Online Appendix

In this appendix, we examine extensions of the model in Section A and present the proofs for the lemmas and propositions in Section B.

A Extensions

We consider three model extensions to establish the robustness of our results. First, in Section A.1, we consider the case in which consumers still have positive reservation value for their offerings ($V_l > 0$) when firms do not succeed. In the main model, we assumed an efficient rationing rule when allocating firms' capacity to consumer demand. In Section A.2, we relax that assumption by considering a proportional rationing rule. Finally, in Section A.3, we look at the incentives of the service provider and endogenize the price of computational capacity, c.

A.1 Positive Low-State Value

In the main model, we assumed that consumers' reservation value for a firm that does not succeed is $v_i = 0$. This assumption could seem strong as it gives monopoly power to the competitor. In this section, we relax this assumption to show the robustness of our results. We assume that the reservation value of consumers for each firm is V_h with probability γ and V_l with probability $1 - \gamma$, where $V_h > V_l > 0$. We show that our counter-intuitive result in Proposition 3 that autoscaling could lower entry becomes even stronger when $V_l > 0$.

Equilibrium Choices without Autoscaling

We use the same techniques as before to solve the pricing subgame, and to calculate the expected profit of Firm i for given capacities. The details of how we solve the pricing subgame are provided in the proof.

The expected profit of Firm i, where $i, j \in \{1, 2\}$, depends on the capacities of the two firms as shown below, assuming $\alpha \leq k_i, k_j \leq 1 - \alpha$.

$$E(\pi_i) = \begin{cases} k_i(-c + \gamma V_h - \gamma V_l + V_l) & \text{if } k_i + k_j \leq 1 \\ \frac{k_i((k_i-1)((\gamma-1)V_l-\gamma V_h)-ck_j)}{k_j} & \text{if } k_i < \frac{V_l}{V_h}k_j \text{ and } k_i + k_j \geq 1 \\ \frac{k_i\left(-ck_j+\gamma^2(-k_j)V_h+\gamma k_j V_h+\gamma^2 k_j V_l+\gamma(-k_j-1)V_l+\gamma^2 V_h+V_l\right)}{k_j} + & \text{if } \frac{V_l}{V_h}k_j < k_i < k_j \text{ and } k_i + k_j \geq 1 \\ \frac{k_i^2\left(\gamma^2(-V_h)+\gamma V_l-V_l\right)}{k_j} + \frac{\gamma^2 k_j^2 V_l-\gamma k_j^2 V_l-\gamma^2 k_j V_l+\gamma k_j V_l}{k_j} & \\ -ck_i + (1-\gamma)\gamma k_i\left(V_h - \frac{V_l(k_i+k_j-1)}{k_j}\right) + & \text{if } \frac{V_l}{V_h}k_i < k_j < k_i \text{ and } k_i + k_j \geq 1 \\ (1-k_j)\left(\gamma^2 V_h + (\gamma-1)^2 V_l\right) + (\gamma-1)\gamma(k_j-1)V_l & \\ (k_j-1)((\gamma-1)V_l - \gamma V_h) - ck_i & \text{if } k_j < \frac{V_l}{V_h}k_i \text{ and } k_i + k_j \geq 1 \end{cases}$$

Given the firms' expected profits in the pricing subgame, we can calculate the equilibrium capacities by comparing the expected profits for each set of capacities. Assuming both firms enter the market, equilibrium capacity choices depend on the probability of success, the low state and high state values, and the cost of capacity as follows:

- If there is a low probability of a successful venture (i.e., $\gamma(V_h - V_l) + V_l < c$), then both firms choose $k_i = 0$.

- If there is a moderate probability of a successful venture (i.e., $(1-\gamma)\gamma(V_h - V_l) > c$), then the firms choose overlapping capacities such that $k_i + k_j > 1$.

- If there is a high probability of a successful venture (i.e., $(1-\gamma)\gamma(V_h - V_l) < c < \gamma(V_h - V_l) + V_l$), then the firms choose separating capacities with the unique symmetric equilibrium being $k_i = k_j = 1/2$.

It is interesting to note that as V_l increases, the region in which firms use separated capacities grows. The region for separated capacities is given by $(1-\gamma)\gamma(V_h - V_l) < c < \gamma(V_h - V_l) + V_l$. Since we have $\frac{\partial((1-\gamma)\gamma(V_h-V_l))}{\partial V_l} < 0$ and $\frac{\partial(\gamma(V_h-V_l)+V_l)}{\partial V_l} > 0$, this region becomes larger as V_l increases. Intuitively, this is because increasing V_l increases direct competition between a high-value firm and a low-value firm when their capacities overlap. To avoid this competition, firms are more likely to choose separated capacities and gain monopoly pricing power for higher V_l. Since autoscaling breaks the firms' ability to dampen competition through limited capacity, as we see in the next section, the competition intensifying effect of autoscaling becomes stronger as V_l increases, and, therefore, autoscaling lowers market entry in a larger region.

Equilibrium Choices with Autoscaling

When autoscaling is available, we show that one or two firms use autoscaling. The analysis when $V_l > 0$ is similar to the analysis in the main model, and, hence, is relegated to the proof. The expected profit of Firm i when both firms use autoscaling is

$$E(\pi_{AA}) = -\alpha c + (2\alpha - 1)\gamma^2(V_h - V_l) + (\alpha - 1)\gamma(V_l - V_h) + \alpha V_l$$

Effect of Low-State Value on Entry

By comparing the cost of entry, F, to firms' profits, we determine how many firms, if any, would choose to enter the market for any given F. Figure A1 shows the effect of increasing V_l on entry. As in Figure 6c, the shaded region A is where autoscaling decreases entry from two firms to one firm. In regions B and D neither firm enters the market unless autoscaling is available. Finally, region C is where autoscaling increases entry from one firm to two firms.

As V_l increases, the shaded region in Figure A1 where autoscaling decreases entry from two firms to one firm expands. This is because increasing V_l results in the expansion of the region for separated capacities without autoscaling. Thus, the region where the *competition intensifying* effect of autoscaling is dominant expands as the low-state value increases, decreasing market entry with autoscaling.

Note that regions B and D in Figure A1 disappear for $V_l > c$. A single entrant using autoscaling always sells to all possible $(1 - \alpha)$ customers when the low-state value is greater than the unit cost of capacity. The reason for this is that marginal cost of selling to each customer becomes less than the charged price, regardless of whether the firm realizes low-state or high-state value. Similarly, without autoscaling, a single entrant would set its capacity to $1 - \alpha$ when $V_l > c$. Therefore, for a single entrant, the profits and entry conditions without autoscaling become the same as those with autoscaling. As such, for $V_l > c$, autoscaling does not affect the condition for which at least one firm enters the market. In other words, Proposition 4, which states autoscaling increases the range of entry costs for which at least one firm enters, holds only for $0 \leq V_l < c$.

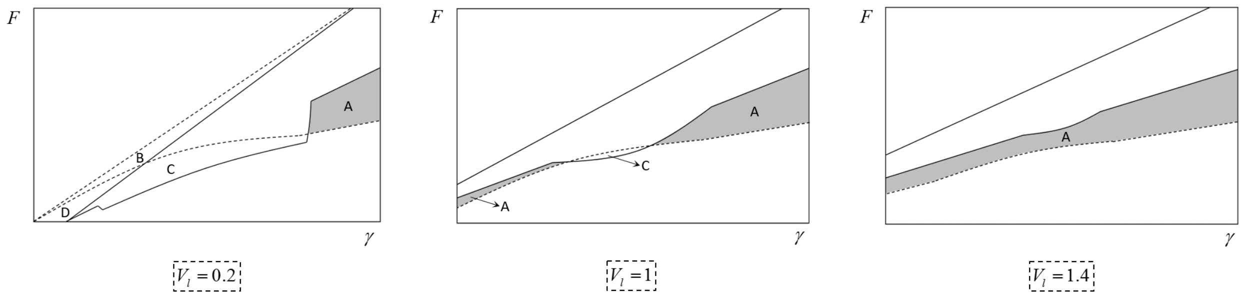

Figure A1: How increasing V_l changes the effect of autoscaling on entry. Parameters used for generating the figure are $V_h = \frac{7}{2}$, $\alpha = 0.25$, $c = 0.5$.

Finally, Figure A1 shows V_l must be low enough for autoscaling to increase entry, since region C only exists for low-state values that are not too high.

A.2 Proportional Rationing

In Section 4.1, we assumed an efficient rationing rule such that, when demand exceeds capacity, Segments 1 and 2 are served before Segment 3. In this extension, we check the robustness of our findings with respect to the rationing rule and solve the model using proportional rationing. We show that our results continue to hold when consumers across all segments arrive uniformly.

Suppose that after prices are set, consumers in Segment 3 prefer Firm i to Firm j. Firm i's capacity is allocated simultaneously to Segment 3 and Segment i, where $i \in \{1, 2\}$. With proportional rationing, the ratio of capacity allocated to each of these segments is relative to the size of that segment:

$$\text{Capacity Allocated to Segment } i = \frac{\text{Size of Segment } i}{\text{Sum of Sizes of Segments } i \text{ and } 3} \times k_i = \frac{\alpha}{1-\alpha} k_i$$

Thus, of the k_i available capacity for Firm i, $\frac{\alpha}{1-\alpha}k_i$ is used by Segment i and $\frac{1-2\alpha}{1-\alpha}k_i$ is used by Segment 3. Note that unlike what we found with efficient rationing, Segment i is not fully satisfied and there are $\alpha(1 - \frac{k_i}{1-\alpha})$ consumers in this segment that are not served.

Once Firm i's capacity is full, the residual demand of Segment 3 (i.e., $(1-2\alpha) - \frac{1-2\alpha}{1-\alpha}k_i$) and the demand from Segment j is satisfied by Firm j, provided it has available capacity. Thus, as long as $k_j < (1-2\alpha) - \frac{1-2\alpha}{1-\alpha}k_i + \alpha$, each of the two firms can sell up to its full capacity without overlapping with the competitor. Otherwise, for $\frac{1-2\alpha}{1-\alpha}k_i + k_j > 1-\alpha$, it is not possible for both firms to sell their maximum capacity. In this case, a pure strategy equilibrium for the pricing subgame does not exist and both firms use mixed strategy pricing.

Note that the condition for overlapping capacities, $\frac{1-2\alpha}{1-\alpha}k_i + k_j > 1-\alpha$, occurs for a greater range of k_i and k_j compared to the overlapping capacities condition with the efficient rationing rule, $k_i + k_j > 1$. In other words, with proportional rationing, there are certain capacities for which $k_i + k_j < 1$ and firms use mixed strategy pricing.

Equilibrium Choices without Autoscaling

Using the same methods as before, we can solve the pricing subgame for given capacities with the new rationing rule. Details of the analysis are provided in the proof. The expected profit of Firm i when $\alpha < k_i, k_j < 1-\alpha$ is as follows.

$$E(\pi_i) = \begin{cases} k_i(\gamma - c) & \text{if } k_i < -\alpha - \frac{(1-2\alpha)k_j}{1-\alpha} + 1 \\ \frac{\gamma^2(-\alpha(\alpha+2k_j-2)+k_j-1)}{\alpha-1} - ck_i - (\gamma-1)\gamma k_i & \text{if } k_i > -\alpha - \frac{(1-2\alpha)k_j}{1-\alpha} + 1 \text{ and} \\ & k_i < k_j < -\frac{(\alpha-1)^2}{2\alpha-1} - k_i \\ k_i\left(\frac{\gamma^2(-\alpha(\alpha+2k_i-2)+k_i-1)}{(\alpha-1)k_j} - c - (\gamma-1)\gamma\right) & \text{if } k_i > -\alpha - \frac{(1-2\alpha)k_j}{1-\alpha} + 1 \text{ and} \\ & k_j < k_i < -\frac{(\alpha-1)^2}{2\alpha-1} - k_j \\ k_i\left(\frac{\gamma^2(-\alpha(\alpha+2k_i-2)+k_i-1)}{(\alpha-1)k_j} - c - (\gamma-1)\gamma\right) & \text{if } k_j > k_i > -\frac{(\alpha-1)^2}{2\alpha-1} - k_j \\ \frac{\gamma^2(-\alpha(\alpha+2k_j-2)+k_j-1)}{\alpha-1} - ck_i - (\gamma-1)\gamma k_i & \text{if } k_i > k_j > -\frac{(\alpha-1)^2}{2\alpha-1} - k_i \end{cases}$$

Comparing the expected profits for each set of capacities, we find the equilibrium capacities:

- If $\gamma < c$, then firms set $k_i = k_j = 0$.
- If $\gamma(1-\gamma) > c$, then the firms choose overlapping capacities such that $\frac{1-2\alpha}{1-\alpha}k_i + k_j > 1-\alpha$.
- If $\gamma(1-\gamma) < c$ and $\gamma > c$, then the firms choose separated capacities with the unique symmetric equilibrium being $k_i = k_j = \frac{(\alpha-1)^2}{2-3\alpha}$.

Thus, the regions for separated and overlapping capacities have the same boundaries as in Proposition 1, when the efficient rationing rule was used. The difference from using a proportional rationing rule only appears in the capacities chosen within each region, not the size of each region.

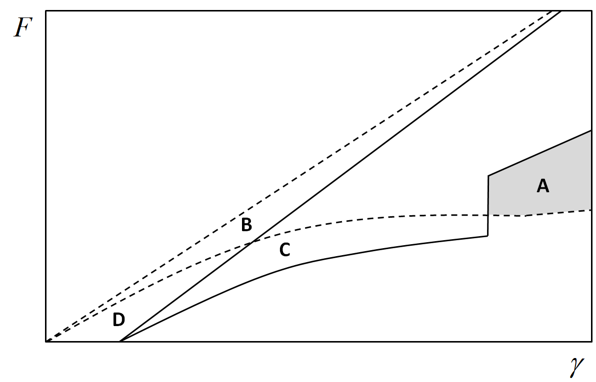

Figure A2: Effect of autoscaling on entry with proportional rationing.

Equilibrium Choices with Autoscaling

When both firms use autoscaling, the rationing rule does not affect the outcome, since neither firm has a capacity constraint. Therefore, each firm's profit is $\pi_{aa} = (1-c)\left(\alpha\gamma^2 + (1-\alpha)(1-\gamma)\gamma\right)$.

