Implications of Market Spillovers

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Abstract

In recent years, several firms have decided to withdraw from profitable markets, believing the move will be beneficial for their overall business. For instance, CVS dropped tobacco products from its shelves in 2014, while Aldi dropped confectionery from its checkout lines in 2016. Findings from consumers' evaluation of such moves suggest there exists a negative market spillover from selling in a market, such that a firm's participation in one market reduces consumers' willingness to pay for the firm's products in other markets. On the other hand, certain socially favorable markets, such as markets for environment-friendly products, have been shown to create a positive market spillover for their sellers, increasing consumers' willingness to pay in other markets the sellers participate in. We build an analytical model of two competing firms to examine how firms react to a market spillover and find the conditions under which different firms would sell in the spillover-producing market. Our analysis of how firms' profits are affected by market spillovers identifies the consumers' reservation value in the spillover-producing market, the relative size of the two markets, and the extent of the market spillover as key factors determining which firms benefit from a market spillover. Interestingly, we find it is possible for both firms to make more profit with a negative market spillover compared to when there is no market spillover, while a positive market spillover can actually result in lower profits for both firms compared to no market spillover.

1. Introduction

In September 2014, all CVS pharmacy locations stopped selling tobacco (Martin and Esterl 2014). While the move was estimated to cost CVS $2 billion from lost cigarette sales (Young 2014), CEO Larry Merlo stated that removing tobacco products could benefit CVS's overall business (Davis 2014). In

January 2016, Aldi announced plans to stop selling unhealthy snacks such as candy bars in their checkout lines, while Target started testing the same policy in select stores in September 2015 (Trotter 2016). Managers of these retailers and industry analysts have speculated that this move can increase profits (Harrison 2014); Senior Vice President of Merchandising for Target, Christina Hennington, viewed the move as “a huge business opportunity” (Stern 2015). This raises several interesting questions: when should a firm withdraw from a profitable market and how is this decision affected by competition among firms competing in multiple markets?

The examples of CVS, Aldi, and Target suggest that for some products, there is a negative market spillover. In other words, by selling in markets like tobacco and unhealthy foods, a firm can damage its ability to sell in other markets such as pharmaceuticals and groceries. Regarding CVS’s decision, many industry experts and analysts have also viewed this move as financially beneficial for CVS; for example, International Strategy and Investment Group has said in a message to investors “We believe the move will be viewed as a positive long-term decision” (Wahba and Steenhuysen 2014). A comparison of CVS’s revenue before and after the ban on tobacco shows a 9.7% increase in revenue as a result of higher pharmacy sales after this move (Calia 2014). Based on this study, the increase in sales of pharmaceuticals had offset the loss of cigarettes sales.[1] A Gallup poll done on consumers’ reaction to CVS’s policy helps explain the increase in revenue: The poll shows that 83% of consumers that stated this policy affected their likelihood of shopping at CVS reported they would have been less likely to shop at this pharmacy if it still sold tobacco (Dvorak and Yu 2014). The results of the consumer survey and profit analysis, coupled with opinions from industry experts, suggest the existence of a market spillover from the tobacco market to the pharmacy market, such that selling in the tobacco market reduces consumers’ willingness to pay for products in the pharmacy market.

[1] As Calia (2014) reports, CVS’ front-of-the-store sales declined 4.5% after dropping tobacco. This shows the overall increase in revenue did not come from the sales of products that replaced cigarettes on the shelves. Similarly, one year later, Egan (2015) reports that while general merchandise sales dropped 8%, prescription sales rose.

Aldi and Target are not the only retailers responding to a negative market spillover effect of selling unhealthy snacks. In Europe, major retailers such as Tesco and Lidl have removed unhealthy products from checkout lines. Tesco, UK's largest retailer, banned all sweets and chocolates from its checkouts in 2014 after a survey of its consumers showed 65% of shoppers supported the removal of confectionery (Smithers 2014). Lidl's decision came after a 10-week trial period, where some of its stores replaced candy at checkout lines. The results of Lidl's trial period suggested that despite the lost sales of confectionery, their stores could benefit from removing these products, as sweet-selling checkouts received 17% fewer customers and 70% of consumers stated they would pick a sweet-free checkout over a sweet-selling one (Poulter 2014). The evidence from the consumer survey and trial results suggest the existence of negative market spillover such that selling confectionery at checkout lines reduces consumers' willingness to pay for other products in the grocery market.

On the other hand, certain products have a positive spillover effect on other markets. For instance, in the fast-food industry, it has been shown that selling food items branded as healthy can lead to customers perceiving other items sold at the store as also healthy (Chandon and Wansink 2007). In addition, a survey of 30,000 consumers shows a majority are willing to pay extra for food they perceive as healthy (Gagliardi 2015). In such cases, selling in one market (i.e., a market for healthy food) can increase consumers' willingness to pay for products in a different market (i.e., the fast food market). Another example of positive market spillovers can be observed in the auto industry, where a brand selling certain environment-friendly or "green" models can create a positive environment-friendly consumer perception, even for its other high-emission vehicles (Reed 2007). In 2009, Chrysler entered the market for small fuel-efficient cars. While Jim Press, Chrysler President at the time, acknowledged the sales limits of the small car market, he predicted the move will help create a better environmental image for the rest of Chrysler's lineup (Snyder 2009). This suggests the existence of a positive market spillover, such that selling in a market perceived as environment-friendly can increase customers' willingness to pay for the firms' other products by enhancing the "green" image of the brand. In this research, we explore whether a positive market spillover may invite entry into previously unprofitable markets.

These examples raise a broader question on how companies should respond to positive and negative market spillovers resulting from shifts in consumer preferences toward socially responsible or healthy products. A study by Havas Media shows more than half of consumers are willing to reward firms selling responsible products by paying a 10% premium (Levick 2012). Also, with consumers becoming more and more health conscious, many are showing a willingness to avoid shopping from stores they perceive as unhealthy (Olenski 2014). This would suggest that market spillovers could arise from products beyond the examples presented so far. For instance, it can be expected that any product that conveys a firm's social responsibility can be a source of positive market spillover, by improving the firm's brand image. Similarly, other unhealthy products, such as alcohol, or socially objectionable products, such as gambling, could also result in a negative market spillover. If the trend of consumer consciousness toward social responsibility and/or heath continues, more and more firms operating in multiple markets may need to decide how to react to the emergence of market spillovers. Thus, it is important to understand the effect market spillovers can have on firms and their consumers.

This paper develops an analytical model to examine the implications of market spillovers. We define the *spillover-producing market* as the market that creates either a positive or a negative market spillover, and the *primary market* as the market affected by the spillover, typically representing the firm's main business. In the case of a negative market spillover, the firm may profitably sell the spillover-producing product (e.g. tobacco, confectionery), but at the cost of reduced willingness to pay from consumers in its primary market (e.g. pharmaceuticals, groceries). This is consistent with our findings from conjoint analyses of the effect of a firm's participation in unhealthy good markets.[2] Conversely, in the presence of a positive market spillover, the firm may participate in an unprofitable spillover-producing market (e.g.

[2] A ratings-based conjoint (Schindler 2011) with 91 (101) subjects on Amazon Mechanical Turk showed consumers would pay an average of \$4.41 (\$0.61) less for travel immunization consulting (a healthy salad) from a pharmacy (grocery store) that sold tobacco (unhealthy snacks at checkouts). The estimates were statistically significant with 95% confidence. Details of the studies are presented in section X of the appendix.

healthy food, “green” small cars) in order to increase consumers’ willingness to pay for its primary market (e.g. fast food, regular cars).

Observations from industries with market spillovers suggest that in competitive settings, firms make different choices regarding participating in spillover-producing markets. For instance, while we reported several firms which have exited spillover-producing markets due to a negative market spillover, there are also several of their competitors that continue to sell in markets that generate negative market spillovers, including pharmacies such as Rite Aid that keep selling tobacco, and grocery stores such as Safeway that keep selling confectionery at checkout lines. We aim to understand the drivers of this difference in different firms’ reaction to market spillovers in competitive markets.

We recognize that competing firms vary in terms of their quality in the primary market and as such the market spillover may not have the same effect on all firms. For instance, one firm may have greater assortment or better trained employees who offer quicker and/or more knowledgeable service. Especially for retailers, research shows the existence of considerable asymmetry among firms, evident from the emergence of “dominant retailers” (Geylani et al. 2007). We incorporate quality asymmetry in our study to see how firms with different quality react to a market spillover. We refer to the firm with the higher quality in the primary market as the *superior* firm and the firm with the lower quality as the *inferior* firm.

When the market spillover is negative, there exist opposing arguments on which firm would sell in the spillover-producing market. On the one hand, one might expect the superior firm to stay in the spillover-producing market, since this would reduce vertical differentiation in the primary market for the horizontally differentiated firms; an outcome consistent with the max-min principle of differentiation (e.g., Ansari, Economides, and Steckel 1998). On the other hand, one might expect the inferior firm to stay in the spillover-producing market, since it has less to lose in the primary market.

Similarly, when the market spillover is positive and the spillover-producing market is unprofitable, it may be expected that the inferior firm would sell in the spillover-producing market in order to close its quality gap with the superior firm. However, one may also expect the inferior firm to not enter the spillover-producing market since it has little to lose from the positive spillover going to the superior firm.

We formalize a model to resolve this issue and examine how a quality difference among firms in the primary market affects firms' profits and prices.

Specifically, this research addresses the following research questions:

1) When will it be optimal for the superior and/or inferior firm to participate in a spillover-producing market?
2) How do market spillovers affect industry profits?
3) How do the superior and the inferior quality firms' profits get affected differently by a market spillover?

To address these research questions, we develop an analytical model of competing firms who can choose whether or not to sell in a spillover-producing market. We consider firms that are both horizontally and vertically differentiated, model consumers with heterogeneous taste for products in both markets, and solve for equilibrium strategies regarding selling in the spillover-producing market.

We find that a market spillover can result in none, one, or both firms selling in the spillover-producing market depending on the market conditions. For negative market spillovers, we show that when the consumers' reservation value for the spillover-producing product and the magnitude of the negative market spillover are high enough, the superior firm will choose to withdraw from the spillover-producing market, while the inferior firm stays. Interestingly, this result shows that although the inferior firm is losing the competition in the primary market to the superior firm, it still prefers to sell in the spillover-producing market and incur the negative market spillover that further weakens its position in the primary market. Also in this case, the superior firm withdraws from a profitable market to avoid the negative market spillover in competition with an inferior firm weakened by the spillover. On the other hand, when a positive market spillover causes firms to choose different market participation strategies, it is the inferior firm that does not sell in the spillover-producing market, allowing the superior firm to enter an unprofitable market and increase its advantage in the primary market.

Regarding the strategic effects of a negative market spillover on profitability, it is not obvious how profits of different firms will be affected, since avoiding the negative market spillover comes at the cost of losing a profitable market. Also, participating in the spillover-producing market lowers consumers' willingness to pay in the firm's primary market. Thus, it may appear that the emergence of a negative market spillover should hurt firms' profits. However, we identify conditions for which the negative market spillover allows both firms to make more profit than they would have made without the market spillover due to strategic forces. Surprisingly, the inferior firm can even earn a higher profit than the superior firm due to a negative market spillover, despite having no advantages in its offerings. The model identifies the strategic mechanism behind this counterintuitive result.

Regarding a positive market spillover's effect on profitability, there can exist a tradeoff such that firms enjoy a higher willingness to pay from the primary market consumers, but this may come at a cost if the spillover-producing market is unprofitable on its own. One may expect firms to earn higher profits with a positive market spillover compared to when the unprofitable market produces no market spillover. However, we show the emergence of a positive spillover can actually reduce both firms' profits under certain market conditions. Our findings help managers assess how much a market spillover benefits or hurts their firm based on its relative quality compared to the competition.

These findings can more broadly inform decisions about umbrella branding, in which two products are sold under the same brand. In umbrella branding, a market spillover can occur when the attributes of a brand extension are associated with its parent brand, affecting the parent brand's image positively or negatively. For instance, inconsistencies between the attributes of the parent brand and the brand extension can result in the dilution of the brand image and have a negative effect on consumers' evaluation of the parent product (Loken and John 1993). On the other hand, extensions with favorable attributes, congruent with the parent brand's attributes, can enhance consumers' evaluation of the parent brand (Lane and Jacobson 1997). Therefore, selling in the market of the brand extension product could impact consumers' evaluation in the parent product market, creating a market spillover. Thus, the implications of our analysis are not limited to social responsibility or health markets, as umbrella branding of products can also be a source of market spillover.

The rest of this paper is organized in the following order. In §2, we relate our paper to the existing literature. Section 3 presents the model setup and in §4 we analyze the model to present the results. Finally, the results are discussed in §5.

2. Previous Literature

Our research considers firms selling in two distinct markets and examines their reaction to a market spillover. There are three bodies of literature most closely related. The literature on umbrella branding and brand extension studies the implications of selling different products under the same brand. Secondly, previous research has considered competition among multiproduct firms, typically offering products with some level of substitutability. Finally, previous research has also considered multimarket competition, where firms face the same competitors in separate markets. In this section, we describe how our paper contributes to each of these literatures.

First, we review the literature on brand extensions and umbrella branding. Our model considers when the selling in one market creates a spillover to another market. This effect has been shown to exist in prior behavioral research on umbrella branding. Loken and John (1993) find that brand extensions can have a negative impact on the brand name when the attributes of the brand extension are inconsistent with the attributes associated with the parent product. Keller and Sood (2003) suggest that even when extensions successfully attract new customers, they may still dilute the parent brand for its loyal users by creating inconsistent messages of what is central to the brand meaning. Lane and Jacobson (1997) study a variety of brands and show that favorably received brand extensions congruent with the parent brand can have a positive feedback effect on consumers' evaluation of the parent brand.

Analytical work on umbrella branding often focuses on a signaling effect. Wernerfelt (1988) considers a model in which consumers can infer the quality of one product by experiencing the other product under the same brand in an adverse selection model. Wernerfelt finds that only firms with two good products decide to use umbrella branding in equilibrium. Choi (1998) finds a similar separating equilibrium for infinite choices of brand extension in a repeated game. Moorthy (2012) argues that when umbrella branding costs less than a new brand, there exists a pooling equilibrium such that a brand extension does not serve as a quality signal. Hakenes and Peitz (2008) and Miklos-Thal (2008) endogenize quality and show umbrella branding remains a signal of high quality. While these previous studies have examined the signaling effect of umbrella branding and the implications of the quality of the brand extension, we are the first to examine what happens when the mere offering of a product in one market has an impact on the valuation of a firm's other product. Unlike Wernerfelt (1988) and subsequent studies of quality signaling through umbrella branding that suggest the high quality firm uses umbrella branding, we find conditions under which the superior firm retreats to only sell in one market, while it is the inferior firm that sells in both markets.

Next, we review related research on multiproduct firms. Anderson and de Palma (1992) use a nested logit model of demand to capture competition among firms over the range of products produced. Grossmann (2005) builds an oligopoly model of multi-product firms and shows that in equilibrium higher quality firms have larger product ranges. Cachon et al. (2008), Kuksov and Villas-Boas (2010), and Liu and Dukes (2013) study multiproduct firms when product evaluation is costly for consumers. Unlike most previous multi-product models, which assume substitutability among products of a firm such that one product's price and quality affects the other products' demand, our model does not require the products from the different markets to have any degree of substitutability. Our model identifies the sign of the market spillover as a key determinant of which firm has a larger product range; while a positive market spillover results in the higher quality firm offering more products than the lower quality firm, a negative market spillover results in the higher quality firm offering fewer products by withdrawing from the spillover-producing market, unlike what Grossmann (2005) suggests.

Finally, we describe related papers examining multimarket competition among firms. Multimarket competition occurs when the same firms compete against each other in more than one market (Karnani and Wernerfelt 1985). The extent of overlap between two firms is represented by their multimarket contact, defined as the aggregation of all contacts between the two firms in all markets (Gimeno and Woo 1996). Previous studies on multimarket competition have looked at diseconomies of scope as an explanation for limits in diversification. Bulow et al. (1985) consider diseconomies of scope in multimarket oligopoly such that increasing quantity sold in one market for a firm negatively affects the optimal quantity of that firm in a second market, but do not allow for market exit or entry. This effect is conceptually related to a negative market spillover. Roller and Tombak (1990) introduce the choice between two types of technology, flexible and dedicated, where a flexible technology allows the firm to produce in multiple markets instead of being dedicated to one market. Dixon (1994) allows firms to choose their diversification strategies in a setting where the marginal cost of production in one market depends on the output level in another market for flexible technologies, causing diseconomies of scope. The results show that when firms choose flexible technology, diseconomies of scope decrease their profits. However, we find that a negative market spillover can increase both firms' profits. The reason for our unique predictions is that unlike previous literature on diseconomies of scope, which assume choice of output in one market affects the other market, in a model of a negative market spillover the choice of participation in the market is what affects the primary market.

