Coordinating Orders in Supply Chains Through Price Discounts

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ABSTRACT

In this paper, we examine the issue of order coordination between a supplier and multiple retailers in a decentralized multi-echelon inventory/distribution system where the supplier provides a product to multiple retailers who experience static demand and standard inventory costs. Specifically, we propose and analyze a new policy where a manufacturer, who outsources production to an OEM, offers a price discount to retailers when they coordinate the timing of their orders with the manufacturer's order cycle. We show that our proposed policy can lead to more efficient supply chains under certain conditions, and present a straightforward method for finding the manufacturer's optimal price discount in this decentralized supply chain. A numerical experiment illustrates the managerial implications of our model as well as conditions when a manufacturer should consider adopting such a policy.

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1. Introduction

Coordination and cooperation issues between manufacturers (suppliers) and retailers (buyers) in decentralized multi-echelon inventory/distribution systems have gained much attention in recent years due to the increasing emphasis on the significance of effective supply chain management (Verity, 1996). A number of recent papers (*e.g.,* Parlar and Wang, 1994; Weng, 1995; Corbett and de Groote, 2000) showed that coordination between suppliers and retailers can result in improved performance and increased profitability to all participants in a supply chain. As an illustration of the benefits which can result from improved supply chain coordination, Dell Computer converted a low-margin direct sales operation into a high profit, high service business by outsourcing much of its production to a reduced number of OEM's and increasing the coordination within its supply chain (McWilliams, 1997).

In this paper, we consider a decentralized inventory/distribution system that consists of a manufacturer who outsources the production of a product to a third party (e.g., an OEM); in turn, the manufacturer supplies the product to a set $J = \{1,...,N\}$ of retailers. Retailers replenish their stock from the manufacturer who, in turn, replenishes his inventory from the OEM at pre-specified order intervals. We assume that demand at the retailers is constant over time and that the retailers incur standard inventory costs. In order to increase profits, the manufacturer offers a price discount to any retailer who places an order which coincides with the beginning of the manufacturer's cycle. We hypothesize that the manufacturer is willing to offer such an arrangement to avoid having to hold these units in stock (*i.e.*, these orders would be cross docked). Retailers who place orders at any other time during the manufacturer's cycle would pay the normal list price per unit.

While much previous research has considered coordination issues in decentralized supply chains, most of these papers suggesting pricing strategies to coordinate supply chain participants have considered either a single (i.e., one-time only) price discount or a discount based on order quantities. In contrast, we suggest a continuing price discount policy offered to all retailers that is based on the time that retailers' place their reorders. Furthermore, we feel that the policies described in this paper are feasible (due to advances in information technology) and cost effective under certain conditions. The objectives of our study are threefold. First, we develop an efficient methodology for finding the optimal discount price and describe conditions which must exist for a manufacturer to consider offering such a discount. Second, we investigate the magnitude of the savings (or gains) which might accrue to a manufacturer who offers such a discount. Finally, we wish to identify the conditions (including the number and characteristics of retailers) when such a scheme would be beneficial to all supply chain participants. We believe that our findings offer significant managerial implications in the area of channel management and supply chain coordination when these systems are decentralized.

The remainder of the paper is organized as follows. Following a description of related research, we describe and analyze our basic model in section 2. In the third section, we discuss two related issues: (1) the relationship between the manufacturer and the OEM and how the manufacturer can increase his profits further by negotiating the OEM reorder interval, and (2) how the manufacturer might proceed if he lacks information about the retailers' cost functions. In the fourth section, we present results from a numerical experiment and discuss the managerial implications. In this section, we also extend our model to the case when all retailers are homogeneous and discuss several interesting implications in this case. In the final section, we summarize the implications of our proposed policy and present several extensions to consider in future research.

1.1 Related Work

An increasing emphasis on improving supply chain performance has resulted in numerous studies of channel coordination in both centralized and decentralized multi-echelon inventory/distribution systems. Jeuland and Shugan (1983) considered the issue of channel coordination from a marketing perspective and discussed several mechanisms (including contracts, joint ownership, quantity discounts, etc.) that could improve channel coordination.

They showed that, under certain conditions, all members of a decentralized distribution channel can earn larger profits when all members coordinate. In a related study, Dolan (1987) specified several different conditions under which quantity discounts could lead to higher profits as a result of improved channel coordination. Crowther (1964) and Lal and Staelin (1984) considered both buyer and seller costs to justify the implementation of a quantity discount schedule. Dada and Srikanth (1987) extended the work by Lal and Staelin and suggested optimal pricing policies as well as a mechanism for allocating the cost savings between the buyer and the seller. Weng (1995) analyzed the effects of joint decision policies on channel coordination in a distribution system which consists of a single supplier and a group of homogeneous buyers when demand is price sensitive and operating costs are functions of order quantities. Unlike previous papers, he showed that quantity discounts alone are not sufficient to guarantee a joint profit maximization; that is, the effective coordination that maximizes joint profit will be achieved by simultaneously implementing both quantity discounts and franchise fees (that is, fixed payments per period from the buyer to the supplier). Parlar and Wang (1994) studied the pricing decision of a supplier and the subsequent ordering decisions of homogeneous buyers as a Stackelberg game and presented conditions in which the supplier will offer quantity discounts. Their work was extended by Wang and Wu (2000) to the case with heterogeneous buyers. Chen *et al*. (2001) considered coordination mechanisms in a decentralized supply chain based on franchise fees and quantity discounts, volume discounts, and frequency discounts. Viswanathan and Piplani (2001) recently considered a policy where a vendor offers a discount to buyers as an incentive for them to place orders only at times specified by the vendor (e.g., Mondays). Unlike our policy, however, Viswanathan and Piplani assume that the vendor follows a lot-for-lot policy and holds no inventory while simply passing the items from the supplier to the buyer.

Some researchers have considered the impact of quantity discounts from the buyer's viewpoint (*e.g.,*Jucker and Rosenblatt, 1985; Sethi, 1984; Rubin *et al.,*1983; and Ladany and Sternlieb, 1974). Monahan (1984) studied a single supplier, single buyer system from the vendor's perspective and showed that the supplier, using an order-for-order policy, could earn higher profits by offering the buyer optimal quantity discounts. Lee and Rosenblatt (1986) generalized Monahan's model to include the case when the buyer places orders more frequently than the supplier. Cachon (1999) studied the impact of scheduled ordering policies on the supply chain variability with a single supplier and multiple retailers. Other researchers have considered pricing and lot sizing decisions as part of a sequential negotiation processes or cooperative games between buyers and sellers (*e.g.*, Kohli and Park, 1989; Goyal, 1977; and Banerjee, 1986).

Our model is also related to some of the previous work on temporary supplier price changes. For example, Tersine and Schwarzkopf (1989) and Tersine and Barman (1995) considered the case when a supplier offers a price reduction for a stated sales period. Other researchers who considered a one-time only price reduction included Arcelus and Srinivasan (1995), Aull-hyde (1992), and Ardalan (1995). In these papers, a buyer reacts to a temporary price reduction that is offered for a defined period; after this time, the price reverts back to a pre-sale level. In contrast, the price reduction in our model is always available to retailers for any orders they place at specified times.

