the cost saving of using this improved policy instead of the standard single base-stock policy is miniscule. For example, let $\lambda_e = \lambda_r = 0.4$, $\mu = 1$, $T = 1$, $h = 1$, $b = 9$, $\alpha_m = 0.9$. When the manufacturer is using a single-level base-stock policy, the retailer's optimal policy is $S_l = 1$, $S_u = 5$, and the manufacturer achieves the desired service level at $S_m = 12$. We note that this is an equilibrium solution in the sense that, given one party's policy, the other has no incentive to change its policy. The manufacturer provides a service level of 90.64% with an average on-hand inventory, $E[S_m - N]^+$ of 8.1685. If the manufacturer uses the improved policy, the optimal policy would be $S_{ml} = 6$, $S_{ip} = 1$, $S_{mh} = 12$, which would provide a service level of 90.07% with an average on-hand inventory of 8.0089. We note that under both scenarios the retailer's optimal policy remains $S_l = 1$, $S_u = 5$ even as its cost increases slightly under the two-level policy at the manufacturer. Our two observations—that the manufacturer's cost reduction is small and the retailer's optimal (S_l, S_u) remains unaffected—are verified across the parameter ranges described in §4.2. This leads us to infer that even in two-way information exchange, the managerial insights drawn earlier will hold.

Intuitively, the two-level policy increases the manufacturer's freedom to set policy, and allows it to deliver the required service level while reducing the base-stock level. However, given that the manufacturer's supply system is a controlled production queue with finite production rate, and not the traditional outside supplier with infinite capacity and fixed lead times, the manufacturer's decisions are limited to "when to stop production" instead of the traditional "when to place each order." This explains the modest size of cost reduction. The manufacturer's decision space is constrained, and so it does not have much room to react to the current information. Given this limitation of the improved policy, we will limit ourselves to a single-level policy at the manufacturer, while developing managerial insights into the manufacturer's actions in the next section.

5.4. Economic Rationale for Reverse Information Sharing

We now investigate reverse information sharing from a purely economic point of view for manufacturers: Are there situations in which reverse information sharing can benefit them? We suggest that sharing reverse information can be a tool to induce the retailer to increase its demand. For instance, if a retailer buys only a portion of its demand from the manufacturer, providing reverse information may give the retailer incentive to transfer a larger portion of its demand to the manufacturer. This will generate more revenues for the manufacturer, but will also increase the congestion in the production system. These outcomes may justify reverse information sharing for the manufacturer. Any such justification must also consider the impact of the retailer's and the manufacturer's policies on walk-in customers. In our study, we consider only those cases in which the service received by walk-in customers remains the same. Therefore, walk-in customers have no reason to change their behavior. We present an example and discuss the manufacturer's incentive for sharing reverse information.

Consider the situation in which the manufacturer experiences an external demand rate of $\lambda_e = 0.4$ and its policy is to set the base-stock S_m to provide a service level of $\alpha_m = 0.9$. Other relevant parameters are $\mu = 1$, $T = 1$, $h = 1$, $b = 9$. The retailer decides its demand rate. In the following, we consider the retailer's problem under two scenarios, with and without reverse information.

To keep the example simple, we assume a specific demand-and-supply model for the retailer. Each of the retailer's customers is assumed to be contributing ε towards a total demand rate of Λ_r. We assume dual supply modes for the retailer. One mode is the manufacturer in our model, and the other is an external source exogenous to our model. The external source charges a higher per-unit price w_e than the manufacturer, but it has unlimited capacity, can deliver directly to the retailer's customers out of its own stock and agrees to compensate for any backorder cost incurred by the retailer due to delayed supplies. The manufacturer charges a smaller price w_m, but has a limited capacity, can only deliver to the retailer with an additional time lag T, and can only commit to a self-specified service level α_m. Both sources require a committed order rate from the retailer. In such a situation, the retailer carries inventory only for the deliveries from the manufacturer, assigns some customers (total demand rate λ_r) to be supplied from its own inventory, and assigns other customers (total demand rate $\Lambda_r - \lambda_r$) to be supplied directly from the external supplier's inventory.