Previous literature on multimarket competition has identified mutual forbearance as a form of tacit collusion among firms involved in multimarket competition that causes firms to decrease competitive attacks against each other because they fear that an attack in one market may be countered in another market (Edwards 1955). Bernheim and Whinston (1990) offer a model of multimarket competition showing that as long as the markets and the firms are considered to be identical, mutual forbearance will not happen. However, if the firms are allowed to have competitive advantages in heterogeneous markets, collusive agreements to avoid competition can be beneficial to both firms. Subsequent research shows mutual forbearance requires observability of firm actions (Thomas and Willig 2006) and coordination mechanisms (Jayachandran et al. 1999). Cai and Raju (2016) show that multimarket competition can cause competing firms to form alliances when entering a new market to benefit from each other's investment in the new market. While most models of mutual forbearance predict dampened competition through collusive agreements between the firms, we find market spillovers can dampen competition not through agreements, but by affecting the consumers' willingness to pay for firms that participate in certain markets.

3. Model

We consider a model where two firms, A and B, may compete in a *primary market* and a *spillover-producing market*. Each firm can decide to participate in the spillover-producing market, but selling in this market results in a spillover to the primary market: When a firm operates in the spillover-producing market, each consumer's valuation of that firm's product in the primary market increases by c if the market spillover is positive, and reduces by c if the market spillover is negative. This assumption is consistent with the anecdotal evidence from the cases of CVS, Aldi's, and Chrysler. To further validate this assumption, we also ran two ratings-based conjoint studies on Amazon Mechanical Turk that showed participants had a lower willingness to pay for travel immunizations (a salad bar) at a pharmacy (grocery store) that sold tobacco (unhealthy snacks at checkout). Procedure details are presented in the appendix.

Each market is represented with a Hotelling model, with the two firms located at opposite ends of the unit lines. In the interest of parsimony, we assume sources of quality advantage can have high impact on the firm's primary market, while having no influence on its spillover-producing market. For instance, CVS is creating a competitive advantage in health service by expanding its accessible clinical services and providing unique health related loyalty programs such as the ExtraCare program (Schmalbruch 2015), which have little impact on the value of tobacco products sold at CVS. As such, we allow the two firms to offer products in the primary market with reservation values, denoted by v_A for firm A and v_B for firm B such that $v_A \geq v_B$, while they offer similar products with the reservation value of v_2 in the spillover-producing market. In the appendix, we demonstrate the results for negative market spillovers when allowing vertical differentiation in both markets.[3] The price of firm i's product in market j is p_{ij}, where $i \in \{A, B\}$ and $j \in \{1, 2\}$.

[3] Allowing superiority in both markets preserves qualitative insights derived from our more parsimonious model, provided the asymmetry in the spillover-producing market is sufficiently low. In fact, there are conditions under which the results can hold even when the superior firm's reservation value advantage is equal in both markets. The proof is presented in section II of the appendix.

In the interest of parsimony, the two markets are assumed to be independent of each other. This will be true if the markets for each product include different customers or the same customers making separate purchase decisions. Relaxing this assumption and considering dependent markets in the presence of a negative market spillover results in similar insights as those of our more parsimonious model.[4]

The size of the primary market is normalized to one and the size of the spillover-producing market is denoted by m, assuming $0 \le m \le 1$ to capture larger market sizes for the primary market relative to the spillover-producing market. The transportation cost of consumers is denoted by t and is assumed constant across markets. The distance of consumers from the location of firm A is denoted by x in the primary market and by y in the spillover-producing market. Both x and y are independently and identically distributed uniformly between 0 and 1. The utility of a consumer located at y in the spillover-producing market buying from firm i is

$$u_{i2}(y) = v_2 - p_{i2} - |y - L_i| t ,$$

where L_i represents the location of firm i such that $L_A = 0$ and $L_B = 1$.

For the primary market, the utility of a consumer located at x buying from firm i depends on whether that firm is also selling in the spillover-producing market or not. Thus we have

$$u_{i1}(x) = v_i - p_{i1} - |x - L_i| t + D_i \times H \times c ,$$

where $D_i = \begin{cases} 1 & \text{if firm } i \text{ sells in the spillover producing market} \\ 0 & \text{if firm } i \text{ does not sell in the spillover producing market} \end{cases}$ and

$$H = \begin{cases} 1 & \text{for a positive market spillover} \\ -1 & \text{for a negative market spillover} \end{cases} .$$

Each firm's objective is to maximize the sum of its profits from the two markets. We denote O as the fixed operating cost of selling in the spillover-producing market. This could be permitting fees, extra insurance, etc. and is only incurred if the firm participates in the market. The operating cost assumption is

[4] The proof is available from the authors upon request.

inconsequential for the negative market spillover results, but plays a critical role in producing novel insights in the case of positive market spillovers. The marginal cost of production is assumed zero for both firms in both markets.

The game has three stages. In stage 1, each firm decides whether to sell in the spillover-producing market. In stage 2, market participation is common knowledge and the firms simultaneously set their prices for their products in the markets in which they operate. In stage 3, consumers in each market decide from which firm to buy, maximizing their utility. Consumers in stage 3 are assumed to be fully informed about the firms' decisions in stages 1 and 2. Table 1 summarizes the notations used in the model.

Table 1. Summary of Notations.

Notation	Definition
Firm A	Superior firm
Firm B	Inferior firm
Market 1	Primary market
Market 2	Spillover-producing market
c	Magnitude of the market spillover
p_{ij}	Price chosen by firm i in market j
q_{ij}	Quantity sold by firm i in market j
π_i	Total profit of firm i
v_A	Consumer reservation value for firm A's primary product
v_B	Consumer reservation value for firm B's primary product
v_2	Consumer reservation value for spillover-producing product
O	Fixed operating cost in the spillover-producing market
m	Size of the spillover-producing market
t	Consumers' per unit transportation cost
x	Consumer's location in the primary market
y	Consumer's location in the spillover-producing market

4. Analysis

In this section, we present the results of the model. We begin by analyzing the effects of a negative market spillover in section 4.1. We find the equilibrium strategies chosen by asymmetric firms in the presence of a negative market spillover and compare firm profitability in the presence of a negative market spillover to firm profitability in the absence of market spillovers. In section 4.2, we study positive market spillovers and how they affect participation strategies and firms' profits. In section 4.3, we analyze a model where some consumers receive a positive market spillover and some receive a negative market spillover. Finally, in section 4.4, we explore an alternative approach to modeling market spillovers in which the market spillover affects the quantity of primary products purchased by each consumer, instead of affecting reservation values.

4.1. The Effects of a Negative Market Spillover

In this section, we study how firms with different levels of quality respond to a negative market spillover. Before a negative market spillover emerges, both firms sell in both profitable markets. We aim to find which of the two firms, if any, stays in the spillover-producing market with the emergence of a negative market spillover.

Note that if the fixed operating cost of the spillover-producing market, O, is high such that this market becomes unprofitable, neither firm has incentive to sell in this market and incur a negative market spillover. Thus, we present our analysis for a spillover-producing market with low fixed operating costs such that selling the spillover-producing product is a plausible strategy for some parameter regions. In order to simplify the analysis, we normalize O to 0 in this section. Since Market 2 is assumed profitable, both firms sell in this market in the absence of a negative market spillover. We solve for the firms' optimal decision on exiting or staying in the spillover-producing market.

This model has four subgames: (1) both firms sell the spillover-producing product (2) only the superior firm (firm A) sells the spillover-producing product, (3) only the inferior firm (firm B) sells the spillover-producing product, and (4) neither firm sells the spillover-producing product. We examine each of them below.

Both firms sell the spillover-producing product. The marginal consumer in the spillover-producing market, who is indifferent between buying from each of the two firms, is located at y^* such that $v_2 - y^* t - p_{A2} = v_2 - (1-y^*)t - p_{B2}$. Similarly in the primary market the marginal consumer's location can be found at x^* such that $v_A - x^* t - p_{A1} - c = v_B - (1-x^*)t - p_{B1} - c$.

$$\begin{aligned} y^* &= (p_{B2} - p_{A2} + t)/2t \\ x^* &= (v_A - v_B + p_{B1} - p_{A1} + t)/2t \end{aligned} \tag{1}$$

Given the size of the spillover-producing market is denoted by *m*, firm A maximizes the sum of its profits, $x^* p_{A1} + m y^* p_{A2}$, with respect to p_{A1} and p_{A2}, while firm B maximizes its profit, $(1 - x^*) p_{B1} + m(1 - y^*) p_{B2}$, with respect to p_{B1} and p_{B2}, where x^* and y^* are defined in equation (1).

Solving the first order conditions, we find the two firms set equal prices in the spillover-producing market and equally share that market; $p_{A2}^{SS} = p_{B2}^{SS} = t$, where the superscript *SS* denotes the subgame when both firms sell the spillover-producing product. These prices will cover the whole spillover-producing market if $v_2 > 3t/2$. Otherwise, for lower v_2, the marginal consumer gets negative utility at these prices and the corresponding equilibrium prices and profits are presented in the appendix. In the primary market, we assume the quality difference to be small enough such that there could still be a portion of consumers in the primary market who prefer to buy from the inferior firm; $v_A - v_B \le 3t$. Thus, firm A chooses $p_{A1}^{SS} = (v_A - v_B)/3 + t$, and firm B chooses $p_{B1}^{SS} = (v_B - v_A)/3 + t$. The equilibrium profits in this subgame are

$$\begin{aligned} \pi_A^{SS} &= ((v_A - v_B)/3 + t)^2 / 2t + mt/2 \\ \pi_B^{SS} &= ((v_B - v_A)/3 + t)^2 / 2t + mt/2 \end{aligned} \tag{2}$$

Neither firm sells the spillover-producing product. Firm A maximizes $x^* p_{A1}$ with respect to p_{A1} and firm B maximizes $(1 - x^*) p_{B1}$ with respect to p_{B1}, where x^* is as defined in equation (1). Subgame equilibrium prices are $p_{A1}^{NN} = (v_A - v_B)/3 + t$ and $p_{B1}^{NN} = (v_B - v_A)/3 + t$, where the *NN* superscript denotes both firms not selling the spillover-producing product. Subgame equilibrium profits are

$$\begin{aligned} \pi_A^{NN} &= ((v_A - v_B)/3 + t)^2 / 2t \\ \pi_B^{NN} &= ((v_B - v_A)/3 + t)^2 / 2t \end{aligned} \tag{3}$$

Only firm B sells the spillover-producing product.[5] Since the market demands are assumed independent of one another, we may separately analyze firm B's pricing decision in the spillover-producing market from the primary market. Consumers for whom $v_2 - (1 - y^*)t - p_{B2} \geq 0$ will buy the spillover-producing product from firm B and the remaining consumers in the spillover-producing market will abstain from purchase. Thus, firm B chooses p_{B2} to maximize $m(1 - y^*)p_{B2}$ subject to the constraint that $y^* \leq 1$ where $y^* = 1 - (v_2 - p_{B2})/t$. Solving the KKT conditions, firm B will choose $p_{B2}^{NS} = v_2/2$ if $v_2 < 2t$, and $p_{B2}^{NS} = v_2 - t$ otherwise. The latter implies that the spillover-producing market is fully covered. In the interest of parsimony, we assume high enough transportation costs such that one firm alone cannot sell to the whole spillover-producing market, and assume $v_2 < 2t$ from here on.

In the primary market, the marginal consumer's location can be found at x^* such that $v_A - x^* t - p_{A1} = v_B - (1 - x^*)t - p_{B1} - c$.

$$x^* = (v_A - v_B + p_{B1} - p_{A1} + c + t)/2t \tag{4}$$

Using equation (4), firm A chooses p_{A1} to maximize $x^* p_{A1}$, while firm B chooses p_{B1} to maximize $(1 - x^*)p_{B1}$. Taking the first order conditions we find that for $c \leq 3t - (v_A - v_B)$ there is an interior solution and both firms still sell in the primary market, but for $c > 3t - (v_A - v_B)$ a corner solution is reached and firm B effectively exits the primary market. For $c \leq 3t - (v_A - v_B)$, the primary market prices of the firms are $p_{A1}^{NSlowc} = (v_A - v_B + c)/3 + t$ and $p_{B1}^{NSlowc} = (v_B - v_A - c)/3 + t$. The corresponding total profits are

$$\begin{aligned} \pi_A^{NSlowc} &= ((v_A - v_B + c)/3 + t)^2 / 2t \\ \pi_B^{NSlowc} &= ((v_B - v_A - c)/3 + t)^2 / 2t + m v_2^{\,2} / 4t \end{aligned} \tag{5}$$

[5] We implicitly assume an entry cost, *F*, associated with setting up a retail environment. For a medium F, a new firm will not choose to enter the spillover-producing market after firm exit (i.e., $F > \hat{\pi}$, where $\hat{\pi} = mt/2$ for $v_2 > 3t/2$, $\hat{\pi} = v_2 - t/2$ for $t < v_2 < 3t/2$, and $\hat{\pi} = v_2^{\,2}/4t$ for $v_2 < t$), but the two firms would have chosen to enter initially before the emergence of the market spillover (i.e., $F < ((v_B - v_A)/3 + t)^2 / 2t + \hat{\pi}$).

For $c > 3t - (v_A - v_B)$, firm A sells exclusively to the primary market, setting the price at $p_{A1}^{NShighc} = v_A - v_B + c - t$. The corresponding total profits are

$$\begin{aligned} \pi_A^{NShighc} &= v_A - v_B + c - t \\ \pi_B^{NShighc} &= m{v_2}^2 / 4t \end{aligned} \tag{6}$$

Only firm A sells the spillover-producing product. We may again examine these two markets independently. As before, the only firm selling the spillover-producing product (firm A in this case) will choose $p_{A2}^{SN} = v_2 / 2$. In the primary market, the marginal consumer's location can be found at x^* such that $v_A - x^* t - p_{A1} - c = v_B - (1 - x^*)t - p_{B1}$.

Similar to the findings for the previous subgame, for $c \le 3t + (v_A - v_B)$ there exists an interior solution and both firms still sell in the primary market, but for $c > 3t + (v_A - v_B)$ there is a corner solution in which firm A exits the primary market. For $c \le 3t + (v_A - v_B)$, the primary market prices are $p_{A1}^{SNlowc} = (v_A - v_B - c)/3 + t$ and $p_{B1}^{SNlowc} = (v_B - v_A + c)/3 + t$. The corresponding total profits are

$$\begin{aligned} \pi_A^{SNlowc} &= ((v_A - v_B - c)/3 + t)^2 / 2t + m{v_2}^2 / 4t \\ \pi_B^{SNlowc} &= ((v_B - v_A + c)/3 + t)^2 / 2t \end{aligned} \tag{7}$$

For $c > 3t + (v_A - v_B)$, firm B sells exclusively to the primary market at $p_{B1} = v_B - v_A + c - t$ and the corresponding total profits are as follows.

$$\begin{aligned} \pi_A^{SNhighc} &= m{v_2}^2 / 4t \\ \pi_B^{SNhighc} &= v_B - v_A + c - t \end{aligned} \tag{8}$$

Examining the profits of the two firms in each subgame solution (see equations (2)-(8)), we solve the game and find the equilibrium spillover-producing market participation of each firm. The proofs of all lemmas and propositions are presented in the appendix.

LEMMA 1. *A negative market spillover affects spillover-producing market participation as follows:*

(a) *When the market spillover is sufficiently large, it reduces participation of firms in the spillover-producing market (i.e., $\exists c'$ s.t. $c > c' \Rightarrow$ {sell, sell} is not an equilibrium).*

(b) *When {sell, sell} is not an equilibrium, both firms leave the spillover-producing market if the consumers' reservation value for the spillover-producing product is low (i.e., $v_2 < v_2'$), otherwise one firm leaves the spillover-producing market.*[6]

This lemma confirms the intuition that the negative market spillover can stop the spillover-producing market from being served, only if the reservation value consumers obtain from the spillover-producing product (i.e., v_2) is sufficiently small. Otherwise, if v_2 is large enough, having monopoly power over the spillover-producing market is attractive enough that no negative market spillover can entirely stop the spillover-producing market from being served.

Now, we examine which of the two firms would choose to leave the market in the asymmetric equilibrium: the inferior firm or the superior firm. When both asymmetric equilibria exist, we use the risk-dominance equilibrium refinement to find the risk-dominant equilibrium. Risk dominance refinement is a mechanism for equilibrium selection introduced by Harsanyi and Selten (1988). This equilibrium selection is based on minimizing losses from the other player's deviation. As Straub (1995) shows, this theory successfully predicts the outcome of different types of games with multiple equilibria.