In the proof, we analyze the case when one firm uses autoscaling and the other chooses a fixed capacity. This case is affected by the rationing rule, which determines how much of the capacity of the firm not using autoscaling is allocated to consumers in Segment 3. We find that the profits are $\pi_{an} = \frac{1}{2}(1-\alpha)(-2\gamma c + c + \gamma)$ for the firm using autoscaling and $\pi_{na} = \frac{(\alpha-1)^2(c-\gamma)^2}{4(1-2\alpha)\gamma^2(1-c)}$ for the firm choosing fixed capacity.

Effect of Autoscaling on Entry with Proportional Rationing

We compare the cost of entry, F, to firms' profits, finding the number of firms that choose to enter the market for any given F. We prove that there exists a cutoff $\tilde{\gamma}$, such that for $\gamma > \tilde{\gamma}$ autoscaling decreases the range of entry costs for which both firms enter the market. This result is similar to what we found about the effect of autoscaling on entry when the efficient rationing rule was applied. Also, the rationing rule does not affect profits and entry conditions when only one firm enters the market. Thus, both Propositions 3 and 4 hold with proportional rationing. Figure A2 shows the effect of autoscaling on market entry with proportional rationing. In region A, autoscaling decreases entry from 2 to 1 firms. In region C, autoscaling increases entry from 1 to 2 firms. In regions B and D, autoscaling increases entry from 0 to 1 and 2 firms respectively. Comparing Figure A2 with Figure 6c, where the efficient rationing rule was used, we see that the insights of our model for market entry remain the same with proportional rationing.

A.3 Service Provider Incentives

To focus on firms' competition, in the main model, we assumed that the cloud provider's decisions were exogenous. In practice, there are many parameters, still exogenous to our model, that affect a cloud provider's decision on how to price capacity (c), and whether to offer autoscaling. For example, a cloud provider may decide to offer autoscaling, or change price c, because other cloud providers are doing so. Furthermore, cloud providers have clients from a wider range of industries with different F's, α's and γ's, but for practical purposes (e.g., cloud capacities are often sold as an off-the-shelf product), they may have to offer the same, or similar, prices and functionalities across many industries. This would again limit the cloud provider's ability to optimize c and the choice of offering autoscaling for a given F, α and γ. For these reasons, when studying firms' competition, an exogenous model for the cloud provider might be a better approximation of the real world. However, from a theoretical point of view, it is interesting to see if and how endogenizing the cloud provider's decisions affects our results. We show that our counter-intuitive result in Proposition 3, that autoscaling could lower market entry, continues to hold when we endogenize the cloud provider's decisions.

Provider's Choice of Capacity Price

In this section, we endogenize the price of capacity, by allowing the cloud provider to choose c as a function of F, α and γ. The cloud provider's profit is

$$\pi^{CP} = c \times (\text{Purchased Capacity}).$$

When autoscaling is available, the provider faces a price-volume tradeoff between c and the number of entrants: increasing c increases the provider's profit per unit of capacity but could also decrease market entry depending on F. We show that the provider is indifferent between one firm or two firms entering the market when F is

$$\hat{F} = \frac{\gamma(-2\alpha\gamma + \alpha + \gamma - 1)^2}{1 - \alpha}.$$

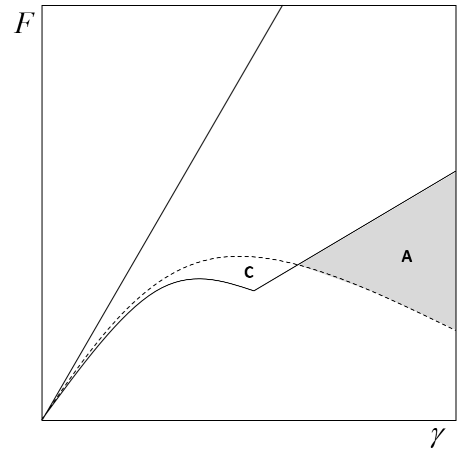

Figure A3: Effect of autoscaling on entry with endogenous c. In region A, autoscaling decreases entry from 2 to 1 firms. In region C, autoscaling increases entry from 1 to 2 firms.

When the cost of entry is low (i.e., $F < \hat{F}$), the provider sets $c = 1 - \frac{F}{\gamma(\alpha(2\gamma-1)-\gamma+1)}$ so that both firms enter the market; otherwise, for $\hat{F} \leq F \leq (1-\alpha)\gamma$, the price of capacity is optimally increased to $c = 1 - \frac{F}{(1-\alpha)\gamma}$, resulting in only one firm entering the market. Finally, for $F > (1-\alpha)\gamma$, neither firm would enter the market for any $c \geq 0$.

When autoscaling is not offered, similar to the case with autoscaling, the provider sets a low c for low F to allow both firms to enter, and increases c for higher F resulting in single entry. The analysis for the optimal price of capacity when autoscaling is not an option is presented in the proof. Figure A3 shows the effect of autoscaling on market entry when c is endogenously chosen by the provider. This figure is a replication of Figure 6c, but with endogenous c. Comparing the two figures, we see that regions B and D from Figure 6c, in which autoscaling increased entry from 0 to 1 or 2 firms, disappear when c becomes endogenous. As stated in Proposition 4, when c was given exogenously, autoscaling increased the range of F for which at least one firm entered the market. The reason for this finding was the *downside risk reducing* effect of autoscaling, allowing a single entrant to only pay for capacity when its demand is high. However, when c is endogenous, the provider sets the price of capacity sufficiently low (when autoscaling is not available) so that still one firm enters. In other words, when autoscaling is not available and c is endogenous, the provider absorbs the firms' downside risk of failure by lowering the price of capacity to encourage entry. Therefore, regions B and D disappear, and our result in Proposition 3, where autoscaling increases entry from 0 to 1 or 2 firms, does not hold for endogenous c.[1] Finally, similar to what we had in Figure 6c, Figure A3 also shows that for high enough γ, autoscaling decreases the range of F for which both firms enter the market (region A). Thus, our counter-intuitive finding in Proposition 4 which stated that autoscaling could lead to fewer firms entering the market still holds when c is endogenous.

Effect of Autoscaling on Provider's Profit

In this section, we find how autoscaling affects the profit of the cloud provider when the provider chooses the price of capacity. Since we do not find closed form solutions for the optimal price under all conditions, we present the results of a numerical comparison of provider profits with and without autoscaling in Figure A4. In Region 1 of Figure A4, autoscaling has no effect on the provider's profit. This is because with or without autoscaling, only one firm enters the market in Region 1 and the provider makes a profit of $(1-\alpha)\gamma - F$. Thus, for autoscaling to change provider's profit, the cost of entry must be low enough so that two firms enter the market.

In Region 2, for medium γ and low enough F, autoscaling increases the provider's profit. In Region 3, for higher γ, autoscaling decreases the provider's profit. Intuitively, medium probabilities of success result in high demand uncertainty. Thus, for medium γ, the *downside risk reducing* and *demand satisfaction* effects of autoscaling are dominant, allowing autoscaling to facilitate market entry and benefit the provider. For high probabilities of success, autoscaling results in a high probability of direct competition between the two firms, which means the provider would in turn have to decrease the price of capacity substantially to create incentive for the firms to enter the market. However, without autoscaling, firms choose capacities that do not overlap for high γ, allowing the provider to choose a higher c. Thus, when γ is high, autoscaling prevents the provider from increasing the price of capacity and earning more profit.

Figure A4 also shows the effect of changing α on the provider's profits. As α increases, the expected purchased capacity of a single entrant, $(1-\alpha)$, decreases, making single entry a less attractive option for the provider and shrinking Region 1.

[1] We should note, however, that there is a caveat in this argument. Here, we are assuming that the provider optimizes c as a function of γ, α and F. In other words, we allow the provider to set different prices for firms with different γ's, α's and F's in the market. If the provider has to keep the price of capacity the same for all (or most) firms, because of other practical concerns (e.g., because it cannot perfectly price discriminate), then the result of Proposition 4 would hold again.

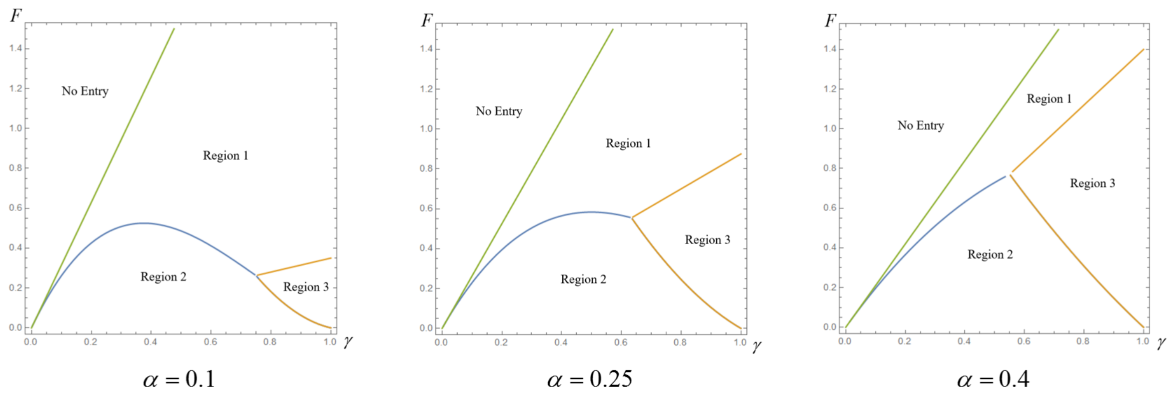

Figure A4: Effect of autoscaling on cloud provider's profit. Region 1 denotes autoscaling does not change providers' profit. Region 2 denotes autoscaling increases provider's profit. Region 3 denotes autoscaling decreases provider's profit.

Figure A4 confirms the intuition that for markets with considerable demand uncertainty, represented by medium γ in Region 2, autoscaling helps increase the provider's profit and impacts the market. Note that while autoscaling may not benefit the provider in certain industries with little uncertainty, the provider makes the choice of offering autoscaling based on the aggregate outcome of multiple industries with varying levels of γ and F. Also, other factors such as competition among providers can be expected to contribute to strengthening the benefit of autoscaling for cloud providers. Observations from the cloud computing industry indicate that autoscaling is offered by all major cloud providers and is not an exclusive offer for only certain markets.[2] A thorough examination of such contributing factors affecting providers' decisions is a potentially rich topic for future research on cloud computing, but outside of the scope of this paper.

In the proof, we show that even when the cloud provider is allowed to choose two separate capacity prices, c_k for fixed pre-purchased capacity and c_a for autoscaling capacity, the insights are similar to those from Figure A4.

B Proofs

B.1 Proof of Proposition 1

If $\gamma < c$, then expected profit is strictly decreasing in capacity for any capacity chosen by the competition. Therefore, each firm optimally chooses zero capacity.

[2]www.knowthecloud.com/Providers/auto-scaling-providers.html, accessed February 2017.

If $\gamma > c$, then for any $k_2 < 1-\alpha$, expected profit is increasing in k_1 for any $k_1 < 1-k_2$. Therefore, we can rule out any $k_1 + k_2 < 1$. First consider $\gamma(1-\gamma) < c$. Suppose there were a symmetric equilibrium in which $k_1 + k_2 > 1$. By definition, this implies that $k_1 > 1/2$ which means Firm 1 earns greater expected profit by deviating downward. Therefore, the only potential symmetric equilibrium requires $k_1 + k_2 = 1$. If Firm 1 deviates upward, its expected change in profit is $\gamma(1-\gamma) - c < 0$.

If $\gamma(1-\gamma) > c$, then $k_1 + k_2 = 1$ is no longer an equilibrium because either firm can profitably deviate to harvest the potential of monopoly power. We can therefore focus our attention on $k_1 + k_2 > 1$. If $k_2 > k_1$, then Firm 2's expected profit is strictly increasing in k_2 until $k_2 = 1-\alpha$ and is strictly decreasing for any $k_2 > 1-\alpha$. If $k_2 > k_1$, Firm 1's expected profit for any $k_1 < 1-\alpha$ is given by $\gamma(1-\gamma) + \gamma^2 \frac{(1-k_1)k_1}{k_2} - k_1 c$, which is concave in k_1 and maximized at $k_1 = \frac{\gamma(1-\alpha(1-\gamma))-c(1-\alpha)}{2\gamma^2}$. Note this value of k_1 is in fact less than $1-\alpha$ if and only if $c > \frac{\gamma(1+\alpha(\gamma-1))}{1-\alpha} - 2\gamma^2$. We verify that Firm 2 cannot benefit from a global deviation undercutting k_1 at this level. Supposing that Firm 2 deviates to a lower capacity than Firm 1, its profit would become $k_2\left(-c+\gamma\left(1-\gamma-\frac{2\gamma^3(-1+k_2)}{c(-1+\alpha)+\gamma(1+(-1+\gamma)\alpha)}\right)\right)$, which is maximized at

$$\tilde{k}_2 = -\frac{c^2(-1+\alpha)+c\gamma(2+2\alpha(-1+\gamma)-\gamma)+\gamma^2\left(-1+\alpha+\gamma-2\alpha\gamma+(-2+\alpha)\gamma^2\right)}{4\gamma^4}.$$

It is easily shown that any values of c that allow for $\tilde{k}_2 < k_1$ will result in Firm 2's profit at $\tilde{k}_2$ (i.e., $\frac{\left(c^2(-1+\alpha)+c\gamma(2-\gamma+2(-1+\gamma)\alpha)+\gamma^2\left(-1+\gamma+\gamma^2(-2+\alpha)+\alpha-2\gamma\alpha\right)\right)^2}{8\gamma^4(c(-1+\alpha)+\gamma(1+(-1+\gamma)\alpha))}$) to be less than Firm 2's profit at our equilibrium.

If $c < \frac{\gamma(1+\alpha(\gamma-1))}{1-\alpha} - 2\gamma^2$, then neither firm benefits from deviating from $k_j = 1-\alpha$. Thus, k^* from Proposition 1 is defined as $k^* = Min[\frac{\gamma(1-\alpha(1-\gamma))-c(1-\alpha)}{2\gamma^2}, 1-\alpha]$.

B.2 Proof for Firms' Decision to Use Autoscaling

In this section, we consider the firms' choice of using autoscaling when autoscaling is available to see how many firms, if any, use autoscaling.