Other related work includes the study of buyer strategies when a supplier announces a future price increase (Taylor and Bradley, 1985) and when discounts are offered for joint replenishment decisions (Chakravarty, 1984; Chakravarty and Martin, 1988). A recent paper by Cheung (1998) analyzes the case when a supplier offers buyers a discount to accept delayed deliveries in order to avoid lost sales. While our discount and Cheung's discount are both designed to improve supply chain performance, the two discount schemes are implemented in very different ways.

2. Basic Supply Chain Coordination Model

We consider a decentralized two-echelon distribution system consisting of a manufacturer who outsources production to an OEM and supplies a set $J = \{1,...,N\}$ of retailers who face a static demand for a single product at a rate of D_j (see Figure 1). The retailers replenish their stock from the manufacturer who in turn is supplied by the OEM at a reorder interval, T_o. The manufacturer, who purchases items from the OEM at unit price c_o, offers the item to all retailers at a standard "list" price c_L (where $c_L > c_o$); the retailers then sell each item at a market price p.

Insert Figure 1 Here

In order to coordinate orders within this supply chain, we propose that the manufacturer offer retailers a reduced purchase price if they place orders at specific times. Specifically, we assume that any retailer who orders at the time which coincides with the beginning of the manufacturer's cycle can purchase items at a reduced price c_D, where $c_o \leq c_D \leq c_L$. In this way, the manufacturer may be able to reduce inventory holding costs (and increase profits) by shipping these units directly to the retailers (*i.e.*, "cross-docking" these items). It should also be noted that the manufacturer must offer the same discount price to all retailers, based on the rulings that the practice of offering differential prices to retailers violates current U.S. antitrust statutes that preclude vendors from "giving different terms to different resellers in the same reseller class" (Robinson-Patman Act of 1936).

We assume that the demand at each retailer is price insensitive; that is, retailer demand is not affected by a lower supplier price. According to Lal and Staelin (1984), this is a reasonably good approximation when the retailer has a fixed requirement for the product or when price is one of the many factors considered in a purchasing decision. We also assume that order leadtimes are negligible, no shortages are allowed, all demand must be met, retailers' order cycles be no longer than T_o, and no quantity discounts are offered. It should be noted that all of these assumptions can be relaxed which will simply complicate the model while not changing our basic result (namely, that offering a "timing" discount may result in a more efficient supply chain).

The manufacturer, who wants to maximize his profit per time period, places orders with an external supplier or OEM at a fixed reorder interval, T_o, and unit price c_o. (In the third section, we relax the assumption that the manufacturer reorder interval is fixed and consider the case when the manufacturer is able to negotiate the value of T_o.) Retailers follow a continuous-review inventory policy and replenish their stock through the manufacturer. We assume that retailers' order cycles be no longer than T_o and that each retailer orders an integer number of orders during the manufacturer's order cycle that results in the well known nested policies (Roundy, 1985; Zipkin, 2000).

Throughout the remainder of this paper, we use the following notation:

c_o = price/unit paid by the manufacturer to the OEM,

c_L = normal "list" price/unit charged by the manufacturer to the retailers,

c_D = discount price/unit charged by the manufacturer if a retailer agrees to purchase at a specific point in time (beginning of the manufacturer's cycle) where $c_o \le c_D \le c_L$,

t_j^D = time between a retailer j's order at the beginning of the manufacturer's cycle and the retailer's next order (at the list price),

t_j^L = time between orders at the normal "list" price for retailer j,

Q_j^D = quantity ordered at the discount price by retailer j,

Q_j^L = quantity ordered at the normal "list" price by retailer j,

T_o = manufacturer reorder interval,

D_j = demand (static) per time period at retailer j,

k_j = number of orders placed at the normal "list" price by retailer j during the manufacturer's reorder cycle T_o,

s_j = fixed reorder cost for retailer j (s_o denotes the manufacturer's reorder cost),

i = inventory carrying cost rate per time period,

$TC\left(k_j, t_j^D, c_D\right)$ = total cost per time period for retailer j, given discount price c_D, and

$z(c_D)$ = manufacturer's profit per time period as a function of the discount price c_D.

2.1 Model Analysis: Retailers' Perspective

Assuming that retailers must meet demand and no shortages are allowed, the inventory behavior at retailer j is indicated in Figure 2.

Insert Figure 2 Here

As indicated, each retailer initially orders quantity Q_j^D ($= D_j\ t_j^D$) at the discount price c_D and then places k_j (where k_j = 0, 1, 2, ...) orders of size Q_j^L ($= D_j\ t_j^L$) at the normal list price c_L within the manufacturer's cycle. Clearly, $T_o = t_j^D + k_j\, t_j^L$. Since $c_D \le c_L$, it follows that $t_j^D \ge t_j^L$ and

$$t_j^D \geq \frac{T_o}{(k_j + 1)} .$$

Since we have assumed that demand (and therefore revenue) is static, it is evident that minimizing retailers' cost is equivalent to maximizing retailer profits. The retailer's cost per time period as a function of k_j and t_j^D can then be defined as follows:

$$TC_j\left(k_j,\, t_j^D, c_D\right) = c_L\, D_j - \frac{(c_L - c_D)\, D_j\, t_j^D}{T_o} + \frac{s_j\,(k_j{+}1)}{T_o} + \frac{i\, D_j}{2T_o}\left\{ c_D \left(t_j^D\right)^2 + \frac{c_L\left(T_o - t_j^D\right)^2}{k_j} \right\} . \qquad (1)$$

For $c_o \leq c_D \leq c_L$ and fixed values of k_j, we can apply first-order optimality conditions with respect to t_j^D to (1) which leads to:

$$t_j^{D*} = \frac{k_j\,(c_L - c_D) + i\, c_L T_o}{i\,(k_j\, c_D + c_L)} . \qquad (2)$$

Note that if $c_D = c_L$ (no discount is offered), $t_j^{D*} = \dfrac{T_o}{k_j + 1}$.

For any value of $c_o \leq c_D \leq c_L$, the cost/time period for retailer j as defined by (1) can then be redefined as a function of k_j and c_D only; that is,

$$f_j(k_j,c_D) = TC_j\left[k_j, t_j^{D*}(k_j), c_D\right] = \frac{s_j\,(k_j{+}1)}{T_o} + c_L\, D_j$$

$$- \frac{D_j\left[k_j\,(c_L - c_D)^2 + 2i\, c_L\, T_o\,(c_L - c_D) - i^2\, c_L c_D\,(T_o)^2\right]}{2i\, T_o\left(k_j c_D + c_L\right)} . \qquad (3)$$

For fixed values of c_D, we show in Appendix A that (3) is discretely convex with respect to k_j. Thus, k_j^* is the largest value of k_j such that

$$\Delta_{k_j} f_j = f_j(k_j + 1, c_D) - f_j(k_j, c_D) \le 0 ,$$

or

$$\left[(k_j + 1) c_D + c_L \right] (k_j c_D + c_L) \le \frac{c_L D_j \left[c_D (1 + i T_o) - c_L \right]^2}{2 i s_j} \quad . \tag{4}$$

Using (3), we can derive a lower bound for the value of c_D. As the manufacturer reduces c_D, the j^{th} retailer will, at some point, order all units at the beginning of the manufacturer's cycle (i.e., the retailer will set $k_j = 0$). The lower bound on c_D for the j^{th} retailer is the largest value of c_D such that $k_j = 0$; that is, the largest value of c_D such that $f_j(1, c_D) > f_j(0, c_D)$. From (3), we have

$$f_j(1, c_D) - f_j(0, c_D) > 0 \quad \Rightarrow \quad \frac{s_j}{T_o} - \frac{D_j \left[i c_D T_o - (c_L - c_D) \right]^2}{2 i T_o (c_L + c_D)} > 0 \, . \tag{5}$$

Rearranging terms in (5), we get

$$\left[\frac{D_j (1 + i T_o)^2}{i s_j} \right] c_D^2 - 2 c_D \left[\frac{D_j c_L (1 + i T_o)}{i s_j} + 1 \right] + \frac{D_j c_L^2}{i s_j} - 2 c_L < 0$$

which can be rewritten as

$$A_j c_D^2 - 2 B_j c_D + E_j < 0;$$

where $A_j = \dfrac{(1 + i T_o)^2 D_j}{i s_j}$, $B_j = \dfrac{D_j c_L (1 + i T_o)}{i s_j} + 1$, and $E_j = \dfrac{D_j c_L^2}{i s_j} - 2 c_L$.

