

Product Diversion to a Direct Competitor

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April 2012

Abstract

A manufacturer will often limit competition among downstream partners by authorizing only a select group of retailers to carry its product. However, it is not uncommon for authorized retailers to create an additional competitor by diverting units to an unauthorized seller. Prior research proposes that this diversion will not be optimal unless the diverted units are sold in new markets, expand demand, or take advantage of price differentials created by channel incentives. However, anecdotal evidence suggests that retailers sell to unauthorized sellers who then steal customers from the diverting retailer. This paper demonstrates in an analytical model that diversion from authorized retailers to an unauthorized direct competitor can occur even without previously viewed necessary conditions. In fact, diversion can represent a prisoner's dilemma whereby retailers diminish their own profit by selling to the unauthorized direct seller. The unauthorized direct seller earns greater profit by procuring fewer units from retailers than it could profitably sell to consumers even though such an order results in an equivalent increase in the quantity sold by the retailers. When authorized retailers are vertically differentiated, the inferior retailer diverts more units and has greater total sales than the higher quality retailer, though it has fewer sales to consumers.

Keywords: Pricing, Gray Markets, Game Theory

¹ I am grateful for the insightful feedback provided by Professors Eyal Biyalogorsky, Theodore Klastorin, and Fabio Caldieraro. I acknowledge funding provided by the Michael G. Foster faculty fellowship.

1. Introduction

In a personal interview, an executive from a producer of a popular trading card game expressed a conundrum his company faced.² The company purposefully authorized sales only by brick-and-mortar retailers who helped build the value of the brand via gaming events, service, and promotions. However, the authorized retailers expressed concern about profit loss due to competition from a low-priced unauthorized online seller. Interestingly, the executive found that retailers themselves were diverting to the direct online competitor. A common cause for this behavior is that a quantity discount incentivizes retailers to buy in bulk to achieve a discount and divert the unneeded units. However, this diversion occurred despite the absence of a quantity discount. In fact, diversion of this company's products occurred under conditions that existing theory would not have predicted. This phenomenon, which has also emerged in markets as diverse as pet medicine, consumer electronics, and salon products, raises the following question addressed in this paper: Why would an authorized retailer divert units to a direct competitor, who serves the same market of consumers, without an obvious incentive to do so?

Manufacturers often sell only to select authorized retailers. Among many reasons for this practice, selective distribution can reduce channel conflict and reward valuable retailers with higher profit (Antia, Bergen, Dutta, Fisher 2006). However, retailers themselves often undermine this process by selling the products to unauthorized direct sellers. For instance, salon product manufacturers often authorize only salons to serve as resellers of the product, yet these salons often divert to online competitors such as drugstore.com (Palmer 2004). Authorized resellers of Pioneer and Harman Kardon electronics also divert to unauthorized online sellers within the same market.³ In the pet flea medicine industry, it is estimated that diversion to unauthorized direct sellers totals as much as \$300 million per year (Luechtefeld 2008). This practice stretches into many industries including sporting goods, scuba gear, and electronics.⁴ While

² Interview with Director of Hobby Sales on November 19, 2008 and May 31, 2011. Company requested anonymity.

³ See "HARMAN'S STRICT WEB POLICY GETS STRICTER." *Consumer Electronics* 16 Aug. 1999. *Academic OneFile*. Web. 25 Apr. 2011.

⁴ For example, speaker company, Velodyne and sporting goods retailer, Tennis Company (see http://www.tenniscompany.com/Question_Price_Match.html) are two retailers who remarked on their websites that direct sellers obtain diverted units from authorized retailers.

diversion to different geographic markets takes advantage of the arbitrage opportunities created by disparity in wholesale prices, the above examples represent diversion within the same market. This is a practice known as *channel flow diversion* (Lowe and Rubin 1986). Interestingly channel flow diversion can occur even if the unauthorized direct sellers ultimately steal customers from the retailer who divert the product in the first place. For instance, it is estimated that this diversion costs salons \$100 million a year in lost sales (Gordon 1997) with one salon estimating a 10% loss in Aveda sales to unauthorized sellers (Palmer 2004).

The purpose of this paper is to resolve the discrepancy between theory and practice regarding channel flow diversion. Specifically, prior research has led to theory that sets out necessary conditions for diversion to occur. Notably, Duhan and Sheffet (1988) propose that diversion requires a price differential occurring due to currency exchange rates, differences in demand, or a segmentation strategy. While there are many examples of diversion that satisfy the necessary conditions (Antia, Bergen, Dutta, and Fisher 2006), the trading card game producer mentioned above observed diversion in the absence of a necessary condition.

To summarize, the purpose of this paper is to answer the following research questions:

1. Why might diversion to a direct competitor occur in the absence of previously viewed necessary conditions?
2. What are the profit implications of the retailers' cost to divert when diversion occurs without previously viewed necessary conditions?
3. What are the implications for profit and diversion quantities of quality asymmetry between retailers when diversion occurs without previously viewed necessary conditions?
4. What is the optimal number of units that the unauthorized direct seller will want to divert from authorized retailers?

Concerning the first research question, the model overturns conventional wisdom and prior research by establishing an additional reason for channel flow diversion. I find that diversion to a competitor can occur even if the competitor does not expand the market and the retailer buys at a constant marginal cost

per unit (i.e., no quantity discount schedule). In fact, retailers may divert to an unauthorized direct seller even if diversion represents a prisoner's dilemma.

The second research question is motivated by the fact that some products are more costly to divert than others. For instance, it costs more to ship heavier bottles of hair products than trading cards. Given the conditions under which diversion occurs, common intuition would suggest that retailers would earn greater profit when they have lower costs. Also, from an operations perspective, logic would suggest that if diversion is going to take place, it would be better for the process to be more efficient and less costly. However, this paper identifies a strategic effect of diversion costs. In fact, the model finds that retailers' profits can be increase with increases in the cost to divert.

Third, retailers selling identical goods may create asymmetric value to customers via the level of service they provide such as qualified sales personnel, faster checkout lines and more parking (Moorthy and Zhang 2006). Therefore, this research aims to offer predictions as to which retailer is more likely to divert to an unauthorized direct seller. On the one hand, a higher quality retailer may be more likely to divert since its quality affords it greater differentiation from the direct seller and thus diminished likelihood of lost sales. On the other hand, the higher quality retailer may be less likely to divert since it commands a higher selling price and thus each sale lost to the direct seller is more costly.

I also aim to determine whether the superior or inferior retailer will suffer greater profit loss from the presence of the unauthorized direct competitor. Intuitively, the superior retailer is more insulated from unauthorized direct seller due to its quality position. However, I show that inferior retailer may not only suffer less from the presence of the unauthorized direct seller, but even outperform the superior retailer.