We begin with the case when only one firm uses autoscaling. Without loss of generality, we assume that Firm 2 adopts autoscaling and Firm 1 chooses capacity k_1. We start by showing that this game does not have a pure strategy pricing equilibrium. Assume for sake of contradiction that the firms use prices p_1 and p_2 in a pure strategy equilibrium. If $p_1 \neq p_2$, then the firm with a lower price can benefit from deviating by increasing its price to $\frac{p_1+p_2}{2}$. If $p_1 = p_2$, then Firm 2 can benefit from deviating by decreasing its price to $p_2 - \varepsilon$, for sufficiently small ε, to acquire all consumers in Segment 3. Therefore, a pure strategy equilibrium cannot exist.

Next, we find a mixed strategy equilibrium for this game. Provided $k_1 \leq 1 - \alpha$, Firm 2 can choose to *attack* with a price that clears its capacity or *retreat* with a price 1 that harvests the value from the $1 - k_1$ consumers that Firm 1 cannot serve due to its capacity constraint. Let z'' be the price at which Firm 2 is indifferent between *attacking* to sell to $1-\alpha$ consumers at price z'' and *retreating* to sell to $1 - k_1$ consumers at price 1. We have $(1 - k_1)(1 - c) = (1 - \alpha)(z'' - c)$ which gives us $z'' = \frac{\alpha c - c k_1 + (k_1 - 1)}{\alpha - 1}$. In equilibrium, both firms use a mixed strategy with prices ranging from z'' to 1. Suppose that $H_i(.)$ is the cumulative distribution function of prices used by Firm i. The profit of Firm 1 earned by setting price x is

$$\pi_1(x) = H_2(x)\alpha x + (1 - H_2(x))k_1 x - k_1 c$$

Using equilibrium conditions, we know that the derivative of this function with respect to x must be zero for $x \in (z'', 1)$. By solving the differential equation we get

$$H_2(x) = \begin{cases} 0 & \text{if } x < z'' \\ \frac{k_1(-ck_1 + (-1+k_1) + \alpha(c-x) + x)}{(-1+\alpha)(\alpha - k_1)x} & \text{if } z'' \leq x < 1 \\ 1 & \text{if } x \geq 1 \end{cases}$$

Similarly, the profit of Firm 2 earned by setting price x is

$$\pi_2(x) = H_1(x)(1 - k_1)(x - c) + (1 - H_1(x))(1 - \alpha)(x - c)$$

By setting the derivative with respect to x to zero for $x \in (z'', 1)$ and solving the differential equation, we get

$$H_1(x) = \begin{cases} 0 & \text{if } x < z'' \\ \frac{-ck_1+(-1+k_1)+\alpha(c-x)+x}{(k_1-\alpha)(x-c)} & \text{if } z'' \le x < 1 \\ 1 & \text{if } x \ge 1 \end{cases}$$

Prior to the pricing game, the optimal capacity k_1 that maximizes expected profit for Firm 1 is given by:

$$k^*_{na} = \frac{\gamma(1-\alpha+\alpha\gamma) - c\left(1+\alpha\left(-1+\gamma^2\right)\right)}{2(1-c)\gamma^2}$$

We may now examine the equilibrium adoption of autoscaling. The payoffs from each possible firm choice of autoscaling or capacity k are summarized in Table 1.

	Firm 2 uses Autoscaling	**Firm 2 uses capacity k**
Firm 1 uses Autoscaling	$\pi_1 = \pi_{AA} = (1-c)(\gamma^2\alpha + \gamma(1-\gamma)(1-\alpha))$ $\pi_2 = \pi_{AA} = (1-c)(\gamma^2\alpha + \gamma(1-\gamma)(1-\alpha))$	$\pi_1 = \pi_{AN} = \gamma^2(1-k)(1-c) + \gamma(1-\gamma)(1-\alpha)(1-c)$ $\pi_2 = \pi_{NA} = \gamma^2 \frac{k(c(k-\alpha)+(1-k))}{1-\alpha} + \gamma(1-\gamma)k - ck$
Firm 1 uses capacity k	$\pi_1 = \pi_{NA} = \gamma^2 \frac{k(c(k-\alpha)+(1-k))}{1-\alpha} + \gamma(1-\gamma)k - ck$ $\pi_2 = \pi_{AN} = \gamma^2(1-k)(1-c) + \gamma(1-\gamma)(1-\alpha)(1-c)$	Profits are the same as in Section 4.1

Table 1: Payoffs from autoscaling adoption strategies assuming both firms enter the market

Proof That At Least One Firm Uses Autoscaling

We show that both firms not using autoscaling cannot be an equilibrium. As seen in Table 1, the expected profit of using autoscaling, when the opponent uses capacity k, is

$$\tilde{\pi}_{AN}(k) = \gamma^2(1-k)(1-c) + \gamma(1-\gamma)(1-\alpha)(1-c).$$

We compare the profit of each firm when neither use autoscaling to the profit from deviating to autoscaling (i.e., $\tilde{\pi}_{AN}(k)$).

First, consider when neither firm uses autoscaling and firms set separated capacities. Each firm's profit is $\frac{1}{2}(\gamma - c)$. A firm's profit from unilaterally using autoscaling is $\tilde{\pi}_{AN}(\frac{1}{2}) = \frac{1}{2}\gamma(2\alpha(\gamma - 1) - \gamma + 2)(1-c)$. For all $\alpha \le \frac{1}{2}$, we have $\tilde{\pi}_{AN}(\frac{1}{2}) > \frac{1}{2}(\gamma - c)$. Thus, each firm has incentive to use autoscaling instead of remaining in the separated capacity equilibrium.

Next, consider the case when firms set overlapping capacities instead of using autoscaling. Based on the proof of Proposition 1, for $c < \frac{\gamma(1+\alpha(\gamma-1))}{1-\alpha} - 2\gamma^2$ each firm sets capacity to $1-\alpha$, earning a profit of $(1-\alpha)((\gamma(1-\gamma)) - c) + \gamma^2\alpha$, which is less than $\tilde{\pi}_{AN}(1-\alpha) = \gamma(\alpha(2\gamma-1) - \gamma + 1)(1-c)$.

For $c > \frac{\gamma(1+\alpha(\gamma-1))}{1-\alpha} - 2\gamma^2$, capacities are $k_i = 1-\alpha$ and $k_j = k^*$. The resulting profits without autoscaling are $\pi_{NNi} = \frac{1}{2}((\alpha-1)c + \gamma(\alpha(\gamma-1)+1))$ and $\pi_{NNj} = \frac{((\alpha-1)c+\gamma(\alpha(\gamma-1)+1))^2}{4(1-\alpha)\gamma^2}$. For all $\frac{\gamma(1+\alpha(\gamma-1))}{1-\alpha} - 2\gamma^2 < c < \gamma(1-\gamma)$, the condition for the asymmetric overlapping capacities, we have $\pi_{NNi} < \tilde{\pi}_{AN}(k^*)$ and $\pi_{NNj} < \tilde{\pi}_{AN}(1-\alpha)$. Therefore, deviating to autoscaling is strictly profitable for each firm and both firms using fixed capacities cannot be an equilibrium.

Intuitively, when both firms set capacity constraints, competition is restricted and firms set high prices. Thus, it is always beneficial to react to a fixed capacity by using autoscaling and undercutting the price of the opposition to win over Segment 3.

Proof for the Choice of Using Autoscaling

Given the fact that at least one firm uses autoscaling in equilibrium, we examine whether both firms use autoscaling or only one, by comparing the profits from Table 1. If Firm 2 adopts autoscaling, Firm 1's best response is to adopt autoscaling if and only if

$$(1-c)(\gamma^2\alpha + \gamma(1-\gamma)(1-\alpha)) \geq \frac{\left(c\left(\alpha\left(\gamma^2-1\right)+1\right) + \gamma(\alpha(-\gamma)+\alpha-1)\right)^2}{4(1-\alpha)\gamma^2(1-c)},$$

where the right hand side of the inequality is Firm 1's profit if it chooses the best possible capacity k, given by $k^*_{na} = \frac{c\left(\alpha\left(\gamma^2-1\right)+1\right)+\gamma(\alpha-1-\alpha\gamma)}{2\gamma^2(c-1)}$, to Firm 2's autoscaling decision.

Thus, only one firm uses autoscaling when $\gamma > (1-\alpha)/(2-3\alpha)$ and $c < \hat{c}$, where $\hat{c}$ is defined as follows:

$$\hat{c} \triangleq \frac{\gamma^2(-3\alpha\gamma + \alpha + 2\gamma - 1)^2}{2(1-\gamma)\sqrt{(1-\alpha)^3\gamma^3(2\alpha\gamma - \alpha - \gamma + 1)} + \gamma\left((2-3\alpha)^2\gamma^3 + ((9-5\alpha)\alpha - 4)\gamma^2 - (\alpha-1)\alpha\gamma + (\alpha-1)^2\right)}$$

For $c > \hat{c}$ or $\gamma < (1-\alpha)/(2-3\alpha)$, both firms will choose autoscaling.

B.3 Proof of Proposition 2

We start by comparing average prices with and without autoscaling in the region for overlapping capacities. Based on the proof of Proposition 1, when $c < \frac{\gamma(1+\alpha(\gamma-1))}{1-\alpha} - 2\gamma^2$, both firms set their capacities equal to $1-\alpha$ without autoscaling. The cumulative distribution function of prices set by each firm is $F(x) = \frac{(1-\alpha)x+\alpha}{(1-2\alpha)x}$.

Thus, the probability density function equals $f(x) = \frac{\alpha}{x^2(1-2\alpha)}$ and the average price of each firm is $\bar{p}_{NN} = \int_z^1 f(x)x\,dx = \frac{\alpha\log(\frac{1-\alpha}{\alpha})}{1-2\alpha}$. Now suppose autoscaling is offered in this region. For $c < \frac{\gamma(1+\alpha(\gamma-1))}{1-\alpha} - 2\gamma^2$, we have $\gamma < (1-\alpha)/(2-3\alpha)$. As shown in the proof of the choice of using autoscaling, when autoscaling is offered and $\gamma < (1-\alpha)/(2-3\alpha)$, both firms use autoscaling. With both firms using autoscaling, the probability density function of the price of each firm equals $g(x) = \frac{\alpha(1-c)}{(1-2\alpha)(x-c)^2}$ and the average price of each firm becomes $\bar{p}_{AA} = \int_{z'}^1 g(x)x\,dx = \frac{(1-2\alpha)c+\alpha\log\left(\frac{1-\alpha}{\alpha}\right)(1-c)}{1-2\alpha}$. Thus we have $\bar{p}_{AA} - \bar{p}_{NN} = c(1 - \frac{\alpha\log(\frac{1-\alpha}{\alpha})}{1-2\alpha})$. We know that $\log(s) < s-1$, for any $s > 1$, . Allowing $s = \frac{1-\alpha}{\alpha}$, we show that $\log(\frac{1-\alpha}{\alpha}) < \frac{1-2\alpha}{\alpha}$, thus $\bar{p}_{AA} > \bar{p}_{NN}$.

Next consider the region for separated capacities. Without autoscaling, $k_1 + k_2 = 1$ and both firms set their price equal to 1. When both firms use autoscaling, the average price equals $\bar{p}_{AA}$. As we showed, $\log(\frac{1-\alpha}{\alpha}) < \frac{1-2\alpha}{\alpha}$. Thus, $\bar{p}_{AA} = \frac{(1-2\alpha)c+\alpha\log\left(\frac{1-\alpha}{\alpha}\right)(1-c)}{1-2\alpha} < 1$.

When only one firm (Firm 2) uses autoscaling and the other (Firm 1) chooses capacity k_1, the probability density function of Firm 1's price equals $h_1(x) = -\frac{(k_1-1)(c-1)}{(\alpha-k_1)(c-x)^2}$ and its average price is $\bar{p}_{NA} = \int_{z''}^1 h_1(x)x\,dx = c + \frac{(1-k_1)(1-c)\log\left(\frac{1-\alpha}{1-k_1}\right)}{k_1-\alpha}$. For all $s > 0$, we know that $s\log(s) > s-1$. Allowing $s = \frac{1-\alpha}{1-k_1}$, we have $\frac{\alpha-k_1}{k_1-1} < \frac{(1-\alpha)Log((1-\alpha)/(1-k_1))}{(1-k_1)}$ and $\bar{p}_{NA} < 1$.

For Firm 2, the probability density function is $h_2(x) = \frac{k_1(-\alpha c+ck_1-k_1+1)}{(\alpha-1)x^2(\alpha-k_1)} + \left(1 - \frac{k_1(c-1)}{(\alpha-1)}\right)\delta(x-1)$ and the average price equals $\bar{p}_{AN} = \int_{z''}^1 h_2(x)x\,dx = \frac{(\alpha-k_1)((\alpha+k_1-1)-ck_1)+k_1(-\alpha c+ck_1-k_1+1)\log\left(\frac{(1-\alpha)}{-\alpha c+ck_1-k_1+1}\right)}{(\alpha-1)(\alpha-k_1)}$. We know that $\log(s) < s-1$, for any $s > 1$. Allowing $s = \frac{(1-\alpha)}{c(k_1-\alpha)+(1-k_1)}$, we find $\log\left(\frac{(1-\alpha)}{-\alpha c+ck_1-k_1+1}\right) < \frac{(1-\alpha)}{-\alpha c+ck_1-k_1+1} - 1$ and therefore, $\bar{p}_{AN} < 1$. Thus, autoscaling decreases average prices in the region for separated capacities.

B.4 Proof of Proposition 3

We compare the cut-offs on F such that both firms enter. Let F_{AA} denote the cut-off for two firms to enter with autoscaling, F_{NA} denote the cut-off for two firms to enter when only one firm uses autoscaling, and F_{NN} denote the cut-off for two firms to enter when autoscaling is not available.

Comparing the profits from Table 1 in the presence of autoscaling, it is straightforward to show $\pi_{NA} < \pi_{AN}$. Thus, if the subgame equilibrium involves only one firm choosing autoscaling, both firms will enter if $F < \pi_{NA}$. Both firms will choose autoscaling if $\pi_{NA} < \pi_{AA}$. As such, both firms will enter if $F < Max[\pi_{NA}, \pi_{AA}]$.