To find the optimal λ_r, the retailer uses a marginal argument. Let $C^*(\lambda_r)$ be the optimal inventory cost if the retailer satisfies an average of λ_r units of its demand from the manufacturer. For any λ_r, $C^*(\lambda_r)$ is computed at the equilibrium policy combination S_m, S_l, S_u, under the condition $S_l = S_u$ to reflect no information sharing. Recall that the manufacturer's policy is to set S_m to provide $\alpha_m = 0.9$, and the retailer uses the optimal single-level base-stock policy. The equilibrium occurs when neither the retailer nor the manufacturer has an incentive to deviate from its policy. Let $\Delta C^*(\lambda_r) = C^*(\lambda_r + \varepsilon) - C^*(\lambda_r)$ be the marginal increase in cost over an increase in the retailer's demand of the manufacturer. The marginal argument suggests that the retailer would assign a customer's demand stream to the manufacturer as long as $w_m\varepsilon + \Delta C^*(\lambda_r) \le w_e\varepsilon$. In this example, we use $\varepsilon = 0.0001$. We can numerically confirm that $\Delta C^*(\lambda_r)$ is positive and increasing in λ_r, and therefore there is a unique optimal solution λ_r^* to the retailer's problem. We note that there is a one-to-one correspondence between the two sources' price difference $w_e - w_m$ and optimal decision λ_r^*. Let us pick one case: $w_e - w_m = 33.0746 \Leftrightarrow \lambda_r^* = 0.4$. The corresponding equilibrium policy combination is $S_m = 10$, $S_l = S_u = 1$. The retailer cost is 2.27743, and the manufacturer has an average finished-goods on-hand inventory, $E[S_m - N]^+$ of 6.4295, to provide a service level of 89.2626%. The number of average walk-in backorders is 0.5717.

Next, suppose reverse information is shared. Again, the retailer will decide its optimal λ_r by searching for the smallest λ_r to satisfy $w_m\varepsilon + \Delta C^*(\lambda_r) \le w_e\varepsilon$ (we keep $w_e - w_m = 33.0746$ to be consistent with the no-information case). Now, $\Delta C^*(\lambda_r)$ is a different function. For any λ_r, $\Delta C^*(\lambda_r)$ is computed at the equilibrium policy combination S_m, S_l, S_u, without enforcing $S_l = S_u$. At the equilibrium, the manufacturer sets S_m to provide α_m at lowest cost, the retailer sets S_l, S_u to minimize its own cost, and given the other party's policy neither has any incentive to deviate from its own policy. We numerically find the optimal decision $\lambda_r^* = 0.46$, with the corresponding equilibrium policy: $S_m = 18$, $S_l = 1$, $S_u = 8$. We assume that Λ_r is large enough not to constrain the retailer's choice of λ_r. The retailer cost is 1.87877 and the manufacturer's average on-hand inventory is 12.0768. Clearly, the manufacturer has to increase S_m to maintain service. The service level α_m is 90.06% and the number of average walk-in backorders is 0.5425.