PROPOSITION 1. *Suppose the negative market spillover and consumers' reservation value for the spillover-producing product are sufficiently high to result in asymmetric spillover-producing market participation.*

(a) *It is a unique equilibrium for only the inferior firm to stay in the spillover-producing market if c and v_2 are not too high, but it is never a unique equilibrium for only the superior firm to stay in the spillover-producing market (i.e., $\exists c''$ and v_2'' s.t. for $c' < c < c''$ and $v_2' < v_2 < v_2''$, {not sell, sell} is the unique equilibrium).* [7]

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[6] The expressions for c' and v_2' are defined in section I of the appendix.

[7] The expressions for c'' and v_2'' are defined in section II of the appendix, where we prove $c' < c''$ and $v_2' < v_2''$.

(b) *The risk-dominant equilibrium involves only the inferior firm staying in the spillover-producing market when both asymmetric equilibria exist (i.e., only the inferior firm stays in the spillover-producing market in the risk-dominant equilibrium if* $c > c'$ *and* $v_2 > v_2'$*).*

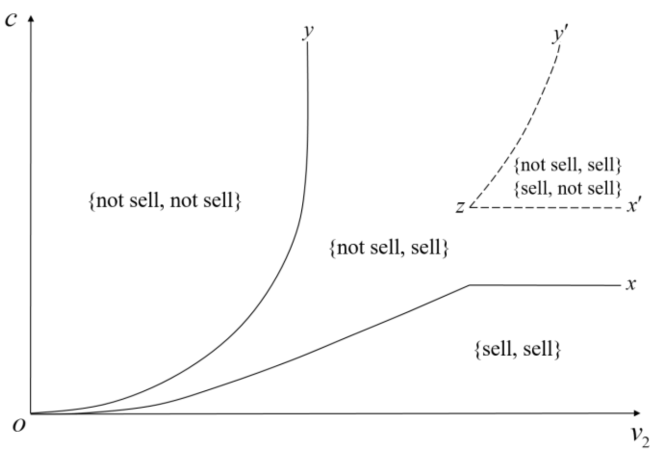
Proposition 1 provides insight into which firm will react to the market spillover by withdrawing from the spillover-producing market. For moderate market spillovers and moderate reservation values in the spillover-producing market, the unique equilibrium calls for only the inferior firm staying in the spillover-producing market, with the superior firm withdrawing. For larger market spillovers and reservation values, either asymmetric equilibrium is possible, but only the inferior firm staying in the spillover-producing market is the risk-dominant equilibrium.

The intuition for why the inferior firm will be the only firm selling in the spillover-producing market is that the inferior firm has less profit to lose in the primary market from the negative market spillover. If the potential profit of the spillover-producing market is high enough, it exceeds the loss incurred in the primary market. The superior firm experiences higher losses in the primary market from the negative market spillover and thus is more inclined to exit the spillover-producing market. This amplifies the inferior firm's incentive to stay in the spillover-producing market and gain monopoly power over this market.

Proposition 1 shows that for horizontally differentiated firms, equilibrium choices of market participation result in a reduction of the inferior firm's value in the primary market, thus increasing the vertical differentiation. This finding is the opposite of what prior literature on differentiation along two dimensions with no market spillover would predict. Ferreira and Thisse (1996) and Ansari, Economides, and Steckel (1998) show that in a single market, firms with maximum horizontal differentiation make quality choices to minimize vertical differentiation. With a negative market spillover, the level of vertical differentiation in the primary market is tied to their presence in the spillover-producing market. Thus, the inferior firm's incentive to decrease vertical differentiation in the primary market becomes dominated by the incentive to profit from the spillover-producing market.

The predictions of the model are depicted in Figure 1. The figure illustrates when both firms will opt to sell in the spillover-producing market, when both firms will opt not to sell in the spillover-producing market, and when only one firm will stay in the spillover-producing market. In the area northwest of the oy curve, there exists an equilibrium where neither firm sells in the spillover-producing market. In the area constrained below the ox curve, both firms sell in the spillover-producing market. The area northeast of yox, representing high enough market spillovers and reservation values for the spillover-producing product, is where the equilibrium with only the inferior firm selling the spillover-producing product exists. Finally, in the area northeast of $y'zx'$ there also exists an equilibrium where the superior firm is the only one staying in the spillover-producing market. This last equilibrium is never unique, and is risk-dominated by the equilibrium with the inferior firm staying in the spillover-producing market. Figure 1 is generated for $m=1$, which means the two markets have equal size. As m decreases and the spillover-producing market becomes smaller relative to the primary market, the ox curve shifts down and the oy curve shifts to the right, expanding the region where neither firm sells in the spillover-producing market and shrinking the region where both firms sell in the spillover-producing market.

Figure 1. Equilibrium Strategies of {Superior, Inferior} Firms Selling the Product Generating a Negative Spillover (WLOG, parameter values $v_A - v_B = 1$, $m=1$, $O=0$, and $t=1$ used for generating figure)



As we summarized the firms' equilibrium response to a negative market spillover, our next question is how the negative market spillover will affect firm profitability.

Firm Profitability under a Negative Market Spillover

In this section we study how the negative market spillover affects firm profitability. This analysis provides insights on whether or not firms and the industry as a whole are hurt by the emergence of a negative market spillover. We start by comparing equilibrium profits made by the firms under a negative market spillover with the profits they would have made in the absence of any market spillover. In the main text, we focus on the interior solutions in which both firms still sell in the primary market in the {not sell, sell} equilibrium and their profits are represented by equation 5. The conditions for the corner solutions in which the inferior firm exits the primary market in the {not sell, sell} equilibrium, with profits represented by equation 6, provide similar insights and are presented in the appendix.

PROPOSITION 2. *Suppose $c > c'$ such that the negative market spillover affects market participation.*

(a) *If the consumers' reservation value for the spillover-producing product and the size of the spillover-producing market are sufficiently high (i.e., $v_2 > \hat{v}_2$ and $m > \hat{m}$), both firms make more profit in the presence of a negative market spillover compared to when there is no market spillover.*[8]

(b) *When only the inferior firm participates in the spillover-producing market, if the consumers' reservation value for the spillover-producing product or the size of the spillover-producing market is sufficiently low, (i.e., $v_2' < v_2 < \hat{v}_2$ or $m < \hat{m}$), the inferior firm makes less profit (and the superior firm makes greater profit) in the presence of a negative market spillover compared to when there is no market spillover.*

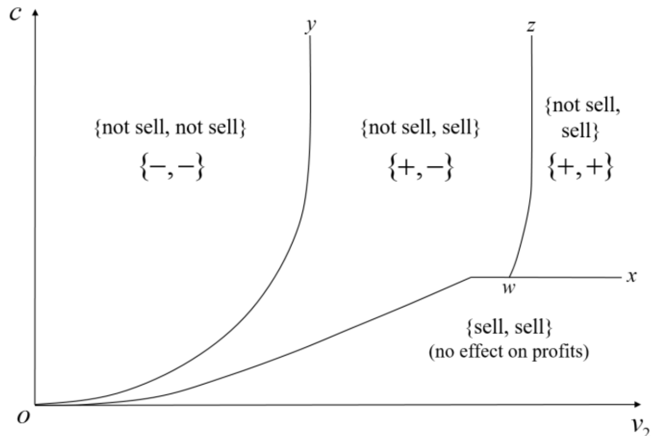
(c) *If neither firm sells in the spillover-producing market (i.e., $v_2 < v_2'$), both firms make less profit in the presence of a negative market spillover compared to when there is no market spillover.*

This proposition shows conditions such that each firm could make more or less profit as a result of the negative market spillover. Consumers' reservation value for the spillover-producing product and the

[8] The expressions for $\hat{v}_2$ and $\hat{m}$ are presented in section III of the appendix.

size of the spillover-producing market are critical factors in determining how the profits of the firms will be affected by the negative market spillover. Figure 2 demonstrates the findings of Proposition 2. The area northeast of the *zwx* curve shows the region described in Proposition 2(a), in which both firms benefit from the emergence of a negative market spillover. The area bounded by the *yowz* curve shows the region for Proposition 2(b), in which only the superior firm benefits from a negative market spillover. Finally, the area northwest of the *oy* curve shows the region described in Proposition 2(c), in which both firms become worse off with the negative market spillover.

Figure 2. The Effect of a Negative Market Spillover on Equilibrium Profits for the {Superior, Inferior} Firms (WLOG, parameter values $v_A - v_B = 1$, $m = 1$, $O = 0$, and $t = 1$ used for generating figure)



Proposition 2 shows the negative market spillover can function as a competition dampening mechanism, resulting in firms becoming better off as a result of its emergence. Interestingly, in the {not sell, sell} equilibrium, the superior firm always earns greater profit than without a negative market spillover even though it loses sales in the spillover-producing market as it exits that market. To help clarify the intuition behind this finding, Table 2 compares firms' prices and demands with a negative market spillover to when no market spillover exists. In the interest of parsimony, the values in the table are derived for $3t/2 < v_2 < 2t$ and $c < 3t - (v_A - v_B)$, though the qualitative results hold for other cases in which the {not sell, sell} equilibrium arises. We also define $K \equiv v_A - v_B$.

Table 2. Price and Demand with a Negative Market Spillover for the {not sell, sell} Equilibrium and without Market Spillover

	Without Market Spillover	With Market Spillover ({not sell, sell} equilibrium)	Change Due to the Negative Market Spillover
Firm A's Price	$p_{A1} = t + K/3$ $p_{A2} = t$	$p_{A1} = t + (K + c)/3$	$\Delta p_{A1} = c/3 > 0$
Firm B's Prices	$p_{B1} = t - K/3$ $p_{B2} = t$	$p_{B1} = t - (K + c)/3$ $p_{B2} = v_2/2$	$\Delta p_{B1} = -c/3 < 0$ $\Delta p_{B2} = v_2/2 - t < 0$
Firm A's Demand	$q_{A1} = 1/2 + K/(6t)$ $q_{A2} = 1/2$	$q_{A1} = (K + c + 3t)/(6t)$ $q_{A2} = 0$	$\Delta q_{A1} = c/(6t) > 0$ $\Delta q_{A2} = -1/2 < 0$
Firm B's Demand	$q_{B1} = 1/2 - K/(6t)$ $q_{B2} = 1/2$	$q_{B1} = (3t - K - c)/(6t)$ $q_{B2} = v_2/(2t)$	$\Delta q_{B1} = -c/(6t) < 0$ $\Delta q_{B2} = (v_2 - t)/(2t) > 0$

Due to the inferior firm's participation in the spillover-producing market, consumers in the primary market have a lower valuation of the inferior firm's primary product and the superior firm consequently gains an even greater advantage in the primary market. Thus, the superior firm's share of the primary market and its price in that market increases in comparison to when there was no negative market spillover. The resulting increase in primary market profit outweighs the loss associated with exiting the spillover-producing market.

Surprisingly, the inferior firm can also earn greater profit as a result of the market spillover when v_2 and m are high enough, even though it incurs a reservation value reduction in its primary market due to the spillover. Using the values in Table 2, the intuition is as follows. The inferior firm gets the spillover-producing market to itself due to the competitor choosing to exit this market as response to the negative spillover. Although the inferior firm's share of the primary market has decreased, this firm can now gain a bigger share of the spillover-producing market. Earning monopoly profit in the spillover-producing market can offset the diminished profitability in the primary market, but only if the consumers' reservation value in the spillover-producing market and the size of this market are high enough.[9]

[9] Note that assuming vertically differentiated firms is not necessary for proposition 2. In the appendix, we show when firms are symmetric such that $v_A = v_B$, it is possible for both firms to make more profit in the presence of a negative market spillover compared to when there is no market spillover, for high enough c and v_2.

Finally, Proposition 2 shows when v_2 is low enough such that both firms exit the spillover-producing market, both firms make less profit as a result of the market spillover. This result was expected, as the negative market spillover causes both firms to lose a profitable spillover-producing market while gaining no additional advantage over the competition in the primary market.

The fact that the negative market spillover can increase the profits of all firms in the industry can be considered the opposite of what would be intuitively expected from a negative spillover effect. For example, in the case of unhealthy good markets, though consumers may avoid buying primary products from sellers of unhealthy goods with the possible intention of punishing them, this avoidance can have the reverse effect and increase the profit of the unhealthy good seller. On the other hand, the firm that exits the unhealthy good market can also get rewarded with more profit despite entirely losing a market.

Comparing the sum of the two firms' profits with and without a negative market spillover, we find the effect of a negative market spillover on total industry profits. When only one firm sells in the spillover-producing market and the negative market spillover is high enough (i.e., $c > c_T$, where c_T is defined in the appendix), the market spillover benefits the industry in total. This is because a high negative market spillover strongly dampens the competition between the two firms, such that even when the inferior firm's profit decreases in the {not sell, sell} equilibrium as a result of the market spillover, the superior firm's profit in the primary market increases by an even larger margin due to lowered competition, resulting in an increase in the total industry profits.

Next, we compare the profits of the two firms in the existence of the negative market spillover to find which of the two firms can benefit more from a negative market spillover.

PROPOSITION 3. *When only the inferior firm stays in the spillover-producing market, for a low negative market spillover (i.e., $c < 3mt/2 - (v_A - v_B)$) and a high reservation value for the spillover-producing product (i.e., $v_2 > 2\sqrt{2t/(3m)}\sqrt{c + (v_A - v_B)}$), the inferior firm's profit is higher than the superior firm's.*

This result shows the firm that is inferior in the primary market may actually earn greater profit than the superior firm in the {not sell, sell} equilibrium. Note that the conditions for Proposition 3 are mutually satisfied if $v_A - v_B$ is not too great.[10]

Although the negative market spillover imposes identical penalties on both firms and both firms face the same set of choices, the inferior firm benefits more from the market spillover even in cases where it is the only firm directly penalized by it. This effect takes place in the equilibrium where only the inferior firm stays in the spillover-producing market. This result is especially interesting because it holds for high v_2, meaning even when the spillover-producing market becomes highly lucrative for the inferior firm, the superior firm may still prefer to exit that market, leaving all the profits to the inferior firm. There are two interesting implications of this finding. First, if quality decisions are fixed prior to the emergence of the market spillover, a negative market spillover can actually reverse the advantage held by the superior firm. Second, if future negative market spillovers are predicted prior to quality investment decisions, it may cause diminished quality investment in the primary market since a quality advantage will actually reduce profitability by diminishing the marginal incentive to participate in a lucrative spillover-producing market.

To understand this result, first consider the market outcome. In equilibrium, the firm that is inferior in the primary market loses a competitor in the spillover-producing market, while it may or may not stay in the primary market depending on the magnitude of the negative market spillover. Though this turns out to be quite lucrative for the inferior firm, this strategy cannot be profitably replicated by the superior firm. Foremost, since the superior firm serves a greater number of consumers in the primary market, it has much to lose in the primary market by staying in the spillover-producing market. Secondly, the superior firm, on the margin, has less to gain from staying in the spillover-producing market because it can at best

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[10] As shown in section IV of the appendix, $c < 3mt/2 - (v_A - v_B)$ holds in the region for the {not sell, sell} equilibrium if $v_A - v_B < 3(\sqrt{4+m} - 2)t$. This makes intuitive sense, as if v_A was much bigger than v_B, then firm A's profit would always be higher than firm B's.

share the market with its competitor, which makes deviation to selling the spillover-producing product not profitable for the superior firm even for high v_2. Thus, the superior firm's advantage in the primary market actually serves as a disadvantage with a negative market spillover because, on the margin, it prevents the firm from participating in the profitable spillover-producing market.

Note that in our analysis, we assume the firms' reservation values to be fixed. One might conjecture that when the superior firm is making less profit than the inferior firm, it would want to lower its quality to become the inferior firm. However, even allowing firm A to costlessly reduce v_A, it would need to reduce it enough to not only become inferior to v_B, but inferior enough to incentivize firm B to leave the spillover-producing market. For $v_B < 9mt^2/(2c) - c/2 - 3t$, it is not possible for firm A to lower v_A enough to regain a profit advantage over firm B.

Next, we study positive market spillovers and find the effect of their emergence on firms' market participation strategies and profitability.

4.2. The Effects of a Positive Market Spillover

In this section, we study positive market spillovers and how they affect firms' decision to enter an unprofitable spillover-producing market. If the operating cost in the spillover-producing market, *O*, is sufficiently low, the positive market spillover intuitively encourages continued market participation and boosts profitability. Our interest is in whether a positive market spillover can attract firms into markets previously viewed as unprofitable. To this end, we consider sufficiently large operating costs, (e.g., $O > mv_2^2/4t$) such that both firms sell only in the primary market in the absence of a positive market spillover. Our aim is to find which of the two firms, if any, participates in the spillover-producing market, after the emergence of a positive market spillover.