First consider the two cases in which $\gamma(1-\gamma) > c$. If $c < \frac{\gamma(1+\alpha(\gamma-1))}{1-\alpha} - 2\gamma^2$, then $0 < c < 1$ requires $\gamma < (1-\alpha)/(2-3\alpha)$. Therefore, both firms will use autoscaling if autoscaling is available. Thus, $F_{AA} - F_{NN} = c(1-\alpha+\gamma(1-\gamma-\alpha+2\alpha\gamma))$, which is decreasing in α and positive at $\alpha = 1/2$ and therefore positive for all $\alpha < 1/2$. If $c > \frac{\gamma(1+\alpha(\gamma-1))}{1-\alpha} - 2\gamma^2$, $F_{AA} - F_{NN}$ is positive for $c > \hat{c}$, where both firms use autoscaling.

Also, $F_{NA} > F_{NN}$ for $\gamma(1-\gamma) > c$. Thus, in the region for overlapping capacities, the cutoff F for two firms entering is bigger with autoscaling.

Now consider when $\gamma > c$ and $\gamma(1-\gamma) < c$. In this case, $F_{AA} - F_{NN} = \gamma(1-c)(1-\alpha-\gamma(1-2\alpha)-(\gamma-c)/2$ which is convex in γ, equal to $-(1-c)(1-2\alpha)/2 < 0$ when evaluated at $\gamma = 1$, decreasing in γ through $\gamma = 1$, and equal to zero at $\hat{\gamma}_{AA} = \frac{(1-2\alpha)-2c(1-\alpha)+\sqrt{8c(1-c)(1-2\alpha)+(1-2c(1-\alpha)-2\alpha)^2}}{4(1-c)(1-2\alpha)}$. Therefore, $F_{AA} - F_{NN}$ is negative for any $\gamma > \hat{\gamma}_{AA}$.

Also for $\gamma > c$ and $\gamma(1-\gamma) < c$, we have $F_{NA} - F_{NN} = \frac{\left(c\left(\alpha\left(\gamma^2-1\right)+1\right)+\gamma(\alpha(-\gamma)+\alpha-1)\right)^2}{4(\alpha-1)\gamma^2(c-1)} - \frac{1}{2}(\gamma-c)$, which is equal to $((-1+2\alpha)(c-1))/(4(-1+\alpha)) < 0$ at $\gamma = 1$. Also, $\frac{\partial(F_{NA}-F_{NN})}{\partial\gamma}$ equals $\frac{(2\alpha-1)(c-1)}{2(\alpha-1)} < 0$ at $\gamma = 1$. We find $F_{NA} = F_{NN}$ has one root between $\gamma = 0$ and $\gamma = 1$, which is $\hat{\gamma}_{NA} = \frac{(1-\alpha)(1-\alpha+\sqrt{1-2\alpha})}{2\alpha^2(c-1)} + \frac{\sqrt{(\alpha-1)\left(\alpha^3(1-2c)^2+\alpha^2\left(4\left(\sqrt{1-2\alpha}-1\right)c^2-4\left(\sqrt{1-2\alpha}-1\right)c+\left(2\sqrt{1-2\alpha}-5\right)\right)+2\left(3-2\sqrt{1-2\alpha}\right)\alpha+2\left(\sqrt{1-2\alpha}-1\right)\right)}}{2\alpha^2(1-c)}$. Therefore, $F_{NA} - F_{NN}$ is negative for any $\gamma > \hat{\gamma}_{NA}$.

Thus, autoscaling decreases the range of F such that both firms enter for $\gamma > \hat{\gamma} = Max[\hat{\gamma}_{AA}, \hat{\gamma}_{NA}]$.

B.5 Proof of Proposition 4

Suppose autoscaling is not available. A firm will enter if its expected profit is greater than its entry cost. A firm's best response to its competitor not entering the market is to enter the market, if and only if $F < (\gamma - c)(1-\alpha)$. Supposing one firm enters the market, the remaining firm's best response to its competitor's entry is to also enter the market provided the expected profit earned when competing is greater than the cost of entry. The anticipated payoffs associated with being one of two firms entering are reported in Section 4.3 and the conditions on F are presented in Lemma 1.

If only one firm enters, this firm will enjoy monopoly power over $(1-\alpha)$ consumers with probability γ. Using autoscaling, the firm's profit equals to $\gamma(1-c)(1-\alpha)$, which is strictly larger than $(\gamma - c)(1-\alpha)$, the profit earned without autoscaling, for any $\gamma > 0$. With autoscaling, at least one firm enters the market if and only if $F < F_A \equiv \gamma(1-c)(1-\alpha)$ whereas without autoscaling at least one firm enters if and only if $F < F_N \equiv (\gamma - c)(1-\alpha)$.

B.6 Proof of Corollary 1

We show that for sufficiently small F (such that both firms enter the market), when $\gamma > c$, $\gamma(1-\gamma) < c$, and $\pi_{NA} < \pi_{AA} < \frac{\gamma-c}{2}$, we have a prisoner's dilemma situation where both firms use autoscaling, even though their profits would be higher if autoscaling was not available.

When $\gamma > c$ and $\gamma(1-\gamma) < c$, firms set separated capacities, and each earn expected profit $\frac{\gamma-c}{2}$, if autoscaling is not available. However, when autoscaling is available, since $\pi_{NA} < \pi_{AA}$, both firms use autoscaling and each earn $\pi_{AA} < \frac{\gamma-c}{2}$ in equilibrium, which creates the prisoner's dilemma. Now, we have to prove that all these conditions can be satisfied at the same time to show that the described region, in which prisoner's dilemma happens, actually exists.

Let $\gamma = \frac{1-\alpha}{2-3\alpha}$. After algebraic simplifications, we have both firms using autoscaling in equilibrium (i.e., $\pi_{NA} < \pi_{AA}$) if and only if $c < \frac{4(\alpha-1)^2}{10\alpha^2-15\alpha+6}$. Furthermore, using algebraic simplifications, the expected profit of autoscaling equilibrium for each firm is lower than that when firms do not use autoscaling (i.e., $\pi_{AA} < \frac{\gamma-c}{2}$) if and only if $c > \frac{(\alpha-1)\alpha}{\alpha^2+2\alpha-2}$. It is easy to see that $\frac{(\alpha-1)\alpha}{\alpha^2+2\alpha-2} < \frac{4(\alpha-1)^2}{10\alpha^2-15\alpha+6}$ for any $\alpha \leq 1/2$. Therefore, when $c \in (\frac{(\alpha-1)\alpha}{\alpha^2+2\alpha-2}, \frac{4(\alpha-1)^2}{10\alpha^2-15\alpha+6})$, the prisoner's dilemma situation holds.

B.7 Proof of Proposition 5

If $c > \hat{c}$, then both firms would choose autoscaling upon entry and consumer surplus is $E[CS_{AA}] = \gamma^2(1-c)(1-2\alpha)$, if $F < (1-c)((1-\alpha)(1-\gamma)\gamma + \alpha\gamma^2)$.

If $c < \hat{c}$, then only one firm would choose autoscaling upon entry and the other chooses $k_{na} = \frac{c\left(\alpha\left(\gamma^2-1\right)+1\right)+\gamma(\alpha(-\gamma)+\alpha-1)}{2\gamma^2(c-1)}$. The expected capacity of the firm using autoscaling is

$$E[k_{an}] = -\frac{(k_{na}-1)k_{na}(c-1)\left(c(c-1)(k_{na}-\alpha) + (c(\alpha-k_{na}) + (k_{na}-1))\log\left(\frac{(k_{na}-1)}{c(\alpha-k_{na})+(k_{na}-1)}\right)\right)}{(\alpha-1)c^2(k_{na}-\alpha)}.$$

Thus, the expected consumer surplus is $E[CS_{NA}] = -c((1-\alpha)(1-\gamma)\gamma + \gamma^2 E[k_{an}] + k_{na}) + (\gamma^2 + (1-\gamma)\gamma((1-\alpha)+k_{na})) - \frac{1}{2}(\gamma(\alpha(\gamma-1)+1) - c(\alpha(\gamma-1)^2 + 2\gamma - 1)) - \frac{(c(\alpha(\gamma^2-1)+1)+\gamma(\alpha(-\gamma)+\alpha-1))^2}{4(\alpha-1)\gamma^2(c-1)}$, when $F < \frac{\left(c\left(\alpha\left(\gamma^2-1\right)+1\right)+\gamma(\alpha(-\gamma)+\alpha-1)\right)^2}{4(\alpha-1)\gamma^2(c-1)}$.

Similarly, when autoscaling is not available, the expected consumer surplus is $E[CS_{NN}] = \gamma^2(1-2\alpha)$ if $c < \frac{\gamma(1+\alpha(\gamma-1))}{1-\alpha} - 2\gamma^2$ and $F < (1-\alpha)(\gamma(1-\gamma)-c) + \alpha\gamma^2$. For $\frac{\gamma(1+\alpha(\gamma-1))}{1-\alpha} - 2\gamma^2 < c < \gamma(1-\gamma)$ and $F < \frac{(\gamma(1-\alpha(1-\gamma))-c(1-\alpha))^2}{4\gamma^2(1-\alpha)}$, we have $E[CS_{NN}] = \frac{(1-\alpha)c^2}{4\gamma^2} + \frac{(\alpha-1)c(2\gamma+1)}{2\gamma} + \frac{(\alpha(\gamma-1)+1)(\alpha\gamma+\alpha-1)}{4(\alpha-1)}$.

We start the comparison of consumer surplus with and without autoscaling in the region for overlapping capacities. For $c < \frac{\gamma(1+\alpha(\gamma-1))}{1-\alpha} - 2\gamma^2$, both firms set their capacity to $1-\alpha$ without autoscaling. Note that $c < \frac{\gamma(1+\alpha(\gamma-1))}{1-\alpha} - 2\gamma^2$ and $0 < c < 1$ require $\gamma < (1-\alpha)/(2-3\alpha)$, therefore it is not possible that only one firm uses autoscaling in this region. Thus, we only compare cases when both firms use autoscaling with cases when autoscaling is not available. If $F < (1-\alpha)(\gamma(1-\gamma)-c)+\alpha\gamma^2$, then both firms enter the market with or without autoscaling and $E[CS_{NN}] > E[CS_{AA}]$. For $(1-\alpha)(\gamma(1-\gamma)-c)+\alpha\gamma^2 < F < (1-c)((1-\alpha)(1-\gamma)\gamma+\alpha\gamma^2)$, $E[CS_{AA}] > E[CS_{NN}] = 0$. Finally, for $F > (1-c)((1-\alpha)(1-\gamma)\gamma+\alpha\gamma^2)$, we have $E[CS_{AA}] = E[CS_{NN}] = 0$.

Next, consider when $\frac{\gamma(1+\alpha(\gamma-1))}{1-\alpha} - 2\gamma^2 < c < \gamma(1-\gamma)$, where one firm sets its capacity to $k^* < 1-\alpha$ and the other chooses $k = 1-\alpha$, when autoscaling is not available. First, we analyze the cases when both firms use autoscaling. We find that $E[CS_{NN}] > E[CS_{AA}]$ when $F < \frac{(\gamma(1-\alpha(1-\gamma))-c(1-\alpha))^2}{4\gamma^2(1-\alpha)}$ and $c < c_M = \frac{\gamma\left(4\alpha\gamma^3 - \sqrt{\gamma}\sqrt{4(1-2\alpha)^2\gamma^5 - 8(\alpha-1)(2\alpha-1)\gamma^3 - 4(\alpha-1)(2\alpha-1)\gamma^2 + (\alpha(13\alpha-20)+8)\gamma + 4(\alpha-1)^2} - 2\alpha\gamma - \alpha - 2\gamma^3 + 2\gamma + 1\right)}{1-\alpha}$. Note that these conditions only hold when both firms use autoscaling, since c_M is positive if and only if $\gamma < (1-\alpha)/(2-3\alpha)$. Also when both firms use autoscaling, $E[CS_{AA}] > E[CS_{NN}] = 0$ if $\frac{(\gamma(1-\alpha(1-\gamma))-c(1-\alpha))^2}{4\gamma^2(1-\alpha)} < F < (1-c)((1-\alpha)(1-\gamma)\gamma+\alpha\gamma^2)$, and $E[CS_{AA}] = E[CS_{NN}] = 0$ if $F > (1-c)((1-\alpha)(1-\gamma)\gamma+\alpha\gamma^2)$.

Next, we compare the consumer surplus without autoscaling for $\frac{\gamma(1+\alpha(\gamma-1))}{1-\alpha} - 2\gamma^2 < c < \gamma(1-\gamma)$, with consumer surplus when only one firm uses autoscaling, which occurs for $\gamma > (1-\alpha)/(2-3\alpha)$ and $c < \hat{c}$. For all $s > 0$, we know that $s\log(s) > s-1$. Assuming $s = \frac{\left(-\alpha c+(\alpha-2)\gamma^2(c-1)+c+(\alpha-1)\gamma\right)}{(c-1)(c(\alpha\gamma^2+\alpha-1)+\gamma((\alpha-2)\gamma-\alpha+1))}$, we find a lower bound for $E[CS_{NA}]$, denoted $E[CS_{NA}]_L$. Thus, we have

$$E[CS_{NA}] > E[CS_{NA}]_L = \frac{\gamma^4(c-1)\left(2(\alpha-1)\alpha c^2+((8-5\alpha)\alpha-4)c+\alpha^2\right)+(\alpha-1)^2c^2}{4(\alpha-1)\gamma^2(c-1)}$$

$$-\frac{(\alpha-1)^2\gamma^2\left(2c^3-4c^2-1\right)+2(\alpha-1)^2c\gamma+2(\alpha-1)^2c\gamma^3(c-2)}{4(\alpha-1)\gamma^2(c-1)}.$$

Comparing $E[CS_{NA}]_L$ and $E[CS_{NN}]$, we find that for $c = 0$, we have $E[CS_{NA}]_L = E[CS_{NN}]$. Also, for $c > 0$, we have $\frac{\partial E[CS_{NA}]_L}{\partial c} > \frac{\partial E[CS_{NN}]}{\partial c}$, when $\gamma > (1-\alpha)/(2-3\alpha)$ and $c < \hat{c}$. Therefore, we have $E[CS_{NA}] > E[CS_{NA}]_L > E[CS_{NN}]$, for any $c > 0$.

Finally, if $c > \gamma(1-\gamma)$, then when autoscaling is not available, both firms either do not enter the market or enter and set their prices equal to 1. Both of these cases result in zero consumer surplus. Therefore autoscaling increases consumer surplus when both firms use autoscaling and $F < (1-c)((1-\alpha)(1-\gamma)\gamma+\alpha\gamma^2)$, or when one firm uses autoscaling and $F < \frac{\left(c\left(\alpha\left(\gamma^2-1\right)+1\right)+\gamma(\alpha(-\gamma)+\alpha-1)\right)^2}{4(\alpha-1)\gamma^2(c-1)}$. Otherwise, autoscaling does not affect consumer surplus.