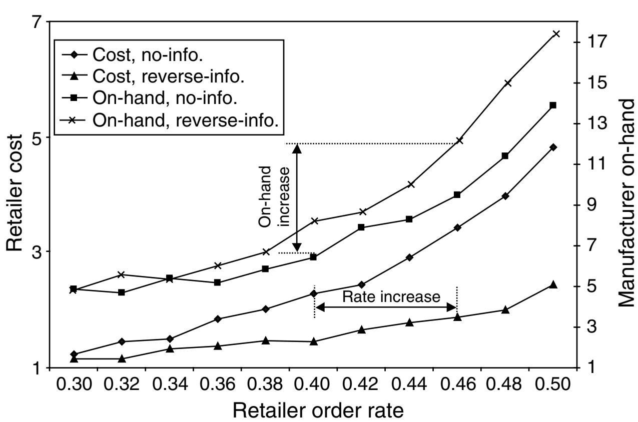
Thus, the retailer's cost rate decreases due to reverse information from $(w_e(\Lambda_r - 0.40) + w_m 0.40 + 2.27743)$ to $(w_e(\Lambda_r - 0.46) + w_m 0.46 + 1.87877)$, resulting in a savings of $((w_e - w_m)0.06 + 0.39866)$. The manufacturer's profit rate changes from $(w_m 0.40 - x_m 0.40 - h_m 6.4295)$ to $(w_m 0.46 - x_m 0.46 - h_m 12.0768)$, where x_m is the production and material cost per unit and h_m is the holding cost rate at the manufacturer's finished-goods inventory. The manufacturer's profit will increase using reverse information sharing if the manufacturer's profit margin per unit $(w_m - x_m)$ is larger than a critical value. This analysis suggests that for any given parameter combination, there may exist a revenue and cost structure that would justify reverse information sharing for the manufacturer while reducing the retailer's cost and keeping the service level for walk-in customers the same. It also suggests that supply chains in which manufacturers have a high profit margin are more likely to use reverse information sharing.

Because we wish to highlight the issue in terms of operating measures rather than exogenous parameters, in this discussion we have not assumed specific values for the per-unit cost parameters, w_e, w_m, x_m. Here, the manufacturer's decision to share information depends on the trade-off between the increase in the retailer's demand rate (benefit of additional revenue) and the increase in finished-goods on-hand inventory (cost of additional holding charges). The increase in the retailer's rate is a consequence of change in the retailer's cost due to information sharing. The increase in the finished-goods on-hand inventory is required to keep the service level fixed so that external customers continue to experience the same service. Now we will focus on this operating measures trade-off for the manufacturer.

Figure 7 illustrates the trade-off by plotting the retailer cost and manufacturer on-hand inventory both before and after reverse information sharing. We note that, for a given λ_r, reverse information sharing not only reduces the retailer's cost, but also reduces the gradient of the retailer cost $(\Delta C^*(\lambda_r)/\varepsilon)$. This motivates the retailer to increase λ_r in order to keep the cost gradient unchanged at $w_e - w_m$. The increase in λ_r (horizontal arrow) measures the benefit of information sharing to the manufacturer, while the corresponding increase in the holding cost (vertical arrow) measures its cost to the manufacturer. Note that the graph is drawn for a fixed λ_e, and any increase in λ_r increases the utilization at the manufacturer, as well as changes the ratio of the two demand rates.

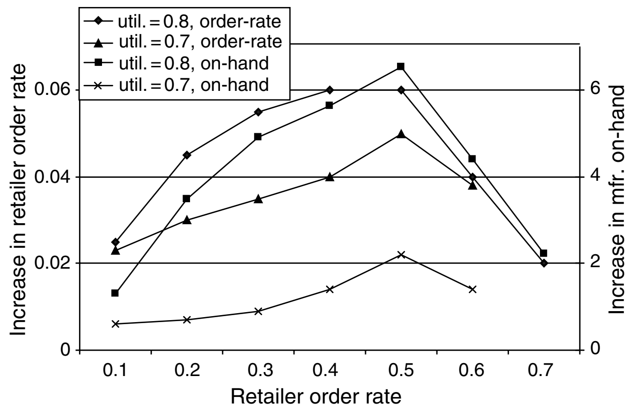
The next graph, Figure 8, focuses on changing the mix of two types of demand rate. The graph is constructed for a constant utilization $(\lambda_e + \lambda_r)/\mu$ while changing the mix of λ_e and λ_r. Consider each point on the horizontal axis in Figure 8 as the retailer's optimal order rate under no information, corresponding to a $w_e - w_m$ value. A larger horizontal axis value reflects

Figure 7 Cost and Benefit of Information Sharing



Note. On-hand: $E[S_m - N]^+$; $\lambda_e = 0.4$, $\mu = 1$, $T = 1$, $h_r = 1$, $b_r = 9$, $\alpha_m = 0.9$.