Letting $X_j = \dfrac{B_j + \sqrt{B_j^2 - A_j E_j}}{A_j}$ and $\hat{X} = \min \left(X_j \right)_{j \in J}$, a lower bound, $LB(c_D)$, on the discount price c_D that will result in all retailers placing a single order during the manufacturer's reorder interval is defined as follows:

If $\hat{X} \le c_o$, then $LB(c_D) = c_o$;

if $c_o < \hat{X} \le c_L$, then $LB(c_D) = \hat{X}$;

if $\hat{X} > c_L$, then $LB(c_D) = c_L$.

2.2. Model Analysis: Manufacturers' Perspective

For a given value of c_D, we have shown how the retailers choose their optimal values of k_j^* to minimize their respective costs. In the manufacturer's case, the manufacturer sets the discount price, c_D, to maximize his profit. Since the manufacturer's profit is a function of his average inventory, we will use I_o to denote the inventory carried by the manufacturer during T_o. Letting,

I = total echelon inventory in the system during T_o, and

I_j = total inventory carried by the j^{th} retailer during T_o,

then,

$$I_o = I - \sum_{j \in J} I_j = \frac{T_o^2}{2} \sum_{j \in J} D_j - \sum_{j \in J} I_j \,, \tag{6}$$

where

$$I_j = \begin{cases} 0.5\, T_o^2\, D_j & \text{if } k_j = 0 \\ \dfrac{\left[T_o\left(T_o - 2\, t_j^D\right) + \left(t_j^D\right)^2 (k_j + 1)\right] D_j}{2\, k_j} & \text{if } k_j \ge 1\,. \end{cases}$$

The total inventory for the manufacturer over the cycle T_o is indicated in Figure 3.

Insert Figure 3 Here

Given the inventory held by the manufacturer for each retailer, we can define the average inventory at the manufacturer (denoted by $\bar{I}_o$) during the period T_o; using (6) and the definition of t_j^{D*} given in (2), we find that

$$\bar{I}_o = \frac{T_o}{2} \sum_{j \in J} D_j \left\{ 1 - \frac{(c_L - c_D)^2 \left[k_j (k_j+1) + 2\, i\, k_j T_o\right] + (iT_o)^2 \left(k_j c_D^2 + c_L^2\right)}{\left[i\, T_o (k_j c_D + c_L)\right]^2} \right\}. \qquad (7)$$

If $k_j = 0$ for any j^{th} retailer, the manufacturer does not hold any inventory for that retailer as expected. Likewise, if $k_j = 0$ for all $j \in J$, then $\bar{I}_o$ equals zero.

From (7), the manufacturer's profit per time period, $z(c_D)$, can be defined as a function of the discount price, c_D, as follows:

$$z\left[c_D, k_j^*(c_D)\right] = (c_L - c_o)\, D_o - \frac{(c_L - c_D)}{T_o} \sum_{j \in J} D_j\, t_j^D - \frac{s_o}{T_o} - i\, c_o\, \bar{I}_o \qquad (8)$$

where $\bar{I}_o$ is defined by (7) and T_o is assumed fixed. The manufacturer's profit defined by (8) can be viewed as the manufacturer's revenue, $(c_L - c_o) D_o$ minus the discount given by the manufacturer to the retailers for coordinating their order cycles ($\frac{(c_L - c_D)}{T_o} \sum_{j \in J} D_j\, t_j^D$) minus the manufacturer's holding cost and fixed ordering cost.

Using (8), we can define a methodology for finding c_D^* (the discount price which maximizes the manufacturer's profit). Our algorithm is based on the calculation of $k_j^*(c_D)$ which we denote as the optimal value of k_j for retailer j for a given discount price c_D. We can find the value of $k_j^*(c_D)$ for each j^{th} retailer using (4). Note that the lower bound for c_D that was previously calculated is equivalent to the value of $k_j^*(c_D)$ when $k_j = 0$.

Conversely, there is a continuum of values of c_D that would result in the same value of k_j for any retailer j. For the manufacturer to maximize his profits, he must set his discount price to the largest possible value for any given value of k_j. Letting $\hat{c}_D(k_j)$ denote the maximum discount price for a given value of k_j, we can find the value of $\hat{c}_D(k_j)$ by setting (4) to be an equality and solving the resultant equation for c_D.

The manufacturer's optimal discount price, c_D^*, must therefore be the value(s) of $\hat{c}_D(k_j)$

that maximizes $z(c_D)$; this leads to the following observations:

Observation 1. The optimal discount price c_D^* is an element of set C, where

$$C = \left\{\hat{c}_D(k_j),\ k_j = 0,\ 1,\ 2,\cdots,k_j(c_L)\ \ \forall\ j \in J\right\}.$$

Observation 2. For all $c_D \in C$, $\dfrac{d\,z(c_D)}{d\,c_D} \geq 0\ \ \forall\ j \in J$ and $k_j = 0,1,\cdots,k_j(c_L)$.

Both observations follow from the definition of manufacturer profits defined by (8) and the fact that $\hat{c}_D(k_j)$ maximizes the manufacturer's profits for any given value of k_j. Based on these observations, we can find c_D^* using the following algorithm:

1) For all retailers $j \in J$, calculate $k_j(c_L)$ using (4).
2) For all $j \in J$ and $k_j = 0, 1, 2, ..., k_j(c_L)\text{-}1$, calculate the elements $\hat{c}_D(k_j)$ of set C.

The optimal discount price c_D^* is then the value of $\hat{c}_D(k_j) \in C$ which maximizes (8).

3. Supply Chain Coordination Issues

There are several additional coordination issues that relate to the decentralized supply chain described in this paper. Two issues, however, are significant and deserve mention. While most of our focus is on the coordination between the manufacturer and retailers, the manufacturer must also coordinate with the OEM or outsourcing organization. If the manufacturer further processes the items from the OEM, there would be additional inventory held at the manufacturer if there were a finite production rate. In this case, however, we assume that manufacturers add little additional processing (perhaps some repackaging) so that this inventory holding cost is small and can be ignored.

