A Decentralized Supply Chain Coordination Policy when Demand is Random

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Abstract

In this paper, we consider the issue of order coordination in a decentralized two-echelon supply chain, where a wholesaler supplies multiple retailers and replenishes its own stock from an external supplier at a pre-specified order interval. Assuming that demand at the retailers is random, we propose a coordination scheme based on a price discount offered by the wholesaler that is related to the timing of retailers' orders. Specifically, retailers are offered a discount price, c_D, if they order at the beginning of the wholesaler's reorder cycle; otherwise, retailers can reorder at any other time at a higher unit price, c_L. We propose an effective ordering policy for the retailers that is an extension of the (R, T) policy, and present a methodology for finding the wholesaler's optimal discount price schedule under this coordination scheme. Using numerical examples, we illustrate the potential managerial implications of our coordination policy and identify conditions when our suggested coordination scheme is beneficial for all stakeholders.

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1. Introduction

Over the last few years, we have witnessed an emergence of new ideas and practices designed to improve the performance of decentralized supply chains. With changes in market sizes, demand patterns, and an increasing emphasis on global outsourcing, new initiatives for improving supply chain performance such as cross-docking and vendor-managed inventory (VMI) have gained attention in both research and practice. In fact, a number of companies (including Wal-Mart, Dell, and Hewlett Packard) have successfully implemented various supply chain coordination practices. Several recent papers (e.g., Cachon, 1999; Moinzadeh, 1997; Weng, 1995) show that many supply chain coordination schemes result in greater profitability for the entire supply chain as well as a lower demand variance for upstream suppliers. The question of how these supply chain coordination schemes should best be structured and implemented remains unclear.

In this paper, we suggest that a timing discount can be an effective coordination mechanism in a decentralized supply chain. Specifically, we consider a two-echelon supply chain with a single wholesaler who supplies multiple retailers who, in turn, have random (and independent) demand. To coordinate the supply chain, we propose a policy where, given a wholesaler's reorder cycle T_0, retailers are offered a discount price, c_D, for all orders placed at the beginning of the wholesaler's reorder cycle. For all orders placed at other times, retailers are charged a standard list price, c_L where $c_L > c_D$.

We show that our suggested policy may result in greater profitability for both the supplier and retailers under some conditions. We assume a nested ordering policy between the retailers and the wholesaler where each retailer orders at the beginning of the wholesaler's cycle when the wholesaler receives a shipment from its supplier. The wholesaler has an incentive to offer a price discount to retailers who order at the beginning of the wholesaler's reorder cycle since this allows the wholesaler to cross-dock the goods and reduce the associated holding costs. As we show in this paper, our coordination scheme may significantly increase the wholesaler's expected profits while not reducing any retailers' profit.

The rest of the paper is organized as follows. In the next section, we describe a heuristic ordering policy for the retailers that is an extension of a (R, T) policy that can be implemented with our coordination scheme when demand is stationary. We then describe a model to analyze our coordination policy and derive the retailer's expected cost function and the wholesaler's profit function based on the proposed policy (in sections 3 and 4, respectively). In section 5, we discuss a search procedure for finding the optimal policy parameters and describe the results and managerial implications of several numerical experiments. Using these numerical experiments, we compare our heuristic ordering policy to a standard (R, T) policy without any price discounts. Finally, we identify conditions under which our proposed order coordination is most effective.

2. Motivation and Model Description

2.1 Previous Research

Previous research has shown that coordinating retailers' orders can lead to lower total supply chain costs (Lee *et al*., 1997; Cachon, 1999). On the other hand, when the retailers' orders are synchronized (such that all retailers order at the same time), the supplier faces a maximum demand variance. Cachon (1999) studied the impact of scheduled ordering policies on the supply chain variability with a single supplier and N retailers who face stochastic demand, and showed that under a balanced ordering policy, supplier's demand variance and the total supply chain cost are reduced when retailers' reorder intervals increase or when the retailers' order sizes decrease.

With respect to other supply chain coordination studies, Jeuland and Shugan (1983) analyzed coordination impacts based on various mechanisms (e.g. contracts, joint ownership, quantity discounts, etc.) from a marketing viewpoint. They showed that coordination among channel members can lead to larger overall profits for the entire supply chain, but that close cooperation among the members is hard to achieve unless every member has sufficient reassurance that all other parties will be forced to coordinate as well. Dolan (1987) further analyzed quantity discounts as a coordination mechanism and specified several conditions where quantity discounts can be profitable for all stakeholders in a supply chain.

Much of the existing research on supply chain coordination has focused on the buyer's perspective (see, for example, Jucker and Rosenblatt, 1985; Sethi, 1984; Rubin, Dilts, and Barron, 1983; and Ladany and Sternlieb, 1974, Crowther, J., 1964). Monahan (1984) developed an optimal quantity discount pricing schedule to maximize vendor profits when there is a single supplier and single buyer. Lee and Rosenblatt (1986) generalized Monahan's model to include the case when a buyer may place orders more frequently than the supplier. More recently, Cheung and Hausman (2000) studied the performance measure focused on the supplier serving multiple retailers in a two-echelon distribution system. Weng (1995) analyzed the effects of joint decision policies on channel coordination in a distribution system which consists of a supplier and a group of homogeneous buyers. Cheung (1998) considered a case where a supplier, faced with a potential stock-out situation, offers a one-time discount to motivate buyers to accept delayed deliveries in order to avoid lost sales. He found that a time based discount significantly reduced the total system costs, although its impact is greater when a significant proportion of customers are willing to have their orders backlogged.

Roundy studied the effectiveness of different types of nested policies in a multi-echelon distribution system. He proposed two lot-sizing policies, the *q-optimal integer-ratio* and the *optimal power-of-two* policies. His paper proved effectiveness of at least 94% and 98% for these two policies respectively. Axsater (1993) also used a nested policy to find an optimal periodic order-up-to-S policy (by assuming that the review period at the warehouse was an integer multiple of the review period at the retailers). Using a virtual allocation policy, he analyzed the system in essentially the same way as a continuous review (S-1, S) policy.

Klastorin *et al*. (2002) first proposed the use of a timing discount for order coordination in a multi-echelon supply chain with deterministic demand at the retailers. In their work, they showed that such discounts could lower system costs (and increase wholesaler profits) when implemented correctly. This paper is an important extension of their work to the case when demand at the retailers is random in order to investigate how such discounts could be implemented (and if they would be beneficial) in more realistic environments.