In contrast to the three retailer-focused research questions, the final research question examines the unauthorized direct seller's decision. Specifically, because the procurement of diverted units occurs before the retail prices are set, an important question becomes whether the unauthorized direct seller will procure as many units as can profitably be sold. Research on quantity choice suggests that a Stackelberg leader will commit to a greater quantity than in a simultaneous move game as a means to encourage the followers to sell a lesser quantity. However, strategic quantity-commitments by all sellers have been

shown to shift competition from Bertrand to Cournot and thus boost profit. Thus it is unclear whether an unauthorized direct seller would choose to 1) procure as many units as can be profitably sold, 2) strategically procure excess units, or 3) strategically procure fewer units. I show that in this situation, the unauthorized direct seller maximizes profit by strategically committing to buy a quantity of diverted units that will *unilaterally* constrain the unauthorized direct seller in subsequent pricing to consumers.

In total, the findings help to explain the intriguing phenomenon of within-market diversion and illustrate the consequences for pricing and profits. They demonstrate to manufacturers that diversion may not be avoided simply by ensuring there are no price differentials between markets. The findings inform manufacturers that lower-quality retailers are more likely to be the culprit of within-market diversion. This paper demonstrates to retailers that technologies that reduce diversion costs do not necessarily increase profit even if diversion occurs and demonstrates to unauthorized direct sellers that the procurement of diverted units has strategic consequences that should be considered.

2. Literature Review

Channel flow diversion is a type of gray market. Duhan and Sheffet (1988) outline necessary conditions for gray market diversion to occur. The necessary condition overturned in the current research is a price differential occurring due to currency exchange rates, differences in demand, or a segmentation strategy. The segmentation strategy is considered viable if there is downward sloping demand and the price differential can attract different segments. The latter condition is shown to increase manufacturer profit by Ahmadi and Yang (2000) and also Xiao, Palekar and Liu (2011).⁵ In my model, I show that these conditions are not necessary for channel flow diversion to occur. In fact, diversion is shown to be an equilibrium strategy even when total demand is fixed and segmentation across consumers results in lower prices without additional units sold. Thus, I find an additional reason for diversion that is not considered in the literature.

⁵ Similarly, Chen and Riordan (2007) find that seller profit in a spokes model of consumer demand can increase with entry of a new seller. This result is driven by the fact that new entry has a market expansion effect.

In this model I examine diversion when a manufacturer limits distribution to a select group of authorized retailers. Fein and Anderson (1997) use transaction cost theory to describe why manufacturers agree to selective distribution with distributors who agree to selective assortment. Dutta, Bergen, and John (1994) illustrate when a reseller will encroach upon another reseller's exclusive territory and when the manufacturer will tolerate some level of this encroachment. Kalnins (2004) empirically demonstrates that the markets of franchisees are often encroached by new franchises approved by the franchisor. In contrast, the current paper establishes that authorized retailers will create their own competition by diverting to an independent, unauthorized direct seller.

Another line of prior research examines the impact of *authorized* direct sellers on brick and mortar retailers. In a seminal paper, Balasubramanian (1998) identifies equilibrium pricing when local retailers face competition from a direct seller and shows that the direct seller may earn greater profit by strategically limiting the number of customers who are aware of its offering. A body of research examines the causes and implications of a manufacturer selling directly to consumers through a vertically integrated channel that competes with its independent resellers (e.g., Chiang, Chhajed, and Hess 2003; Vinhas and Anderson 2005; Kumar and Ruan 2006; Yoo and Lee 2011). Another body of research looks at multi-channel "bricks and clicks" retailers who sell online and in stores (Zhang 2009; Viswanathan 2005; Ofek, Katona, Sarvary 2011). Several papers empirically examine pricing competition between online sellers and traditional retailers (Brynjolfsson and Smith 2000; Brynjolfsson, Hu, and Rahman 2009; Overby and Jap 2009, Forman, Ghose, and Goldfarb 2009). In contrast, this paper focuses on situations in which the manufacturer does not authorize a direct channel, yet the authorized retailers endogenously choose to divert to an unauthorized direct seller.

Transshipment literature finds that a retailer who experiences demand at the location exceeding the supply on-hand will be able to fulfill the order by buying from another retailer. For instance, a Macy's in Chicago facing an order from a customer will be able to satisfy this order by getting a Macy's in a nearby suburb to transship to the store in Chicago. A body of transshipment research examines ordering and inventory decisions when transshipments between resellers are coordinated centrally (e.g., Lee 1987;

Robinson 1990; Axsater 1990). Companies using centralized transshipment benefit from risk pooling and hence offset the costs associated with transshipment. Transshipment between retailers can also occur when inventory orders and transshipment purchase decisions are made locally by the independent resellers (e.g., Rudi, Kapur and Pyke 2001; Hu, Duenyas, and Kapuscinski 2007; Huang and Susic 2010). Whereas the focus in transshipment literature is how firms cooperatively adjust regional supply to match regional demand, diversion between non-cooperative players occurs in the model presented below.

The finding that retailers may divert to a direct competitor even if the end result is a reduction in profit contributes to the growing body of research identifying prisoners' dilemmas that firms face in a broad range of marketing contexts. Prior research finds that prisoner's dilemmas arise in targeted coupons (e.g., Shaffer and Zhang 1995; Chen, Narasimhan, and Zhang 2001), channel-partner choice (Coughlan 1985), loss-leader pricing with rain checks (Hess and Gerstner 1987), advertising (e.g., Chen, Joshi, Raju, and Zhang 2009), and exchange programs (Desai, Purohit, and Zhou 2011). This research demonstrates when retailers cannot govern themselves from engaging in self-defeating diversion to a direct seller.

Finally, I identify the optimal number of diverted units ordered by the unauthorized direct seller. Kreps and Scheinkman (1983) find that capacity commitment can move undifferentiated competitors from Bertrand (and hence zero profit) to Cournot (and hence positive profit) competition. Similarly, Maggi (1996) finds that differentiated competitors can mitigate competition via quantity commitments. Thus competition intensity can be reduced when *both firms* commit to quantity before choosing pricing. When one firm commits to quantity, Daughety (1990) finds a Stackelberg leader will commit to a *greater* quantity than in a simultaneous move game. In contrast, I find a scenario in which one seller (the unauthorized direct seller) commits to a *lesser* quantity even though its competitors are not only unconstrained but also sell greater quantities than if no quantity commitment were made.

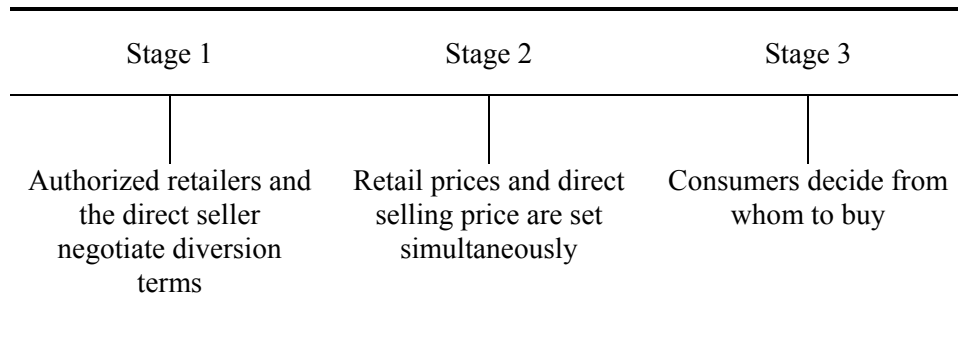
In summary, I make several main contributions to the existing literature. First, I demonstrate how diversion can occur without the previously assumed necessary conditions. This occurs even if diversion results in less profit for the authorized retailers who divert. Secondly, this is the first research to show that the profit of a retailer who diverts is increasing in the per-unit cost to divert. Third, this is the first

research to show how diversion to an unauthorized direct competitor differentially affects asymmetric retailers. Finally, I am the first to show how a unilateral commitment to a lower quantity by the unauthorized direct seller can improve profit even when it results in an equivalent increase in quantity sold by the competing retailers.