In the interest of parsimony, we assume the quality difference between the firms to be small enough such that even when the superior firm benefits from a positive market spillover, a portion of consumers in the primary market still prefer to buy from the inferior firm; $v_A - v_B \leq 3t - c$.

Similar to our analysis of the negative market spillover, we compare the firms' profits from the four subgames representing firms' decisions of participation in the spillover-producing market. We then find the conditions for each firm's equilibrium market participation strategy, depending on O, c, and v_2. Analysis is presented in the appendix.

LEMMA 2. *Suppose the fixed operating cost of the spillover-producing market is high enough (i.e.,* $O > mv_2^2 / 4t$*). The effect of a positive market spillover on market participation is as follows:*

(a) *When the market spillover and the consumers' reservation value for the spillover-producing product are sufficiently high (i.e.,* $c > c'_+$ *and* $v_2 > \tilde{v}$*), both firms enter the spillover-producing market.*

(b) *When {sell, sell} is not an equilibrium, neither firm enters the spillover-producing market if the consumers' reservation value for the spillover-producing product is low (i.e.,* $v_2 < \dot{v}_2$*), otherwise one firm enters the spillover-producing market.*[11]

This lemma shows a high enough positive market spillover can result in firms choosing to incur a loss in the spillover-producing market in order to increase their value in the primary market. As expected, when the consumers' reservation value for the spillover-producing product is low enough, the spillover-producing market is highly unprofitable even for a monopoly firm. Thus, both firms forgo the benefits of the positive spillover to avoid losses in the unprofitable spillover-producing market.

Next, we find which of the two firms enters the spillover-producing market in the region where only one firm sells the spillover-producing product.

PROPOSITION 4. *Suppose* $O > mv_2^2 / 4t$ *and firms choose asymmetric spillover-producing market participation strategies.*

(a) *It is a unique equilibrium for only the superior firm to enter the spillover-producing market unless c is too low and* v_2 *is too high, but it is never a unique equilibrium for only the inferior*

[11] The expressions for c'_+ and $\dot{v}_2$ are defined in the appendix.

firm to enter the spillover-producing market (i.e., $\exists c''_+$ *and* $\ddot{v}_2$ *s.t. {sell, not sell} is the unique equilibrium for* $c''_+ < c < c'_+$ *and* $\dot{v}_2 < v_2 < \ddot{v}_2$*,).* [12]

(b) *The risk-dominant equilibrium involves only the superior firm entering the spillover-producing market when both asymmetric equilibria exist.*

Proposition 4 shows that when the market spillover is positive, it is the superior firm that will be the only firm selling in the spillover-producing market. This is the opposite of the asymmetric equilibrium with a negative market spillover, where the inferior firm was the one selling in the spillover-producing market. The reason for this finding is that the inferior firm has less profit to lose in the primary market due to its quality disadvantage. Thus, the inferior firm's savings from not entering the unprofitable spillover-producing market exceed its loss in the primary market due to the superior firm's additional advantage from the positive spillover. The inferior firm's absence in the spillover-producing market reduces the superior firm's profit loss in this market. This results in an overall profit gain for the superior firm from incurring a loss in the spillover-producing market and benefiting from the positive spillover in the primary market.[13]

The findings from Proposition 4 are depicted in Figure 3.[14] The rs curve represents $v_2 = \dot{v}_2$ and the ywx curve represents $c = c'_+$. This figure is generated for a fixed operating cost of the spillover-producing market; $O = 0.6$. As O increases, the spillover-producing market becomes less attractive to firms and the rs and ywx curves shift upwards, expanding the region where neither firm sells in the spillover-producing market.

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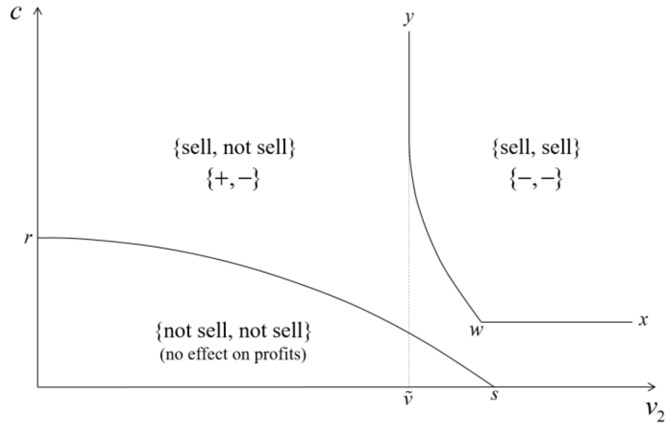
[12] The expressions for c''_+ and $\ddot{v}_2$ are defined in the appendix, where we prove $c''_+ < c'_+$ and $\dot{v}_2 < \ddot{v}_2$.

[13] Note that assuming the superior firm has a higher reservation value in both markets only strengthens the result that the {sell, not sell} equilibrium occurs, since there would be more incentive for the superior firm to enter the spillover-producing market under this assumption.

[14] Note that Figure 3 only shows the risk-dominant equilibrium strategy (i.e., the {sell, not sell} equilibrium) in regions where both {sell, not sell} and {not sell, sell} equilibria exist.

Figure 3. Risk-Dominant Equilibrium Strategies of {Superior, Inferior} Firms Selling the Product Generating a Positive Spillover (WLOG, parameter values $v_A - v_B = 1$, $O = 0.6$, $m = 1$, and $t = 1$ used for generating figure)



Next, we examine how a positive market spillover affects firm profitability.

Firm Profitability under a Positive Market Spillover

We compare firms' profits under a positive market spillover with the profits in the absence of any market spillover. Without a market spillover, neither firm sells in the spillover-producing market when the fixed operating cost in the spillover-producing market is high enough (i.e., $O > mv_2^{\,2} / 4t$).

PROPOSITION 5. *Suppose* $v_2 > \dot{v}_2$ *and* $O > mv_2^{\,2} / 4t$*, such that the positive market spillover increases market participation.*

(a) *If the positive market spillover and the consumers' reservation value for the spillover-producing product are sufficiently high such that both firms sell in the spillover-producing market (i.e.,* $c > c'_+$ *and* $v_2 > \tilde{v}$*), both firms make less profit in the presence of a positive market spillover compared to when there is no market spillover.*

(b) *If the positive market spillover or the consumers' reservation value for the spillover-producing product are low (i.e.,* $c < c'_+$ *or* $v_2 < \tilde{v}$*) such that only the superior firm sells in the spillover-producing market, the superior (inferior) firm makes more (less) profit in the presence of a positive market spillover compared to when there is no market spillover.*

Interestingly, this proposition shows a positive market spillover can decrease both firms' profits, by preventing them from avoiding an unprofitable market. Without a positive market spillover, both firms would have sold only in the primary market for high enough O, avoiding a profit loss in the other market. However, with a high enough positive market spillover, not entering the spillover-producing market creates an opportunity for the competitor to gain a large advantage in the primary market by selling the spillover-producing product and benefitting from the positive spillover. As a result, both firms enter the spillover-producing market, incurring a loss in this market, while neither firm gains an advantage over the competitor in the primary market from the positive spillover. Thus, although the positive market spillover increases a firm's value in the primary market, its introduction can actually make both firms worse off.

Proposition 5 also shows that in the {sell, not sell} equilibrium, the positive market spillover increases the superior firm's profit and decreases the inferior firm's profit. In this equilibrium, the positive market spillover magnifies the firms' differences in the primary market, putting the inferior firm at an even bigger disadvantage in the primary market. The effect of a positive market spillover on each firm's profit in each equilibrium is shown in Figure 3.

Next, we consider the effect of a positive market spillover on total industry profits. When only the superior firm is selling in the spillover-producing market and the magnitude of the positive market spillover is high (i.e., $c > c_T'$, where c_T' is defined in the appendix), the gap between the consumers' valuation for the two firms in the primary market expands due to the positive spillover, resulting in less competition in this market. Thus, while the inferior firm becomes worse off as a result of the market spillover, the increase in the superior firm's profit more than makes up for this loss, resulting in higher overall industry profits.

In sections 4.1 and 4.2, we assumed a fixed sign of the market spillover for all primary market consumers. In the following section, we relax this assumption and consider a model in which the sign of the market spillover may vary across consumers in the primary market.

4.3. A Model of Consumer Heterogeneity in the Sign of the Market Spillover

So far, we have assumed all consumers in the primary market are affected similarly by a market spillover. In particular, we assumed as a result of a firm's participation in the spillover-producing market, all consumers' willingness to pay for the firm's primary product either decreases by c (for a negative market spillover) or increases by c (for a positive market spillover). In this section, we allow for positive and negative market spillovers to exist simultaneously in the primary market. This model accounts for cases where consumers vary in how they perceive the firm's participation in the spillover-producing market; some consumers view this participation positively and receive a positive spillover while the rest view it negatively and receive a negative spillover.

We model a combination of positive and negative market spillovers by assuming an α portion of consumers in the primary market receive a positive spillover and the rest of the $1-\alpha$ consumers receive a negative spillover. Thus, when firm *i* participates in the spillover-producing market, willingness to pay for firm *i*'s primary product increases by c_{pos} for α consumers and decreases by c_{neg} for the rest of the primary market consumers.

In the interest of parsimony, we assume the quality difference between the firms is small enough such that each firm sells to a portion of the primary market regardless of the participation strategies in the spillover-producing market; $v_A - v_B \leq 3t - Max\{c_{pos}, c_{neg}\}$.

Each firm's profit in the primary market is a weighted average of the firm's profits from the consumers with a positive spillover and the consumers with a negative spillover. We find the firms' equilibrium participation strategies regarding the spillover-producing market as a function of the relative magnitude of the positive and negative spillovers and the relative sizes of the primary market consumer segments receiving each type of market spillovers.

Lemma 3. *Suppose the consumers in the primary market can receive either a positive or a negative market spillover.*

(a) *For high fixed operating costs in the spillover-producing market (i.e.,* $O > O_h$*), neither firm sells the spillover-producing product.*

(b) *For medium fixed operating costs in the spillover-producing market (i.e.,* $O_l < O < O_h$*), when the proportion of the consumers receiving a positive spillover is low enough (i.e.,* $\alpha < \dfrac{c_{neg}}{c_{pos} + c_{neg}}$*), only the inferior firm sells the spillover-producing product in the risk-dominant equilibrium. Otherwise, for* $\alpha > \dfrac{c_{neg}}{c_{pos} + c_{neg}}$*, only the superior firm sells the spillover-producing product in the risk-dominant equilibrium.*

(c) *For low fixed operating costs in the spillover-producing market (i.e.,* $O < O_l$*), both firms sell the spillover-producing product.*[15]

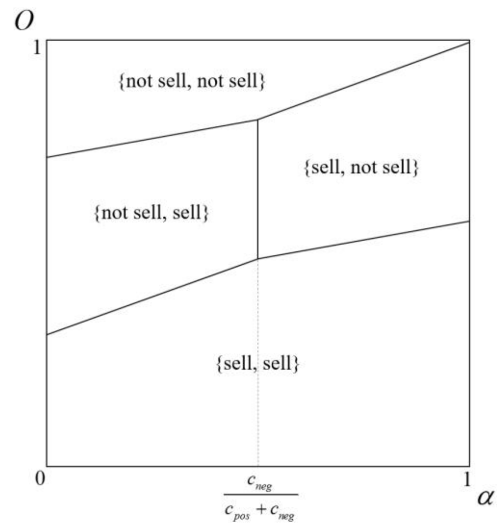
The condition $\alpha < c_{neg} / (c_{pos} + c_{neg})$, which determines which firm sells in the spillover-producing market for medium O, can be rewritten as $\alpha c_{pos} < (1-\alpha) c_{neg}$. Thus, Lemma 3 shows that for medium O, only the inferior (superior) firm participates in the spillover-producing market when the magnitude of a market spillover weighted by the segment size receiving that spillover is higher for the negative (positive) spillover. Intuitively, when the negative market spillover has a larger magnitude and affects more consumers than the positive market spillover, its effects dominate the effects of the positive market spillover. Thus, the superior firm, which has more to lose in the primary market from the stronger effects of the negative market spillover, responds primarily to the consumer segment that receives a negative market spillover and does not sell in the spillover-producing market. On the other hand, for $\alpha c_{pos} > (1-\alpha) c_{neg}$, the effects of the positive market spillover are dominant and firms choose their participation strategies primarily based on the consumer segment that receives a positive market spillover.

[15] The expressions for O_l and O_h are defined in the appendix.

Thus, similar to the result of the model in Section 4.2, only the superior firm, which has more to lose from not receiving the positive market spillover, sells in the spillover-producing market.

The findings of Lemma 3 are depicted in Figure 4, which shows how the fixed operating costs in the spillover-producing market and the proportion of consumers with a positive spillover determine equilibrium participation strategies. Note that this figure only shows the risk-dominant equilibrium strategies in regions with multiple equilibria.

Figure 4. Risk-Dominant Equilibrium Strategies of {Superior, Inferior} Firms Selling the Spillover-Producing Product, When Both Positive and Negative Market Spillovers Exist (WLOG, parameter values $c_{pos} = c_{neg} = 0.4$, $v_A - v_B = 1$, $v_2 = 1.8$, $m = 1$, and $t = 1$ used for generating figure)



4.4. A Model of Market Spillover as a Change in Purchase Quantity Per Consumer

An alternative way to capture the effect of market spillover is to assume participation of a firm in the spillover-producing market results in each of the firm's consumers purchasing more (fewer) products from that firm, when the market spillover is positive (negative). While our conjoint analysis shows that there are cases where participation in the spillover-producing market changes the primary product value for consumers, it is possible that in other cases, instead of this change in value, market spillover appears in the form of change in the number of primary products each customer buys.

In this model the location of the marginal consumer indifferent between buying the primary product from firm A or firm B is denoted by $\hat{x}$, such that $v_A - p_{A1} - \hat{x}t = v_B - p_{B1} - (1-\hat{x})t$, regardless of whether or not each firm sells in the spillover-producing market. However, if a firm sells in the spillover-producing market, the quantity of the primary product bought by each consumer from that firm will be multiplied by a constant g for a positive spillover and divided by g for a negative spillover, where $g \geq 1$. Thus, the quantity of primary products demanded from firm i depends on whether firm i sells the spillover-producing product, as shown below:

$$q_{i1} = g^{D_i \times H} \times \hat{x} \tag{10}$$

where $D_i = \begin{cases} 1 & \text{if firm } i \text{ sells in the spillover producing market} \\ 0 & \text{if firm } i \text{ does not sell in the spillover producing market} \end{cases}$,

$H = \begin{cases} 1 & \text{for a positive market spillover} \\ -1 & \text{for a negative market spillover} \end{cases}$, and $g \geq 1$.

Solving for optimal prices, we derive the profit solutions for each subgame, presented in Table 3. Comparing the profit outcomes in this table, we determine the equilibrium strategies of firms regarding participation in the spillover-producing market.

Table 3. Profit Outcomes for Market Spillover as a Change in Purchase Quantity Per Consumer ($H = 1$ for a positive market spillover and $H = -1$ for a negative market spillover)

	B: Sell Spillover-Producing Product	B: Not Sell Spillover-Producing Product
A: Sell Spillover-Producing Product	$If\ \frac{3t}{2} < v_2 < 2t: \begin{cases} \pi_A^{SShighv_2} = g^H((v_A - v_B)/3 + t)^2/2t + mt/2 \\ \pi_B^{SShighv_2} = g^H((v_B - v_A)/3 + t)^2/2t + mt/2 \end{cases}$ $If\ t < v_2 < \frac{3t}{2}: \begin{cases} \pi_A^{SSmedv_2} = g^H((v_A - v_B)/3 + t)^2/2t + m(v_2 - t/2)/2 \\ \pi_B^{SSmedv_2} = g^H((v_B - v_A)/3 + t)^2/2t + m(v_2 - t/2)/2 \end{cases}$ $If\ 0 < v_2 < t: \begin{cases} \pi_A^{SSlowv_2} = g^H((v_A - v_B)/3 + t)^2/2t + mv_2^2/4t \\ \pi_B^{SSlowv_2} = g^H((v_B - v_A)/3 + t)^2/2t + mv_2^2/4t \end{cases}$	$\begin{cases} \pi_A^{SN} = g^H((v_A - v_B)/3 + t)^2/2t + mv_2^2/4t \\ \pi_B^{SN} = ((v_B - v_A)/3 + t)^2/2t \end{cases}$
A: Not Sell Spillover-Producing Product	$\begin{cases} \pi_A^{NSlowc} = ((v_A - v_B)/3 + t)^2/2t \\ \pi_B^{NSlowc} = g^H((v_B - v_A)/3 + t)^2/2t + mv_2^2/4t \end{cases}$	$\begin{cases} \pi_A^{NN} = ((v_A - v_B)/3 + t)^2/2t \\ \pi_B^{NN} = ((v_B - v_A)/3 + t)^2/2t \end{cases}$

When the market spillover is negative, only the inferior firm sells the spillover-producing product in the risk-dominant equilibrium for high v_2 and g. This result is similar to our finding in proposition 1.