Figure 8 Effect of Retailer Size on Information Sharing



Note. Util.: ρ; on-hand: $E[S_m - N]^+$, $\lambda_e = \rho\mu - \lambda_r$, $\mu = 1$, $T = 1$, $h_r = 1$, $b_r = 9$, $\alpha_m = 0.9$.

a larger $w_e - w_m$. Now, for the manufacturer sharing information will lead to a benefit—increase in λ_r—and a cost—increase in holding cost. The benefit and cost pair is plotted for two different utilization values.

As the portion of the retailer's demand in the total grows, both the cost and the benefit of information sharing first grow and then decline. The graph suggests that if the holding cost were negligible, the manufacturer should share information with retailers that make up about half of the workload. If the holding cost is nonnegligible, we take the separation between the cost and the benefit lines as a comparative measure, and this suggests that retailers that constitute less than half of the workload are better candidates for information sharing than are those that constitute more than half of the workload. This makes intuitive sense because, as we noted in §4.2.2, the percentage cost reduction is smaller for the retailer when its demand is a larger part of the overall system. This suggests that the benefit to the manufacturer (increase in order rate) will be smaller if the retailer is larger. We do note, however, that while larger customers may be able to exercise more pressure for sharing reverse information, the manufacturer's economic interest is better served by sharing it with smaller customers.

The graph shows another pair of cost benefit lines for a lower utilization. We observe that the separation is wider, and therefore that reverse information sharing may make more sense for manufacturers with lower utilization. Again, this makes intuitive sense because the manufacturer's cost will increase less rapidly with utilization when utilization is small.

The example and analysis above highlight the manufacturer's economic motivation to share information: releasing the information may result in the retailer increasing its order rate. More detailed supply and demand models for the retailer may suggest different mechanisms for the increase in order rate. The retailer may lower its price to acquire new demand or, in the case of two identical supply modes, may shift orders from the mode that does not share reverse information to the one that does. From the manufacturer's perspective, however, the end result is the same: an increase in the retailer's order rate. For the manufacturer, reverse information sharing is a way to differentiate itself from its competitors, and thus attract larger demand. It is also a tool that can be used to achieve nonprice discrimination among its customers, in favor of those who have the ability to increase their demand rate.

6. Extensions and Conclusion

This paper analyzes the implications of a growing business practice called reverse information sharing for different parties in a supply chain. We build and analyze a manufacturer-retailer model that avoids a limiting assumption made in earlier works. We demonstrate that a simple policy will enable the retailer to take advantage of reverse information to reduce its inventory cost. We also demonstrate how reverse information sharing may increase the manufacturer's profits. The model leads to insights that provide guidance to managers on when to share reverse information and how to use it.

On the modeling side, two questions stand out for future research. First, what if the retailer has access to more detailed current information, for example, if the retailer knows the number of orders outstanding at the manufacturer, N, at the time of ordering? Second, what if the manufacturer shares current information with all customers? We believe that the infrastructure we have developed in this model can be extended to address these questions and others. It may also be extended to provide detailed models of the retailer's static knowledge of the supply system. In this paper we assume that the retailer possesses sufficient knowledge to develop π_{ij} and to use our algorithm. An interesting avenue for future research is the study of situations in which the retailer does not have this knowledge.

Appendix

This appendix provides brief sketches of the steps in the analysis in §3 and of the proofs. Complete details are available in the online appendix.