3. The Model

In this section, I describe the players of the game and their payoff functions. All notations are summarized in Table 1. There are four types of players in the game. There are horizontally differentiated authorized retailers who procure units from the manufacturer. There is an unauthorized direct seller who only has access to units if it can buy them from the authorized retailers. There are consumers who are heterogeneous in their preference for sellers. There is a manufacturer who produces a single product and relies on intermediaries to reach consumers. Figure 1 establishes the timing of the model and the following subsections further describe the decisions and payoffs for each of the players.

Figure 1: Timing of the Model



3.1. Firms

Authorized Retailers

There are N authorized retailers distributed evenly along the Salop circle (with location of retailer j denoted by x_j) who procure products from the manufacturer at a per-unit rate of w in a single-period. Retailer j chooses a retail price p_j to charge to consumers. The quantity sold to consumers by retailer j is q_j . Retailer j may also sell $q_{jt} \geq 0$ units to the unauthorized direct seller at a per-unit diversion price

w_d . Authorized retailers incur an additional cost, c , for each unit diverted to the unauthorized direct seller. Examples of this cost include the opportunity cost of time spent in preparing each unit for diversion and any costs paid to third-party shippers. Profit for the j th authorized retailer is $\pi_j = (p_j - w)q_j + (w_d - w - c)q_{jt}$.

While I establish results at the retailer level for any given number of authorized retailers, N , and any wholesale price set by the manufacturer, it is implicitly assumed that N and w are optimal choices from a strategic manufacturer. A finite value of N will arise in equilibrium if there is a retailer entry cost or a fixed cost to the manufacturer of developing a contractual relationship with a retailer. A greater entry cost will lead to a smaller value of N .⁶ As will be shown, my results hold for any value of w , and thus the results also hold for an endogenously chosen w .

Unauthorized Direct Seller

There is a direct seller located in the center of circle. In this paper, I focus on the scenario when the manufacturer does not authorize the direct seller. Reasons often cited for selective distribution of this kind include diminished incentive for retailers to provide service because of a direct seller's free-riding (Carlton and Chevalier 2001), a higher incidence of product returns in direct channels,⁷ diminished manufacturer control of product information (Luechtefeld 2008), and protection of the brand name value.⁸ While the model parsimoniously abstracts from many of these factors, I prove analytically in Appendix B that market-expanding service by retailers with free-riding on this service by the direct seller can lead the manufacturer to optimally leave the direct seller unauthorized. The unauthorized direct seller incurs a

⁶ Consider, for example, a market already in pure retail entry equilibrium prior to the emergence of the unauthorized direct seller as also considered by Balasubramanian (1998). From Tirole (1992), p. 283, the equilibrium number of retailers when incurring an entry cost K_r will be $N^* = \sqrt{t / K_r}$. The equilibrium N will be the same if instead of the retailer incurring an entry cost, the manufacturer incurs a fixed cost per retailer with whom an authorized relationship is established (denoted by K_r) and can extract all profit from each retailer.

⁷ See "HARMAN'S STRICT WEB POLICY GETS STRICTER." *Consumer Electronics* 16 Aug. 1999. *Academic OneFile*. Web. 25 Apr. 2011.

⁸ For example, Escort Inc.'s Passport series products are sold only via high-performance car dealers and car audio specialists or from the manufacturer. A manager from Escort Inc. describes the importance of protecting the brand value on http://www.netenforcers.com/pdfs/Net%20Enforcers%20Case%20Study_ESCORT%20Inc.pdf.

fixed cost K_d of being in business. It is assumed that K_d is sufficiently high such that only one direct seller enters the market. The qualitative insights are robust to this assumption because if diversion occurs with one unauthorized seller, it will occur with two unauthorized sellers or else the second unauthorized seller would not enter the marketplace.

The unauthorized direct seller only has access to units if it can buy them from the authorized retailers at a diversion price, w_d . It should be noted that gray market activity is a legal practice. Moreover, it is well documented that manufacturer efforts to deter diversion are often unsuccessful (Antia, Bergen, Dutta, and Fisher 2006). The current model applies to situations in which either the manufacturer passively allows diversion or is unsuccessful in any enforcement attempts occurring outside of this model.

It should also be noted that this characterization of the unauthorized seller represents an extreme case in which its presence does not expand demand. I consider this extreme case for two reasons. First, *previous research suggests that diversion will not occur in this extreme case* and thus its consideration in this paper adds to the literature. Second, if the direct seller significantly expanded demand then the manufacturer would either authorize the direct seller or embrace diversion.⁹ Since there are manufacturers who have expressed discontent with diversion to unauthorized direct sellers (e.g., electronics manufacturers, Yamaha and Philips,¹⁰ hair care manufacturer John Paul Mitchell Systems,¹¹ pet medicine manufacturer Fort Dodge¹²), there are cases in which other forces are at play and consideration of this extreme case allows for isolation of these forces. I comment on the sensitivity of the model to the characterization of the direct seller in the discussion.

⁹ A manufacturer may embrace diversion when the unauthorized seller expands the market rather than authorize each seller in order to limit the number of channel partners with whom costly negotiations are required. I thank Eyal Bialagorsky for this comment. Alternatively, a manufacturer may embrace diversion since it allows retailers to earn a profit on diverted units and facilitates price discrimination via market-expanding low prices from the unauthorized direct seller.

¹⁰ See Rovito, M. 2002. Etailing: The Unauthorized Sale *Dealerscope*, February 2002. Downloaded from <http://www.dealerscope.com/article/e-tailing-the-unauthorized-sale-14555/1> on April 18, 2012

¹¹ See Palmer (2004).

¹² See Luechtefeld (2008).

Manufacturer

The focus of this paper is the diversion from authorized retailers to the unauthorized direct seller. While the manufacturer's choice of w is treated as exogenous, the results will be shown to hold for any w . Thus analysis of the manufacturer's pricing decision is omitted. While the manufacturer's choice of N is treated as a parameter in this model, the equilibrium value of N depends on the retailer's entry cost. The manufacturer's decision not to authorize the direct seller is also treated as given; though in Appendix B, it is shown how the decision by the manufacturer not to authorize the direct seller may be optimal.