The length of the OEM-manufacturer reorder interval, T_o, has an impact on the costs incurred by the retailers as well as the manufacturer profits. If possible, the manufacturer may want to negotiate this interval with the OEM. To investigate the impact of changing the value of

T_o, we can treat the k_j values as continuous to simplify the analysis; if there are a sufficiently large number of retailers, this approximation will be relatively close to the solution with integer values of k_j. Applying first order optimality conditions to (1), we find that

$$k_j^* = \beta_j\,(i\,T_o + 1) - \left(\frac{c_L}{c_D}\right)(\beta_j + 1) \tag{9}$$

where $\beta_j = \sqrt{\dfrac{c_L\,D_j}{2\,i\,s_j}}$. The values of k_j^* defined by (9) are nonnegative if and only if

$$(i\,T_o + 1)\,c_D\,D_j \;\geq\; c_L\,D_j + \sqrt{2\,i\,s_j\,c_L\,D_j}\;,$$

that can be written as:

$$(i\,T_o + 1)\,c_D\,D_j \;\geq\; c_L\,D_j\,(1 + i\,t_j) \tag{10}$$

where $t_j = \sqrt{\dfrac{2s_j}{i\,D_j\,c_L}}$ is the optimal cycle time of retailer j in an uncoordinated supply chain. Thus, the j^{th} retailer will set her corresponding value of k_j greater than zero if it is less costly for her to order (and hold) items at the list price than at the discount price; otherwise, she will set k_j equal to zero.

The inequality defined in (10) indicates that the manufacturer may benefit by renegotiating the value of T_o as the value of c_D changes. To investigate this relationship, we can use (9) to find an expression for the manufacturer's optimal value of T_o^*. Since the manufacturer incurs no holding cost for retailers who set $k_j = 0$ (since we are treating the values of k_j as continuous), the marginal profit earned by the manufacturer for these retailers is independent of c_D. Thus, letting J' = $\{j \in J \mid k_j > 0\}$ and applying FOC to (8), we find that

$$T_o^* = \sqrt{\frac{2}{i\,c_o D_o}}\left\{(c_L - c_D)\,W - \frac{c_o}{c_L}\,V + \frac{c_o}{c_L c_D}\sum_{j\in J'} s_j\left[\beta_j\,(c_L - c_D) + c_L\right] + s_o\right\}^{\frac{1}{2}} \tag{11}$$

where $$W = \frac{2}{c_L} \sum_{j \in J'} s_j \left[\left(\frac{c_L}{c_D}\right) \beta_j \left(\beta_j - 1\right) - \beta_j^2 \right]$$ and

$$V = \sum_{j \in J'} s_j \left\{ \left(\frac{c_L}{c_D}\right) \left(\beta_j - 1\right) \left[\left(\frac{c_L}{c_D}\right) \left(\beta_j - 1\right) - 2\beta_j \right] + \beta_j^2 \right\} .$$

Assuming that the list price, c_L, is fixed by the market and the order price, c_o, is set by the OEM who supplies the manufacturer, the inequality in (10) implies that the manufacturer's optimal reorder interval is determined by the discount price, c_D. Furthermore, we can show that the manufacturer's profit, defined by (8), is a concave function of the reorder interval T_o for any fixed vector $(k_1, k_2, ..., k_N) \geq 0$ (this result also holds for the case when k_j's are restricted to integer values).

Thus, the manufacturer can increase his profits if he can negotiate a reorder interval T_o that satisfies (11). (This may be difficult to achieve, however, as the optimal value of T_o is a function of the number of retailers, N, assuming a fixed market demand. As the number of retailers varies, the manufacturer would have to renegotiate the value of T_o which the OEM may be reluctant to change due to his ordering and/or manufacturing costs.)

When the retailers are homogeneous (such that $k_j = k$ for all $j \in J$), we can also show that the manufacturer's profit function is maximized when $T_o = \sqrt{2}\, t$ where $t = t_j = \sqrt{\frac{2s_j}{i D_j c_L}}$ (results that are consistent with those in Roundy, 1985). In this case, a cycle time of $T_o = \sqrt{2}\, t$ results in $k = 0$ which implies that the manufacturer would not offer a coordination discount to maximize his profit. In our numerical experiments, we found that the manufacturer could increase his profits by an average of 8.47 percent if he could negotiate T_o (compared to the case when he offered a coordination discount to the retailers but could not negotiate T_o).

3.1. Imperfect Information About the Retailers

A second issue is whether the manufacturer will have sufficient information about the retailers in order to determine their optimal reordering behavior for various values of c_D. While the assumption of complete information has been made by various other researchers (e.g., Viswanathan and Piplani, 2001; Chen *et al,*2001), it is possible that the manufacturer may not have access to such information. (For a case when the supplier lacks perfect information about a buyer's cost structure, see Corbett and de Groote, 2000.)

If the manufacturer lacks information about retailers' cost structures, he could still implement our suggested policy by initially offering a small discount (i.e., letting $c_D = c_L - \varepsilon$), observing the retailers' resultant behaviors, and computing his profit. He could then reduce the discount price and repeat the process until he observes no further increase in profits. Since the retailers only change their values of k_j if their costs are reduced, retailers will never be worse off as the manufacturer reduces the discount price, c_D. However, since the manufacturer's profit function is not concave, such a scheme may converge at a local optimum (nevertheless, the manufacturer's profits would still be improved).

4. Numerical Experiments and Managerial Implications

Our proposed timing discount policy can be viewed as both a profit maximization scheme for the manufacturer as well as a profit sharing scheme for the retailers. In our model, the manufacturer offers a discount price c_D ($\leq c_L$) to the retailers in order to reduce holding costs and increase profits. Since the retailer demand is static and price inelastic, retailers' profits can never decrease (and are likely to increase) when the manufacturer offers a discount price c_D. Furthermore, retailers' profits vary inversely with the magnitude of this discount price c_D, based on the definition of retailers' costs in (1) and the necessary condition for optimality given in (4). Thus, the use of a discount price to influence order timing decisions can only increase the profits of all stakeholders in the decentralized supply chain described in the previous section.

4.1. Numerical Experiments

To study the impact of varying parameters on the manufacturer's profits and the retailers' willingness to coordinate their reorder cycles (and accept the discount price), we ran a number of numerical experiments. These experiments provided similar results with respect to the potential benefits from our coordination scheme; to illustrate these results, we used the following parameters:

$c_o = \$15$,

$c_L = \$20$,

$i = 0.1, 0.2, 0.3$,

$s_j = s_o = \$10$ for all $j \in J$,

$D_o = \sum_{j \in J} D_j = 2000$,

and varied the number of retailers (N = 2, 5, 10, 20), and the degree of retailer homogeneity. We set the manufacturer's reorder cycle, T_o, equal to 0.4.

To vary the degree of retailer homogeneity, we varied the degree of demand variation among the N retailers (recall that the demand at each specific retailer is assumed to be static). Since we held the total demand D_o constant, demand at the N retailers could range from an equal allocation of demand (i.e., demand at each retailer is equal to $\frac{D_o}{N}$) to the case where one retailer has D_o units and the remaining (N-1) retailers have zero demand. To vary the distribution of demand among the N retailers, we defined a weight w_j for each retailer $j \in J$ such that $D_j = w_j D_o$, where $w_j = \frac{\omega_j}{\sum_{j \in J} \omega_j}$ and ω_j follows a uniform distribution defined over the interval $\left[\frac{1}{N} - x, \frac{1}{N} + x\right]$. Since ω_j has a mean of $\frac{1}{N}$ and a standard deviation of $\frac{x}{\sqrt{3}}$, the coefficient of variation (denoted by cv) is then $\frac{N\,x}{\sqrt{3}}$; by varying the value of cv, we changed the values of x and, hence, ω_j and D_j. We generated four different types of allocations by varying cv as follows:

1) no demand variation among retailers; i.e., cv = 0 $\Rightarrow D_j = \frac{D_o}{N}$ for all $j \in J$,
2) low demand variation among retailers; i.e., cv = 0.2,
3) medium demand variation among retailers; i.e., cv = 0.4, and
4) maximum demand variation; i.e., cv = $\frac{1}{\sqrt{3}}$.