Also, similar to previous findings, we find that in the {not sell, sell} equilibrium, for low enough g and high enough v_2, the inferior firm can earn more profit than the superior firm.

Interestingly, we find the result presented in proposition 2 stating both firms can earn more profit as a result of a negative market spillover in the {not sell, sell} equilibrium does not hold in this model. Here, with the introduction of the negative market spillover, the profit of the firm that exits the spillover-producing market is less than what it was without market spillover. What drives this difference between the two models is that when market spillover causes a reduction in purchase quantity per consumer, the firm that exits the spillover-producing market does not benefit competitively from the competing firm's participation in the spillover-producing market, since the reservation values for the primary products remain the same. Thus, in the {not sell, sell} equilibrium, firm A loses the spillover-producing market, while gaining no new competitive advantage over firm B in the primary market as a result of the negative market spillover.

When the market spillover is positive, in the risk-dominant equilibrium for high v_2 and low g, only the superior firm enters the spillover-producing market. This result is similar to what we found in proposition 4. However, the finding from proposition 5 that states both firms earn less profit as a result of the positive spillover in the {sell, sell} equilibrium no longer holds. In contrast with our previous finding, the emergence of a positive market spillover in this model increases both firms' profits when it causes both firms to enter the spillover-producing market. This is because in the previous model, where the positive market spillover caused an upward shift in both firms' primary market reservation value, neither firm gained a competitive advantage and their profits in the primary market remained unchanged. However, when the positive market spillover increases the quantity purchased, both firms sell more units in the primary market and earn higher profits.

As shown above, considering a model of market spillover in which a firm's participation in the spillover-producing market changes the quantity of primary products purchased by each consumer from the firm results in equilibrium strategies similar to those chosen when market spillover was modeled as a change in the value of the primary product. However, analysis of firms' profits results in new findings: It is not possible for both firms to earn more (less) profit as a result of a negative (positive) market spillover.

5. Discussion

In this paper we looked at market spillovers that may occur if the firm decides to participate in a spillover-producing market that lowers or increases consumers' willingness to pay for the firm's product in its primary market. Examples of a negative market spillover can be observed in pharmacy and grocery industries, while examples of positive spillovers occur in environment-friendly industries. Such market spillovers are becoming more prevalent as consumers become more health and socially conscious.

Previous findings on umbrella branding provide a basis for understanding the underlying behavioral mechanisms causing market spillovers. According to the umbrella branding literature, once a brand introduces a brand extension, consumers may associate the parent brand with the attributes of the extension. Thus, positive attributes of the brand extension can be extended to the parent brand as well. For instance, when a brand extension is perceived as environment-friendly, consumers may associate the parent brand with the environment-friendliness attribute as well. This would explain the existence of a positive market spillover from the extension market to the parent brand market. Negative attributes of a brand extension can also be extended to the parent brand. For example, a brand extension with a perceived image of being unhealthy can cause consumers to associate the parent brand with this unhealthy image as well, resulting in what we observe as a negative market spillover from an unhealthy good market. Thus, it can be speculated that consumers' association of a firm's primary product with the attributes of that firm's spillover-producing product is an underlying mechanism for the existence of a market spillover.

Firms active in industries with a negative market spillover face an interesting dilemma: should they withdraw from a profitable market to avoid lowering their value in the primary market? On the other hand, in industries with a positive market spillover, firms must decide between benefiting from the positive spillover and avoiding an unprofitable spillover-producing market. We modeled this phenomenon by considering two competing firms that can decide whether or not to participate in a second market that produces a market spillover to their primary market.

Analyzing the equilibrium strategies of competing firms and their profitability under a negative market spillover, we found multiple interesting results from the model. First, we found that for sufficiently large market spillovers and consumers' reservation value for spillover-producing products, there is an equilibrium where the inferior firm stays in the spillover-producing market while the superior firm exits this market. Interestingly, we show that in this equilibrium both the inferior and the superior firm can be better off with the negative market spillover compared to before its emergence, when the consumers' reservation value for the spillover-producing product and the size of the spillover-producing market are high enough. Otherwise, when only the superior firm exits the market and consumers' reservation value for the spillover-producing product or the size of the spillover-producing market is not too high, only the superior firm makes more profit with the negative market spillover than without it. Also, when consumers' reservation value in the spillover-producing market is low enough such that both firms exit that market, both firms become worse off as a result of the negative market spillover. Finally, we find when the inferior firm is the only one selling the spillover-producing product, it can even make more profit than the superior firm.

These results show the opposite effect of what may be expected from a negative market spillover; even though firms get penalized through the market spillover, there are conditions for which they can still end up making more profit. The superior firm may exit the spillover-producing market and, in spite of entirely losing one market, become better off since it earns more profit in the primary market as a consequence of negative market spillover for the inferior firm for staying in the spillover-producing market. On the other hand, the inferior firm can also be better off when consumers' reservation value for the spillover-producing product and the size of the spillover-producing market are high enough, despite getting penalized in the primary market, because it now has monopoly power in the lucrative spillover-producing market. The negative market spillover thus can work as a competition dampening mechanism resulting in higher profits for firms. As we mentioned, even more surprisingly the inferior firm can earn higher profit than the superior firm in this equilibrium if the consumers' reservation value for the spillover-producing product is high enough. The inferior firm enjoys monopoly power over the spillover-producing market, but the superior firm has no incentive, on the margin, to participate in this market since that market won't be as lucrative with two participants and it would lose its advantage in the primary market.

Our analysis of positive market spillovers shows firms' market participation strategies depend on the sign of the market spillover. For a positive market spillover, it is the superior firm that decides to sell in the spillover-producing market, while the inferior firm avoids this market, when the magnitude of the positive spillover is not too high. This is the opposite of what we found with a negative market spillover, where the inferior firm was the one selling the spillover-producing product. Intuitively, the inferior firm has less to lose from the additional disadvantage in the primary market due to the positive market spillover and prefers avoiding the loss in the unprofitable spillover-producing market. Thus, the positive market spillover can magnify the firms' quality difference in the primary market. Analyzing firms' profits under a positive market spillover, we find the emergence of a positive market spillover can actually result in lower profits for both firms compared to when no market spillover exists. Surprisingly, this result occurs when the positive market spillover is high enough, creating a strong incentive for entering the spillover-producing market. Intuitively, when both firms enter the spillover-producing market, neither gains an advantage in the primary market. But both firms still decide to sell in the unprofitable spillover-producing market to not allow the competition to get an advantage from the positive spillover. Thus, firms' profits would have been higher if the positive market spillover had not emerged.

We also examine a model in which some primary market consumers receive positive spillovers and some receive negative spillovers from a firm's decision to sell in the spillover-producing market. Our findings show when the fixed operating cost in the spillover-producing market is not too low or too high, the firms' market participation strategies depend on the relative magnitude of the positive and the negative market spillovers and the relative size of the consumer segments receiving each type of market spillover: If more (fewer) consumers receive a positive spillover and the magnitude of the positive spillover is higher (lower) than the negative spillover, only the superior (inferior) firm sells the spillover-producing product. Intuitively, firms choose their participation strategies based on the more dominant market spillover, which has a stronger effect and impacts more consumers.

Furthermore, we analyze an alternative approach of modeling market spillovers, where selling in the spillover-producing market increases or decreases the quantity of the primary products purchased from that firm by each consumer. We show that the findings and insights from the previous model hold, except that both firms cannot be better (worse) off as a result of a negative (positive) market spillover when market spillover is modeled as a change in purchase quantity.

Our analysis has clear implications for managers involved in industries with market spillovers. Our research shows when a manager for a firm should react to a negative market spillover by withdrawing from the spillover-producing market. Similarly, we help inform decisions on entering unprofitable markets that generate a positive market spillover. Managers should also consider the effect that the emergence of a market spillover can have on the firms' profits, by evaluating the critical factors of consumers' reservation value for the spillover-producing product, size of the spillover-producing market, and the magnitude of the market spillover. Based on our findings, it may actually be counterproductive for managers to try to resist and fight negative market spillovers, as they can increase the profits of all competing firms through lowered competition. On the other hand, encouraging positive market spillovers can be counterproductive, as they may lower both firms' profits. Future studies on market spillovers may explore the effect of allowing firms to change the magnitude of the market spillover through investment in advertisement. We also provide managers insights on which of the high quality or low quality firms benefit from a market spillover. This result suggests when investing in quality, managers should consider whether market spillovers are to be expected and decide accordingly. Modeling quality investment decisions can be another interesting topic for future research on market spillovers.

In summary, this paper studied the concept of market spillovers. We developed an analytical model that identified when firms would exit the market generating a negative market spillover or enter the market generating a positive market spillover. Our comparison between firms' profits before and after the introduction of the market spillover shows that under some conditions both firms can benefit from a negative market spillover, while a positive market spillover may hurt both firms. The results provide implications for managers considering participation in markets such as unhealthy good markets, which could generate negative market spillovers, or environment-friendly markets, which could generate positive market spillovers.

References

Anderson, S. P. and De Palma, A. 1992. Multiproduct Firms: A Nested Logit Approach. *The Journal of Industrial Economics*, 40(3), 261-276.

Ansari, A., Economides, N. and Steckel, J., 1998. The Max- Min- Min Principle of Product Differentiation. *Journal of Regional Science*, *38*(2), 207-230.

Bernheim, B. and M. Whinston. 1990. Multimarket Contact and Collusive Behavior. *The RAND Journal of Economics*, 21(1), 1-26.

Bulow, J.I., Geanakoplos, J.D. and Klemperer, P.D., 1985. Multimarket Oligopoly: Strategic Substitutes and Complements. *Journal of Political economy*, 93(3), 488-511.

Cachon, G. P., Terwiesch, C., and Xu, Y. 2008. On the Effects of Consumer Search and Firm Entry in a Multiproduct Competitive Market. *Marketing Science*, 27(3), 461-473.

Cai, C. J. and Raju, J. S. 2016. Keeping Your Enemies Closer: When Market Entry as an Alliance with Your Competitor Makes Sense. *Marketing Science,* forthcoming.

Calia, M. 2014. CVS Revenue Rises on Strength at Pharmacy. *The Wall Street Journal*, 4, Nov.

Chandon, P. and Wansink, B. 2007. Multiproduct Firms: A Nested Logit Approach. *Journal of Consumer Research*, 34(3), 301-314.

Choi, J.P., 1998. Brand Extension as Informational Leverage. *The Review of Economic Studies*, 65(4), 655-669.

Davis, S. 2014. CVS CEO: It's a "Contradiction" to Help People Be Healthier and Sell Tobacco Products. *CBS News*, 5, Feb.

Dixon, H.D., 1994. Inefficient Diversification in Multi-Market Oligopoly with Diseconomies of Scope. *Economica*, 61, 213-219.

Dvorak, N. and Yu, D. 2014. Why CVS May Not Get Burned by Its Tobacco Decision. *Gallup.com*, 18, Mar.

Edwards, C. 1955. Conglomerate Bigness as a Source of Power. Business Concentration and Price Policy. *Princeton University Press*, 331-359.

Egan, M. 2015. CVS Banned Tobacco. Now its Sales are Hurting. *CNN.com*, 4, Aug.

Ferreira, R.D.S. and Thisse, J.F., 1996. Horizontal and Vertical Differentiation: The Launhardt Model. *International Journal of Industrial Organization*, *14*(4), 485-506.

Gagliardi, N. 2015. Consumers Want Healthy Foods--And Will Pay More For Them. *Forbes*, 18, Feb.

Geylani, T., Dukes, A., and Srinivasan, K. 2007. Strategic Manufacturer Response to a Dominant Retailer. *Marketing Science,* 26(2), 164-178.

Gimeno, J. and Woo, C. 1996. Hypercompetition in a Multimarket Environment: The Role of Strategic Similarity and Multimarket Contact in Competitive De-Escalation. *Organization Science,* 7(3), 322-341.

Grossmann, V. 2007. Firm Size and Diversification: Multiproduct Firms in Asymmetric Oligopoly. *International Journal of Industrial Organization*, 25(1), 51-67.

Hakenes, H. and Peitz, M., 2008. Umbrella Branding and the Provision of Quality. *International Journal of Industrial Organization*, 26(2), 546-556.

Harrison, A. 2014. Checkout Bans Could Kick Confectionery While It's Down: Euromonitor. *confectionerynews.com,* 2, Jun.

Harsanyi, J. C., and Selten, R. 1988. A General Theory of Equilibrium Selection in Games. *MIT Press Books*, 1.

Jayachandran, S., J. Gimeno, and P. Varadarajan. 1999. The Theory of Multimarket Competition: A Synthesis and Implications for Marketing Strategy. *The Journal of Marketing*, 63(3), 49-66.

Karnani, A. and B. Wernerfelt. 1985. Multiple Point Competition. *Strategic Management Journal*, 6(1), 87-96.

Keller, K.L. and Sood, S., 2003. Brand Equity Dilution. *MIT Sloan Management Review*, *45*(1), 12-15.

Kuksov, D. and Villas-Boas, J. M. 2010. When More Alternatives Lead to Less Choice. *Marketing Science,* 29(3), 507-524.

Lane, V. and Jacobson, R., 1997. The Reciprocal Impact of Brand Leveraging: Feedback Effects from Brand Extension Evaluation to Brand Evaluation. *Marketing Letters*, 8(3), 261-271.

Levick, R. 2012. Corporate Social Responsibility for Profit. *Forbes*, 11, Jan.

Liu, L. and Dukes, A. 2013. Consideration Set Formation with Multiproduct Firms: The Case of Within-Firm and Across-Firm Evaluation Costs. *Management Science*, 59(8), 1871-1886.

Loken, B. and John, D.R., 1993. Diluting Brand Beliefs: When do Brand Extensions Have a Negative Impact? *The Journal of Marketing*, 57(7), 71-84.

Martin, T. and M. Esterl. 2014. CVS to Stop Selling Cigarettes. *The Wall Street Journal*, 5, Feb.

Miklos-Thal, J., 2008. Linking Reputations: The Signaling and Feedback Effects of Umbrella Branding.

Moorthy, S., 2012. Can Brand Extension Signal Product Quality? *Marketing science*, 31(5), 756-770.

Olenski, S. 2014. Healthy Is the New Green: Businesses and Brands Tapping Into The Healthy Food Market. *Forbes*, 23, May.

Poulter, S. 2014. Lidl Leads the Way with Sweet and Crisps Ban at Checkout. *dailymail.co.uk*, 13, Jan.

Reed, J. 2007. Closing the Gap in the Race to Be Green. *Financial Times*, 26, Sep.

Röller, L.H. and Tombak, M.M., 1990. Strategic Choice of Flexible Production Technologies and Welfare Implications. *The Journal of Industrial Economics*, 38, 417-431.

Schindler, R. M. 2011. Pricing Strategies: A Marketing Approach. *Sage*, 11, Oct.

Schmalbruch, S. 2015. Why I Love the CVS Rewards Program. *Business Insider*, 4, Feb.

Smithers, R. 2014. Tesco Bans Sweets from Checkouts in All Stores. *The Guardian*, 21, May.

Snyder, J. 2009. Fiat Alone Can't Save Chrysler. *Automotive News Europe*, 18, May.

Stern, M. 2015. Is Target Becoming a Wellness Dictator? *Forbes*, 27, Sep.

Straub, P. G. 1996. Risk Dominance and Coordination Failures in Static Games. *The Quarterly Review of Economics and Finance*, 35(4), 339-363.

Thomas, C. and R. Willig. 2006. The Risk of Contagion from Multimarket Contact. *International Journal of Industrial Organization*, 24(6), 1157-1184.

Trotter, G. 2016. Aldi Putting More Granola, Less Chocolate in Checkout Lanes. *Chicago Tribune*, 12, Jan.
Wahba, P. and Steenhuysen, J. CVS Becomes First Big U.S. Drugstore Chain to Drop Tobacco. *Reuters*, 5, Feb.
Wernerfelt, B., 1988. Umbrella Branding as a Signal of New Product Quality: An Example of Signaling by Posting a Bond. *The RAND Journal of Economics*, 19(3), 458-466.
Young, J. 2014. Why Is CVS Walking Away from $2 BILLION A Year? *The Huffington Post*, 12, Sep.