2.2. Model Description

We consider a two-echelon supply chain with a single supplier (wholesaler) and a set J = {1, 2, ···· , N} of homogeneous retailers where demand follows a Poisson process with a mean rate, λ. Demand at each retailer is independent of the demand at any other retailer. We will let $D(t)$ denote the demand at any retailer over an interval *t*.

We assume that both the retailers and the wholesaler use a periodic review system, and that the wholesaler's order cycle, T_0, is longer than the retailers' cycle such that each retailer typically orders more than once during the wholesaler's reorder interval. Furthermore, we assume a nested ordering schedule between the wholesaler and retailers; that is, each retailer places an order at the beginning of the wholesaler's review period (as well as an integer number of orders during the wholesaler's review period). Nested ordering policies have been widely used in previous research and have been shown to be good approximations to observed practice (Lee and Rosenblatt 1986, Roundy 1985, Axsater 1993). When retailers order, they replenish items from the wholesaler according to a (R, T) policy; that is, every T time units, each retailer orders up to level R. We also assume a zero shipping delay from the wholesaler to retailers (although this assumption can be easily relaxed) and that all shortages at retailers are backordered.

The wholesaler offers a discounted price c_D for all units purchased at the beginning of its order cycle; at any other time, retailers purchase items at a standard list price c_L where $c_D \le c_L$. Since the discount price will cause retailers to order larger amounts at the beginning of the wholesaler's reorder cycle, the order-up-to level (R^D) at the beginning of the wholesaler's cycle will be greater than or equal to the order-up-to level R^L ($\therefore R^D \ge R^L$) at other times during the wholesaler's cycle. Retailers' ordering behavior is illustrated in Figure 1.

Insert Figure 1 about here

To analyze our proposed policy, we will use the following notation for any retailer:

R^D: Order-up-to level when replenishing at the discount price, c_D.

R^L: Order-up-to level when replenishing at the list price, c_L.

t : The length of retailer's reorder cycle.

k : Number of times orders can be placed at c_L during the wholesaler's cycle, T_0, such that

$T_0 = t(k+1)$.

s : (Fixed) order cost

π : Unit backorder cost,

h : Unit holding cost/time,

$p(x_j;\lambda t) = e^{-\lambda t} (\lambda t)^{x_j} / x_j! \quad for \ all \ j \in J \ and \ x_j = 0,1,2,...$

$P(y;\mu) = \sum_{x=y}^{\infty} p(x;\mu)$

For the wholesaler, we will use the following notation.

R_0 : Order-up-to level (at the beginning of the wholesaler's cycle).

h_0 : Unit holding cost/time period

s_0 : (Fixed) order cost

Given the wholesaler's cycle time, T_0, each retailer must decide the order-up-to-levels (R^D and R^L), while the wholesaler determines its order-up-to level, R_o, and the discount price, c_D. We assume that the (normal) list price c_L is exogenously determined by the market.

3. Analysis of a Retailer's Ordering Policy

A retailer's typical ordering behavior is indicated in Figure 2. Using an extension of an (R, T) policy, a retailer would not place an order at any review point within the wholesaler's cycle if the inventory level immediately prior to the review point is equal to or greater than the order-up-to level, R^L. .

Insert Figure 2 about here

Retailers set their order-up-to-levels to minimize their expected total cost per time period $EC(R^D, R^L)$, which is defined by the sum of the expected purchasing, ordering, holding costs and backorder costs as indicated below.

$$EC[R^D, R^L] = \frac{1}{T_0}[c_D(PUR_D) + c_L(PUR_L) + s(OR) + hE(OH) + \pi E(BO)] \tag{1}$$

where

PUR_D, PUR_L = expected purchase amount at c_D and c_L respectively, over T_0,

OR = expected number of orders placed over T_0,

$E(OH)$ = expected on-hand inventory over T_0,

$E(BO)$ = expected backorder amount over T_0.

3.1. Expected Replenishment Cost

Let τ denote the time between the beginning of the wholesaler's reorder cycle and the time when the inventory level initially reaches R^L within T_0. To derive the distribution of τ, we will let $f_\tau(x)$ and $F_\tau(x)$ be the density and cumulative distribution functions of τ, respectively. Since the demand at a retailer is Poisson, $f_\tau(x)$ follows an Erlang distribution. Thus, we have

$$F_\tau(\gamma) = P(\tau > \gamma) = \sum_{x=0}^{R^D - R^L} (\lambda\gamma)^x e^{-\lambda t} / x!$$

$$P(\tau \le \gamma) = 1 - \sum_{x=0}^{R^D - R^L} (\lambda\gamma)^x e^{-\lambda t} / x! = \int_0^\gamma f_\tau(t)dt$$

which implies that

$$f_\tau(t) = \frac{d}{dt} P(\tau \le t) = \frac{(\lambda t)^{R^D - R^L} e^{-\lambda t}}{(R^D - R^L)!} \lambda = \lambda \cdot p(R^D - R^L; \lambda t) \text{ for } t \ge 0,$$

We obtain the replenishment amount over T_0 by conditioning on the events $\{\tau > T_0 - t\}$ and $\{\tau \le T_0 - t\}$. If $\{\tau > T_0 - t\}$ or $\{D(T_0 - t) \le R^D - R^L\}$, then the inventory level remains at or above R^L during $T_0 - t$ and the retailer only orders $D(T_0)$ at the beginning of the wholesaler's cycle at price c_D. If, on the other hand, $\{\tau \le T_0 - t\}$ or $\{D(T_0 - t) > R^D - R^L\}$, the inventory level at the retailer falls below R^L and the retailer purchases:

(i) $D(t) + (R^D - R^L)$ at c_D and

(ii) $D(T_0 - t) - (R^D - R^L)$ at c_L.

Thus, the expected replenishment cost over T_0 can be expressed as:

$$c_D(PUR_D) + c_L(PUR_L)$$
$$= c_D[\sum_{y=0}^{R^D-R^L} yp(y;\lambda(T_0 - t)) + \lambda t \cdot \sum_{y=0}^{R^D-R^L} p(y;\lambda(T_0 - t))] + c_D[\lambda t + (R^D - R^L)] \cdot \sum_{y=R^D-R^L+1}^{\infty} p(y;\lambda(T_0 - t))$$
$$+ c_L[\sum_{y=R^D-R^L+1}^{\infty} yp(y;\lambda(T_0 - t)) - (R^D - R^L) \cdot \sum_{y=R^D-R^L+1}^{\infty} p(y;\lambda(T_0 - t))]$$

$$= c_D \lambda T_0 + (c_L - c_D)[\lambda(T_0 - t) \cdot P(R^D - R^L; \lambda(T_0 - t)) - (R^D - R^L) \cdot P(R^D - R^L + 1; \lambda(T_0 - t))] \quad (2)$$

3.2. Expected Ordering Cost

At the beginning of the wholesaler's cycle, each retailer can place an order to bring its inventory level up to R^D. After its initial order, a retailer can order at any subsequent review point if its inventory level is lower than the order-up-to level, R^L (recall that there are *k* potential ordering opportunities within the wholesaler's cycle that are *t* time periods apart). Of these *k* potential ordering opportunities, a retailer will place its first order at the m^{th} opportunity, where $m = \left\lceil \frac{\tau}{t} \right\rceil$.