In our numerical experiments, we evaluated a total of 48 realizations; for each parameter set, we generated 500 runs in order to obtain expected values. For each run, we calculated the manufacturer's profit, the optimal discount price, and the manufacturer's gain due to channel coordination. The results of the numerical experiment with the inventory carrying cost rate set equal to 0.2 are reported in the following section.

4.2 Timing Discount Experimental Results

As indicated in Figure 4, our experiments show that the manufacturer's profits generally increase as the number of retailers increase. Since total demand was fixed, an increase in the number of retailers results in a lower average demand at each retailer. From a retailer's perspective, a smaller demand would result in a larger "natural" cycle time $t_j^* = \sqrt{\frac{2s_j}{iD_j c_L}}$; retailers with larger "natural" cycle times would be more willing to coordinate with a manufacturer who offers a discount pricing policy in order to reduce their holding costs. Thus, we observed an increase in the number of units that were cross-docked as well as the manufacturer's profits as more retailers were added to the supply chain.

Insert Figure 4 Here

As previously noted, a manufacturer can never reduce his profits by offering a discount for retailers to coordinate with his order cycle. However, we would like to find how much the manufacturer can increase his profits by offering a discount price and under what circumstances our coordination scheme is most effective. To analyze these issues, we calculated the profit that the manufacturer would earn if no discount price were offered; that is, the non-coordinated case when each retailer operates at their optimal cycle length (where $t_j^* = \sqrt{\frac{2s_j}{iD_j c_L}}$). (Recall that manufacturer revenues are constant and equal to $(c_L - c_o)D_o$ so that maximizing manufacturer profit is equivalent to minimizing manufacturer costs.) As indicated in Figure 5, the manufacturer could increase his profits up to 4.83 percent by adopting our coordination scheme, depending on the number of retailers and their degree of homogeneity. In general, it appears that the gain from offering a timing discount improves when there are more retailers in the system due to the cross-docking effect (especially if the degree of retailer homogeneity is relatively low). It should also be noted that even in those cases when the percent gain was relatively small, such an increase could be gained at no additional cost to either the retailers or the manufacturer by adopting our channel coordination scheme.

Insert Figure 5 Here

Clearly, the holding cost rate and the fixed order cost impact the costs in the supply chain system. As indicated in (4), the values of k_j are monotonically nonincreasing as the holding cost rate (i) decreases, which implies that retailers are more willing to take advantage of the price discount (and hold items longer) as the holding cost decreases (thereby resulting in increased manufacturer profits). This observation was supported by our numerical results; the average manufacturer profit (for all N = 2, 5, 10, 20) was \$8773 when the holding cost rate (i) was 0.3. When the holding cost rate was reduced to 0.2, the average manufacturer profit increased to \$9298; when the holding cost rate was further reduced to 0.1, the manufacturer's average profit increased to \$9738. This reduction in the holding cost rate from 0.3 to 0.1 resulted in an average increase in manufacturer profits of eleven percent.

Furthermore, as the number of retailers became large and demand at each retailer was reduced (since total demand was fixed), each retailer's optimal cycle length ($\sqrt{\frac{2\, s_j}{i\, D_j\, c_L}}$) approached T_o. When this occurs, the manufacturer would only have to offer a minimal (or zero) discount to induce retailers' coordination.

4.3. Homogeneous Retailer Case

If all retailers are homogeneous (i.e., $D_j = \frac{D_o}{N}$ and $s_j = s \; \forall \; j \in J$), the analysis can be simplified and some additional insights become evident. Using the same parameters in the previous section (i.e., setting cv = 0), the results indicated in Table 1 show that manufacturer profits increase with the number of retailers; in this case, profits increased from $8,993 (with 2 retailers) to $9,375 (with 20 retailers). When retailers are homogeneous, we can also show that the retailers' costs decrease (and manufacturer profits increase) monotonically as the quantity ($c_L - c_D$) increases; that is, retailers will always take advantage of a price discount to some extent and will do better as the manufacturer reduces the discount price.

Insert Table 1 Here

We also investigated the possibility that the manufacturer may offer a discount price that is sufficiently low such that retailers would set their respective values of $k^* = 0$; that is, retailers would order all items at the beginning of the manufacturer's cycle only. (This price is equal to the lower bound $LB(c_D)$ calculated in section 2.1.) Since all items in this case would be cross-docked and none would be held in stock, the manufacturer would be able to eliminate all fixed (and variable) costs associated with operating a warehouse. The manufacturer's profits in this case are generally less than his maximum profits; as indicated in Table 1, the manufacturer's profits are reduced by an average of approximately nine percent for the four cases with 2, 5, 10, and 20 retailers. However, a manufacturer who considers this strategy would be able to eliminate his warehouse and all associated costs; effectively, he would become an "e-manufacturer" who serves only as an information broker to coordinate shipments between the OEM and the retailers. In the long run, this might be the manufacturer's most effective strategy.

5. Conclusions and Extensions.

In this paper, we suggest a new mechanism for coordinating orders that suppliers might use to increase their profits in a decentralized supply chain when the manufacturer outsources production to an OEM. To analyze our proposed channel coordination policy, we developed a model based on the assumption that the manufacturer (supplier) has a fixed reorder cycle, T_o, with the OEM and the retailers place an integer number of orders within this cycle (*i.e.,* they follow a nested policy).

To test the assumption that retailers would be willing to adopt a nested policy, we computed the costs that retailers would incur if they placed orders at their EOQ or "natural" ordering cycles and compared these costs to the retailers' costs incurred using a nested policy. If no discount is offered by the manufacturer (the non-coordinated case), retailers clearly incur higher costs if they follow a nested policy. For all cases we analyzed in this study, we found that retailer costs increased by an average of 0.06 percent if retailers follow a nested policy and no coordination discounts are offered. However, when the manufacturer offered a discount price $c_D < c_L$, retailers' costs were consistently lower (by a small amount) than their non-nested costs. Thus, our assumption that retailers would be willing to follow a nested policy appears to be a reasonable one.

With the continuing development of information technology, an increasing amount of information about retailers is both accessible and less costly to manufacturers and suppliers (and vice versa). For example, EDI (Electronic Data Interchange) has made sales and cost data readily available to supply chain stakeholders; in other cases, retailer demand data is simply posted on the world wide web. The result is that coordination between manufacturers and retailers has become more prevalent as reported in the literature.

The supply chain coordination mechanism suggested in this paper can be extended in numerous ways. At the present time, we are investigating the cases when backorders are allowed and the case when the demand at the retailers is stochastic (we assume that retailers use a periodic review inventory policy in this case). Initial investigations indicate that the concept of a coordination "discount price" under these conditions remains a viable alternative.