B.8 Proofs of Section A.1

Equilibrium Strategies without Autoscaling

In outcomes in which both firms have the same reservation value, the solution to the pricing subgame is similar to that of Section 4.1, replacing $v_i = 1$ with v_i equal to V_h or V_l. We focus on the analysis of the case when one firm has high value V_h while the other firm has low value V_l. We denote the capacity chosen by the high-value firm k_h and the capacity of the low-value firm k_l. Assuming the firms' capacities overlap such that $k_l + k_h > 1$, the pricing subgame has no pure strategy equilibrium and both firms choose mixed strategy pricing.

Suppose Firm i's range of prices in the mixed strategy equilibrium is $[{p_i}^{Min}, v_i]$, where ${p_i}^{Min}$ is the lowest price in the support of the price distribution chosen by Firm i. Similarly, suppose Firm j's range of prices is $[{p_j}^{Min}, v_j]$. We prove that in equilibrium $v_i - {p_i}^{Min} = v_j - {p_j}^{Min}$: Assume to the contrary that $v_i - {p_i}^{Min} > v_j - {p_j}^{Min}$. In a mixed strategy equilibrium, Firm i gains the same expected profit from any price $p_i \in [{p_i}^{Min}, v_i]$. Firm i's expected profit from setting $p_i = {p_i}^{Min}$ is ${p_i}^{Min}(1-\alpha)$, since we have supposed $v_i - {p_i}^{Min} > v_j - p_j$ for any $p_j \in [{p_j}^{Min}, v_j]$ and all consumers in Segment 3 prefer Firm i to Firm j. Now, suppose Firm i chooses $p_i = {p_i}^{Min} + \varepsilon$, where ε is an infinitely small positive number. Still, all consumers in Segment 3 prefer Firm i to Firm j, since $v_i - ({p_i}^{Min} + \varepsilon) > v_j - p_j$ for any $p_j \in [{p_j}^{Min}, v_j]$. Thus, the expected profit of Firm i becomes $({p_i}^{Min} + \varepsilon)(1-\alpha)$. Since $\varepsilon > 0$, Firm i's expected profit from $p_i = {p_i}^{Min} + \varepsilon$ is strictly larger than the profit from $p_i = {p_i}^{Min}$. This contradicts the assumption that both prices $p_i = {p_i}^{Min}$ and $p_i = {p_i}^{Min} + \varepsilon$ are included in Firm i's mixed strategy pricing equilibrium. Thus, both firms must have the same length of price interval, where the length of price interval for Firm i is defined as $v_i - {p_i}^{Min}$.

Let z_l be the price at which the low-value firm is indifferent between *attacking* to sell to k_l consumers at price z_l and *retreating* to sell to $1-k_h$ consumers at price V_l. We have $z_l = \frac{V_l(1-k_h)}{k_l}$. Similarly, $z_h = \frac{V_h(1-k_l)}{k_h}$ is the price at which the high-value firm is indifferent between *attacking* and *retreating*. The firms' price intervals in the mixed strategy equilibrium are such that $v_i - {p_i}^{Min} = v_j - {p_j}^{Min}$ and the high-value and low-value firms' prices are greater than z_h and z_l respectively. Thus, the length of the price intervals of both firms must be equal to $Min[V_l - z_l, V_h - z_h]$. For $k_l > \frac{V_l}{V_h} k_h$, we have $V_l - z_l < V_h - z_h$ and therefore, the length of the price intervals of both firms is $V_l - z_l$. Thus, for $k_l > \frac{V_l}{V_h} k_h$, the low-value firm chooses the price range $p_l \in (z_l, V_l)$ and the high-value firm chooses the price range $p_h \in (V_h - (V_l - z_l), V_h)$. For $k_l < \frac{V_l}{V_h} k_h$, we have $V_l - z_l > V_h - z_h$ and therefore, the the length of the price intervals of both firms is $V_h - z_h$. Thus, for $k_l < \frac{V_l}{V_h} k_h$, the high-value firm chooses the price range $p_h \in (z_h, V_h)$ and the low-value firm chooses the price range $p_l \in (V_l - (V_h - z_h), V_l)$.

When $V_l - p_l > V_h - p_h$, the low-value firm sells all of its capacity, k_l, leaving the remaining $1 - k_l$ consumers for the high-value firm. When $V_l - p_l < V_h - p_h$, the high-value firm sells all of its capacity, k_h, and the low-value firm sells to the remaining $1 - k_h$ consumers. Suppose that $F_l(.)$ is the cumulative distribution function of the price of the low-value firm and $F_h(.)$ is the cumulative distribution function of the price of the high-value firm. Thus, excluding the sunk cost of capacity, the profit of the low-value firm setting price x is $\pi_l(x) = x((1-k_h)F_h((V_h - V_l) + x) + k_l(1 - F_h((V_h - V_l) + x)))$. Similarly, the profit of the high-value firm is $\pi_h(x) = x(k_h(1 - F_l(x - (V_h - V_l))) + (1 - k_l)F_l(x - (V_h - V_l)))$.

We first solve the pricing subgame without autoscaling for $k_l < \frac{V_l}{V_h} k_h$, where price ranges are $p_l \in (V_l - (V_h - z_h), V_l)$ and $p_h \in (z_h, V_h)$. We set the derivative of the profit functions $\pi_l(x)$ and $\pi_h(x)$ to zero and use the boundary conditions $F_l(V_l - (V_h - z_h)) = 0$ and $F_h(z_h) = 0$. We find $F_l(x) = \frac{V_h(k_h + k_l - 1) + k_h(x - V_l)}{(k_h + k_l - 1)(V_h - V_l + x)}$ and $F_h(x) = \frac{k_l(-k_h x - k_l V_h + V_h)}{k_h(k_h + k_l - 1)(V_h - V_l - x)}$. Note that $F_h(x)$ jumps from $\frac{k_l V_h}{k_h V_l}$ to 1 at $x = V_h$, which means the high-value firm uses price V_h with probability $1 - \frac{k_l V_h}{k_h V_l}$. Thus, when $k_l < \frac{V_l}{V_h} k_h$, the profits excluding sunk costs are

$$\pi_l = k_l \left(V_l - \frac{V_h(k_h + k_l - 1)}{k_h} \right) \text{ and } \pi_h = V_h - k_l V_h.$$

Next, we analyze the pricing subgame without autoscaling when $k_l > \frac{V_l}{V_h} k_h$, where price ranges are $p_l \in (z_l, V_l)$ and $p_h \in (V_h - (V_l - z_l), V_h)$. We set the derivative of the profit functions to zero, using the boundary conditions $F_l(z_l) = 0$ and $F_h(V_h - (V_l - z_l)) = 0$. We find $F_l(x) = \frac{k_h((k_h-1)V_l + k_l x)}{k_l(k_h+k_l-1)(V_h - V_l + x)}$ and $F_h(x) = \frac{\frac{(k_h-1)V_l}{-V_h+V_l+x} + k_l}{k_h+k_l-1}$. Note that $F_l(V_l) = \frac{k_h V_l}{k_l V_h}$, implying the low-value firm sets its price to V_l with probability $1 - \frac{k_h V_l}{k_l V_h}$. Thus, when $k_l > \frac{V_l}{V_h} k_h$, the profits excluding sunk costs are

$$\pi_l = V_l - k_h V_l \text{ and } \pi_h = k_h \left(V_h - \frac{V_l(k_h + k_l - 1)}{k_l} \right).$$

Assuming each firm realizes high value with a probability γ, expected profits are derived as follows.

$$E(\pi_i) = \begin{cases} k_i(-c + \gamma V_h - \gamma V_l + V_l) & \text{if } k_i + k_j \leq 1 \\ \frac{k_i((k_i-1)((\gamma-1)V_l - \gamma V_h) - c k_j)}{k_j} & \text{if } k_i < \frac{V_l}{V_h} k_j \text{ and } k_i + k_j \geq 1 \\ \frac{k_i\left(-c k_j + \gamma^2(-k_j)V_h + \gamma k_j V_h + \gamma^2 k_j V_l + \gamma(-k_j - 1)V_l + \gamma^2 V_h + V_l\right)}{k_j} + & \text{if } \frac{V_l}{V_h} k_j < k_i < k_j \text{ and } k_i + k_j \geq 1 \\ \frac{k_i^2\left(\gamma^2(-V_h) + \gamma V_l - V_l\right)}{k_j} + \frac{\gamma^2 k_j^2 V_l - \gamma k_j^2 V_l - \gamma^2 k_j V_l + \gamma k_j V_l}{k_j} & \\ -c k_i + (1-\gamma)\gamma k_i \left(V_h - \frac{V_l(k_i + k_j - 1)}{k_j} \right) + & \text{if } \frac{V_l}{V_h} k_i < k_j < k_i \text{ and } k_i + k_j \geq 1 \\ (1 - k_j)\left(\gamma^2 V_h + (\gamma-1)^2 V_l\right) + (\gamma - 1)\gamma(k_j - 1)V_l & \\ (k_j - 1)((\gamma-1)V_l - \gamma V_h) - c k_i & \text{if } k_j < \frac{V_l}{V_h} k_i \text{ and } k_i + k_j \geq 1 \end{cases}$$

We find the optimal capacities that maximize the firms' expected profits. For $k_i + k_j < 1$, we have $\frac{\partial E(\pi_i)}{\partial k_i} = -c + \gamma V_h - \gamma V_l + V_l$, which is negative for $\gamma < \frac{c - V_l}{V_h - V_l}$. Thus, for $c > \gamma(V_h - V_l) + V_l$, equilibrium capacities are $k_i = k_j = 0$.

For $k_j < \frac{V_l}{V_h} k_i$ and $k_i + k_j \geq 1$, we have $\frac{\partial E(\pi_i)}{\partial k_i} = -c$. Therefore, there can be no equilibrium where capacities are so far apart such that $k_i < \frac{V_l}{V_h} k_j$, since the high capacity firm would have incentive to decrease its capacity until $k_i < \frac{V_l}{V_h} k_j$ no longer holds.

For $\frac{V_l}{V_h} k_j < k_i < k_j$ and $k_i + k_j \geq 1$, the optimal capacities are

$$k_i = k^* = \frac{(\gamma - 1)\gamma V_l(-c + \gamma(\gamma+1)V_h + (\gamma-2)(\gamma-1)V_l)}{c^2 + 2(\gamma-1)\gamma c(V_h - V_l) + (\gamma-1)\gamma\left((\gamma-1)\gamma {V_h}^2 + 2\gamma(\gamma+1)V_h V_l + (\gamma-4)(\gamma-1){V_l}^2\right)}$$

$$k_j = k^{**} = \frac{\left(\gamma^2 V_h - \gamma V_l + V_l\right)(c + (\gamma-1)\gamma(V_h + V_l))}{c^2 + 2(\gamma-1)\gamma c(V_h - V_l) + (\gamma-1)\gamma\left((\gamma-1)\gamma {V_h}^2 + 2\gamma(\gamma+1)V_h V_l + (\gamma-4)(\gamma-1){V_l}^2\right)}$$

For this equilibrium to satisfy $k_i + k_j > 1$, we must have $c < (1-\gamma)\gamma(V_h - V_l)$. Otherwise, for $(1-\gamma)\gamma(V_h -$

$V_l) < c < \gamma(V_h - V_l) + V_l$, equilibrium capacities are such that $k_i + k_j = 1$.

Comparing these capacities with $1-\alpha$, we find the higher capacity reaches its ceiling (i.e., $k^{**} > 1-\alpha$) for $c < c^* = \frac{\sqrt{\gamma^4 {V_h}^2 - 2(\gamma-1)\gamma^2 V_h V_l (4(2\alpha^2-3\alpha+1)\gamma+1) + (\gamma-1)^2 {V_l}^2 (8(2\alpha^2-3\alpha+1)\gamma+1)} - 2\alpha\gamma^2 V_h + 2\alpha\gamma V_h + \gamma^2 V_h - 2\gamma V_h + (\gamma-1)V_l(2(\alpha-1)\gamma+1)}{2(\alpha-1)}$.

For $c < c^{**} = \frac{-3\alpha\gamma^2 V_h + \alpha\gamma V_h + 2\gamma^2 V_h - \gamma V_h + \alpha\gamma^2 V_l + \alpha\gamma V_l - 2\alpha V_l - \gamma^2 V_l + V_l}{\alpha - 1}$, we have $k^* > 1 - \alpha$ and thus both firms set their capacity equal to $1 - \alpha$.