Thus, the retailer's expected ordering cost over the wholesaler's cycle is:

$$s \cdot [1 + \sum_{m=1}^{k+1} \sum_{y=0}^{R^D-R^L} \sum_{x=(R^D-R^L-y)+1}^{\infty} (k + 1 - m) \cdot p(y;\lambda(m-1)t) \cdot p(x;\lambda t)]. \quad (3)$$

3.3. Expected On-hand Inventory Cost over T_0

Following Moinzadeh (1997a) and Moinzadeh and Nahmias (1986), we assume a fixed holding cost per unit per time period (*h*) at each retailer. Since τ is the time when a retailer's inventory level first falls below R^L, the expected on-hand inventory *E(OH*) for any retailer over the wholesaler's cycle[1] can be defined as

$$\begin{aligned} E(OH) &= E(OH, \tau \geq T_0) + E(OH, \tau < T_0) \\ &= E(OH \mid \tau \geq T_0) \cdot \Pr(\tau \geq T_0) + E(OH \mid \tau < T_0) \cdot \Pr(\tau < T_0) \end{aligned} \tag{4}$$

which can be rewritten as follows:

$$\begin{aligned} E(OH) &= (R^D - \frac{1}{2}\lambda T_0) \cdot T_0 \cdot \sum_{y=0}^{R^D - R^L} p(y; \lambda T_0) \\ &+ \sum_{m=1}^{k+1} [(R^D - \frac{1}{2}\lambda t m) \cdot m \cdot t + (k+1-m) \cdot (R^L - \frac{1}{2}\lambda t) \cdot t] \cdot \sum_{y=0}^{R^D - R^L} \sum_{x=(R^D - R^L - y)+1}^{\infty} p(y; \lambda (m-1)t) \cdot p(x; \lambda t) \end{aligned} \tag{5}$$

The expected holding cost for each retailer over the wholesaler's cycle is then $h \cdot E(OH)$.

3.4. Expected Backorder Cost over T_0

A retailer cannot experience a stockout until the retailer's inventory level equals R^L. When the inventory level equals R^L during a retailer's m^{th} reorder cycle (where m = 1, 2, …, k +1), the retailer may experience shortages in the m^{th} cycle or any of the remaining $(k+1-m)$ cycles. The expected number of backordered units at a retailer during the wholesaler's cycle can then be defined as follows:

$$E(BO) = E(BO, \tau \geq T_0) + E(BO, \tau < T_0)\,. \tag{6}$$

which can be rewritten as

[1] Note that we are using net inventory to approximate on-hand inventory; for more information, see Hadley and Whitien, 1963.

$$E(BO)=\sum_{m=1}^{k+1}\Big[\{\int_{(m-1)t}^{mt}E(BO\mid\tau=\gamma)\cdot f(\gamma)d\gamma\}\cdot\sum_{y=0}^{R^D-R^L}\sum_{x=(R^D-R^L-y)+1}^{\infty}p(y;\lambda(m-1)t)\cdot p(x;\lambda t)\Big]. \tag{7}$$

For a given $m = 1, 2, \ldots, k+1$, we know that

$$E(BO\mid\tau=\gamma)=\sum_{z_1=R^L+1}^{\infty}(z_1-R^L)p(z_1;\lambda(mt-\gamma))+(k+1-m)\cdot\sum_{z_2=R^L+1}^{\infty}(z_2-R^L)p(z_2;\lambda t) \tag{8}$$

that can be simplified as follows:

$$\begin{aligned}E(BO|\tau=\gamma)&=\lambda(mt-\gamma)\cdot P(R^L-1;\lambda(mt-\gamma))-R^L\cdot P(R^L;\lambda(mt-\gamma))\\&\quad+(k+1-m)\cdot[\lambda t\cdot P(R^L-1;\lambda t)-R^L\cdot P(R^L;\lambda t)]\ .\end{aligned}$$

The expected number of backorders for any retailer during the wholesaler's cycle is then

$$\begin{aligned}&\sum_{m=1}^{k+1}\Big[\int_{(m-1)t}^{mt}\{\lambda(mt-\gamma)\cdot P(R^L-1;\lambda(mt-\gamma))-R^L\cdot P(R^L;\lambda(mt-\gamma))\\&+(k+1-m)\cdot[\lambda t\cdot P(R^L-1;\lambda t)-R^L\cdot P(R^L;\lambda t)]\}f(\gamma)d\gamma\Big]\cdot\sum_{y=0}^{R^D-R^L}\sum_{x=(R^D-R^L-y)+1}^{\infty}p(y;\lambda(m-1)t)\cdot p(x;\lambda t).\end{aligned} \tag{9}$$

We can now define a retailer's expected total cost function $EC(R^D,R^L)$ based on (2), (3), (5) and (9). The function $EC(R^D,R^L)$ is not jointly convex over all values of R^D and R^L; however, our numerical results indicate that $EC(R^D,R^L)$ appears to be convex for large values of R^D and/or small values of R^L. In all cases we tested, however, we were able to show that the expected cost function was convex in R^L for fixed values of R^D.

4. The Wholesaler's Problem

The wholesaler places an order with an external supplier every T_o time units; this order is sufficiently large to increase the wholesaler's inventory level up to its order-up-to level, R_0. Orders placed by the retailers' at the beginning of the wholesaler's cycle are shipped out immediately. If the wholesaler should experience a stockout at any time during its cycle, we assume that the wholesaler will place an expedited order at each of the remaining retailers' order points with a third party supplier and pay a premium unit price, c_E, as well as incurring an emergency ordering cost, s_E (where $c_E > c_0$ and $s_E > s_0$). These expedited orders will be filled immediately to satisfy retailers' demand, such that the operations at the retailers will not be disrupted and they will be able to meet their customers' demands in timely fashion. We assume that the costs associated with these emergency orders (both c_E and s_E) are sufficiently high to warrant a reasonably large order-up-to level (R_0) for the wholesaler. As a result, stockouts at the wholesaler would only occur—if at all—near the end of its review cycle. It should also be noted that if a stockout occurs and the wholesaler places an emergency order, the wholesaler would not incur any additional holding cost until the beginning of its next replenishment cycle.