APPENDIX

I) Proof of Lemma 1:

The subgame equilibrium prices for $3t/2 < v_2 < 2t$ are derived in the main text, with profits defined by equations (2)-(8). With these prices, the spillover-producing market is fully covered if $v_2 > 3t/2$. If $v_2 < t$, both firms are local monopolies and set prices of $v_2/2$, earning profits of $mv_2^2/4t$.

CLAIM: For $t < v_2 < 3t/2$, in the {sell, sell} subgame, each firm charges the highest price possible in the spillover-producing market that will cover half of the market, which is $v_2 - t/2$. PROOF: No firm can deviate to higher or lower prices profitably. The profit of charging $v_2 - t/2$ is $m(v_2 - t/2)/2$ for both firms. If firm A increases its price, demand becomes $m(v_2 - p_{A2})/t$. The profit function becomes $m(v_2 p_{A2} - p_{A2}^2)/\mathrm{t}$, which is decreasing in p_{A2} for $p_{A2} \geq v_2 - t/2$. If firm A decreases its price, the demand will be derived from $v_2 - yt - p_{A2} = v_2 - (1-y)t - (v_2 - t/2)$ and profit would be $m(t/2 + v_2 - p_{A2})p_{A2}/t$, which is diminished by decreasing p_{A2} from $\mathrm{v}_2 - t/2$. Thus, $p_{A2} = v_2 - t/2$ is the equilibrium price. Similar analysis can be used to show $p_{B2} = v_2 - t/2$. □

We summarize the payoffs from each possibility in Tables A1, A2, and A3.

Table A1. Profit Outcomes if $c < 3t - (v_A - v_B)$

	B: Sell Spillover-Producing Product	B: Not Sell Spillover-Producing Product
A: Sell Spillover-Producing Product	$If\ \frac{3t}{2} < v_2 < 2t: \begin{cases} \pi_A^{SShighv_2} = ((v_A - v_B)/3 + t)^2 / 2t + mt/2 \\ \pi_B^{SShighv_2} = ((v_B - v_A)/3 + t)^2 / 2t + mt/2 \end{cases}$ $If\ t < v_2 < \frac{3t}{2}: \begin{cases} \pi_A^{SSmedv_2} = ((v_A - v_B)/3 + t)^2 / 2t + m(v_2 - t/2)/2 \\ \pi_B^{SSmedv_2} = ((v_B - v_A)/3 + t)^2 / 2t + m(v_2 - t/2)/2 \end{cases}$ $If\ 0 < v_2 < t: \begin{cases} \pi_A^{SSlowv_2} = ((v_A - v_B)/3 + t)^2 / 2t + mv_2^2 / 4t \\ \pi_B^{SSlowv_2} = ((v_B - v_A)/3 + t)^2 / 2t + mv_2^2 / 4t \end{cases}$	$\begin{cases} \pi_A^{SNlowc} = ((v_B - v_A + c)/3 - t)^2 / 2t + mv_2^2 / 4t \\ \pi_B^{SNlowc} = ((v_B - v_A + c)/3 + t)^2 / 2t \end{cases}$
A: Not Sell Spillover-Producing Product	$\begin{cases} \pi_A^{NSlowc} = ((v_A - v_B + c)/3 + t)^2 / 2t \\ \pi_B^{NSlowc} = ((v_A - v_B + c)/3 - t)^2 / 2t + mv_2^2 / 4t \end{cases}$	$\begin{cases} \pi_A^{NN} = ((v_A - v_B)/3 + t)^2 / 2t \\ \pi_B^{NN} = ((v_B - v_A)/3 + t)^2 / 2t \end{cases}$

Table A2. Profit Outcomes if $3t - (v_A - v_B) < c < 3t + (v_A - v_B)$

	B: Sell Spillover-Producing Product	B: Not Sell Spillover-Producing Product
A: Sell Spillover-Producing Product	Same as Table A1	Same as Table A1
A: Not Sell Spillover-Producing Product	$\begin{cases} \pi_A^{NShighc} = v_A - v_B + c - t \\ \pi_B^{NShighc} = mv_2^2 / 4t \end{cases}$	Same as Table A1

Table A3. Profit Outcomes if $c > 3t + (v_A - v_B)$

	B: Sell Spillover-Producing Product	B: Not Sell Spillover-Producing Product
A: Sell Spillover-Producing Product	Same as Table A1	$\begin{cases} \pi_A^{SNhighc} = mv_2^2 / 4t \\ \pi_B^{SNhighc} = v_B - v_A + c - t \end{cases}$
A: Not Sell Spillover-Producing Product	$\begin{cases} \pi_A^{NShighc} = v_A - v_B + c - t \\ \pi_B^{NShighc} = mv_2^2 / 4t \end{cases}$	Same as Table A1

If $c < 3t - (v_A - v_B)$, then all subgames have interior solutions and the equilibrium is found from Table A1. If $3t - (v_A - v_B) < c < 3t + (v_A - v_B)$, then the {not sell, sell} subgame reaches a corner solution, resulting in Table A2. Finally if $c > 3t + (v_A - v_B)$, then both subgames with asymmetric strategies have corner solutions, shown in Table A3.

We start by analyzing the profits in Table A1 when $3t/2 < v_2 < 2t$, denoting $K \doteq v_A - v_B$. Comparing profits, $\pi_A^{SShighv_2} < \pi_A^{NSlowc}$ if $c > c_{NS/SS\sim H} \equiv -K - 3t + \sqrt{9\,mt^2 + (K+3t)^2}$. Recall our bound on c for Table A1 and note that $c_{NS/SS\sim H} < 3t - K$ iff $K < 3t(\sqrt{4-m} - 1)$. Therefore in Table A1, $\pi_A^{SShighv_2} < \pi_A^{NSlowc}$ if $K < 3t(\sqrt{4-m} - 1)$ and $c > c_{NS/SS\sim H}$. Using similar logic we find the cutoffs for each range of v_2. For $t < v_2 < 3t/2$, $\pi_A^{SSmedv_2} < \pi_A^{NSlowc}$ if $K < -3t + 3\sqrt{t((8+m)t/2 - mv_2)}$ and $c > c_{NS/SS\sim M} = -3t - K + \sqrt{(9mt(v_2 - t/2) + (3t+K)^2)}$. Similarly for $v_2 < t$, $\pi_A^{SSlowv_2} < \pi_A^{NSlowc}$ if $K < -3t + 3\sqrt{4t^2 - mv_2^{\,2}/2}$ and $c > c_{NS/SS\sim L} = -3t - K + \sqrt{9mv_2^{\,2}/2 + (3t+K)^2}$.

Also, for high v_2, $\pi_B^{SNlowc} > \pi_B^{SShighv_2}$ if $c > c_{SN/SS\sim H} = -3t + K + \sqrt{9\,mt^2 + (3t-K)^2}$ and $K < t(3 - \sqrt{3m})$. For $t < v_2 < 3t/2$, $\pi_B^{SNlowc} > \pi_B^{SSmedv_2}$ if $K < 3t - \sqrt{3mt(v_2 - t/2)}$ and $c > c_{SN/SS\sim M} = -3t + K + \sqrt{9mt(v_2 - t/2) + (3t-K)^2}$. Similarly for $v_2 < t$, $\pi_B^{SNlowc} > \pi_B^{SSlowv_2}$ if $K < 3t - (\sqrt{3m/2})v_2$ and $c > c_{SN/SS\sim L} = -3t + K + \sqrt{9mv_2^{\,2}/2 + (3t-K)^2}$.

Also, $\pi_A^{SNlowc} > \pi_A^{NN}$ for $v_2 > v_{2SN/NN} \equiv \sqrt{2c(6t + 2K - c)/(9m)}$. Finally, $\pi_B^{NSlowc} > \pi_B^{NN}$ for $v_2 > v_{2NS/NN} \equiv \sqrt{2c(6t - 2K - c)/(9m)}$. Both $v_{2SN/NN}$ and $v_{2NS/NN}$ have real values in for $c < 3t - K$.

Next we analyze profits in Table A2, when $3t - K < c < 3t + K$. For high v_2, $\pi_A^{SShighv_2} > \pi_A^{NShighc}$ if $K > 3t(\sqrt{4-m} - 1)$ and $3t - K < c < c_{NS/SS2\sim H} = (3+m)t/2 + K(K - 12t)/18t$; otherwise $\pi_A^{SShighv_2} < \pi_A^{NShighc}$. For medium v_2, $\pi_A^{SSmedv_2} > \pi_A^{NShighc}$ if $K > -3t + 3\sqrt{t((8+m)t/2 - mv_2)}$ and $3t - K < c < c_{NS/SS2\sim M} = (6-m)t/4 + mv_2/2 + K(K - 12t)/18t$; otherwise $\pi_A^{SSmedv_2} < \pi_A^{NShighc}$. Similarly, for low v_2, we have $\pi_A^{SSlowv_2} > \pi_A^{NShighc}$ if $K > -3t + 3\sqrt{4t^2 - mv_2^{\,2}/2}$ and $3t - K < c < c_{NS/SS2\sim L} = mv_2^{\,2}/4t + (3t - K)(1/2 - K/18t)$; otherwise $\pi_A^{SSlowv_2} < \pi_A^{NShighc}$. Comparing π_B^{SS} and π_B^{SN} is similar to Table A1, but with different conditions on K: $3t - K < c_{SN/SS\sim H} < 3t + K$ if

$K > t(3-\sqrt{3m})$, $3t - K < c_{SN/SS\sim M} < 3t + K$ if $K > 3t - \sqrt{3mt(v_2 - t/2)}$, and $3t - K < c_{SN/SS\sim L} < 3t + K$ if $K > 3t - \sqrt{3m/2}v_2$. We also find $\pi_A^{SNlowc} > \pi_A^{NN}$ if $v_2 > v_{2SN/NN} \equiv \sqrt{2c(6t+2K-c)/(9m)}$, and $\pi_B^{NShighc} > \pi_B^{NN}$ if $v_2 > \sqrt{2/m}(3t-K)/3$.

Finally, we analyze Table A3 when $c > 3t + K$. For all $0 < K < 3t$, we know that $\pi_B^{SNhighc} > \pi_B^{SNhighc}\big|_{c\to 3t+K} = 2t$ and $\pi_B^{SSlowv_2}, \pi_B^{SSmedv_2}, \pi_B^{SShighv_2} < \pi_B^{SShighv_2}\big|_{K\to 0} = (m+1)t/2 < t$, which means in this table we always have $\pi_B^{SNhighc} > \pi_B^{SS}$, where π_B^{SS} could be any of the three profits $\pi_B^{SSlowv_2}$, $\pi_B^{SSmedv_2}$, or $\pi_B^{SShighv_2}$. Also for $0 < K < 3t$, we know $c_{NS/SS\sim H}, c_{NS/SS\sim M}, c_{NS/SS\sim L} < 3t + K$, which means in Table 3 we always have $\pi_A^{NShighc} > \pi_A^{SS}$. The inequality $\pi_A^{SNhighc} > \pi_A^{NN}$ holds if $v_2 > \sqrt{2/m}(3t+K)/3$, and $\pi_B^{NShighc} > \pi_B^{NN}$ holds if $v_2 > \sqrt{2/m}(3t-K)/3$.

We compare the condition for firm A not deviating from {sell, sell} with the condition for firm B not deviating to see which condition is stricter. Starting with high v_2, we show that for $K > 0$, $c_{NS/SS\sim H} < c_{SN/SS\sim H}$; taking the derivatives of $c_{NS/SS\sim H}$ and $c_{SN/SS\sim H}$ with respect to K, we find $\partial c_{NS/SS\sim H}/\partial K = -1 + (K+3t)/\sqrt{(K+3t)^2 + 9mt^2} < 0$ and $\partial c_{SN/SS\sim H}/\partial K = 1 - (K-3t)/\sqrt{(K-3t)^2 + 9mt^2} > 0$. Also note that $c_{NS/SS\sim H}\big|_{K\to 0} = c_{SN/SS\sim H}\big|_{K\to 0}$. Thus, $c_{NS/SS\sim H}$ and $c_{SN/SS\sim H}$ have the same value at $K = 0$, but $c_{NS/SS\sim H}$ is decreasing in K, while $c_{SN/SS\sim H}$ is increasing. Thus for all $K > 0$, $c_{NS/SS\sim H} < c_{SN/SS\sim H}$. For $K > 3t(\sqrt{4-m} - 1)$, we have $c_{NS/SS2\sim H} < c_{NS/SS\sim H}$ which also implies $c_{NS/SS2\sim H} < c_{SN/SS\sim H}$. Using similar logic for other ranges of v_2, we show that $c_{NS/SS\sim M} < c_{SN/SS\sim M}$ and for $K > -3t + 3\sqrt{t((8+m)t/2 - mv_2)}$ we have $c_{NS/SS2\sim M} < c_{NS/SS\sim M} < c_{SN/SS\sim M}$. Finally, $c_{NS/SS\sim L} < c_{SN/SS\sim L}$ and for $K > -3t + 3\sqrt{4t^2 - mv_2^2/2}$ we have $c_{NS/SS2\sim L} < c_{NS/SS\sim L} < c_{SN/SS\sim L}$.

This proves that the {sell, sell} equilibrium exists only for $c < c'$. Let $r \in \{L, M, H\}$ represent the region to which v_2 belongs, such that $r = L$ requires $0 < v_2 < t$, $r = M$ requires $t < v_2 < 3t/2$, and

$r = H$ requires $3t/2 < v_2 < 2t$. The definition of c' is such that for v_2 belonging to the region $r \in \{L, M, H\}$, $c' = c_{NS/SS\sim\mathrm{r}}$ if $K < K_r^*$, and $c' = c_{NS/SS2\sim\mathrm{r}}$ if $K > K_r^*$, where $K_L^* = -3t + 3\sqrt{4t^2 - mv_2^2/2}$, $K_M^* = -3t + 3\sqrt{t((8+m)t/2 - mv_2)}$, and $K_H^* = 3t(\sqrt{4-m} - 1)$.

For the {not sell, not sell} equilibrium, we compare the $\pi_A^{NN} > \pi_A^{SN}$ and $\pi_B^{NN} > \pi_B^{NS}$ conditions. For $c < 3t - K$, we have $v_{2NS/NN} < v_{2SN/NN}$. For $3t - K < c < 3t + K$, the minimum of $v_{2SN/NN}$ occurs at $c = 3t - K$ and is equal to $\sqrt{2(3t-K)(3t+3K)/(9m)}$ which is greater than $\sqrt{2}(3t-K)/(3\sqrt{m})$. Thus, we have $v_{2SN/NN} > \sqrt{2}(3t-K)/(3\sqrt{m})$. Finally, for $c > 3t + K$ the threshold for $\pi_B^{NN} > \pi_B^{NS}$, $v_2 = \sqrt{2/m}(3t-K)/3$, is less than the threshold for $\pi_A^{NN} > \pi_A^{SN}$, $v_2 = \sqrt{2/m}(3t+K)/3$. Thus the {not sell, not sell} equilibrium exists iff $v_2 < v_2'$, where $v_2' = v_{2NS/NN}$ for $c < 3t - K$ and $v_2' = \sqrt{2/m}(3t-K)/3$ for $c > 3t - K$. *Q.E.D.*

II) Proof of Proposition 1:

Based on the proof of Lemma 1, the {not sell, sell} equilibrium exists for $c > c'$ and $v_2 > v_2'$. Also the {sell, not sell} equilibrium exists for $c > c''$ and $v_2 > v_2''$. Let v_2 belong to the region $r \in \{L, M, H\}$. We define $c'' = c_{SN/SS\sim\mathrm{r}}$, $v_2'' = v_{2SN/NN}$ for $c < 3t + K$, and $v_2'' = \sqrt{2/m}(3t+K)/3$ for $c > 3t + K$. We showed $v_2' < v_2''$ and thus the region for $\pi_A^{SN} > \pi_A^{NN}$ is a subset of the region for $\pi_B^{NS} > \pi_B^{NN}$. We also showed $c' < c''$ and thus the region for $\pi_B^{SN} > \pi_B^{SS}$ is a subset of the region for $\pi_A^{NS} > \pi_A^{SS}$. Thus, the {not sell, sell} equilibrium is unique for $c' < c < c''$ and $v_2' < v_2 < v_2''$. This proves Proposition 1(a).