Thus, the equilibrium capacities and profits are derived as shown below.

$$\begin{cases} k_i = k_j = 0 \text{ and } \pi_i = \pi_j = 0 & \text{if } c > \gamma(V_h - V_l) + V_l \\ k_i = k_j = \frac{1}{2} \text{ and } \pi_i = \pi_j = \frac{1}{2}(-c + \gamma V_h - \gamma V_l + V_l) & \text{if } (1-\gamma)\gamma(V_h - V_l) < c < \gamma(V_h - V_l) + V_l \\ k_i = k^*, k_j = k^{**} \text{ and} & \text{if } c^* < c < (1-\gamma)\gamma(V_h - V_l) \\ \pi_i = -\frac{(\gamma-1)\gamma V_l (c - \gamma V_h + (\gamma-1)V_l)}{c + (\gamma-1)\gamma(V_h + V_l)}, \pi_j = -\frac{\left((\gamma-1)V_l - \gamma^2 V_h\right)(c - \gamma V_h + (\gamma-1)V_l)}{c - \gamma(\gamma+1)V_h - (\gamma-2)(\gamma-1)V_l} & \\ k_i = k^*, k_j = 1 - \alpha \text{ and } \pi_i = {\pi^*}_{asym}, \pi_j = {\pi^{**}}_{asym} & \text{if } c^{**} < c < c^* \\ k_i = k_j = 1 - \alpha \text{ and } \pi_i = \pi_j = {\pi^*}_{sym} & \text{if } 0 < c < c^{**} \end{cases}$$

Where we have:

$${\pi^*}_{asym} = \frac{(\alpha-1)^2 c^2 - 2(\alpha-1)c((\gamma-1)(-\alpha\gamma V_h + (\alpha-1)\gamma V_l + V_l) - \gamma V_h) + \gamma^2 {V_h}^2 (\alpha(\gamma-1)+1)^2}{4(\alpha-1)\left((\gamma-1)V_l - \gamma^2 V_h\right)} +$$

$$\frac{2(\gamma-1)\gamma V_h V_l \left((\alpha-1)\alpha\gamma^2 + (\alpha-3)\alpha\gamma + \alpha + \gamma - 1\right) + (\gamma-1)^2 {V_l}^2 ((\alpha-1)\gamma(\alpha(\gamma-4) - \gamma + 2) + 1)}{4(\alpha-1)\left((\gamma-1)V_l - \gamma^2 V_h\right)}$$

$${\pi^{**}}_{asym} = \frac{(\alpha-1)^2 c^2 - 2(\alpha-1)c((\gamma-1)(-\alpha\gamma V_h + (\alpha-1)\gamma V_l + V_l) - \gamma V_h) + \gamma^2 {V_h}^2 (\alpha(\gamma-1)+1)^2}{2((\alpha-1)c + \gamma V_h - (\gamma-1)(-\alpha\gamma V_h + (\alpha-1)\gamma V_l + V_l))} +$$

$$\frac{2(\gamma-1)\gamma V_h V_l \left((\alpha-1)\alpha\gamma^2 + (\alpha-3)\alpha\gamma + \alpha + \gamma - 1\right) + (\gamma-1)^2 {V_l}^2 ((\alpha-1)\gamma(\alpha(\gamma-4) - \gamma + 2) + 1)}{2((\alpha-1)c + \gamma V_h - (\gamma-1)(-\alpha\gamma V_h + (\alpha-1)\gamma V_l + V_l))}$$

$${\pi^*}_{sym} = (\alpha-1)c + (2\alpha-1)\gamma^2(V_h - V_l) + (\alpha-1)\gamma(V_l - V_h) + \alpha V_l$$

Thus, both firms enter the market without autoscaling, if and only if $F < Min[\pi_i, \pi_j]$.

Equilibrium Strategies when Both Firms Use Autoscaling

We solve the case where both firms use autoscaling, assuming one firm has high value and the other has low value. For $V_l < c$, the low-value firm does not benefit from selling to any consumers with autoscaling, and thus the high-value firm sells to $1 - \alpha$ consumers at the price of V_h. Next, suppose $V_l > c$ such that the low-value firm has incentive to sell with autoscaling. There is no pure strategy equilibrium for the pricing subgame and firms use mixed strategy prices. The price at which the firm is indifferent between *attacking* and *retreating* is $z_l = \frac{-2\alpha c + c + \alpha V_l}{1-\alpha}$ for the low-value firm, and $z_h = \frac{-2\alpha c + c + \alpha V_h}{1-\alpha}$ for the high-value firm. It is easy to show $V_h - z_h > V_l - z_l$. Thus, using a logic similar to our analysis of the case without autoscaling, we show the length of the price interval chosen by both firms in the mixed strategy equilibrium should be $V_l - z_l$. This means the low-value firm chooses the price range $p_l \in (z_l, V_l)$ and the high-value firm chooses the price range $p_h \in (V_h - (V_l - z_l), V_h)$.

Given the price distributions, the profits are $\pi_l(x) = (x-c)((1-\alpha)(1-F_h((V_h-V_l)+x))+\alpha F_h((V_h-V_l)+x))$ for the low-value firm and $\pi_h(x) = (x-c)((1-\alpha)(1-F_l(x-(V_h-V_l)))+\alpha F_l(x-(V_h-V_l)))$ for the high-value firm. Using boundary condition $F_h(V_h-(V_l-z_l)) = 0$, we find $F_h(x) = \frac{(2\alpha-1)c+(\alpha-1)V_h-\alpha(2V_l+x)+V_l+x}{(2\alpha-1)(c+V_h-V_l-x)}$, resulting in $\pi_l = \alpha(V_l-c)$. Similarly, with boundary condition $F_l[z_l] = 0$, we have $F_l(x) = \frac{(2\alpha-1)c-\alpha(V_l+x)+x}{(2\alpha-1)(c-V_h+V_l-x)}$, where $F_l(x)$ jumps from $\frac{c-V_l}{c-V_h}$ to 1 at $x = V_l$. This distribution results in $\pi_h = -\alpha(c + V_h - 2V_l) + V_h - V_l$.

Considering the different possible values for each firm, the expected profit of Firm i when both firms use autoscaling is $E(\pi_{AA}) = -\alpha c + (2\alpha - 1)\gamma^2(V_h - V_l) + (\alpha - 1)\gamma(V_l - V_h) + \alpha V_l$.

Equilibrium Strategies when Only One Firm Uses Autoscaling

Consider the case where only one firm uses autoscaling and the two firms have different values. The pricing strategy of each firm depends on 1) whether it is using autoscaling, and 2) whether its value is V_l or V_h. Let $z_{an}(.)$ be the price at which the firm using autoscaling is indifferent between *attacking* to sell to $1-\alpha$ consumers at price $z_{an}(.)$ and *retreating* to sell to $1 - k_{na}$ consumers at price v_{an}, where v_{an} is the value of the firm using autoscaling and k_{na} is the capacity chosen by the firm not using autoscaling. We have $z_{an}(v_{an}) = \frac{c(k_{na}-\alpha)+(1-k_{na})v_{an}}{1-\alpha}$. Similarly, $z_{na}(v_{na}) = \frac{\alpha v_{na}}{k_{na}}$ is the price at which the firm not using autoscaling is indifferent between *attacking* and *retreating*, where v_{na} is the value of the firm not using autoscaling.

First, suppose the firm using autoscaling has low value V_l and the firm with fixed capacity has high value V_h. Assume $V_l > c$ so that the low-value firm has incentive to sell with autoscaling. We have $V_l - z_{an}(V_l) < V_h - z_{na}(V_h)$. Thus, the low-value firm chooses the price range $p_l \in (z_{an}(V_l), V_l)$ and the high-value firm chooses the price range $p_h \in (V_h - (V_l - z_{an}(V_l)), V_h)$.

Next, suppose the firm using autoscaling has high value V_h and the firm with fixed capacity has low-value V_l. For $V_h - V_l > c$ and $k_{na} > \frac{(\alpha-1)V_l}{c-V_h}$, we have $V_h - z_{an}(V_h) > V_l - z_{na}(V_l)$; otherwise, $V_h - z_{an}(V_h) < V_l - z_{na}(V_l)$. Thus, for $V_h - V_l > c$ and $k_{na} > \frac{(\alpha-1)V_l}{c-V_h}$, the low-value firm chooses the price range $p_l \in (z_{na}(V_l), V_l)$ and the high-value firm chooses the price range $p_h \in (V_h - (V_l - z_{na}(V_l)), V_h)$. Otherwise, the low-value firm chooses the price range $p_l \in (V_l - (V_h - z_{an}(V_h)), V_l)$ and the high-value firm chooses the price range $p_h \in (z_{an}(V_h), V_h)$.

Using the same steps as before, we derive the pricing distribution for each firm. First, suppose the firm that uses autoscaling has V_l, such that $V_l > c$, and the firm with fixed capacity has V_h. We find

$F_h(x) = \frac{(k_{na}-1)(V_h-x)}{(\alpha-k_{na})(c+V_h-V_l-x)}+1$ and $F_l(x) = \frac{k_{na}(c(k_{na}-\alpha)-k_{na}V_l+V_l+(\alpha-1)x)}{(\alpha-1)(k_{na}-\alpha)(V_h-V_l+x)}$, where $F_l(x)$ jumps from $\frac{k_{na}(c-V_l)}{(\alpha-1)V_h}$ to 1 at the price of $x = V_l$. Corresponding profits are $\pi_l = (k_{na}-1)(c-V_l)$ for the firm using autoscaling and $\pi_h = \frac{k_{na}(-ck_{na}+\alpha(c+V_h-V_l)+k_{na}V_l-V_h)}{\alpha-1}$ for the firm with fixed capacity, excluding sunk costs.

Next, consider the case where the firm using autoscaling realizes V_h and the firm with fixed capacity realizes V_l. If $V_h - V_l < c$ or $k_{na} < \frac{(\alpha-1)V_l}{c-V_h}$, the price ranges are $p_l \in (V_l - (V_h - z_{an}(V_h)), V_l)$ and $p_h \in (z_{an}(V_h), V_h)$. The price distributions are $F_l(x) = \frac{(k_{na}-1)(V_l-x)}{(\alpha-k_{na})(c-V_h+V_l-x)}+1$ and $F_h(x) = \frac{k_{na}(c(\alpha-k_{na})+(k_{na}-1)V_h-\alpha x+x)}{(\alpha-1)(k_{na}-\alpha)(V_h-V_l-x)}$, where $F_h(x)$ jumps from $\frac{k_{na}(c-V_h)}{(\alpha-1)V_l}$ to 1 at the price of $x = V_h$. The profits are $\pi_l = \frac{k_{na}(-ck_{na}+\alpha(c-V_h+V_l)+k_{na}V_h-V_l)}{\alpha-1}$ for the firm that sets capacity and $\pi_h = (k_{na}-1)(c-V_h)$ for the firm that uses autoscaling.

Finally, we assume the firm using autoscaling has V_h, the other firm has V_l, and we have $V_h - V_l > c$ and $k_{na} > \frac{(\alpha-1)V_l}{c-V_h}$. Price ranges are $p_l \in (z_{na}(V_l), V_l)$ and $p_h \in (V_h - (V_l - z_{na}(V_l)), V_h)$. The firms use the price distributions $F_h(x) = \frac{k_{na}(-V_h+V_l+x)-\alpha V_l}{(k_{na}-\alpha)(-V_h+V_l+x)}$ and $F_l(x) = \frac{(\alpha-1)(k_{na}x-\alpha V_l)}{k_{na}(k_{na}-\alpha)(c-V_h+V_l-x)}$, where $F_l(x)$ jumps from $\frac{(\alpha-1)V_l}{k_{na}(c-V_h)}$ to 1 at the price of $x = V_l$. The profits are $\pi_l = \alpha V_l$ for the firm that sets capacity and $\pi_h = \frac{(\alpha-1)(k_{na}(c-V_h+V_l)-\alpha V_l)}{k_{na}}$ for the firm that uses autoscaling.

We denote $E(\pi_{AN})$ as the expected profit of the firm using autoscaling and $E(\pi_{NA})$ as the expected profit of the firm setting fixed capacity.

For $0 < V_l < c$, the firm using autoscaling does not sell to any consumers if it realizes low value and we have

$$E(\pi_{AN}) = \gamma\left(\gamma(k_{na}-1)(c-V_h) + \frac{(\alpha-1)(\gamma-1)(\alpha V_l - k_{na}(c-V_h+V_l))}{k_{na}}\right)$$

$$E(\pi_{NA}) = \frac{\gamma^2 k_{na}(c(\alpha-k_{na})+(k_{na}-1)V_h)}{\alpha-1} - ck_{na} - \gamma(\gamma-1)k_{na}V_h + (\gamma-1)^2 k_{na}V_l - \alpha\gamma(\gamma-1)V_l$$

For $c < V_l < V_h - c$ and $k_{na} > \frac{(\alpha-1)V_l}{c-V_h}$, we have

$$E(\pi_{AN}) = \gamma^2(k_{na}-1)(c-V_h) + \frac{(\alpha-1)\gamma(\gamma-1)(\alpha V_l - k_{na}(c-V_h+V_l))}{k_{na}} +$$

$$(\gamma-1)^2(k_{na}-1)(c-V_l) - \gamma(\gamma-1)(k_{na}-1)(c-V_l)$$

$$E(\pi_{NA}) = \frac{\gamma\left(-\alpha(k_{na}(c-V_h+V_l)+V_l) + k_{na}(ck_{na}-(k_{na}-2)V_l-V_h) + \alpha^2 V_l\right)}{\alpha-1} +$$

$$\frac{\gamma^2(-(k_{na}-\alpha))(ck_{na}-k_{na}V_h-\alpha V_l+V_l) - (k_{na}-1)k_{na}(c-V_l)}{\alpha-1}$$

Finally, for $V_l > V_h - c$ or $k_{na} < \frac{(\alpha-1)V_l}{c-V_h}$, expected profits are

$$E(\pi_{AN}) = (k_{na}-1)(c-\gamma V_h + (\gamma-1)V_l)$$

$$E(\pi_{NA}) = -\frac{(k_{na}-1)k_{na}(c-\gamma V_h+(\gamma-1)V_l)}{\alpha-1}$$

The firm not using autoscaling sets the optimal capacity, k^*_{na}, such that $\frac{\partial E(\pi_{NA})}{\partial k_{na}} = 0$:

$$k^*_{na} = \begin{cases} \frac{c\left(\alpha\left(\gamma^2-1\right)+1\right)+\gamma V_h(\alpha(-\gamma)+\alpha-1)+(\alpha-1)(\gamma-1)^2V_l}{2\gamma^2(c-V_h)} & \text{if } 0 < V_l < c \\ \frac{c(\alpha(\gamma-1)\gamma+1)+\gamma(-V_h)+(\gamma-1)(-\alpha\gamma V_h+(\alpha-1)\gamma V_l+V_l)}{2(c((\gamma-1)\gamma+1)+\gamma^2(-V_h)+(\gamma-1)V_l)} & \text{if } c < V_l < V_h - c \\ \frac{1}{2} & \text{if } V_l > V_h - c \text{ and } V_l > c \end{cases}$$

Inserting k^*_{na} into $E(\pi_{NA})$ and $E(\pi_{AN})$, we derive the optimal profits when only one firm uses autoscaling.

Note that we have $E(\pi_{NA}) = \frac{k_{na}}{1-\alpha}E(\pi_{AN}) < E(\pi_{AN})$, for $V_l > c$. Therefore, the condition for both firms entering the market, when only one firm uses autoscaling, is $F < Min[E(\pi_{NA}), E(\pi_{AN})] = E(\pi_{NA})$. Thus, when autoscaling is offered, both firms enter the market if and only if $F < Max[E(\pi_{AA}), E(\pi_{NA})]$.