Since retailers are homogeneous with Poisson demand, the overall demand to the system is Poisson with rate $\lambda_0 = N \cdot \lambda$. Thus, $p(\cdot\,;\lambda_0 t)$ and $P(\cdot\,;\lambda_0 t)$ denote the probability mass function and complementary distribution function, respectively, of the Poisson demand to the entire system. Since the retailers place orders only at their review points, the wholesaler can only experience a stockout at these k review (ordering) points. The wholesaler would not, however, place an expedited order after the last review point as long as its inventory level immediately after this review point is positive (i.e., even though the sum of retailers demand over time T_0 may exceed R_0, system demand occurring during the final order interval within T_0 will be filled by the wholesaler at the beginning of its next reorder cycle).

Let β denote the time until the $(R_o + 1)^{th}$ demand to the system. Then, β follows an Erlang distribution with parameters λ_0 and $R_0 + 1$. Moreover, the retailer's order interval within T_0 when the system demand exceeds R_0, q, is defined as:

$$q = \left\lceil \frac{\beta}{t} \right\rceil$$

where q = 1, 2, …, k +1 (e.g., if q = 1, the system demand exceeds R_0 during the first retailer cycle of length t).

The wholesaler's expected profit $Z(c_D, R_0)$ with N homogeneous retailers can be defined as:

$$
\begin{aligned}
Z(c_D, R_0) = \frac{N}{T_0} & [\, c_D \lambda T_0 + (c_L - c_D)\{\lambda(T_0 - t)\cdot P(R^D - R^L; \lambda(T_0 - t)) \\
& - (R^D - R^L)\cdot P(R^D - R^L + 1; \lambda(T_0 - t))\,] \\
& - \frac{1}{T_0}[c_0 \lambda_0 T_0 + \sum_{q=1}^{k}\{s_E + c_E \cdot \{\lambda_0 qt \cdot P(R_0 - 1; \lambda_0 qt) - R_0 \cdot P(R_0; \lambda_0 qt)\} \\
& + (k-q)\cdot(s_E + \lambda_0 t c_E)\}\cdot P(R_0 + 1; \lambda_0 qt)] - h_0 E(OH_W) \qquad (10)
\end{aligned}
$$

where $E(OH_W)$ = the expected installation inventory at the wholesaler.

The expected on-hand inventory $E(OH_W)$ at the wholesaler (approximated by the net inventory) is the difference between the expected echelon inventory at the wholesaler and the expected inventory carried by the retailers (see Figure 3); the expected on-hand inventory $E(OH_W)$ is defined as[2]:

$$
\begin{aligned}
E(OH_W) = (R_0 - \frac{1}{2}\lambda_0 T_0) - \frac{N}{T_0}[\,(R^D - \frac{1}{2}\lambda T_0)\cdot T_0 \cdot \sum_{y=0}^{R^D - R^L} p(y; \lambda T_0) \\
+ \sum_{m=1}^{k+1}[(R^D - \frac{1}{2}\lambda t m)\cdot m \cdot t + (k+1-m)\cdot(R^L - \frac{1}{2}\lambda t)\cdot t]\cdot \sum_{y=0}^{R^D - R^L} \sum_{x=(R^D - R^L - y)+1}^{\infty} p(y; \lambda(m-1)t)\cdot p(x; \lambda t)]
\end{aligned}
$$

Insert Figure 3 about here

After the wholesaler sets the prices, the retailers order to minimize their expected costs $EC(R^D, R^L)$. If the wholesaler sets a discount price c_D that is sufficiently low to offset the retailers' holding cost, the retailers will set a high value of R^D and order sufficient inventory at the beginning of the wholesaler's cycle to bypass subsequent order points. Thus, the discount price c_D will determine the wholesaler's on-hand inventory as well as the retailers' order-up-to-levels and the cross-docked quantities.

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[2] For details see Clark and Scarf (1960)

To find the values of c_D and c_L that maximize the wholesaler's profit, we varied the values of c_D and c_L and solved for the retailers' optimal order-up-to-levels, R^D and R^L. Clearly, the discount price, c_D, must be set in the interval $c_0 \le c_D \le c_L$. (When $c_D = c_L$, the coordination policy becomes a standard (*R, T*) policy where $R = R^L = R^D$ and $T = t = \frac{T_0}{k+1}$.) For values of $c_D < c_L$, the retailers set their values of R^D and R^L to minimize their expected total cost. Since the retailers' expected cost function is not jointly convex, we used a heuristic search procedure that searched for the best solution in the relevant range for each variable. To guarantee that a sufficient range is covered in the search for the higher order-up-to level, R^D, we extended the search limit to $3 \cdot \lambda T_0$. For each value of R^D, we then found the minimum cost value of R^L over the range of [0, R^D] and retained the values of R^D and R^L that minimized the retailers' expected total cost.

5. Numerical Results and Managerial Implications

The objective of the numerical study was twofold: First, we wanted to study how retailers behaved as various parameters changed with respect to the wholesaler's discount price as well as the retailers' parameters. Second, we wanted to identify the conditions when our suggested timing discount resulted in the greatest expected gain for the wholesaler and the resulting impact on the retailers' costs. (Clearly, retailers will never be worse off when a wholesaler offers a timing discount since they can ignore it if they wish). In addition, we wanted to investigate the impact of our order coordination policy by examining the wholesaler's performance (e.g. profits) with and without the existence of order coordination; that is, we compared the wholesaler profits using our proposed coordination scheme versus the wholesaler profits under a standard (*R, T*) policy when no price discounts are offered.

In order to consider a range of conditions, we considered used the following parameter values and studied the various combinations:

c_L = \$40

c_0 = \$20

c_E = \$25, 30

$s = s_0$ = \$10

s_E=\$50

T_0 =3.0

$h_0/h = 0.5$

$\lambda_0 = N \cdot \lambda = 50$

i = 0.1, 0.2, 0.3 (or h = \$4, 8, 12)

π/h = 1, 5, 10

N = 2, 5, 10, 20

k = 1, 2, 3

$R_0 = 0.8 \cdot \lambda_0 T_0$, $1.0 \cdot \lambda_0 T_0$, $1.5 \cdot \lambda_0 T_0$ $2.0 \cdot \lambda_0 T_0$

With respect to changes in the retailers' cost parameters, it appears that changes in the holding cost (h) have a significant impact on the ordering policies in the supply chain. When the holding cost rate is high, both retailers and the wholesaler do not want to hold items for a long period of time, prompting the retailers to set lower values for order-up-to levels (R^D and R^L). As a consequence, the wholesaler may end up carrying larger amount of on-hand inventory resulting in reduced profits for the wholesaler (see Figure 4). While there may be an incentive on the part of the wholesaler to offer a deeper discount in an effort to raise the retailer's order-up-to level (especially R^D) and to increase the amount of cross-docked items, this may often lead to a decrease in the wholesaler's expected profit.