3.2. Consumers

Total consumer demand is inelastic (i.e., the reservation value is sufficiently high). The discussion section remarks upon the robustness of the findings to this assumption. Utility of buying from retailer j for consumer i , who is located at θ_i , is equal to $u - t|x_j - \theta_i| - p_j$ where u is the reservation utility from owning the product and t is the transportation cost. Consumers buy at most one unit of the product and their locations are uniformly distributed along the unit circle. A proportion α of consumers will derive utility from buying from the unauthorized direct seller equal to $(u - \mu - p_d)$ where μ is a measure of the lack of fit with the unauthorized direct seller. The lack of fit from the unauthorized direct seller, μ , captures the inconvenience of waiting for delivery, the inability to talk to a live salesperson, the cost of shipping, or the perceived reliability of the direct seller. Thus, they will buy from the unauthorized direct seller if $u - t|x_j - \theta_i| - p_j < (u - \mu - p_d) \forall j$ and will purchase from retailer j that maximizes $u - t|x_j - \theta_i| - p_j$ otherwise. Notice that although μ is constant across consumers, the formulation of the model captures the empirically validated phenomenon that preference for buying online versus buying in the store depends on the consumer's location (Forman, Ghose, and Goldfarb 2009). The remaining $(1 - \alpha)$ consumers will not consider the unauthorized direct seller and will purchase from retailer j that maximizes $u - t|x_j - \theta_i| - p_j$. Potential reasons why a consumer may not consider the direct seller include: lack of awareness of the direct seller's existence, concern regarding purchase through an unauthorized seller, significant impatience in receiving the product, or reliance on live salespersons.

Table 1: Parameters and Decision Variables

Symbol	Definition
w	Marginal cost to authorized retailer per unit acquired
c	Marginal cost to authorized retailer per unit diverted
x_j	Location of retailer j
p_j	Retail price for retailer j
p_d	Selling price for unauthorized direct seller
w_d	Per-unit diversion price paid by unauthorized direct seller to retailers
u	Consumer reservation utility of owning the product
t	Transportation cost for consumers
θ_i	Consumer i 's location
α	Proportion of consumers who consider buying from unauthorized direct seller
q_d	Total quantity sold to consumers by unauthorized direct seller
q_t	Quantity of units sold to unauthorized direct seller by each diverting retailer
q_j	Quantity sold to consumers by firm j
N	Number of authorized retailers

4. Equilibrium Diversion

In this section, I explore whether diversion from authorized sellers to an unauthorized seller can occur in equilibrium when there is a fixed market size. To this end, I impose no assumptions on the process by which the diversion agreement is established. Instead, I solve generally for whether a mutual diversion agreement can be made between parties. For any given quantity of diverted units from each authorized retailer, q_t , I examine whether the unauthorized direct seller's willingness to pay per unit is greater than the lowest acceptable price that an authorized retailer would accept in exchange for diverting the units.

The order of analysis is as follows. I first establish equilibrium pricing and payoffs for the following subgames: The unauthorized seller is unable to procure units or there are S authorized retailers ($S \in [1, N]$) that each divert q_t units to the unauthorized direct seller. I then determine whether the minimum diversion price such that all authorized retailers divert and no authorized retailer would

unilaterally deviate is less than or equal to the diversion price such that the unauthorized direct seller would be willing to buy from the N firms. I then remark upon the manufacturer's stage one decisions.

First consider the instance in which no retailers divert. This boils down to the standard result that retail prices are $p^* = w + t / N$ and the authorized retailers earn profit t / N^2 . When diversion does occur, there are two cases to consider in the pricing game: either the unauthorized direct seller is unconstrained by the quantity procured from the retailers through diversion or the unauthorized direct seller is constrained by the quantity procured from the retailers. I solve for each case separately while generally allowing for any $S \leq N$ number of retailers to each divert a quantity q_t .

Lemmas 1 and 2 in Appendix A describe the prices and payoffs for each seller that will occur for any given set of diversion agreements $\{q_t, w_d\}$. I summarize when diversion will occur and define the set of possible per-retailer diversion quantities and diversion prices in the following proposition. Proof of all Lemmas and Propositions are available in the Appendix.

Proposition 1 *If $\mu < \frac{t(3N - 4 + N\alpha)}{2N^2(1 + \alpha)} - c$, each retailer will divert q_t^* units at a diversion price w_d^* .*

The set of feasible contracts $\{q_t^, w_d^*\}$ are such that $q_t^* \leq \frac{\alpha(t(3N - 4 + N\alpha) - 2N^2(1 + \alpha)(c + \mu))}{(2N - 1)t(N - 2\alpha + 3N\alpha)}$ and*

$$\underline{w}_d(q_t^*) \leq w_d \leq \overline{w}_d(q_t^*) \quad \text{where} \quad \underline{w}_d(q_t) \equiv w + c + \frac{t(2 + (1 - 2N)q_t)}{N^2(1 + \alpha)} \quad \text{and}$$

$$\overline{w}_d(q_t) \equiv w - \mu + \frac{\alpha t(3 + \alpha) - (-1 + 2N)t(1 + 3\alpha)q_t}{2N\alpha(1 + \alpha)}.$$

Proposition 1 demonstrates that it can be an equilibrium strategy for retailers to fuel their direct competition even if the diversion does not expand the market or allow the authorized retailers to tap into new markets. This result is new to the literature. The specific agreement between a retailer and the unauthorized direct seller depends on the bargaining process, though the proposition establishes a range of diversion quantities and diversion prices such that the retailer and the unauthorized direct seller will prefer to reach an agreement. Interestingly, diversion occurs in equilibrium even though the number of

diverted units by each retailer in the diversion equilibrium is equal to the number of customers the direct seller steals from each authorized retailer (i.e., $\forall j, q_j^* = \frac{1}{N} - q_t^*$). Moreover, it is feasible that the diversion price is below the selling price that would arise without diversion (i.e., $\underline{w}_d(q_t^*) < t + 1/N$). In other words, diversion can occur even when every sale to the unauthorized direct retailer comes at a lower price than the resulting lost sale to consumers.

While it is counterintuitive for a retailer to sell its stock to a direct competitor at a lower price than could be sold to consumers, the finding makes sense when one considers the strategic interaction between players. The profit loss associated with an authorized retailer stocking the competition with diverted units is distributed across all competing authorized sellers. Therefore, the marginal loss to each firm from the additional competitor is less than the gain from the trade. Moreover, each firm recognizes that if they do not cooperate with the unauthorized direct seller, the competing authorized retailers will. Thus diversion within a fixed market size can still occur even if the diversion price is less than the retail price of the sale that is lost to the direct seller.

Notice that the number of diverted units does not depend on the wholesale price or fixed fee charged by the manufacturer. Neither does the range of potential diversion prices (i.e., $\frac{\partial[\overline{w}_d(q_t) - \underline{w}_d(q_t^*)]}{\partial w} = 0$). Therefore, Proposition 1 holds generally regardless of the elements of a linear contract between the manufacturer and authorized retailers.

To this point I have shown that diversion can occur in equilibrium even if there is a fixed market size. To determine the specific quantity and diversion price that is agreed upon between the unauthorized direct seller and N authorized sellers requires additional assumptions regarding how the agreement is made. In the following section, I examine implications for a case in which the unauthorized direct seller has the power to make take-it-or-leave-it offers to the retailers.