B.9 Proofs of Section A.2

Equilibrium Strategies without Autoscaling

Consider overlapping capacities without autoscaling. If the price of Firm i is higher than its competitor's, Firm i sells to Segment i and what is left of Segment 3 after Firm j sells all of its capacity. Thus, Firm i sells to $(\alpha + (1-2\alpha) - \frac{(1-2\alpha)k_j}{1-\alpha})$ consumers if $p_i > p_j$ and to k_i consumers if $p_i < p_j$. Suppose $F_j(.)$ is the cumulative distribution function of the price set by Firm j when the overlapping condition on capacities holds. The profit of Firm i is derived as $\pi_i(x) = x\left(k_i(1-F_j(x)) + F_j(x)\left(\alpha + (1-2\alpha) - \frac{(1-2\alpha)k_j}{1-\alpha}\right)\right)$.

Let z_i be the price at which Firm i is indifferent between *attacking* to sell to k_i consumers at price z_i and *retreating* to sell to $((1-\alpha) - \frac{(1-2\alpha)k_j}{1-\alpha})$ consumers at price 1. We have $z_i = \frac{\left(-\alpha-\frac{(1-2\alpha)k_j}{1-\alpha}+1\right)}{k_i}$. As shown in the proof of Section A.1, the length of the price intervals chosen by both firms must be the same in equilibrium to satisfy the necessary condition of mixed strategy equilibrium. Also Firm i does not choose any price lower than z_i. Thus, both firms choose prices in the range of $p \in (Max[z_1, z_2], 1)$. Without loss of generality, suppose $k_1 < k_2$. For $k_1 + k_2 > -\frac{(\alpha-1)^2}{2\alpha-1}$, the price range is $(z_2, 1)$; otherwise, the price range is $(z_1, 1)$.

We first solve the pricing subgame for $k_1 > -\frac{(\alpha-1)^2}{2\alpha-1} - k_2$, for which both firms' price range in the mixed strategy equilibrium is $(z_2, 1)$. Solving the differential equation $\frac{\partial \pi_2}{\partial x} = 0$ and using the boundary condition $F_1(z_2) = 0$, we have $F_1(x) = \frac{\left((\alpha-1)^2+(2\alpha-1)k_1\right)+(\alpha-1)k_2x}{x((2\alpha-1)k_1+(\alpha-1)(\alpha+k_2-1))}$. Also, solving $\frac{\partial \pi_1}{\partial x} = 0$ and using the boundary condition $F_2(z_2) = 0$ results in $F_2(x) = \frac{k_1\left(\left((\alpha-1)^2+(2\alpha-1)k_1\right)+(\alpha-1)k_2x\right)}{k_2x((\alpha-1)^2+(\alpha-1)k_1+(2\alpha-1)k_2)}$. Note that $F_2(1)$ equals $\frac{k_1((2\alpha-1)k_1+(\alpha-1)(\alpha+k_2-1))}{k_2((\alpha-1)^2+(\alpha-1)k_1+(2\alpha-1)k_2)}$, which is less than 1 for $k_1 + k_2 > -\frac{(\alpha-1)^2}{2\alpha-1}$. This means Firm 2 is setting its price to 1 with a probability of $1 - \frac{k_1((2\alpha-1)k_1+(\alpha-1)(\alpha+k_2-1))}{k_2((\alpha-1)^2+(\alpha-1)k_1+(2\alpha-1)k_2)}$. Firms' profits, excluding costs of capacity, when $k_1 > -\frac{(\alpha-1)^2}{2\alpha-1} - k_2$, are

$$\pi_1 = \frac{k_1\left(-(\alpha-1)^2 - 2\alpha k_1 + k_1\right)}{(\alpha-1)k_2} \text{ and } \pi_2 = \frac{\left(-(\alpha-1)^2 - 2\alpha k_1 + k_1\right)}{\alpha - 1}.$$

Next, we find the solution for $-\alpha - \frac{(1-2\alpha)k_2}{1-\alpha} + 1 < k_1 < -\frac{(\alpha-1)^2}{2\alpha-1} - k_2$, where both firms' price range in the mixed strategy equilibrium is $(z_1, 1)$. This time, we use the boundary conditions $F_1(z_1) = 0$ and $F_2(z_1) = 0$ to solve $\frac{\partial \pi_2}{\partial x} = 0$ and $\frac{\partial \pi_1}{\partial x} = 0$. We find $F_1(x) = \frac{k_2\left((\alpha-1)k_1 x + \left((\alpha-1)^2 + (2\alpha-1)k_2\right)\right)}{k_1 x((2\alpha-1)k_1 + (\alpha-1)(\alpha+k_2-1))}$ and $F_2(x) = \frac{(\alpha-1)k_1 x + \left((\alpha-1)^2 + (2\alpha-1)k_2\right)}{x((\alpha-1)^2 + (\alpha-1)k_1 + (2\alpha-1)k_2)}$, where $F_1(x)$ jumps from $\frac{k_2\left((\alpha-1)^2 + (\alpha-1)k_1 + (2\alpha-1)k_2\right)}{k_1((2\alpha-1)k_1 + (\alpha-1)(\alpha+k_2-1))}$ to 1 at $x = 1$. Excluding sunk costs, firms' profits when $-\alpha - \frac{(1-2\alpha)k_2}{1-\alpha} + 1 < k_1 < -\frac{(\alpha-1)^2}{2\alpha-1} - k_2$ are

$$\pi_1 = \frac{\left((\alpha-1)^2 + 2\alpha k_2 - k_2\right)}{1-\alpha} \text{ and } \pi_2 = \frac{k_2\left((\alpha-1)^2 + 2\alpha k_2 - k_2\right)}{(1-\alpha)k_1}.$$

Finally, when $k_1 < -\alpha - \frac{(1-2\alpha)k_2}{1-\alpha} + 1$, each firm has a local monopoly and sets its price at $p_i = 1$, earning a profit of k_i. Note that for $-\frac{(\alpha-1)(\alpha+k_2-1)}{2\alpha-1} < k_1 < -\alpha - \frac{(1-2\alpha)k_2}{1-\alpha} + 1$, there is no capacity overlap for $p_2 < p_1$, but there is capacity overlap for $p_1 < p_2$. In both these cases, regardless of what p_2 is, Firm 1 can sell to k_1 consumers. Thus Firm 1 chooses $p_1 = 1$. In order to sell all of its capacity, Firm 2 needs to choose $p_2 < p_1$, and does so by setting $p_2 = 1 - \varepsilon$, where ε is an infinitely small positive number. Therefore, the equilibrium outcome for $-\frac{(\alpha-1)(\alpha+k_2-1)}{2\alpha-1} < k_1 < -\alpha - \frac{(1-2\alpha)k_2}{1-\alpha} + 1$ is $\pi_1 = k_1$ and $\pi_2 = k_2$, the same as when $k_1 < -\frac{(\alpha-1)(\alpha+k_2-1)}{2\alpha-1}$.

Assuming Firm i realizes $v_i = 1$ with probability γ, expected profits for $\alpha < k_i, k_j < 1 - \alpha$ are

$$E(\pi_i) = \begin{cases} k_i(\gamma - c) & \text{if } k_i < -\alpha - \frac{(1-2\alpha)k_j}{1-\alpha} + 1 \\ \frac{\gamma^2(-\alpha(\alpha+2k_j-2)+k_j-1)}{\alpha-1} - ck_i - (\gamma-1)\gamma k_i & \text{if } k_i > -\alpha - \frac{(1-2\alpha)k_j}{1-\alpha} + 1 \text{ and} \\ & k_i < k_j < -\frac{(\alpha-1)^2}{2\alpha-1} - k_i \\ k_i\left(\frac{\gamma^2(-\alpha(\alpha+2k_i-2)+k_i-1)}{(\alpha-1)k_j} - c - (\gamma-1)\gamma\right) & \text{if } k_i > -\alpha - \frac{(1-2\alpha)k_j}{1-\alpha} + 1 \text{ and} \\ & k_j < k_i < -\frac{(\alpha-1)^2}{2\alpha-1} - k_j \\ k_i\left(\frac{\gamma^2(-\alpha(\alpha+2k_i-2)+k_i-1)}{(\alpha-1)k_j} - c - (\gamma-1)\gamma\right) & \text{if } k_j > k_i > -\frac{(\alpha-1)^2}{2\alpha-1} - k_j \\ \frac{\gamma^2(-\alpha(\alpha+2k_j-2)+k_j-1)}{\alpha-1} - ck_i - (\gamma-1)\gamma k_i & \text{if } k_i > k_j > -\frac{(\alpha-1)^2}{2\alpha-1} - k_i \end{cases}$$

Given the expected profits for each set of capacities, we solve for equilibrium capacity choices. For $k_i < -\alpha - \frac{(1-2\alpha)k_j}{1-\alpha} + 1$, we have $\frac{\partial E(\pi_i)}{\partial k_i} = \gamma - c$. Thus for $\gamma < c$, equilibrium capacities are $k_i = k_j = 0$.

For $k_i > -\alpha - \frac{(1-2\alpha)k_j}{1-\alpha} + 1$ and $k_i < k_j < -\frac{(\alpha-1)^2}{2\alpha-1} - k_i$, we have $\frac{\partial E(\pi_i)}{\partial k_i} = \gamma(1-\gamma) - c$. Thus, for $\gamma(1-\gamma) < c$, the firm with the lower capacity in this region, decreases its capacity until $k_i + \frac{(1-2\alpha)k_j}{1-\alpha} = 1 - \alpha$.

The symmetric equilibrium requires $k_i = k_j = \frac{(\alpha-1)^2}{2-3\alpha}$, resulting in each firm earning a monopoly profit of $\pi^*{}_{nn} = \frac{(\alpha-1)^2}{2-3\alpha}(\gamma - c)$.

For $k_i > k_j > -\frac{(\alpha-1)^2}{2\alpha-1} - k_i$, we have $\frac{\partial E(\pi_i)}{\partial k_i} = \gamma(1-\gamma) - c$. Thus, when $\gamma(1-\gamma) > c$, for $k_i + k_j > -\frac{(\alpha-1)^2}{2\alpha-1}$, the firm with the higher capacity increases its capacity to $1-\alpha$. When $k_i = 1-\alpha$, Firm j maximizes its profit by choosing $k_j = k^* = \frac{(\alpha-1)^2(c-\gamma)}{2(2\alpha-1)\gamma^2}$. Thus, profits become $\pi_i = \frac{1}{2}(\alpha-1)(c-\gamma)$ and $\pi_j = -\frac{(\alpha-1)^2(c-\gamma)^2}{4(2\alpha-1)\gamma^2}$.

For $c < \frac{\gamma(-4\alpha\gamma+\alpha+2\gamma-1)}{\alpha-1}$, we have $k^* > 1-\alpha$. Therefore, if $c < \frac{\gamma(-4\alpha\gamma+\alpha+2\gamma-1)}{\alpha-1}$, both firms set their capacity at $1-\alpha$, and each earn a profit of $(\alpha-1)c + \gamma(\alpha(2\gamma-1) - \gamma + 1)$.

Equilibrium Strategies when Only One Firm Uses Autoscaling

Suppose one firm uses autoscaling, setting the price p_{an}, while the other firm chooses a fixed capacity of k_{na} and sets the price p_{na}. When $p_{an} < p_{na}$, the firm using autoscaling sells to $(1-\alpha)$ consumers and the firm with fixed capacity sells to α consumers. When $p_{an} > p_{na}$, the firm with fixed capacity sells to k_{na} consumers, of whom $\frac{1-2\alpha}{1-\alpha}k_{na}$ come from Segment 3. Thus, the firm using autoscaling can only sell to $(1-2\alpha) - \frac{1-2\alpha}{1-\alpha}k_{na} + \alpha$ consumers. Suppose $F_{an}(.)$ is the cumulative distribution function of price for the firm using autoscaling and $F_{na}(.)$ is the cumulative distribution function of price for the firm with fixed capacity. The profits of the two firms, excluding sunk costs, are $\pi_{na}(x) = x(k_{na}(1-F_{an}(x)) + \alpha F_{an}(x))$ for the firm with fixed capacity and $\pi_{an}(x) = (x-c)\left(F_{na}(x)\left(\alpha + (1-2\alpha) - \frac{(1-2\alpha)k_{na}}{1-\alpha}\right) + (1-\alpha)(1-F_{na}(x))\right)$ for the firm using autoscaling.

The firm setting capacity is indifferent between *attacking* and *retreating* at the price of $z_{na} = \frac{\alpha}{k_{na}}$. The firm using autoscaling is indifferent between *attacking* and *retreating* at the price of $z_{an} = \frac{c(k_{na}-2\alpha k_{na})}{(\alpha-1)^2} + \frac{(\alpha-1)^2+(2\alpha-1)k_{na}}{(\alpha-1)^2}$. As shown in the proof of Section A.1, the condition for mixed strategy equilibrium is that the length of the price interval for both firms should be equal. If $k_{na} > \frac{(\alpha-1)^2-\sqrt{(\alpha-1)^2(4\alpha(1-2\alpha)c+(1-3\alpha)^2)}}{2(2\alpha-1)(c-1)}$, we have $z_{an} > z_{na}$ and both firms choose prices in the range $(z_{an}, 1)$; otherwise, prices are set in the range $(z_{na}, 1)$.

First, suppose $k_{na} < \frac{(\alpha-1)^2-\sqrt{(\alpha-1)^2(4\alpha(1-2\alpha)c+(1-3\alpha)^2)}}{2(2\alpha-1)(c-1)}$. Solving $\frac{\partial \pi_{na}}{\partial x} = 0$ and using the boundary condition $F_{an}(z_{na}) = 0$ returns $F_{an}(x) = \frac{k_{na}x-\alpha}{k_{na}x-\alpha x}$ and $\pi_{na} = \alpha$. Thus, the expected profit of the firm choosing capacity is $E(\pi_{na}) = -ck_{na} + \gamma(\alpha\gamma - \gamma k_{na} + k_{na})$. We have $\frac{\partial E(\pi_{na})}{\partial k_{na}} > 0$ for $c < \gamma(1-\gamma)$. Thus, the firm choosing capacity increases its capacity until $k_{na} = \frac{(\alpha-1)^2-\sqrt{(\alpha-1)^2(4\alpha(1-2\alpha)c+(1-3\alpha)^2)}}{2(2\alpha-1)(c-1)}$ and there is no equilibrium for $k_{na} < \frac{(\alpha-1)^2-\sqrt{(\alpha-1)^2(4\alpha(1-2\alpha)c+(1-3\alpha)^2)}}{2(2\alpha-1)(c-1)}$.