Insert Figure 4 about here

With respect to the retailer's backorder costs, we observed that a high backorder cost (i.e., *10*h*) generally resulted in a higher order-up-to level (R^D) at the retailers for a given k. This, in turn, resulted in a reduction in the wholesaler's on-hand inventory level due to increased shipments at the beginning of the wholesaler's cycle (items that are cross-docked). In general, the wholesaler's profits increased monotonically as the retailers' backorder costs increased (see Figure 5).

When we varied the number of orders placed by the retailers (*k*), we obtained somewhat different results with high backorder costs. In this case, retailers placed more frequent orders with shorter time intervals (that is, a higher value of *k* resulted in a lower value of R^D) to protect against stockouts. As indicated in Figure 6, retailer costs decreased in general as *k* increased for higher values of backorder costs (i.e., $\pi = 5 \cdot h$ and $10 \cdot h$), which was not necessarily the case when backorder costs were low (i.e., $\pi = 1 \cdot h$). With respect to the wholesaler, she had lower expected profits as values of k increased and backorder costs decreased (since smaller order sizes from the retailers resulted in more cross-docked items).

Insert Figures 5 and 6 about here

For a constant average system demand, a decrease in the number of retailers results in a higher average demand at each retailer; consequently, retailers are more likely to place larger orders or increase order-up-to-levels R^D and R^L as the stockout cost increases. Furthermore, when retailers face a higher demand rate, they were more likely to place larger orders at the beginning of the wholesaler's cycle (i.e., set a higher order-up-to-level R^D) and bypass some of the subsequent *k* ordering opportunities within the wholesaler's cycle. Moreover, the wholesaler had greater expected profits when it supplied fewer retailers in the system due to the holding cost savings attained through cross-docking (see Figure 7).

Insert Figure 7 about here

To judge the benefits of our proposed order coordination scheme, we calculated the percent increase in the expected wholesaler profits when our proposed timing discount was implemented. (We obtained results from the non-coordination case by setting the discount price equal to the list price, and assumed that retailers implemented a standard (*R, T*) policy.) As indicated in Figure 8, our proposed coordination scheme consistently resulted in increased wholesaler profits over the non-coordination case. For the case illustrated in Figure 8 (i = 0.20, π=5*h, k = 1, c_E=\$30), the timing discount policy resulted in an average increase of 4.47 percent in the wholesaler profits over the non-coordinated case. Our analysis also indicates that our coordination scheme appears to be more beneficial for the wholesaler when operating with settings that generally result in relatively low wholesaler profitability. In particular, Figure 9 indicates that the wholesaler experienced a greater percent increase in profits using the order coordination scheme when it operated under (i) a higher holding cost rate (i = 0.30), and (ii) supplied a relatively greater number of retailers (N = 20). Stockout costs and retailers' order frequency did not appear to have a significant impact on the wholesaler's profitability. For the case illustrated in Figure 9 ($\pi = 1*h$, k = 3, c_E=\$30), the wholesaler gained an average profit increase of 5.6 percent over the non-coordination case when N = 20, an average increase of 3.9 percent when i = 0.30, and an average increase of 9.5 percent when N = 20 and i = 0.30.

Insert Figures 8 and 9 about here

Finally, we examined the impact of changing the retailers' ordering frequency (k) on the wholesaler profits. In general, we found that it was beneficial for the wholesaler to supply retailers less frequently (i.e., a smaller value of k) as it increased the amount of cross-docked items and decreased the inventory level at the wholesaler. As indicated in Figure 10, we observed that the wholesaler profits were monotonically decreasing in k. This observation suggests that dynamically updating the value of k as the wholesaler incrementally increases the discount price may be an interesting extension to our suggested coordination scheme.

Insert Figure 10 about here

6. Summary and Extensions

In this paper, we suggested an order coordination scheme for a decentralized two-echelon supply chain when demand is random. Our scheme is based on the wholesaler offering a price discount to the retailers who place orders that correspond to the wholesaler's reorder cycle. We proposed an extension of a (R, T) policy as the retailer's replenishment policy, which incorporates the incentive mechanism of the coordination scheme. We developed and analyzed the coordination model from the retailer and the wholesaler's perspectives respectively. Each retailer faces the problem of determining its order-up-to levels (R^D and R^L) for a given order frequency (k+1)

within T_0; whereas the wholesaler determines the discount price, c_D, and its order-up-to level, R_0. Finally, we showed that both the wholesaler and the retailers can benefit significantly by exploiting this timing discount appropriately.

In the numerical study, we illustrated the impact of varying cost parameters (the backorder costs and the holding costs) on the wholesaler profits as well as the retailer's ordering policy and found that a relatively lower holding cost rate and a higher retailer backorder penalty resulted in increased wholesaler profits. However, as we increased the value of *k*, we observed that the retailers protected themselves against a high backorder penalty by placing orders more frequently, which reduced the wholesaler's profits due to the smaller order sizes that were cross-docked. To assess our proposed policy, we compared the profitability of the wholesaler under our scheme to a standard (R, T) replenishment policy without a price discount. In general, we observed that our suggested coordination policy results in greater wholesaler profits as the wholesaler supplies more retailers and incurs high holding costs.