Next we prove the risk-dominance of the {not sell, sell} equilibrium over the {sell, not sell} equilibrium. The condition for the risk-dominance of the {not sell, sell} equilibrium is $RD = (\pi_A^{NS} - \pi_A^{SS})(\pi_B^{NS} - \pi_B^{NN}) - (\pi_A^{SN} - \pi_A^{NN})(\pi_B^{SN} - \pi_B^{SS}) > 0$. For low v_2, there are three tables to consider. For Table A1, $RD = 2c^3K/81t^2 > 0$. For Table A2, $\partial(RD)/\partial v_2 = mv_2(c - 3t + K)/6t$ is positive, meaning RD is minimized with respect to v_2 at $v_2 = 0$. $\partial(RD|_{v_2 \to 0})/\partial K$ is continuous in K

and never zero for $0 < K < 3t$, which means $RD\big|_{v_2 \to 0}$ is monotonic for $0 < K < 3t$. Thus, the minimum of $RD\big|_{v_2 \to 0}$ with respect to K is at one of the corners of the region $0 \le K \le 3t$. Since $RD\big|_{v_2 \to 0, K \to 0} > 0$ and $RD\big|_{v_2 \to 0, K \to 3t} > 0$, the minimum of $RD\big|_{v_2 \to 0}$ is positive, thus $RD\big|_{v_2 \to 0} > 0$ and $RD > 0$. For Table A3, $RD = (6(2c - 5t)t + 3mv_2^{\,2} - 2K^2)K / 18t > 0$ for $c > \hat{c} = (-3mv_2^{\,2} + 30t^2 + 2K^2) / 12t$, where $\hat{c} < 3t + K$ for all $K < 3t$. Using similar logic, we show that for higher values of v_2, $RD > 0$. *Q.E.D.*

Proof for when Firm A is Superior in Both Markets:

We assume firm A's spillover-producing product value is $v_2 + K_2$, while firm B's is v_2. We show solutions for $v_2 < t - K_2 / 2$, when both firms can have local monopolies in the spillover-producing market. Table A4 shows profits when c is low enough.

Table A4. Profit Outcomes if Firm A is Superior in the Spillover-Producing Market by K_2

	B: Sell Spillover-Producing Product	B: Not Sell Spillover-Producing Product
A: Sell Spillover-Producing Product	$\begin{cases} \pi_A^{\ SSK_2} = ((v_A - v_B)/3 + t)^2 / 2t + m(v_2 + K_2)^2 / 4t \\ \pi_B^{\ SSK_2} = ((v_B - v_A)/3 + t)^2 / 2t + mv_2^{\,2} / 4t \end{cases}$	$\begin{cases} \pi_A^{\ SNK_2} = ((v_B - v_A + c)/3 - t)^2 / 2t + m(v_2 + K_2)^2 / 4t \\ \pi_B^{\ SNK_2} = ((v_B - v_A + c)/3 + t)^2 / 2t \end{cases}$
A: Not Sell Spillover-Producing Product	$\begin{cases} \pi_A^{\ NSK_2} = ((v_A - v_B + c)/3 + t)^2 / 2t \\ \pi_B^{\ NSK_2} = ((v_A - v_B + c)/3 - t)^2 / 2t + mv_2^{\,2} / 4t \end{cases}$	$\begin{cases} \pi_A^{\ NNK_2} = ((v_A - v_B)/3 + t)^2 / 2t \\ \pi_B^{\ NNK_2} = ((v_B - v_A)/3 + t)^2 / 2t \end{cases}$

We find $\pi_A^{\ SSK_2} > \pi_A^{\ NSK_2}$ for $c < c_{NS/SSK_2} = -3t - K + \sqrt{9m(v_2 + K_2)^2 / 2 + (3t + K)^2}$. Also, $\pi_B^{\ SSK_2} > \pi_B^{\ SNK_2}$ for $c < c_{SN/SSK_2} = -3t + K + \sqrt{9mv_2^{\,2} / 2 + (3t - K)^2}$. We find $c_{NS/SSK_2} < c_{SN/SSK_2}$ for $K_2 < K_2^{\,*} = \sqrt{v_2^2 + 8K\left(-3t + K + \sqrt{9mv_2^2 / 2 + \left(3t - K\right)^2}\right) / 9m} - v_2$. Similarly from Table A4 we know $\pi_A^{\ NNK_2} > \pi_A^{\ SNK_2}$ for $c > c_{SN/NNK_2} = 3t + K - \sqrt{(3t + K)^2 - 9m(K_2 + v_2)^2 / 2}$. Also $\pi_B^{\ NNK_2} > \pi_B^{\ NSK_2}$ for $c > c_{NS/NNK_2} = 3t - K - \sqrt{(3t - K)^2 - 9mv_2^{\,2} / 2}$. We find that $c_{NS/NNK_2} > c_{SN/NNK_2}$ for $K_2 < K_2^{\,**} = \sqrt{v_2^2 + 8K\left(3t - K - \sqrt{\left(3t - K\right)^2 - 9mv_2^2 / 2}\right) / 9m} - v_2$. For $K > 0$, we have $K_2^{\,*} < K_2^{\,**}$. Thus as long as $K_2 < K_2^*$, the {sell, not sell} equilibrium is a subset of the {not sell, sell} equilibrium. When $K_2 = K$, for $K > \hat{K} = (2(8-9m)v_2 - 4\left(6t + \sqrt{36t^2 + 48tv_2 + 2(-8+9m)v_2^2}\right)) / (-16+9m)$ we have $K_2^* > K$. We show $\hat{K} < 3t$ for $m < 32v_2^2 / (9(3t+2v_2)^2)$. *Q.E.D.*

III) Proof of Proposition 2:

If no negative market spillover exists, then the two markets are independent. The firms' profits would be the same as in the {sell, sell} subgame shown in Table A1. We denote these profits $\pi_A{}^Z$ and $\pi_B{}^Z$. In the {not sell, sell} equilibrium we have $\pi_A{}^Z = \pi_A{}^{SS} < \pi_A{}^{NS}$. In Table A1, for $3t/2 < v_2 < 2t$, we have $\pi_B{}^{Zhighv_2} < \pi_B{}^{NSlowc}$ if $v_2 > v_{2NS/Zhigh} = \sqrt{2(9mt^2 + 6ct - c^2 - 2cK)/(9m)}$. For $t < v_2 < 3t/2$, we have $\pi_B{}^{Zmedv_2} < \pi_B{}^{NSlowc}$ if $v_2 > v_{2NS/ZMed} = t + \sqrt{2c(6t - c - 2K)/(9m)}$. For $v_2 < t$, $\pi_B{}^{Zlowv_2} > \pi_B{}^{NSlowc}$. We find $3t/2 < v_{2NS/Zhigh} < 2t$ requires $c(6t - 2K - c)/(9t^2) < m < 8c(6t - 2K - c)/(9t^2)$. Also $t < v_{2NS/ZMed} < 3t/2$ requires $m > 8c(6t - 2K - c)/(9t^2)$. We know $m > 8c(6t - 2K - c)/(9t^2)$ results in $v_{2NS/Zhigh} > v_{2NS/ZMed}$. Thus, $\pi_B{}^Z < \pi_B{}^{NS}$ for $v_2 > Min\{v_{2NS/Zhigh}, v_{2NS/ZMed}\}$ and $m > c(6t - 2K - c)/(9t^2)$. Similarly, for tables A2 and A3, we find $\pi_B{}^Z < \pi_B{}^{NS}$ if $v_2 > Min\{\sqrt{2(9(1+m)t^2 - K(6t-K))/(9m)}, t + \sqrt{2}(3t-K)/(3\sqrt{m})\}$ and $m > (3t-K)^2/(9t^2)$. Thus,

$$\hat{v}_2 = \begin{cases} Min\{\sqrt{2(9mt^2 + 6ct - c^2 - 2cK)/(9m)}, t + \sqrt{2c(6t - c - 2K)/(9m)}\} & if\ c < 3t - K \\ Min\{\sqrt{2(9(1+m)t^2 - K(6t-K))/(9m)}, t + \sqrt{2}(3t-K)/(3\sqrt{m})\} & if\ c \geq 3t - K \end{cases} \quad \text{and}$$

$$\hat{m} = \begin{cases} c(6t - 2K - c)/(9t^2) & if\ c < 3t - K \\ (3t-K)^2/(9t^2) & if\ c \geq 3t - K \end{cases}. \quad Q.E.D.$$

Proof for Symmetric Firms:

The analysis is similar to our proof for Lemma 1, with $K = 0$. If $c < 3t$, there's an interior solution and if $c > 3t$, we get a corner solution in both asymmetric subgames. When $K = 0$, we find that $v_2'|_{K\to 0} = v_2''|_{K\to 0}$ and $c'|_{K\to 0} = c''|_{K\to 0}$. Thus, for $v_2 > v_2'|_{K\to 0}$ and $c > c'|_{K\to 0}$, {not sell, sell} equilibrium and {sell, not sell} equilibrium both exist. *F*or $c < 3t$, we have $\pi_B^{\ Z} < \pi_B^{\ NS}$ if $v_2 > Min\{\sqrt{2(9mt^2 + 6ct - c^2)/(9m)}, t + \sqrt{2c(6t - c)/(9m)}\}$ and $m > c(6t - c)/(9t^2)$. For $c > 3t$, for all $0 < m < 1$ we have $\pi_B^{\ Z} > \pi_B^{\ NS}$. Also, in the {not sell, sell} equilibrium, $\pi_A^{\ Z} < \pi_A^{\ NS}$. *Q.E.D.*

Proof for Total Industry Profits:

We find the region where the market spillover increases the sum of the two firms' profits. We show the proof for Table A1. The proof for Tables A2 and A3 follows similar steps. We have $(\pi_A^{\ Zhighv_2} + \pi_B^{\ Zhighv_2}) < (\pi_A^{\ NSlowc} + \pi_B^{\ NSlowc})$ for $c > c_{T1} = \sqrt{4\left(K^2 + 9mt^2\right) - 9mv_2^2}\,/\,2 - K$. Similarly, we have $(\pi_A^{\ Zmedv_2} + \pi_B^{\ Zmedv_2}) < (\pi_A^{\ NSlowc} + \pi_B^{\ NSlowc})$ for $c > c_{T2} = \sqrt{4K^2 - 18mt^2 + 9mv_2\left(4t - v_2\right)}\,/\,2 - K$, and $(\pi_A^{\ Zlowv_2} + \pi_B^{\ Zlowv_2}) < (\pi_A^{\ NSlowc} + \pi_B^{\ NSlowc})$ for $c > c_{T3} = \sqrt{4K^2 + 9mv_2^2}\,/\,2 - K$. Thus, the market spillover increases total profits in the {not sell, sell} equilibrium for $c > Max\{c_T, c'\}$, where

$$c_T = \begin{cases} c_{T1} & \text{if } 3t/2 < v_2 < 2t \\ c_{T2} & \text{if } t < v_2 < 3t/2 \\ c_{T3} & \text{if } 0 < v_2 < t \end{cases} \quad . \quad Q.E.D.$$

IV) Proof of Proposition 3:

In Table A1, $\pi_A^{\ NS} < \pi_B^{\ NS}$ holds true if $v_2 > 2\sqrt{2/3m}\sqrt{ct + Kt}$, which guarantees $v_2 > v_{2NS/NN}$ and thus satisfies the {not sell, sell} equilibrium condition. Also, $2\sqrt{2/3m}\sqrt{ct + Kt}$ is below $v_2 = 2t$ if $c < 3mt/2 - K$, which can be satisfied in Table A1, since $3mt/2 - K$ is bigger than $c_{NS/SS\sim H}$ for $K < 3(\sqrt{4 + m} - 2)t$. Thus the region where $\pi_A^{\ NS} < \pi_B^{\ NS}$ can satisfy the conditions $c > c'$ and $v_2 > v_2'$ required for the {not sell, sell} equilibrium in Table A1. Considering Tables A2 and A3, $\pi_A^{\ NS} < \pi_B^{\ NS}$ requires $v_2 > 2\sqrt{t/m}\sqrt{c - t + K}$. The threshold $2\sqrt{t/m}\sqrt{c - t + K}$ is larger than $v_2 = 2t$ for all $c > 3t - K$. Thus, only Table A1 can result in $\pi_A^{\ NS} < \pi_B^{\ NS}$.

Assuming firm A can costlessly reduce v_A to $\tilde{v}_A < v_A$, $\tilde{K} = v_B - \tilde{v}_A$ must become large enough such that $c > \tilde{c}_{NS/SS\sim H} \equiv -\tilde{K} - 3t + \sqrt{9\,mt^2 + (\tilde{K}+3t)^2}$, in order to cause firm B to leave the spillover-producing market. Thus, we must have $\tilde{v}_A < v_B - (9mt^2/2c - c/2 - 3t)$, which is not possible for $v_B < 9mt^2/2c - c/2 - 3t$. *Q.E.D.*

V) Proof of Lemma 2:

The firms' profits in the four subgames are shown in the table below.

Table A5. Profits with a positive spillover (assuming $K \le 3t - c$)

	B: Sell Spillover-Producing Product	B: Not Sell Spillover-Producing Product
A: Sell Spillover-Producing Product	$If\ \frac{3t}{2} < v_2 < 2t: \begin{cases} \pi_A^{SShighv_2} = (K/3+t)^2/2t + mt/2 - O \\ \pi_B^{SShighv_2} = (t-K/3)^2/2t + mt/2 - O \end{cases}$ $If\ t < v_2 < \frac{3t}{2}: \begin{cases} \pi_A^{SSmedv_2} = (K/3+t)^2/2t + m(v_2 - t/2)/2 - O \\ \pi_B^{SSmedv_2} = (t-K/3)^2/2t + m(v_2 - t/2)/2 - O \end{cases}$ $If\ 0 < v_2 < t: \begin{cases} \pi_A^{SSlowv_2} = (K/3+t)^2/2t + m{v_2}^2/4t - O \\ \pi_B^{SSlowv_2} = (t-K/3)^2/2t + m{v_2}^2/4t - O \end{cases}$	$\pi_A^{SN} = ((K+c)/3+t)^2/2t + m{v_2}^2/4t - O$ $\pi_B^{SN} = ((K+c)/3-t)^2/2t$
A: Not Sell Spillover-Producing Product	$\pi_A^{NS} = ((K-c)/3+t)^2/2t$ $\pi_B^{NS} = ((c-K)/3+t)^2/2t + m{v_2}^2/4t - O$	$\pi_A^{NN} = (K/3+t)^2/2t$ $\pi_B^{NN} = (t-K/3)^2/2t$

Comparing the profits in Table A5, for $c > c_{NS/SS\sim H} \equiv 3t + K - \sqrt{(K+3t)^2 + 9t(mt - 2O)}$ we find $\pi_A^{SShighv_2} > \pi_A^{NS}$. Also, $\pi_A^{SSmedv_2} > \pi_A^{NS}$ if $v_2 > \tilde{v}_{NS\sim M} = (9t(4O + (m-2)t) - 2K(K+6t))/(18mt)$ and $c > c_{NS/SS\sim M} = 3t + K - \sqrt{(K+3t)^2 - 9t(4O + mt - 2mv_2)/2}$. Finally, we have $\pi_A^{SSlowv_2} > \pi_A^{NS}$ if $v_2 > \tilde{v}_{NS\sim L} = \sqrt{2/m}\sqrt{9t(2O-t) - K(K+6t)}/3$ and $c > c_{NS/SS\sim L} = 3t + K - \sqrt{(K+3t)^2 + 9(mv_2^2 - 4Ot)/2}$.

Considering firm B, we find $\pi_B^{SShighv_2} > \pi_B^{SN}$ for $c > c_{SN/SS\sim H} \equiv 3t - K - \sqrt{(3t-K)^2 + 9t(mt - 2O)}$. Also, we have $\pi_B^{SSmedv_2} > \pi_B^{SN}$ if $v_2 > \tilde{v}_{SN\sim M} = (2K(-K+6t) + 9t(4O + (m-2)t))/(18mt)$ and

$c > c_{SN/SS\sim M} = 3t - K - \sqrt{(-K+3t)^2 - 9t(4O + mt - 2mv_2)/2}$. Finally, we have $\pi_B^{SSlowv_2} > \pi_B^{SN}$ if $v_2 > \tilde{v}_{SN\sim L} = \sqrt{2/m}\sqrt{9(2O-t)t + K(6t-K)}/3$ and $c > c_{SN/SS\sim L} = 3t - K - \sqrt{(-K+3t)^2 + 9(mv_2^2 - 4Ot)/2}$.