5. Implications of Diversion

In this section, I make an additional assumption regarding the sequence of events. I assume that the unauthorized direct seller makes symmetric take-it-or-leave-it offers simultaneously to each of the N authorized retailers regarding the quantity to buy from the retailer and the diversion price to pay per unit (i.e., $\{q_t, w_d\}$). The N authorized sellers simultaneously decide whether or not to accept the offer. The unauthorized direct seller and N authorized retailers simultaneously set prices. The timing of the interactions between the retailers and the direct seller are presented in Figure 2 where the key modification relative to Figure 1 are Stages 1a and 1b. While the sequence of events contributes to the parsimony of the subsequent analysis and findings, the key findings are robust to the assumptions, though the specific prices, profits, and quantities are sensitive to the order of the game.

Figure 2: Timing of Retailer-Direct Seller Interactions

Stage 1a	Stage 1b	Stage 2	Stage 3
Unauthorized direct seller offers to buy q_t at price w_d from each retailer	Retailers simultaneously choose to accept or reject $\{q_t, w_d\}$	Retail prices and direct selling price are set simultaneously	Consumers decide from whom to buy

The equilibrium outcome is solved via backward induction and is described in Lemma 3.

Lemma 3 *When the unauthorized direct seller makes take-it-or-leave-it to the N retailers, then diversion from the N authorized retailers to the unauthorized direct seller will occur in equilibrium if the consumer cost of shopping from the direct seller is not too great. Specifically, each authorized retailer will divert*

$$q_t^* = \frac{\alpha(t(3N - 4 + N\alpha) - 2N^2(1 + \alpha)(c + \mu))}{2t(2\alpha - 4N\alpha + N^2(1 + 3\alpha))} \text{ units if } \underline{\mu} \leq \mu < \hat{\mu} \equiv \frac{t(3N - 4 + N\alpha)}{2N^2(1 + \alpha)} - c \text{ where } \underline{\mu} \text{ is}$$

defined in the Appendix.

Lemma 3 identifies the specific quantity of diverted units that each authorized seller will provide. I restrict attention to values of μ such that the authorized retailers do not maximize profit over the $(1 - \alpha)$ consumers who only consider authorized retailers and forsake the consumers who also consider the unauthorized direct seller (i.e., $\mu \geq \underline{\mu}$ as defined in the Appendix). The equilibrium prices, quantities, and profits are presented in Table 2. I next look at the unauthorized direct seller's ordering decision.

Table 2: Equilibrium Prices, Quantities, and Profits

$\mu < \left(\frac{t(3N - 4 + N\alpha)}{2N^2(1 + \alpha)} - c \right)$	$\mu \geq \left(\frac{t(3N - 4 + N\alpha)}{2N^2(1 + \alpha)} - c \right)$
Diversion Occurs	No Diversion Occurs

Retail Price for each Authorized Retailer

$$p_r^* = w + \frac{1}{N + N\alpha} \left(t + \frac{N\alpha(-t(3N - 4 + N\alpha) + 2N^2(1 + \alpha)(c + \mu))}{2(2\alpha - 4N\alpha + N^2(1 + 3\alpha))} \right) \quad p_r^* = w + \frac{t}{N}$$

Unauthorized Direct Seller Price

$$p_d^* = w + \frac{1}{(4N(1 + \alpha)(2\alpha - 4N\alpha + N^2(1 + 3\alpha)))} [4t\alpha(3 + \alpha) + 2N^3(1 + 4\alpha + 3\alpha^2)(c - \mu) - 4N(t(-1 + 3\alpha + 2\alpha^2) + 2\alpha(1 + \alpha)\mu) + N^2(t(3 + 10\alpha + 3\alpha^2) + 16\alpha(1 + \alpha)\mu)] \quad \text{N/A}$$

Quantity sold to Consumers by Each Authorized Retailer

$$q_r^* = \frac{4t\alpha - 4Nt\alpha + N^2t(2 + 3\alpha - \alpha^2) + 2N^3\alpha(1 + \alpha)(c + \mu)}{2Nt(2\alpha - 4N\alpha + N^2(1 + 3\alpha))} \quad q_r^* = \frac{1}{N}$$

Unauthorized Direct Seller Quantity

$$q_d^* = \frac{N\alpha(t(3N - 4 + N\alpha) - 2N^2(1 + \alpha)(c + \mu))}{2t(2\alpha - 4N\alpha + N^2(1 + 3\alpha))} \quad 0$$

Contribution for each Authorized Retailer

$$\pi_r^* = \frac{1}{4t(1 + \alpha)(2\alpha - 4N\alpha + N^2(1 + 3\alpha))^2} [t(-1 + \alpha)\alpha + 2N^2\alpha(1 + \alpha)(c + \mu) - N(t(-2 - 3\alpha + \alpha^2) + 2\alpha(1 + \alpha)(c + \mu))]^2 \quad \pi_r^* = \frac{t}{N^2}$$

Unauthorized Direct Seller Contribution

$$\pi_d^* = \frac{\alpha(2cN^2(1 + \alpha) - t(-4 + N(3 + \alpha)) + 2N^2(1 + \alpha)\mu)^2}{8Nt(1 + \alpha)(2\alpha - 4N\alpha + N^2(1 + 3\alpha))} \quad 0$$

Proposition 2 *The unauthorized direct seller strategically buys a quantity of diverted units that leaves it unilaterally constrained by quantity in setting selling price. In other words, the equilibrium quantity sold by the unauthorized direct seller is less than the quantity that would be sold in the unconstrained pricing game (i.e., $q_d^* < q_d^{**}$).*

In the pricing subgame, the unauthorized direct seller can sell no more than the number of units diverted by the authorized retailers in the previous stage. Recall from Lemmas 1 and 2 that the equilibrium prices (and resulting quantity sold) depend on whether or not the unauthorized direct seller procures sufficient supply to satisfy the resulting demand. If the unauthorized direct seller has limitless supply, the equilibrium prices will lead to direct sales equal to

$$q_d^{**} = \frac{N\alpha(t(3N-4+N\alpha) - 2N^2(1+\alpha)(c+\mu))}{2t(\alpha - 2N\alpha + N^2(1+2\alpha))}. \text{ However, the unauthorized direct seller will limit}$$

itself to procuring $q_d^* < q_d^{**}$ units. While a typical game in which a firm can Stackelberg lead in its choice of quantity will lead to a higher choice of quantity than a simultaneous move game (Daughety 1990), here the unauthorized direct seller boosts profit by strategically buying fewer units than would subsequently be optimal to sell. In my model, the inability to procure units directly from the manufacturer presents the unauthorized direct seller an opportunity to credibly constrain capacity in its offer to buy from authorized retailers. By buying a lesser quantity from the authorized retailers, the unauthorized direct seller mitigates price competition and raises equilibrium prices by all sellers. This is counter-intuitive given that the *unilateral* commitment to a lower quantity results in a greater quantity sold by the authorized resellers who could still choose to drive down prices.

This capacity constraint, though, limits the unauthorized direct seller's incentive to cut price because it cannot serve demand beyond its supply of units. Also, the diversion price, w_d , paid to the authorized retailers is higher with the constrained capacity, further diminishing the unauthorized direct seller's incentive to cut price. As a consequence, the competing retailers can be less aggressive on price and still

achieve sales of a given quantity. Thus, the unauthorized direct seller's unilateral commitment to divert fewer units than would subsequently be optimal to sell softens price competition and benefits all sellers.