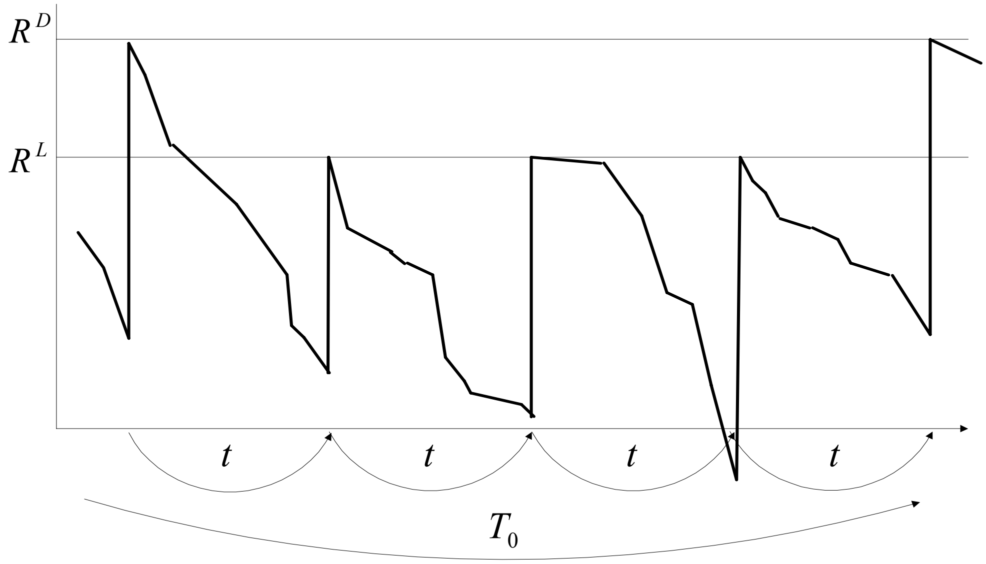

Figure 1. Extended periodic order-up-to-R policy at a retailer

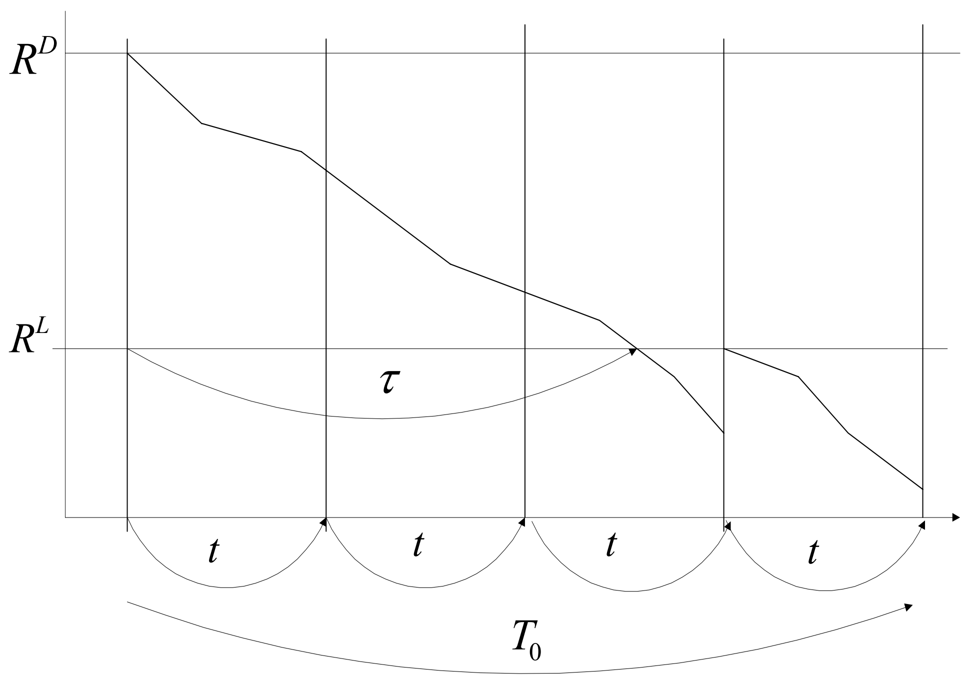

Figure 2. Inventory path at a generic retailer

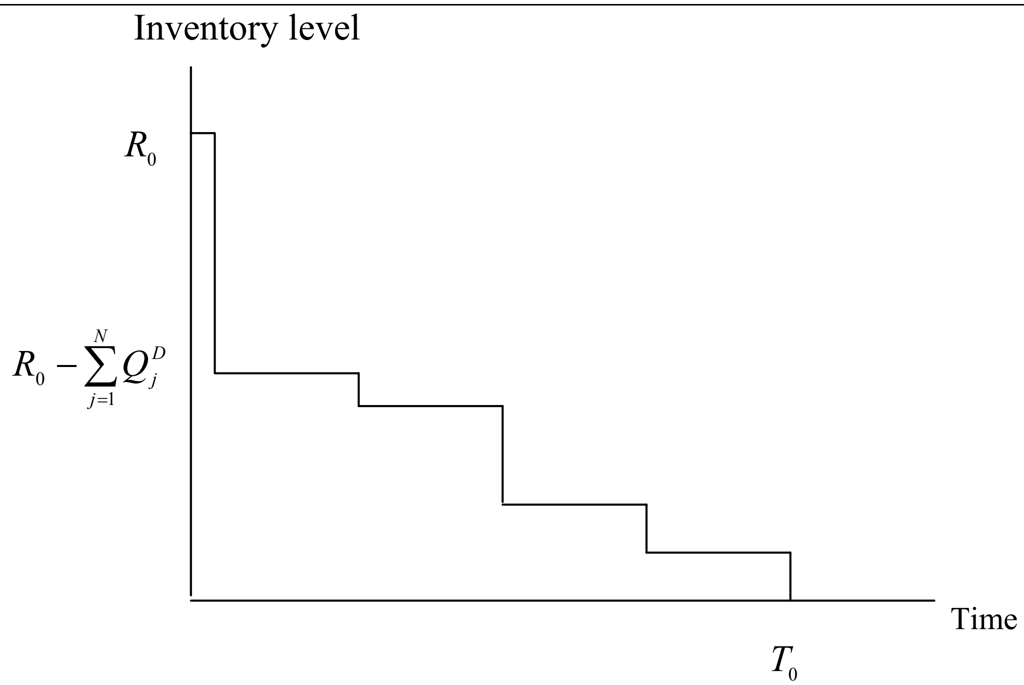

Figure 3. On hand inventory at the wholesaler

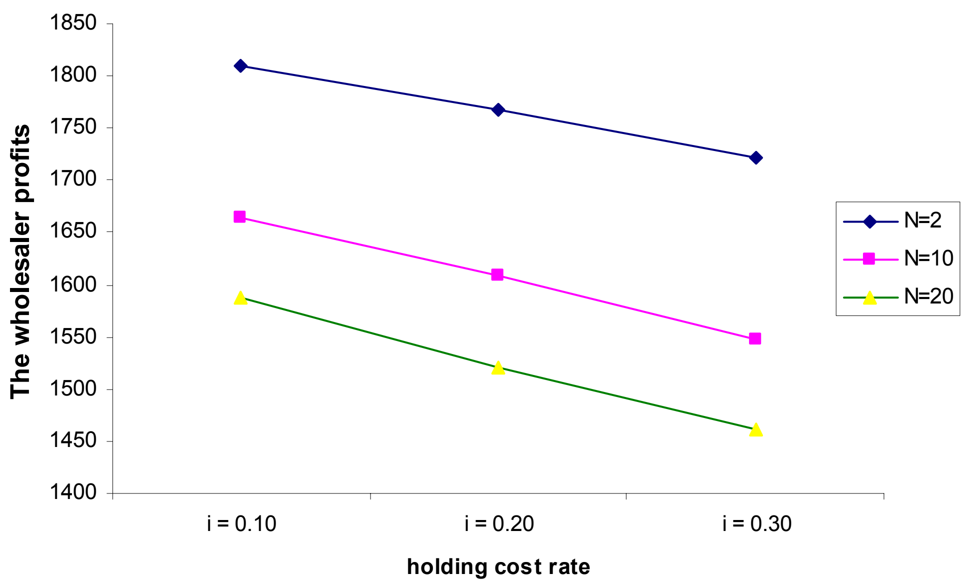

Figure 4. The wholesaler profits vs. holding cost rate

(π = 1*h, k = 1, c_E = \$25)