For $k_{na} \geq \frac{(\alpha-1)^2-\sqrt{(\alpha-1)^2(4\alpha(1-2\alpha)c+(1-3\alpha)^2)}}{2(2\alpha-1)(c-1)}$, we solve differential equations $\frac{\partial \pi_{an}}{\partial x} = 0$ and $\frac{\partial \pi_{na}}{\partial x} = 0$ using the boundary conditions $F_{an}(z_{an}) = 0$ and $F_{na}(z_{an}) = 0$. We find $F_{na}(x) = \frac{(2\alpha-1)ck_{na}+\left(-(\alpha-1)^2-2\alpha k_{na}+k_{na}\right)+(\alpha-1)^2x}{(2\alpha-1)k_{na}(c-x)}$ and $F_{an}(x) = \frac{k_{na}\left((2\alpha-1)ck_{na}+\left(-(\alpha-1)^2-2\alpha k_{na}+k_{na}\right)+(\alpha-1)^2x\right)}{(\alpha-1)^2x(k_{na}-\alpha)}$, where $F_{an}(x)$ jumps from $\frac{(2\alpha-1)k_{na}^2(c-1)}{(\alpha-1)^2(k_{na}-\alpha)}$ to 1 at $x = 1$. The profit of the firm using autoscaling is $\frac{(c-1)\left((\alpha-1)^2+(2\alpha-1)k_{na}\right)}{\alpha-1}$ and the profit of the other firm is $\frac{k_{na}\left(c(k_{na}-2\alpha k_{na})+\left((\alpha-1)^2+(2\alpha-1)k_{na}\right)\right)}{(\alpha-1)^2}$. The expected profit of the firm using autoscaling is $E(\pi_{an}) = \frac{\gamma(c-1)\left((\alpha-1)^2+(2\alpha-1)\gamma k_{na}\right)}{\alpha-1}$ and the expected profit of the firm setting capacity is $E(\pi_{na}) = k_{na}\left(-\frac{(2\alpha-1)\gamma^2 k_{na}(c-1)}{(\alpha-1)^2} - c + \gamma\right)$, which is maximized at $k_{na} = k^*{}_{na} = -\frac{(\alpha-1)^2(c-\gamma)}{2(2\alpha-1)\gamma^2(c-1)}$. Inserting $k_{na} = k^*{}_{na}$ into $E(\pi_{an})$ and $E(\pi_{na})$, we find

$$\pi^*{}_{an} = \frac{1}{2}(1-\alpha)(-2\gamma c + c + \gamma) \text{ and } \pi^*{}_{na} = \frac{(\alpha-1)^2(c-\gamma)^2}{4(1-2\alpha)\gamma^2(1-c)}.$$

Effect of Autoscaling on Entry

Next, we find the region where autoscaling decreases entry. We have $\pi^*{}_{nn} > \pi^*{}_{na}$ for

$$\gamma > \tilde{\gamma}_{NA} = \frac{(2-3\alpha) - \sqrt{3\alpha-2}\sqrt{16(2\alpha-1)c^2 + 16(1-2\alpha)c + (3\alpha-2)}}{8(2\alpha-1)(c-1)}.$$

Similarly, denoting $\pi^*{}_{aa}$ as the profit of each firm when both firms use autoscaling, which is the same as what we had with the efficient rationing rule, we have $\pi^*{}_{nn} > \pi^*{}_{aa}$ for

$$\gamma > \tilde{\gamma}_{AA} = \frac{(3\alpha-2)\sqrt{\frac{(\alpha-1)^2((-15\alpha^2+16\alpha-4)c^2+2(6\alpha^2-7\alpha+2)c+(1-2\alpha)^2)}{(2-3\alpha)^2}} + \left(3\alpha^2-5\alpha+2\right)c + \left(-2\alpha^2+3\alpha-1\right)}{2(2\alpha-1)(3\alpha-2)(c-1)}.$$

Thus, autoscaling decreases entry for $\gamma > \tilde{\gamma} = Max[\tilde{\gamma}_{AA}, \tilde{\gamma}_{NA}]$ and $Max[\pi^*{}_{aa}, \pi^*{}_{na}] < F < \pi^*{}_{nn}$.

B.10 Proofs of Section A.3

When autoscaling is available, the condition for at least one firm entering the market is $F < (1-c)\gamma(1-\alpha)$. This means at least one firm enters as long as $c \leq \frac{F}{(\alpha-1)\gamma} + 1$. The cloud provider's profit, given the single entrant's probability of success, is $\gamma c(1-\alpha)$. Thus, the provider maximizes c and sets $c = \frac{F}{(\alpha-1)\gamma} + 1$. The provider's expected profit is $\pi_a{}^{CP} = \gamma(\frac{F}{(\alpha-1)\gamma} + 1)(1-\alpha)$.

Both firms enter the market using autoscaling for $F < (1-c)(\alpha\gamma^2 + (1-\alpha)(1-\gamma)\gamma)$. In this region, the total capacity purchased is 1, if both entrants realize high value. If only one firm realizes high value, the total capacity purchased is $1-\alpha$. Thus, the provider's profit is $c(2(1-\alpha)(1-\gamma)\gamma+\gamma^2)$ and c is set to its maximum, $1-\frac{F}{\gamma(\alpha(2\gamma-1)-\gamma+1)}$. Thus, the provider's expected profit is $\pi_{aa}{}^{CP} = (1-\frac{F}{\gamma(\alpha(2\gamma-1)-\gamma+1)})(2(1-\alpha)(1-\gamma)\gamma+\gamma^2)$.

We know one firm uses autoscaling and the other sets $k_{na} = \frac{c\left(\alpha\left(\gamma^2-1\right)+1\right)+\gamma(\alpha(-\gamma)+\alpha-1)}{2\gamma^2(c-1)}$, when $c < \hat{c}$. The cumulative distribution functions of price for the firms are $F_{na}(x) = \frac{(k_{na}-1)(x-1)}{(c-x)(k_{na}-\alpha)} + 1$ and $F_{an}(x) = \frac{k_{na}(c(k_{na}-\alpha)-k_{na}+1+(\alpha-1)x)}{(\alpha-1)x(k_{na}-\alpha)}$. The purchased capacity of the firm using autoscaling is $(1-k_{na})F_{na}(x) + (1-\alpha)(1-F_{na}(x)) = \frac{(k_{na}-1)(c-1)}{x-c}$. Thus, the capacity of this firm depends on the probability density function of its price, which is $f_{an}(x) = \frac{k_{na}(c(\alpha-k_{na})+(k_{na}-1))}{(\alpha-1)x^2(k_{na}-\alpha)}$. We derive the expected capacity of the firm using autoscal-

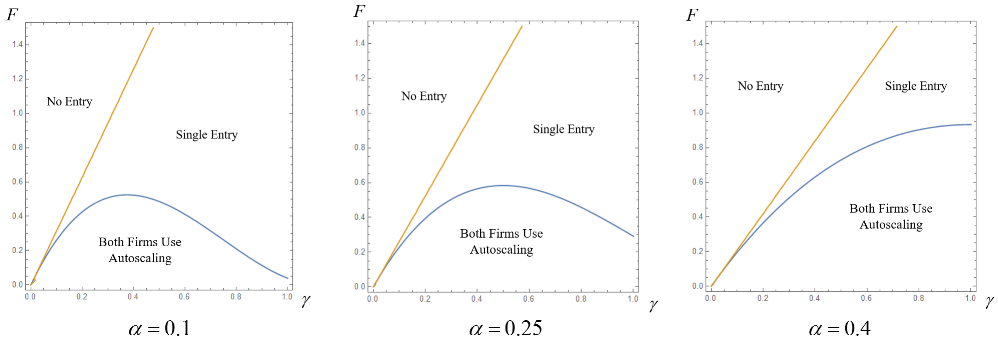

Figure A5: Entry decisions with endogenous price of capacity when autoscaling is available.

ing as $E[k_{an}] = \int_{z''}^{1} \frac{f_{an}(x)((k_{na}-1)(c-1))}{x-c}\, dx = \frac{(k_{na}-1)k_{na}(c-1)\left(c(c-1)(k_{na}-\alpha)+(c(\alpha-k_{na})+(k_{na}-1))\log\left(\frac{(k_{na}-1)}{c(\alpha-k_{na})+(k_{na}-1)}\right)\right)}{(1-\alpha)c^2(k_{na}-\alpha)}$.

The expected profit of the provider when both firms enter but only one firm uses autoscaling is ${\pi_{na}}^{CP} = c\left((1-\alpha)(1-\gamma)\gamma + \gamma^2 E[k_{an}] + k_{na}\right)$. Maximizing ${\pi_{na}}^{CP}$ with respect to c does not have a closed form solution. However, we can numerically compare the providers' maximum profit when one firm or both firms use autoscaling and determine what c would be set by the provider. The results are shown in Figure A5.

Without autoscaling, at least one firm enters the market for $F < (1-\alpha-1)(\gamma-c)$. The provider can increase the price to $c = \frac{F}{\alpha-1} + \gamma$, so only one firm enters and purchases $1-\alpha$ capacity. The provider makes a profit of $(1-\alpha)\gamma - F$. Note that this profit is equal to ${\pi_a}^{CP}$, the provider's profit when autoscaling is offered and only one firm enters the market.

When $\gamma(1-\gamma) < c < \gamma$, both firms enter the market, each firm purchasing a capacity half the size of the market, if $F < \frac{\gamma-c}{2}$. The provider sets $c = \gamma - 2F$ and makes a profit of $\gamma - 2F$.

Next, consider the overlapping capacities equilibrium, where Firm i has $1-\alpha$ capacity and Firm j has a capacity of ${k_j}^* = \frac{(\alpha-1)c+\gamma(\alpha(\gamma-1)+1)}{2\gamma^2}$. Firm j's profit equals $\frac{((\alpha-1)c+\gamma(\alpha(\gamma-1)+1))^2}{4(1-\alpha)\gamma^2}$ and the highest c for which Firm j enters the market in this equilibrium is $c = \frac{\gamma\left(\alpha(-\gamma)+\alpha+2\sqrt{(1-\alpha)F}-1\right)}{\alpha-1}$. The profit of the provider is $c({k_j}^* + (1-\alpha))$. This profit is maximized at $c = \frac{\gamma(\alpha\gamma+\alpha-2\gamma-1)}{2(\alpha-1)}$, resulting in a profit of $\frac{(\alpha\gamma+\alpha-2\gamma-1)^2}{8(1-\alpha)}$ for the provider.

Finally, for $c < \frac{\gamma(\alpha(3\gamma-1)-2\gamma+1)}{1-\alpha}$, both firms purchase $(1-\alpha)$ capacity and earn a profit of $(\alpha-1)c + (2\alpha-1)\gamma^2 - (\alpha-1)\gamma$. The provider's profit is $2c(1-\alpha)$, where the maximum c for which both firms enter is $c = \frac{F+\gamma(-2\alpha\gamma+\alpha+\gamma-1)}{\alpha-1}$. Thus, the provider's expected profit is $2(\gamma(2\alpha\gamma-\alpha-\gamma+1)-F)$

We compare the provider's maximum profit for different γ, F, and α and present the results in Figure A6.

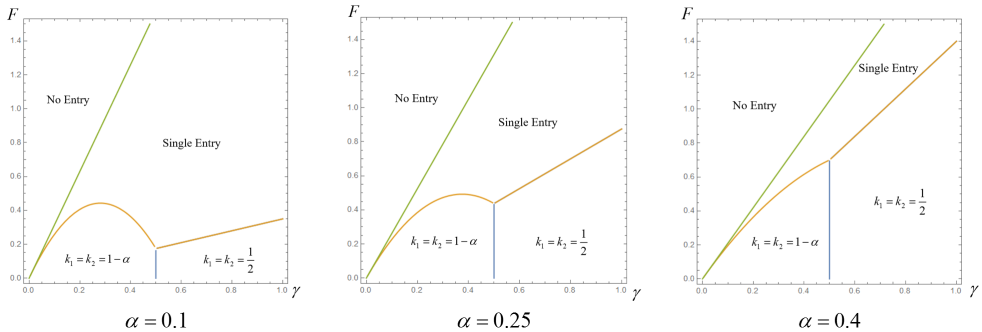

Figure A6: Entry decisions with endogenous price of capacity without autoscaling.

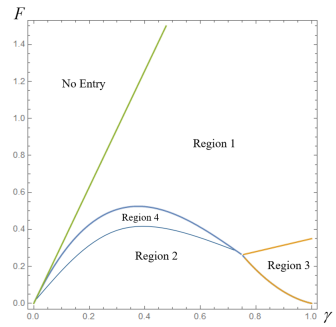

Figure A7: Entry decisions with separate prices for pre-purchased and autoscaling capacities.

Separate Endogenous Prices for Autoscaling Capacity and Fixed Capacity

We assume the provider charges c_k for a unit of fixed capacity and c_a for a unit of capacity purchased through autoscaling. This distinction between the two capacity prices only affects the provider's optimal profit when one firm uses autoscaling and the other enters the market without autoscaling. The capacity of the firm not using autoscaling is calculated in the same way as before and equals $k_{na} = \frac{-\alpha c_k + c_k + \gamma(\alpha\gamma c_a + (\alpha(-\gamma)+\alpha-1))}{2\gamma^2(c_a-1)}$. The expected capacity of the firm using autoscaling is derived as $E[k_{an}] = \frac{(k_{\text{na}}-1)k_{\text{na}}(c_a-1)(c_a(k_{\text{na}}-\alpha)+(-k_{\text{na}})+1)\left(\frac{(\alpha-1)c_a}{c_a(\alpha-k_{\text{na}})+(k_{\text{na}}-1)}+\log\left(\frac{(k_{\text{na}}-1)}{c_a(\alpha-k_{\text{na}})+(k_{\text{na}}-1)}\right)-c_a\right)}{(\alpha-1)c_a^2(k_{\text{na}}-\alpha)}$.

Thus, the expected profit of the provider when only one firm uses autoscaling becomes ${\pi_{na}}^{CP} = c_a\left((1-\alpha)(1-\gamma)\gamma + \gamma^2 E[k_{an}]\right) + c_k k_{\text{na}}$. We solve numerically for the maximum ${\pi_{na}}^{CP}$ with respect to c_k and c_a, accounting for a possible deviation from the firm using autoscaling to purchasing fixed capacity. The results are shown in Figure A7. Region 4 is where the provider sets prices such that both firms enter but only one firm uses autoscaling, resulting in autoscaling increasing the provider's profit. Regions 1-3 represent similar equilibria as in Figure A4.