Finally, we note that there are numerous other coordination policies available to suppliers in decentralized supply chains, including the use of franchise fees, quantity discounts, volume discounts, and frequency discounts (Chen *et al*., 2001). While some of these policies may result in greater supply chain profits than the policy we suggest in this paper, we note that these policies are not mutually exclusive; that is, our policy can be implemented in conjunction with these other policies. Given that implementation costs associated with our policy are relatively small, the marginal gains may be significant. In addition, our policy generally leads to smaller peak inventory levels at the manufacturer level than policies based on quantity discounts, thereby reducing storage requirements and associated (fixed) warehousing costs.

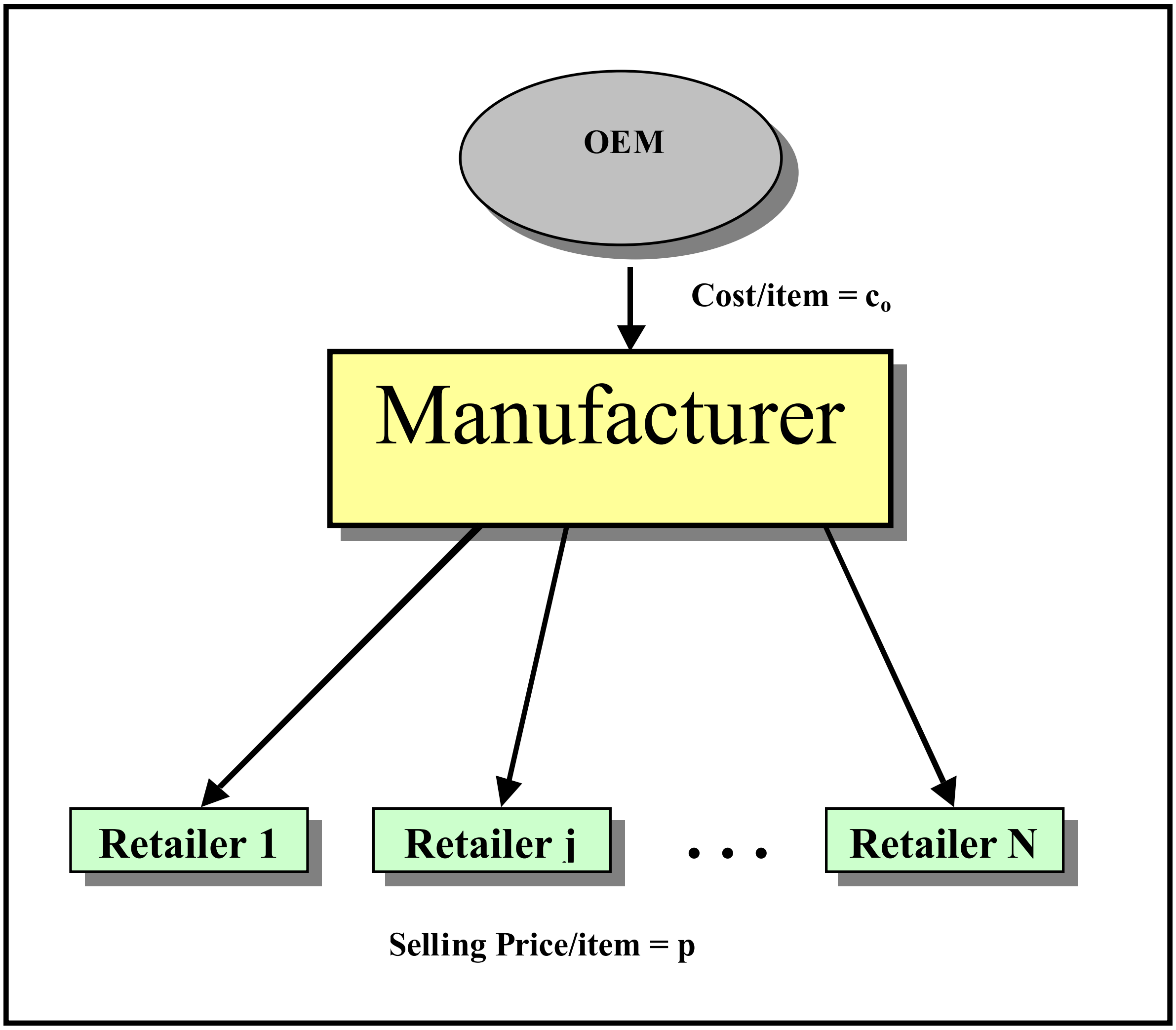

Figure 1. A Decentralized Supply Chain

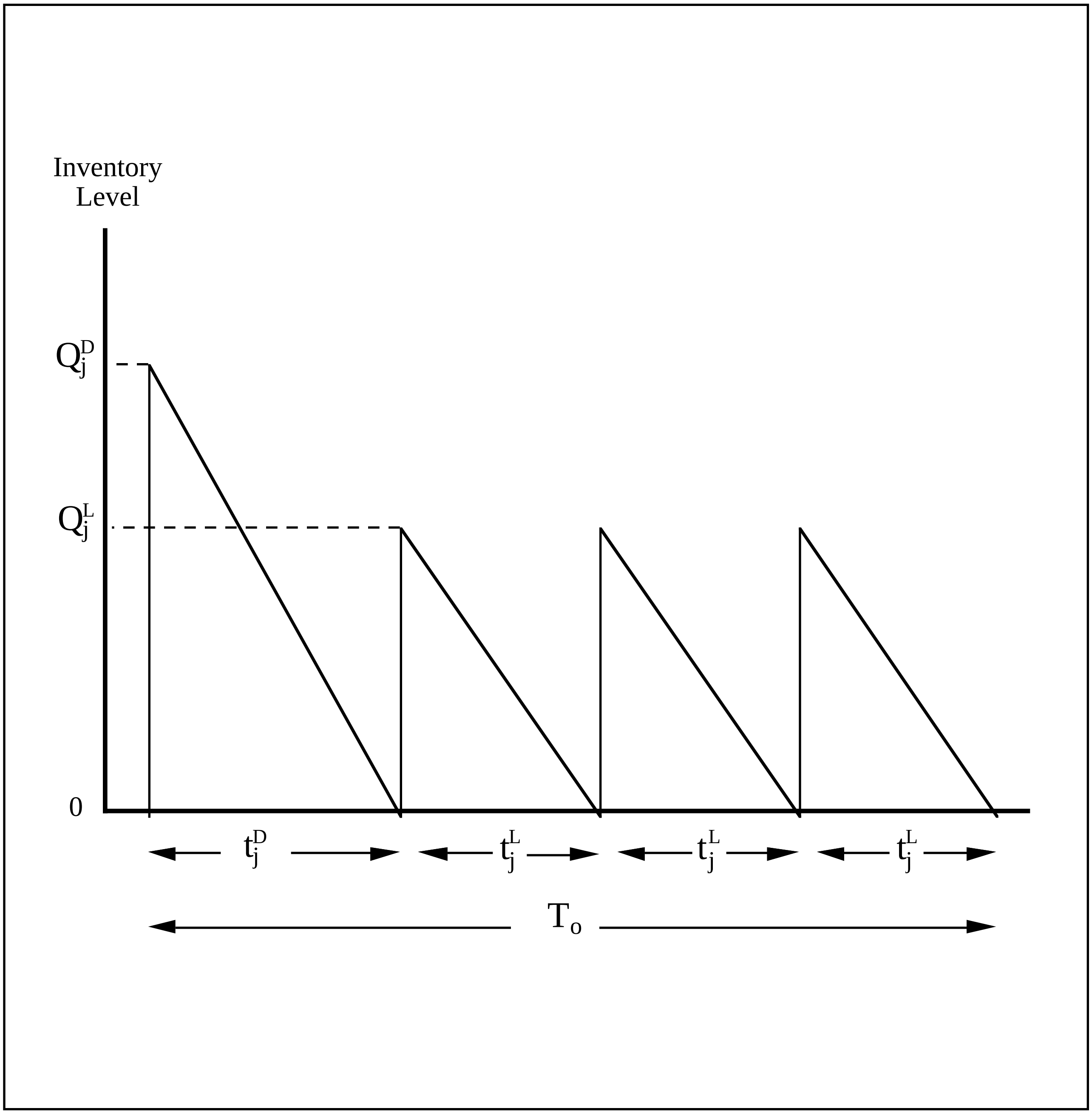

Figure 2. Inventory Levels for Retailer j.