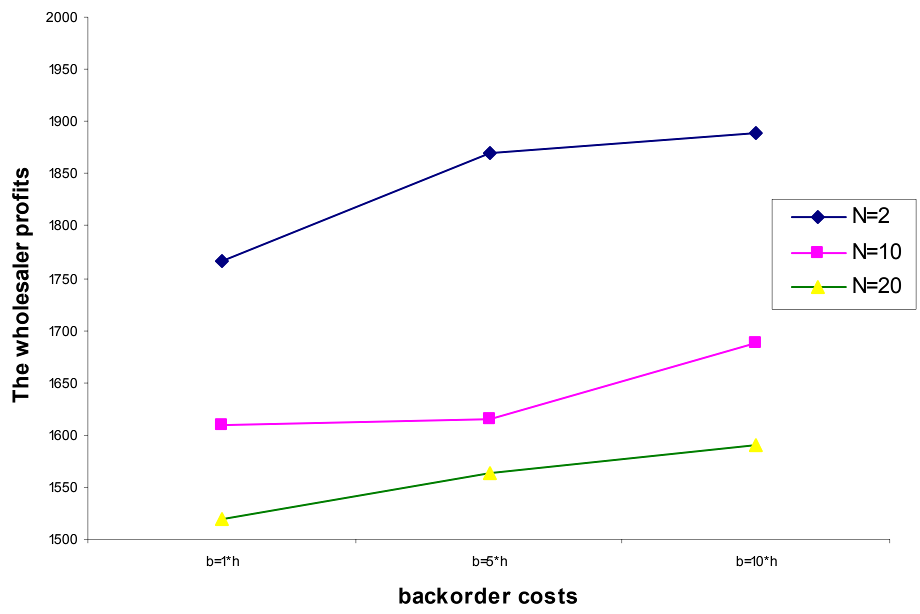

Figure 5. The wholesaler profits vs. backorder costs

($i = 0.2$, $k = 1$, $c_E = \$25$)

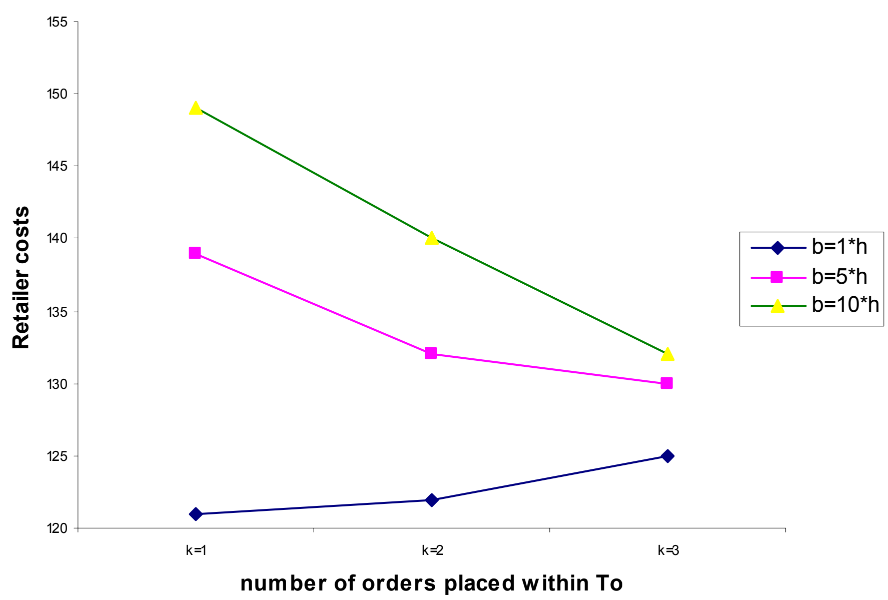

Figure 6. Retailer costs vs. number of orders placed within T_0

($N = 20$, $i = 0.20$, $c_E = \$30$)

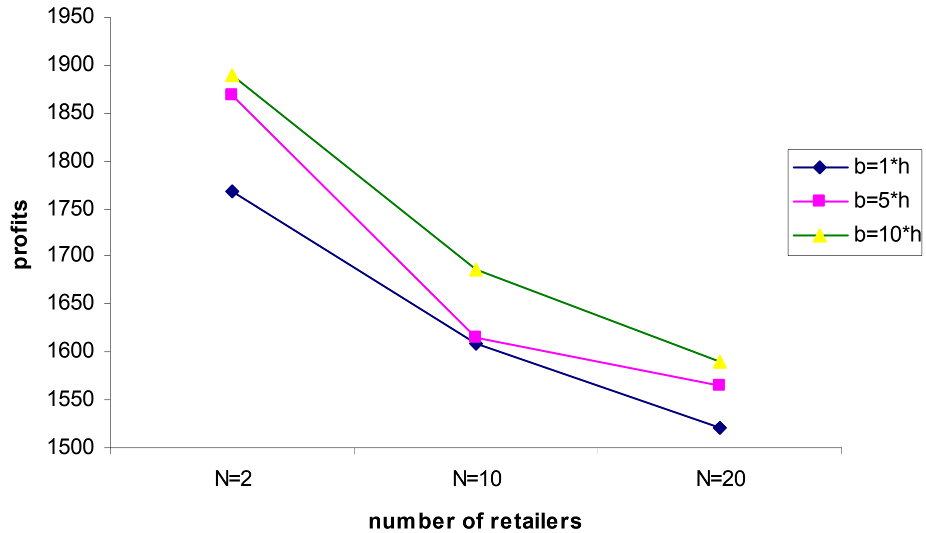

Figure 7. The wholesaler profits vs. number of retailers

(i = 0.20, k = 1, c_E = \$25)