Step 1. Defining the Restricted Process. The restricted version observes the process only over states for which $N \le S_m$. The state-transition-rate diagram is shown in Figure 2. All the transition rates except those out of states $\{i, S_m\}$ remain the same as in the unrestricted process. To determine the transition rates for $\{i, S_m \mid i < \Delta\} \to \{i, S_m\}$ and $\{i, S_m \mid i < \Delta\} \to \{\Delta, S_m\}$, first define

f_{ij}: Starting in a state $\{i, j \mid i < \Delta,\ j \ge S_m\}$ and conditioned on the next transition being due to external arrival, the probability that the unrestricted process will return to state $\{i, j\}$ for the first time before it reaches state $\{\Delta, S_m\}$.

Using conditioning arguments, we can write the recursion $f_{ij} = q\sum_{k=0}^{\infty}(pf_{i.\,j+1})^k$ where $q = \mu/(\lambda_e + \lambda_r + \mu)$ and $p = \lambda_e/(\lambda_e + \lambda_r + \mu)$. Next, note that $f_{ij} = f_{i.\,j+1}$ and therefore, $f_{ij} = (1 - \sqrt{1-4pq})/(2p) = f$. The transition rate attached to $\{i, S_m\} \to \{i, S_m\}$ for $0 \le i < \Delta$ will be $\lambda_e f$ and the rate for $\{i, S_m\} \to \{\Delta, S_m\}$ will be $\lambda_r + \lambda_e(1-f)$. The rate for $\{\Delta, S_m\} \to \{\Delta, S_m\}$ will be $\lambda_e + \lambda_r$. We will normalize these rates by dividing them by μ.

Steps 2, 3, *and* 4. In Steps 2 and 3 we formulate and solve second-order homogeneous difference equations to obtain Equations (1) to (6). Further algebraic manipulation of the balance equations yields Equation (7) in Step 4.

Proof of Theorem 1. We obtain this proof by showing that Equations (1) to (7) in §3 satisfy all the balance equations of the original process. □

Proof of Theorem 2. We first prove the following identities (a) to (e).

$$\text{(a)}\quad \sum_{j=0}^{n}\pi_{1.\,j} = \sum_{i=1}^{\Delta}\pi_{i.\,n+1} - \rho_e\sum_{i=1}^{\Delta}\pi_{i.\,n} \quad \text{for } 0 \le n \le S_m - 1.$$

$$\text{(b)}\quad \rho_r\sum_{j=0}^{S_m-1}\pi_{m.\,j} = (\rho - z_2)\sum_{i=0}^{m-1}\pi_{i.\,S_m}, \quad \text{for } 1 \le m \le \Delta.$$

$$\text{(c)}\quad \rho - z_2 = \frac{\rho_r}{(1-z_2)}.$$

$$\text{(d)}\quad (1-\rho)\sum_{i=0}^{\Delta-1}\sum_{j=S_m}^{\infty}\pi_{i.\,j} = \frac{(1-\rho)}{(1-z_2)}\sum_{i=0}^{\Delta-1}\pi_{i.\,S_m}.$$

$$\text{(e)}\quad (1-\rho)\sum_{j=S_m}^{\infty}\pi_{\Delta.\,j} = \pi_{\Delta.\,S_m} + (\rho - z_2)\sum_{i=0}^{\Delta-1}(\Delta - i)\pi_{i.\,S_m} + \frac{(\rho - z_2)}{(1-z_2)}\sum_{i=0}^{\Delta-1}\pi_{i.\,S_m}.$$

The last step in the proof begins with writing the balance equations for the set of states $\{(0, j) \mid 0 \le j \le n\}$ for all $0 \le n \le S_m - 1$ and then using (a) to (e) to show $\sum_{i=0}^{\Delta}\pi_{i.\,0} = (1-\rho)$. □

Proof of Theorem 3. We separately prove the convexity of $C[i, j, k]$ for $j \ge S_m$ and of $C[0, 0, 1]$. Combining the two results proves the convexity of $C_{S_l.\,\Delta}$ in S_l. □