It should be noted that this result occurs when the diversion agreements are established *before* the selling prices. However, logic suggests it is robust to the assumption that the unauthorized direct seller makes take-it-or-leave-it offers. This is because, in a bargaining game, the equilibrium quantity and wholesale price will lie somewhere between the retailers' preferred values and the unauthorized direct seller's preferred values. Thus, if the unauthorized direct seller has sufficient bargaining power, the resulting quantity will be close to the take-it-or-leave offer equilibrium and thus will still constrain the pricing game.

I now turn attention to retailer profitability when diversion occurs. The following proposition demonstrates that the presence of the unauthorized direct seller can reduce retailer profit even though the retailers have the power to collectively starve the direct seller.

Proposition 3 *The opportunity to divert to the unauthorized direct seller can represent a prisoner's dilemma whereby each authorized seller earns less profit than if diversion were not feasible.*

The key insight from Proposition 3 is that improved profitability is not a necessary condition for diversion. In fact, retailers may choose to help a direct competitor come into existence even if the diversion results in diminished profit. The reason is that the unauthorized direct seller will be able to come to a mutually beneficial agreement with at least one authorized retailer who recognizes that of the q_i units diverted only q_i / N will steal from that retailer's customer base. Each firm would prefer to sell to the direct competitor than have the direct competitor's request fulfilled by the remaining authorized retailers. Therefore, even in the parameter range such that the existence of the direct competitor reduces retailer profit, diversion will occur in equilibrium.

It should be noted that Proposition 3 is not meant to suggest that diversion always represents a prisoner's dilemma. In this model, market size is fixed. While this assumption helps demonstrate a novel strategic reason for diversion in the absence of previously viewed necessary conditions, it is possible that

the unauthorized direct seller instead helps to expand the market. Logically, the profitability of diversion for the authorized retailers (relative to when diversion is infeasible) is increasing in degree to which the unauthorized direct seller attracts new consumers to the market. Proposition 3 is robust to small levels of market-expansion by the unauthorized direct seller since the profit difference between when diversion is feasible and when it is not feasible is strictly negative with a fixed market size.

Next I consider the impact of retailer diversion costs on profitability. Recall that retailers face a per-unit cost of transshipping units to the unauthorized direct seller. These costs include administrative hours in processing, packing and reshipping, as well as shipping charges paid to a third party. In the following proposition, I describe how these diversion costs affect profitability.

Proposition 4 *The profit retailers earn when diverting to the unauthorized direct seller is increasing in the per-unit cost to the retailers of diverting (i.e., $\partial \pi_r^* / \partial c > 0$).*

Proposition 4 demonstrates the counter-intuitive result that a reduction in costs would actually reduce profit. This result is driven by the strategic implications of diversion costs. When a retailer incurs a greater cost per unit to divert, it will demand a greater diversion price from the unauthorized direct seller. The greater diversion price has two implications for the unauthorized direct seller. First, the unauthorized direct seller passes these costs on to consumers in the form of higher prices. Second, the unauthorized retailer will prefer to buy fewer units from the retailers than with a lower diversion price. These effects result in two benefits to the retailers: a greater number of units sold to consumers *and* a higher retail price. So, while the direct effect of the diversion cost is a lower profit from diversion, the strategic effects boost profit via higher prices and retail quantities.

In the following section, I explore the implications of vertical differentiation between retailers. Though the value of the product itself remains the same across retailers, retailer actions may positively affect the consumer's willingness to pay in buying from that specific retailer. I identify the equilibrium diversion strategy and profit implications with this additional consideration.

6. Implications of Retailer Asymmetry

In this extension, I consider the diversion behavior by vertically differentiated retailers. There are two objectives of this extension. First, it demonstrates that diversion to a direct competitor can occur when firms are asymmetric. Second, it identifies the impact of a retailer's vertical position on its diversion behavior and profit. On the one hand, a higher quality retailer may be more likely to divert since its quality affords it greater differentiation from the direct seller and thus diminished likelihood of lost sales. On the other hand, the higher quality retailer may be less likely to divert since it commands a higher selling price and thus each sale lost to the direct seller is more costly. To identify the impact of vertical differentiation on diversion behavior, I make several modifications to the main model.

In the interest of parsimony, I consider $N=2$ firms. The reservation value for consumers purchasing from retailer 1 located at $x_1 = 0$ is $u + \delta$ whereas the reservation value for consumers purchasing from retailer 2 located at $x_2 = 1/2$ is u . Thus, the parameter δ captures the difference in retailer quality. Without loss of generality, consumers are homogeneous in their preference for quality and the consumer location, θ , captures consumer heterogeneity in preferences between retailers.

It should be noted that the product sold by each retailer has equivalent value (and thus are undifferentiated when diverted to the unauthorized seller), but the utility from purchasing from the higher quality retailer is enhanced by the retailer service outputs (see also Moorthy and Zhang 2006). As in Kuksov and Lin (2010), I allow higher quality to come at a higher marginal cost. Thus, in addition to the wholesale price that each retailer pays per unit acquired, the higher quality retailer has an additional cost per unit sold to consumers equal to c_v . I consider the case when the cost difference is less than the value difference (i.e., $c_v < \delta$) and the consumer located at the midpoint between two retailers will get greater disutility from buying from a retailer than from buying from the unauthorized direct seller (i.e., $t > 4\mu$). It should be noted that the results are preserved when firms have the same marginal costs (i.e., $c_v = 0$).

The unauthorized direct seller simultaneously offers to buy q_{1t} units from firm 1 and q_{2t} units from firm 2, each at constant per-unit transfer price w_d .

To facilitate interpretation of the results and the impact of δ , I further assume that the retailers' marginal cost of diversion, c , is equal to zero. The remaining elements of the game, including timing, remain the same as in section 5. I limit attention to intermediate values of μ such that the retailers do not cede to the unauthorized direct seller all consumers who consider it and such that the direct seller can profitably be in business.

The game is solved via backward induction. Proposition 5 establishes the equilibrium diversion result.

Proposition 5 *The lower quality retailer diverts a greater quantity than the higher quality retailer. For lesser quality differences between retailers (i.e., $\delta < c_v + \frac{\alpha(3+\alpha)(t-4\mu)}{2(8+15\alpha+\alpha^2)}$), both retailers will divert.*

For greater quality differences between retailers (i.e., $\delta \geq c_v + \frac{\alpha(3+\alpha)(t-4\mu)}{2(8+15\alpha+\alpha^2)}$), only the lower quality retailer will divert.

The intuition behind Proposition 5 relies on the incentives for each firm to divert. The marginal benefit to a retailer of diverting is the sale to the direct seller that may come at the expense of the retail competitor. The marginal loss to a retailer of diverting is a function of the difference in value between the selling price to consumers and the diversion price charged to the direct seller. For the higher quality retailer, the marginal loss is greater. Thus the quantity of diverted units such that the marginal benefit equals the marginal loss is less for the high quality retailer than for the low quality retailer. This effect trumps the fact that the direct seller is a more distant competitor with the high quality retailer than the low quality retailer (as measured by the quality differential). In the following Proposition, I compare profit and total sales from the high quality retailer to the low quality retailer.