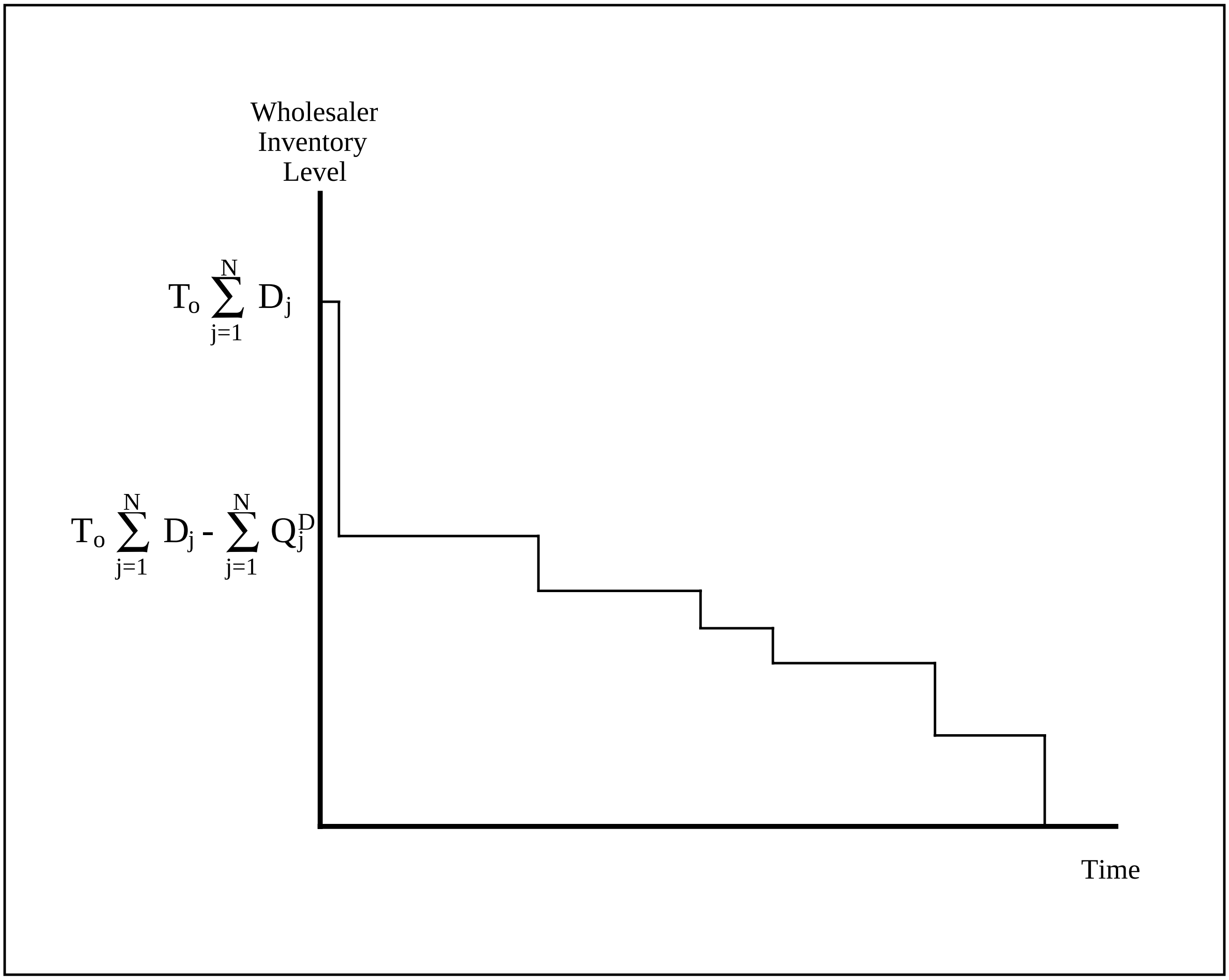

Figure 3. Manufacturer Inventory Levels

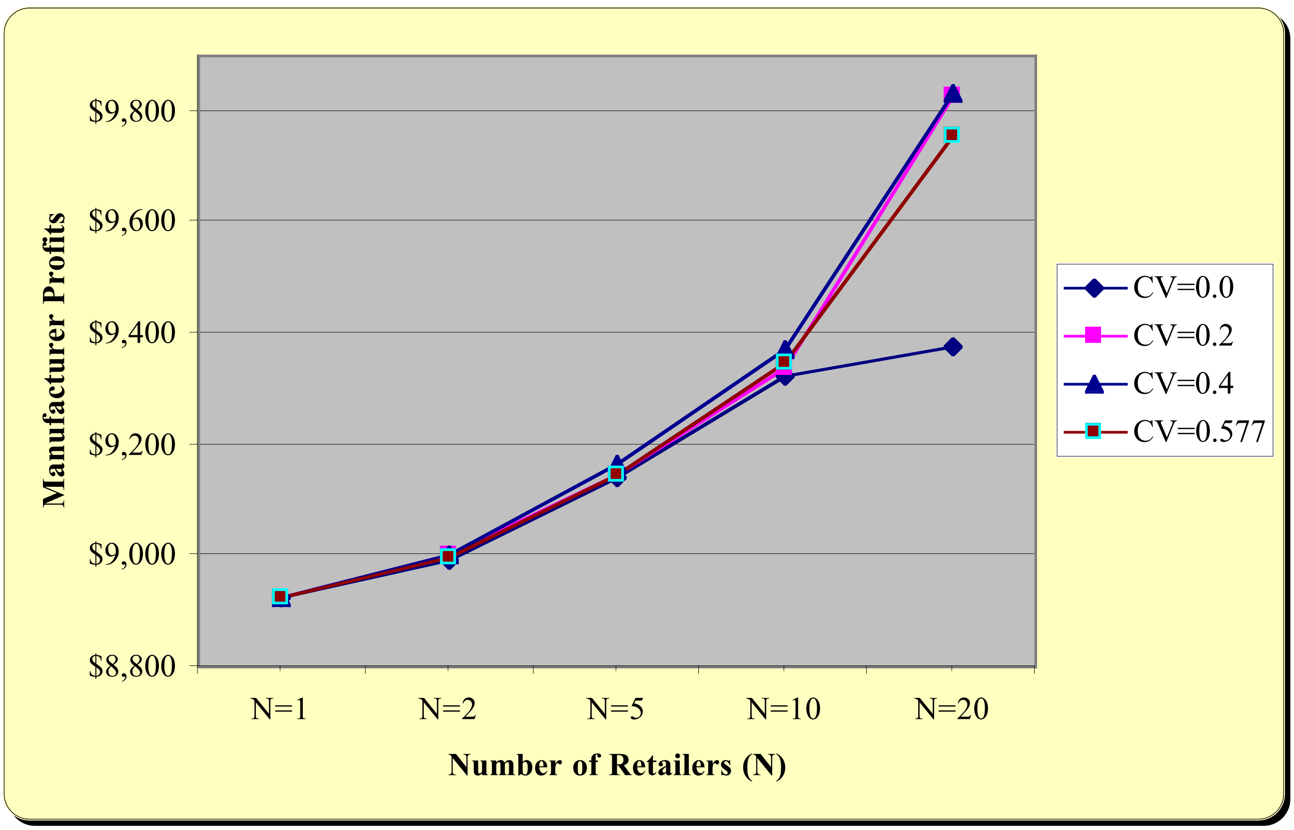

Figure 4. Manufacturer Profits versus Number of Retailers (N)

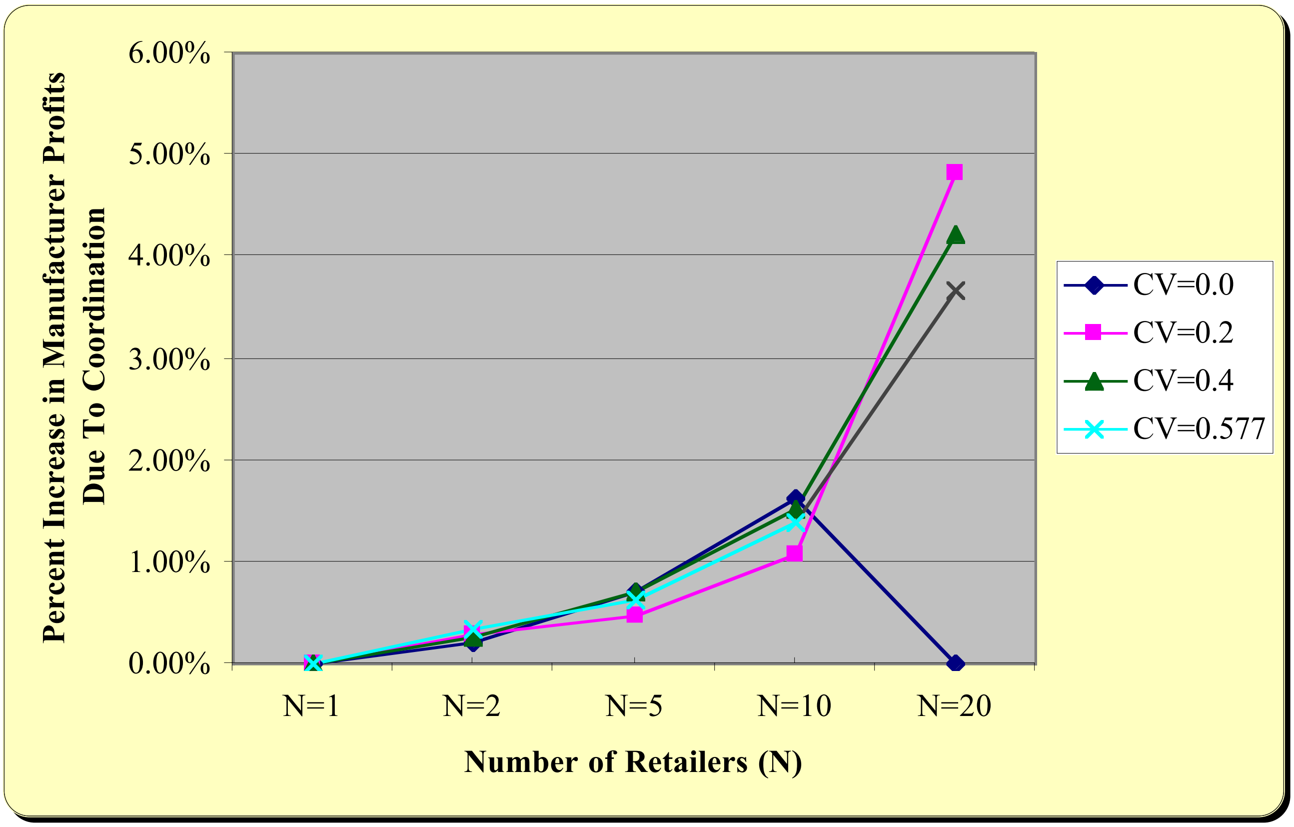

Figure 5. Percent Increase in Manufacturer's Profits due to Order Coordination

No. of Retailers (N)	2	5	10	20
Optimal discount price (c_D)	\$19.95	\$19.95	\$19.95	\$20.00
Manufacturer's Profit (with coordination)	\$8,993	\$9,140	\$9,324	\$9,375
Manufacturer's Profit (no coordination)	\$8,975	\$9,075	\$9,175	\$9,375
Percent Profit Increase with coordination	0.20%	0.72%	1.62%	0.00%
Manufacturer's Profit ($k_j = 0$ for all $j \in J$)	\$7,742	\$8,170	\$8,654	\$9,345
Discount price ($k_j = 0$ for all $j \in J$)	\$18.88	\$19.09	\$19.34	\$19.68

Table 1. Homogeneous Retailers Example.

Appendix A

Lemma: For a given discount price, c_D, the retailer's cost function $f_j(k_j,c_D)$ is discretely convex in k_j where

$$f_j(k_j,c_D) = \frac{s_j(k_j+1)}{T_o} + c_L D_j - \frac{D_j\left[k_j(c_L - c_D)^2 + 2i\, c_L T_o (c_L - c_D) - i^2 c_L c_D (T_o)^2\right]}{2i\, T_o (k_j c_D + c_L)} .$$