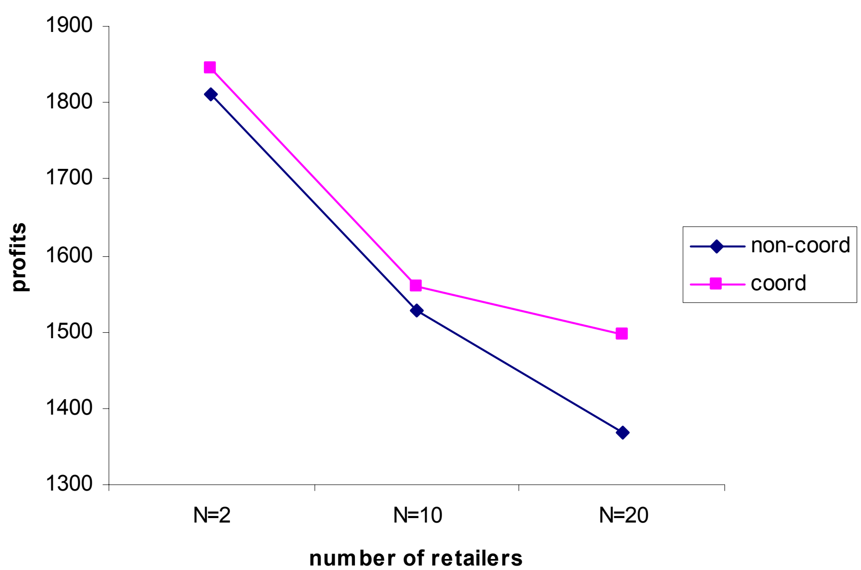

Figure 8. Coordination vs. non-coordination

(i = 0.20, π = $5*h$, k = 1, c_E = \$30)

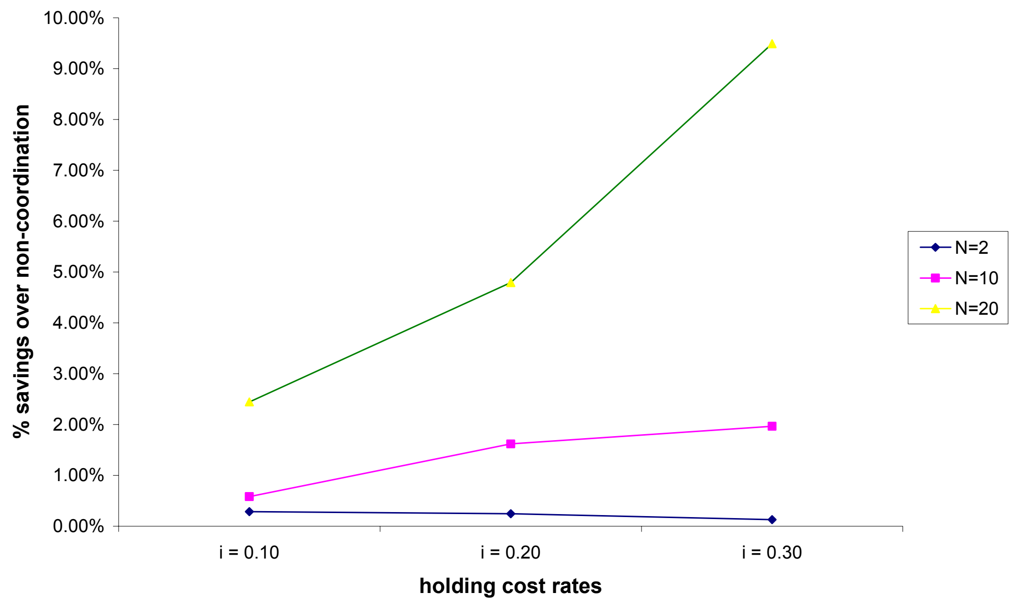

Figure 9. % savings over non-coordination

($\pi = 1{*}h$, $k = 3$, c_E=\$30)

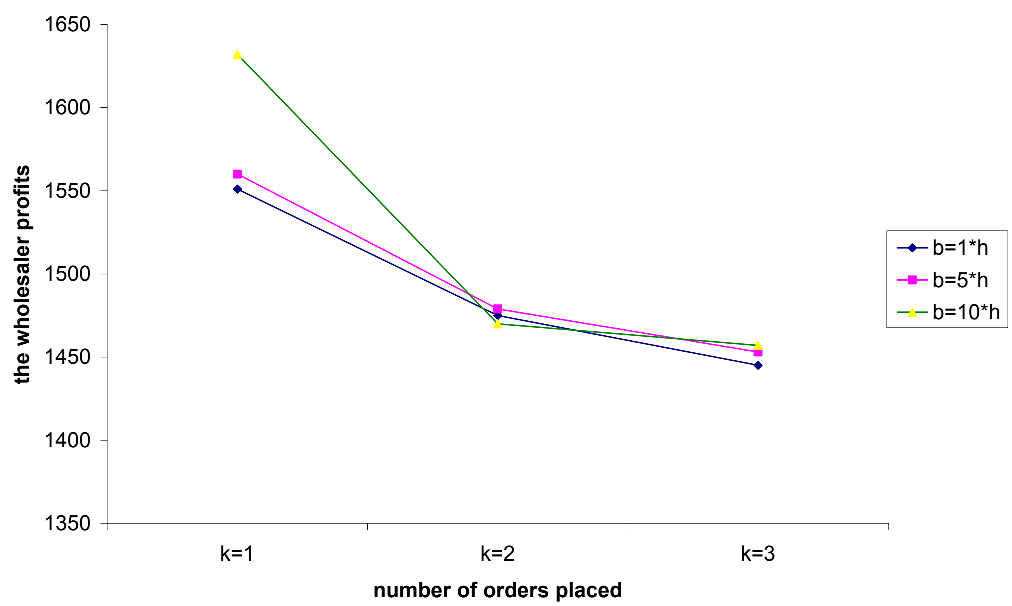

Figure 10 The wholesaler profits vs. number of orders placed within T_0

($N = 10$, $i = 0.20$, $c_E = \$30$)

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