Proposition 6 *When the unauthorized direct seller makes take-it-or-leave offers to purchase diverted units from both retailers (i.e., $\delta < c_v + \frac{\alpha(3+\alpha)(t-4\mu)}{2(8+15\alpha+\alpha^2)}$), the high quality retailer earns less profit than*

the low quality retailer even if the retailers share the same marginal cost of sales. The total quantity of sales by the high quality retailer (retail sales plus diverted units) is less than the total quantity of sales by the low quality retailer.

Proposition 6 highlights the strategic effect of product diversion on asymmetric firms. The high quality firm earns a higher margin per unit sold to consumers and sells more units to consumers than the low quality retailer. However, this blessing turns out to be a curse in the presence of an unauthorized direct seller. Given the value to the retailer of the authorized sales to consumers, the high quality retailer is more hesitant to divert to the direct competitor. However, the low-quality retailer is more eager to divert to the direct competitor. The profit result is then driven by the total sales by each player.

The comparison of total sales by each player is inversely related to the comparison of sales when there is no diversion. When there is no unauthorized direct channel, the high quality retailer sells a greater quantity than the low quality firm. However, the low quality retailer has a much higher incentive, on the margin, to sell diverted units. The high quality retailer's resistance, on the margin, to eroding profit margin on sales via diversion to the direct competitor makes the low quality firm a more attractive business partner to the unauthorized direct seller. Hence while the high quality retailer sells more units to consumers, the low quality retailer more than makes up for this difference with the number of additional diverted units.

Proposition 6 has implications for manufacturers who wish to clamp down on diversion. Without diversion, the high quality retailer will order a greater quantity than the low quality retailer. However, this is reversed in the presence of diversion. Whereas monitoring in the symmetric case is made difficult by the fact that each retailer orders the same quantity, order quantity in the asymmetric retailer case can be a good indicator of diversion. Thus, manufacturers can identify a retailer who is diverting when there are quality differences by determining if the retailer's order quantity is counter to its quality position.

7. Discussion

This paper demonstrates how diversion to a direct competitor can occur even without market expansion or non-linear wholesale pricing. As a result, previously viewed necessary conditions for diversion need

not hold. There is evidence that suggests retailers will sell to unauthorized dealers even though the action intensifies competition and reduces profit. The model provides a reason for why retailers might take this detrimental action.

When channel flow diversion occurs, retailers actually earn greater profit when incurring a greater cost of diverting. For example, if the product being sold is bulky and expensive to ship to the unauthorized direct seller, then profit will be greater than if the product is easy and inexpensive to ship. While higher costs have a negative direct effect on retailer profit, the strategic implications are higher retail prices *and* higher quantities sold to consumers. In this context, the strategic effects outweigh the direct effect and result in greater profitability.

While diversion may be an equilibrium strategy for authorized retailers, it is shown that this equilibrium strategy may result in lower profit than if diversion were not possible. Thus channel flow diversion can represent a prisoner's dilemma if the direct seller does not significantly expand the market for the product. This finding contributes to the growing body of marketing research that identifies marketing variables implemented by firms that reduce profit in equilibrium.

It should be noted that the model abstracts from the fact that, in some industries, the unauthorized direct seller has the ability to expand the market of consumers. This is a positive direct effect and logically the prisoner's dilemma could be transformed to a conditional effect depending on the level of market expansion by the unauthorized direct seller. The model also examines a linear wholesale contract. As described in previous literature, the combination of a quantity-discount schedule and a market-expanding unauthorized channel are ripe conditions for diversion and would serve as another direct positive effect of diversion. These conditions would also amplify the negative direct effect of diversion cost on retailer profits. As such both results are conditional on the extent to which the unauthorized seller expands the market and the discount schedule offered by the manufacturer.

Moreover, the stylized main model suggests that all retailers divert whereas this is rarely seen in practice. Asymmetry in retailer quality is one way in which the symmetric results are overturned. The vertical differentiation among retailers demonstrates why the low quality retailers are more likely to divert

than the high quality retailers. It also shows how this diversion can actually lead the lower quality retailer to earn greater profit than the higher quality retailer. The high quality retailer's hesitance, on the margin, to divert units provides an opportunity for the lower quality retailer to make up for the consumer sales differential with more sales to the unauthorized direct seller. This finding has implications for manufacturers attempting to monitor diversion behaviors.

Managers should continue to recognize that retailers in closed markets have incentive to engage in channel flow diversion to unauthorized direct sellers even without quantity discounts. Thus, actions to carefully write retailer agreements and monitor and enforce those agreements may be necessary steps to reducing diversion. Retailers should welcome very strict policy intervention that can crack down on diversion, especially in industries in which the diversion can be done with little cost to the retailers. Unauthorized direct sellers should show restraint and buy fewer diverted units than could ultimately be sold in order to soften competition.

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APPENDIX A: PROOF OF STATEMENTS

Lemma 1 When S authorized sellers each divert q_t units and the unauthorized direct seller is not

constrained by supply of diverted units (i.e., $S \cdot q_t \geq \frac{\alpha(t(3+\alpha) + 2N(1+\alpha)(w-\mu) - 2N(1+\alpha)w_d)}{2(t+2t\alpha)}$),

then each authorized retailer charges $p^* = \frac{t(2-\alpha) + 2N(w+w\alpha+\alpha\mu) + 2N\alpha w_d}{2(N+2N\alpha)}$ and the

unauthorized direct seller charges $p_d^{**} = \frac{t(3+\alpha) + 2N(1+\alpha)(w-\mu) + 2(N+3N\alpha)w_d}{4(N+2N\alpha)}$. The N - S

retailers that do not divert earn profit equal to $\frac{(1+\alpha)(t(-2+\alpha) + 2N\alpha(w-\mu) - 2N\alpha w_d)^2}{4t(N+2N\alpha)^2}$. The S

retailers that divert earn profit equal to $\frac{(1+\alpha)(t(-2+\alpha) + 2N\alpha(w-\mu) - 2N\alpha w_d)^2}{4t(N+2N\alpha)^2} + q_t(w_d - w - c)$.

The unauthorized direct seller earns profit equal to $\frac{\alpha(t(3+\alpha) + 2N(1+\alpha)(w-\mu) - 2N(1+\alpha)w_d)^2}{8Nt(1+2\alpha)^2}$.

Lemma 2 When S authorized sellers each divert q_t units to the unauthorized direct seller and the

unauthorized direct seller is constrained by the supply of diverted units (i.e.,

$S \cdot q_t \leq \frac{\alpha(t(3+\alpha) + 2N(1+\alpha)(w-\mu) - 2N(1+\alpha)w_d)}{2(t+2t\alpha)}$), each authorized retailer charges

$p^* = w + \frac{t(1-Sq_t)}{N+N\alpha}$ and the unauthorized direct seller charges $p_d^* = w - \mu + \frac{\alpha t(3+\alpha) - St(1+3\alpha)q_t}{2N\alpha(1+\alpha)}$.