We prove $\pi_B^{SS} > \pi_B^{SN}$ guarantees $\pi_A^{SS} > \pi_A^{NS}$. We know $c_{NS/SS\sim r}\Big|_{K\to 0} = c_{SN/SS\sim r}\Big|_{K\to 0}$ for $r \in \{L, M, H\}$. We also have $\frac{\partial c_{NS/SS\sim r}}{\partial K} < \frac{\partial c_{SN/SS\sim r}}{\partial K}$ for $r \in \{L, M, H\}$, when $K > 0$. Thus, for $K > 0$, we have $c_{NS/SS\sim r} < c_{SN/SS\sim r}$. We also have $\tilde{v}_{SN\sim M} > \tilde{v}_{NS\sim M}$ and $\tilde{v}_{SN\sim L} > \tilde{v}_{NS\sim L}$. Thus, for $c > c'_+ = c_{SN/SS\sim r}$ and $v_2 > \tilde{v} = \tilde{v}_{SN\sim r}$, we have $\pi_B^{SS} > \pi_B^{SN}$ and $\pi_A^{SS} > \pi_A^{NS}$, and therefore {sell, sell} is an equilibrium.

Considering the profits when neither firm sells the spillover-producing product, we find $\pi_A^{NN} > \pi_A^{SN}$ for $v_2 < v_{2SN/NN} = \sqrt{2/m}\sqrt{-2cK - c^2 - 6ct + 18Ot}/3$. Similarly, we find $\pi_B^{NN} > \pi_B^{NS}$ for $v_2 < v_{2NS/NN} = \sqrt{2/m}\sqrt{2cK - c^2 - 6ct + 18Ot}/3$. We have $v_{2SN/NN} < v_{2NS/NN}$. Thus $\pi_A^{NN} > \pi_A^{SN}$ guarantees $\pi_B^{NN} > \pi_B^{NS}$ and {not sell, not sell} is an equilibrium for $v_2 < \dot{v}_2 = v_{2SN/NN}$. *Q.E.D.*

VI) Proof of Proposition 4:

Based on the proof of Lemma 2, assuming $v_2 > \tilde{v}$, the {sell, not sell} equilibrium exists for $c < c'_+ = c_{SN/SS\sim r}$ and $v_2 > \dot{v}_2 = v_{2SN/NN}$. Also the {not sell, sell} equilibrium exists for $c < c''_+ = c_{NS/SS\sim r}$ and $v_2 > \ddot{v}_2 = v_{2NS/NN}$. We showed $\dot{v}_2 < \ddot{v}_2$ and thus the region for $\pi_B^{NS} > \pi_B^{NN}$ is a subset of the region for $\pi_A^{SN} > \pi_A^{NN}$. We also showed $c'_+ > c''_+$ and thus the region for $\pi_A^{NS} > \pi_A^{SS}$ is a subset of the region for $\pi_B^{SN} > \pi_B^{SS}$. Thus, the {sell, not sell} equilibrium is unique for $c''_+ < c < c'_+$ and $\dot{v}_2 < v_2 < \ddot{v}_2$.

The condition for the {sell, not sell} equilibrium risk-dominating the {not sell, sell} equilibrium is $RD_{SN} = (\pi_A^{SN} - \pi_A^{NN})(\pi_B^{SN} - \pi_B^{SS}) - (\pi_A^{NS} - \pi_A^{SS})(\pi_B^{NS} - \pi_B^{NN}) > 0$. For $3t/2 < v_2 < 2t$, we find $RD_{SN} = cK(4c^2 - 18mt^2 + 9mv_2^2)/162t^2$. Since $3t/2 < v_2 < 2t$, we have $RD_{SN} > 0$ and thus the {sell, not sell} equilibrium is risk-dominant. The proof for low and medium values of v_2 follows similar steps. *Q.E.D.*

VII) Proof of Proposition 5:

If no negative market spillover exists, then the firms' profits for $O > mv_2^{\,2}/4t$ would be the same as in the {not sell, not sell} subgame shown in Table A5. We denote these profits $\pi_A^{\,Z}$ and $\pi_B^{\,Z}$. In the {sell, not sell} equilibrium, we have $\pi_A^{\,Z} = \pi_A^{\,NN} < \pi_A^{\,SN}$. Considering the inferior firm, we have $\pi_B^{\,SN} = \left(3t - K - c\right)^2/18t < \pi_B^{\,Z} = \left(3t - K\right)^2/18t$. In the {sell, sell} equilibrium, we have $\pi_A^{\,Z} > \pi_A^{\,SS}$ and $\pi_B^{\,Z} > \pi_B^{\,SS}$, since $O > mv_2^{\,2}/4t$. *Q.E.D.*

Proof for Total Industry Profits: We find the region where the market spillover increases the sum of the two firms' profits. From Table A5, We find $(\pi_A^{\,NN} + \pi_B^{\,NN}) < (\pi_A^{\,SN} + \pi_B^{\,SN})$ for $c > c_T' = \sqrt{K^2 + 9Ot - 9mv_2^2/4} - K$. Thus, the positive market spillover increases total profits in the {sell, not sell} equilibrium for $c > c_T'$. *Q.E.D.*

VIII) Proof of Lemma 3

Firms' profits from each of the subgames are presented in Table A6.

Table A6. Profits with both positive and negative spillovers (assuming and $3t/2 < v_2 < 2t$)

	B: Sell Spillover-Producing Product	B: Not Sell Spillover-Producing Product
A: Sell Spillover-Producing Product	$If\ \frac{3t}{2} < v_2 < 2t: \begin{cases} \pi_A^{\,SShighv_2} = (K/3+t)^2/2t + mt/2 - O \\ \pi_B^{\,SShighv_2} = (t-K/3)^2/2t + mt/2 - O \end{cases}$ $If\ t < v_2 < \frac{3t}{2}: \begin{cases} \pi_A^{\,SSmedv_2} = (K/3+t)^2/2t + m(v_2 - t/2)/2 - O \\ \pi_B^{\,SSmedv_2} = (t-K/3)^2/2t + m(v_2 - t/2)/2 - O \end{cases}$ $If\ 0 < v_2 < t: \begin{cases} \pi_A^{\,SSlowv_2} = (K/3+t)^2/2t + mv_2^{\,2}/4t - O \\ \pi_B^{\,SSlowv_2} = (t-K/3)^2/2t + mv_2^{\,2}/4t - O \end{cases}$	$\pi_A^{\,SN} = \alpha((K + c_{pos})/3 + t)^2/2t +$ $(1-\alpha)((K - c_{neg})/3 + t)^2/2t +$ $mv_2^{\,2}/4t - O$ $\pi_B^{\,SN} = \alpha((K + c_{pos})/3 - t)^2/2t +$ $(1-\alpha)((K - c_{neg})/3 - t)^2/2t$
A: Not Sell Spillover-Producing Product	$\pi_A^{\,NS} = \alpha((K - c_{pos})/3 + t)^2/2t +$ $(1-\alpha)((K + c_{neg})/3 + t)^2/2t$ $\pi_B^{\,NS} = \alpha((-K + c_{pos})/3 + t)^2/2t +$ $(1-\alpha)((-K - c_{neg})/3 + t)^2/2t +$ $mv_2^{\,2}/4t - O$	$\pi_A^{\,NN} = (K/3+t)^2/2t$ $\pi_B^{\,NN} = (-K/3+t)^2/2t$

Comparing the profits from Table A6, we find the conditions for each equilibrium. We show the proof for $3t/2 < v_2 < 2t$. The proof for lower v_2 includes similar steps. We have $\pi_A^{SShighv_2} > \pi_A^{NS}$ for $O < O_{lA} = (2K(\alpha c_{pos} + (\alpha - 1)c_{neg}) + \alpha(c_{pos} + c_{neg})(-c_{pos} + c_{neg} + 6t) - c_{neg}^{\ 2} - 6c_{neg}t + 9mt^2)/18t$. Also for $O < O_{lB} = (-2K(\alpha c_{pos} + (\alpha - 1)c_{neg}) + \alpha(c_{pos} + c_{neg})(-c_{pos} + c_{neg} + 6t) - c_{neg}^{\ 2} - 6c_{neg}t + 9mt^2)/18t$, we have $\pi_B^{SShighv_2} > \pi_B^{SN}$. Thus, the {sell, sell} equilibrium exists for $O < O_l = Min\{O_{lA}, O_{lB}\}$. Note that for $\alpha < c_{neg}/(c_{pos} + c_{neg})$, we have $O_{lA} < O_{lB}$; otherwise, we have $O_{lA} > O_{lB}$.

Next, we find the condition for the {not sell, not sell} equilibrium. We have $\pi_A^{NN} > \pi_A^{SN}$ for $O > O_{hA} = (2\left(2K(\alpha c_{pos} + (\alpha - 1)c_{neg}) + \alpha(c_{pos} + c_{neg})(c_{pos} - c_{neg} + 6t) + c_{neg}(c_{neg} - 6t)\right) + 9mv_2^2)/36t$. Also, for $O > O_{hB} = (2\left(-2K(\alpha c_{pos} + (\alpha - 1)c_{neg}) + \alpha(c_{pos} + c_{neg})(c_{pos} - c_{neg} + 6t) + c_{neg}(c_{neg} - 6t)\right) + 9mv_2^2)/36t$, we have $\pi_B^{NN} > \pi_B^{NS}$. Thus, the {not sell, not sell} equilibrium exists for $O > O_h = Max\{O_{hA}, O_{hB}\}$. Note that for $\alpha < c_{neg}/(c_{pos} + c_{neg})$, we have $O_{hA} < O_{hB}$; otherwise, we have $O_{hA} > O_{hB}$.

The {not sell, sell} equilibrium exists for $O_{lA} < O < O_{hB}$ and the {sell, not sell} equilibrium exists for $O_{lB} < O < O_{hA}$. Thus, both equilibria exist when $Max\{O_{lA}, O_{lB}\} < O < Min\{O_{hA}, O_{hB}\}$. For this region, we find the risk dominant equilibrium. The condition for the risk dominance of the {sell, not sell} equilibrium is $RD_{SN} = (\pi_A^{SN} - \pi_A^{NN})(\pi_B^{SN} - \pi_B^{SS}) - (\pi_A^{NS} - \pi_A^{SS})(\pi_B^{NS} - \pi_B^{NN}) > 0$. We find $RD_{SN} = K(\alpha c_{pos} - (1-\alpha)c_{neg})\left(4\alpha c_{pos}^{\ 2} + 4(1-\alpha)c_{neg}^{\ 2} - 18mt^2 + 9mv_2^2\right)/162t^2$. For $v_2 > 3t/2$, we have $RD_{SN} > 0$ if and only if $\alpha > c_{neg}/(c_{pos} + c_{neg})$. Similarly, we find $RD_{NS} = -RD_{SN} > 0$ if and only if $\alpha < c_{neg}/(c_{pos} + c_{neg})$. *Q.E.D.*

IX) Proof for the Model of Market Spillover as a Change in Purchase Quantity Per Consumer:

We show the proof of analysis for high v_2, $v_2 > 3t/2$.

When the market spillover is negative: Comparing profits from Table 3, we find $\pi_A^{SS} > \pi_A^{NS}$ for $g < g_{NS/SS} = (3t + K)^2/((3t + K)^2 - 9mt^2)$. Also, we find $\pi_B^{SS} > \pi_B^{SN}$ for

$g < g_{SN/SS} = (3t-K)^2/((3t-K)^2 - 9mt^2)$. For all positive values of $g_{NS/SS}$ and $g_{SN/SS}$, we have $g_{SN/SS} > g_{NS/SS}$. Similarly, we find $\pi_A^{SN} > \pi_A^{NN}$ for $v_2 > v_{SN-} = \sqrt{2-2/g}\left(K+3t\right)/3\sqrt{m}$. Also, $\pi_B^{NS} > \pi_B^{NN}$ for $v_2 > v_{NS-} = \sqrt{2-2/g}\left(-K+3t\right)/3\sqrt{m}$. We have $v_{NS-} < v_{SN-}$. Thus, the {sell, not sell} equilibrium is a subset of the {not sell, sell} equilibrium region. When both equilibria exist, the condition for the risk-dominance of the {not sell, sell} equilibrium is $RD_{NS} = \left(g-1\right)m\left(v_2^2 - 2t^2\right)\left(v_A - v_B\right)/(6gt) > 0$. Thus, for $v_2 > 3t/2$, the {not sell, sell} equilibrium is risk dominant.

When no market spillover exists, we have $\pi_A^{Z} = (K/3+t)^2/2t + mt/2$ and $\pi_B^{Z} = (-K/3+t)^2/2t + mt/2$. Thus, $\pi_A^{NS} = \pi_A^{Z} - mt/2 < \pi_A^{Z}$. Finally, comparing the two firms' profits, $\pi_B^{NS} > \pi_A^{NS}$ for $g < (3t-K)^2/((3t+K)^2 - 9mv_2^{\,2}/2)$. Considering the necessary condition $g > g_{NS/SS}$, we find $v_2 > \sqrt{8Kt(3t+K)^2/3m + 2t^2(3t-K)^2}\,/(3t+K)$ must also hold. *Q.E.D.*

When the market spillover is positive: For $g > g_{NS/SS} = 1 + 9t(2O - mt)/\left(K+3t\right)^2$, we have $\pi_A^{SS} > \pi_A^{NS}$. Also, $\pi_B^{SS} > \pi_B^{SN}$ for $g > g_{SN/SS} = 1 + 9t(2O - mt)/\left(3t-K\right)^2$. Thus, we have $g_{SN/SS} > g_{NS/SS}$. We find $\pi_A^{SN} > \pi_A^{NN}$ for $v_2 > v_{SN+} = \sqrt{2/m}\sqrt{9t(-gt+2O+t) - (g-1)K\left(K+6t\right)}/3$. Also, $\pi_B^{NS} > \pi_B^{NN}$ for $v_2 > v_{NS+} = \sqrt{2/m}\sqrt{(g-1)K\left(-K+6t\right) + 9t(-gt+2O+t)}/3$. Since $g > 1$, we have $v_{NS+} > v_{SN+}$. Thus, the {not sell, sell} equilibrium is a subset of the {sell, not sell} equilibrium region. For the region where both equilibria exist, the condition for the risk dominance of the {sell, not sell} equilibrium is $RD_{SN} = \dfrac{(g-1)m\left(v_2^2 - 2t^2\right)K}{6t} > 0$, which holds for $v_2 > 3t/2$.

Without any market spillover, neither firm sells the spillover-producing product and we have $\pi_A^{Z} = (3t+K)^2/18t$ and $\pi_B^{Z} = (3t-K)^2/18t$. We have $\pi_A^{SS} > \pi_A^{Z}$ for $g > g_{NS/SS}$. Also, $\pi_B^{SS} > \pi_B^{Z}$ for $g > g_{SN/SS}$. Thus, in the region for the {sell, sell} equilibrium, where $g > g_{SN/SS} > g_{NS/SS}$, firms earn higher profits with a positive market spillover than they would have earned without a market spillover. *Q.E.D.*

X) Ratings-Based Conjoint Studies:

We designed two ratings-based conjoint studies (see Schindler (2011), pp. 56-62) where we varied prices and whether the seller also sold unhealthy goods. In study 1, we studied the effect of a pharmacy selling tobacco on the subjects' willingness to pay for unrelated products from that pharmacy. We 91 participants on Amazon Mechanical Turk were told that "Pharmacy A sells cigarettes and other tobacco products in addition to medical drugs" and "Pharmacy B sells medical drugs, but does NOT sell tobacco products." They were then asked to state their likelihood of purchasing travel immunization consulting from each of the pharmacies at the prices of \$10 and \$12 on a scale of 0 to 10. We regressed the ratings of likelihood to purchase on price and a dummy variable for the presence of tobacco. The results are shown in Table A7.

Table A7. Regression Results for Study 1

	Coefficients	Standard Error
Price	-0.59*	0.14
Tobacco Sale (dummy)	-2.62*	0.29
Intercept	14.05*	1.59

** p-value < 0.001*

Table A8. Regression Results for Study 2

	Coefficients	Standard Error
Price	-1.84*	0.12
Confectionery Sale (dummy)	-1.11*	0.20
Intercept	13.73*	0.51

** p-value < 0.001*

As described in Schindler (2011), the negative value of selling tobacco is calculated by

$\frac{|\text{coefficient of dummy variable}|}{|\text{coefficient of price}|}$. This gives the estimate of a negative market spillover of \$4.41.

Study 2 analyzed the effect of a grocery store selling confectionery at checkout lines on subjects' willingness to pay for unrelated products at that store. In this study, 101 Amazon Mechanical Turk participants were told that "Grocery store A sells candy, chocolates, and other sugar-filled treats at their checkout lines" and "Grocery store B has removed candy, chocolates, and other sugar-filled treats from their checkout lines, replacing them instead with nuts, dried fruit, trail mixes, water, and other healthy snacks." Then, they were asked to state their likelihood of purchasing a healthy salad from each store's salad bar at the prices of \$3, \$4, and \$5 on a scale of 0 to 10. The results of the regression are shown in Table A8. The estimated negative value of selling confectionery is \$0.61.