Proof:

Deriving the first difference,

$$\Delta_{k_j} f_j = f_j(k_j, c_D) - f_j(k_j - 1, c_D)$$

$$= \frac{s_j}{T_o} - \frac{i\, c_L D_j}{2\, T_o} \frac{\left[c_D(1 + i\, T_o) - c_L\right]^2}{i^2 (k_j c_D + c_L)\left[(k_j - 1) c_D + c_L\right]} \leq 0$$

which leads directly to equation (3) for finding k_j^*. To prove convexity of the cost function $f(k_j,c_D)$, it is sufficient to show that the second order condition with respect to k_j is satisfied. The second difference,

$$\Delta_{k_j}^2 f_j = \Delta f_j(k_j, c_D) - \Delta f_j(k_j - 1, c_D) \geq 0 ,$$

becomes

$$= \frac{c_L D_j}{T_o} \left\{ \frac{c_D\left[c_D(1 + i\, T_o) - c_L\right]^2}{(k_j c_D + c_L)\left[(k_j - 1) c_D + c_L\right]\left[(k_j - 2) c_D + c_L\right]} \right\}$$

which is nonnegative for positive values of T_o, c_D, c_L, k_j, and i. Thus, the retailer's cost function $f_j(k_j,c_D)$ is convex in k_j and the value of k_j^* minimizes the total cost function for retailer j.

Q.E.D.

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