Proof of Theorem 4. We will use the symbol $\pi_{\Sigma.j}$ to denote $\Pr[N^{RI}=j]=\sum_{i=0}^{\Delta}\pi_{i.j}=\pi_{\Sigma.j}$. In NI, $\Pr[N^{NI}=j]=(1-\rho)\rho^j$. We need to show that: $\Pr[N^{RI}\le j]\le \Pr[N^{NI}\le j]$. From Theorem 2, we have $\pi_{\Sigma.0}=1-\rho$. Using the aggregation/disaggregation approach to analyze Markov chains (see Schweitzer 1991), we can write: $\pi_{\Sigma.1}=(\rho_e+\rho_r(\pi_{0.0}/\pi_{\Sigma.0}))\pi_{\Sigma.0}<\rho\pi_{\Sigma.0}$. We also show that $\pi_{\Sigma.j}<\rho\pi_{\Sigma.j-1}$ for $j\le S_m$ and that $\pi_{\Sigma.S_m+1}=\rho\pi_{\Sigma.S_m}$. Therefore, $\Pr[N^{RI}\le j]\le \Pr[N^{NI}\le j]$ for $j\le S_m+1$.

Next, consider the balance equation for the group of states $\{\pi_{i.j}\mid 0\le i\le\Delta;\ j\le S_m\}$, which gives: $\pi_{\Sigma.S_m+2}=\rho\pi_{\Sigma.S_m+1}+\rho_r\sum_{i=0}^{\Delta}\pi_{i.S_m}>\rho\pi_{\Sigma.S_m+1}$. Similarly, $\pi_{\Sigma.S_m+j}>\rho\pi_{\Sigma.S_m+j-1}$ for $j\ge 2$. Therefore, $\pi_{\Sigma.S_m+j}>\rho^{j-k}\pi_{\Sigma.S_m+k}$ for $j>k\ge 1$.

We will prove the rest by contradiction. Assume there exists:

$$j^*=\mathrm{Min}\left\{k\,\Big|\sum_{j=0}^{k}\pi_{\Sigma.j}\ge \Pr[N^{NI}\le k];\ k\ge S_m+2\right\}.$$

From the definition of j^*: $\Pr[N^{RI}=j^*]=\pi_{\Sigma.j^*}>\Pr[N^{NI}=j^*]=(1-\rho)\rho^{j^*}$. Now,

$$\begin{aligned}\lim_{j\to\infty}\Pr[N^{RI}\le j]&=\sum_{j=0}^{j^*}\pi_{\Sigma.j}+\sum_{j=j^*+1}^{\infty}\pi_{\Sigma.j}\\&>\Pr[N^{NI}\le j^*]+\sum_{j=j^*+1}^{\infty}\rho^{j-j^*}\Pr[N^{NI}=j^*]\\&=(1-\rho)\frac{(1-\rho^{j^*+1})}{(1-\rho)}+(1-\rho)\rho^{j^*}\frac{\rho}{(1-\rho)}=1.\end{aligned}$$

This is impossible. Therefore, no such j^* exists and $\Pr[N^{RI}\le j]\le \Pr[N^{NI}\le j]$ for all $j\ge 0$. □

Proof of Proposition 5. Recall from Theorem 4 that $\Pr[N^{RI}=j]\le\Pr[N^{NI}=j]$ for $j\le S_m$. Thus

$$\begin{aligned}E[S_m-N]^{+^{RI}}&=\sum_{j=0}^{S_m-1}(S_m-j)\Pr[N^{RI}=j]\\&\le\sum_{j=0}^{S_m-1}(S_m-j)\Pr[N^{NI}=j]=E[S_m-N]^{+^{NI}}.\quad\square\end{aligned}$$

Two extensions of the basic model, discussed in §2 and §5.3, are available in the online appendix (Jain and Moinzadeh 2005).

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