The N - S retailers that do not divert earn profit equal to $\frac{t(1-Sq_t)^2}{N^2(1+\alpha)}$. The S firms that divert earn profit

equal to $\frac{t(1-Sq_t)^2}{N^2(1+\alpha)} + q_t(w_d - w - c)$. The unauthorized direct seller earns profit equal to

$$\frac{Sq_t(\alpha(t(3+\alpha) + 2N(1+\alpha)(w-\mu) - 2N(1+\alpha)w_d) - St(1+3\alpha)q_t)}{2N\alpha(1+\alpha)}.$$

Proof of Lemma 1 and Lemma 2

Consider retailer **A** that diverts. Contribution (profit gross of any fixed costs) for retailer **A** is $\pi_A = (p_A - w)q_A + (w_d - w - c)q_t$. Contribution for any retailer **r** adopting a common price of p_r is $\pi_r = (p_r - w)q_r + X_r$ where $X_r = (w_d - w - c)q_t$ if the retailer diverts and $X_r = 0$ otherwise. Since $\{w_d, q_t\}$ are set in the prior stage and w is the same for all retailers, the pricing game is symmetric with respect to retailers.

There are α consumers who will consider the unauthorized direct seller. Because the pricing game is symmetric with respect to retailers, I can consider the interval $\theta_i \in [x_A - 1/(2N), x_A + 1/(2N)]$ for which retailer **A** is the closest option to consumers. A consumer located a distance $y = |\theta_i - x_A|$ from retailer **A** is indifferent between buying from retailer **A** and buying from the unauthorized direct seller if $p_A + ty = p_d + \mu$. Thus consumers in the interval $\theta_i \in [x_A - y, x_A + y]$ will buy from retailer **A**, where $y = (p_d - p_A + \mu)/t$. Consumers in the intervals $\theta_i \in [x_A - 1/(2N), x_A - y]$ and $\theta_i \in [x_A + y, x_A + 1/(2N)]$ will buy from the unauthorized direct seller. Equivalently in any interval $\theta_i \in [x_r - 1/(2N), x_r + 1/(2N)] \forall r$, demand for the direct seller is equal to $\frac{1}{N} - 2\frac{p_d - p_r + \mu}{t}$.

Therefore, the total quantity sold by the unauthorized direct seller is equal to

$$q_d = \alpha\left(\frac{1}{N} - 2\frac{p_d - p_A + \mu}{t}\right) + \alpha(N-1)\left(\frac{1}{N} - 2\frac{p_d - p_r + \mu}{t}\right).$$

There are $(1-\alpha)$ consumers who will NOT consider the unauthorized direct seller. Consider the consumer indifferent between buying from retailer **A** at price p_A and buying from the adjacent retailer

located at $x_A + 1/N$ at price p_r . Comparing utilities, this consumer is located at

$\theta = x_A + \frac{p_r - p_A}{2t} + \frac{1}{2N}$. By symmetry, consumers in the interval

$\theta_i \in [x_A - (\frac{p_r - p_A}{2t} + \frac{1}{2N}), x_A + (\frac{p_r - p_A}{2t} + \frac{1}{2N})]$ buy from retailer **A** rather than from one of the

adjacent retailers. As shown previously, the α consumers who consider the unauthorized direct seller

will buy from retailer **A** if $\theta_i \in [x_A - y, x_A + y]$. Thus demand for retailer **A** as a function of its price and

the other firm prices is $q_A = [(1 - \alpha)(p_r + t/N - p_A) + 2\alpha(p_d - p_A + \mu)]/t$.

Suppose S retailers each sell q_t to the unauthorized direct seller. The total units available to the direct seller will be equal to Sq_t . The direct seller has the following constrained optimization problem where λ is the Lagrange multiplier.

$$\begin{aligned} \max_{p_d, \lambda} L(p_d, \lambda) &= (p_d - w_d)q_d + \lambda(Sq_t - q_d) \\ \text{s.t. } \lambda &\geq 0, (Sq_t - q_d) \geq 0, \lambda(Sq_t - q_d) = 0 \end{aligned}$$

Since the retailer pricing game is symmetric, I solve for the retail price $p_A^* = p_r^*$ and p_d^* such that $\partial \pi_A / \partial p_A = 0$, $\partial \pi_d / \partial p_d = 0$, $\lambda(Sq_t - q_d) = 0$ and $\lambda \geq 0$. The resulting prices and profit when

$$\lambda^* = \frac{-2St(1 + 2\alpha)q_t + \alpha(t(3 + \alpha) + 2N(1 + \alpha)(w - \mu) - 2N(1 + \alpha)w_d)}{2N\alpha(1 + \alpha)} \leq 0 \text{ are simplified in Lemma 1.}$$

Lemma 2 presents the resulting simplified prices and profit when $\lambda^* = \frac{-2St(1 + 2\alpha)q_t + \alpha(t(3 + \alpha) + 2N(1 + \alpha)(w - \mu) - 2N(1 + \alpha)w_d)}{2N\alpha(1 + \alpha)} > 0$.

For completeness, it must also be shown that retailer **A** will not choose to ignore the α consumers who consider the unauthorized direct seller. If all other retailers choose p^* , the best profit retailer **A** can earn, not including revenue from diversion, by ignoring the unauthorized direct seller is

$\max_{p_A} \pi_d^I \equiv (1 - \alpha)(p_A - w)(p^* + t/N - p_A)/t$. At the equilibrium in Lemma 1, this profit is

$\frac{(1-\alpha)(2+\alpha-Sq_t)(t(2+\alpha(2+\alpha))-2N\alpha(1+\alpha)(w-\mu)+St(1+2\alpha)q_t+2N\alpha(1+\alpha)w_d)}{4N^2(1+\alpha)^2(1+2\alpha)}$. At the

equilibrium in Lemma 2, this profit is $\frac{t(1-\alpha)(2+\alpha-Sq_t)^2}{4N^2(1+\alpha)^2}$. Therefore, no retailer would prefer to

ignore the consumers who consider the unauthorized direct seller at the equilibrium in Lemma 1 if

$\frac{(1-\alpha)(2+\alpha-Sq_t)(t(2+\alpha(2+\alpha))-2N\alpha(1+\alpha)(w-\mu)+St(1+2\alpha)q_t+2N\alpha(1+\alpha)w_d)}{4N^2(1+\alpha)^2(1+2\alpha)}$ is less

than $\frac{(1+\alpha)(t(-2+\alpha)+2N\alpha(w-\mu)-2N\alpha w_d)^2}{4t(N+2N\alpha)^2}$. Simplifying the comparison of profits, no retailer

would prefer to ignore the consumers who consider the unauthorized direct seller at the equilibrium in

Lemma 2 if $(1-\alpha)(2+\alpha-Sq_t)^2 < 4(1+\alpha)(1-Sq_t)^2$.

Proof of Proposition 1

Consider when the quantity of diverted units supplied to the unauthorized direct seller is equal to or less than the quantity of units that would be sold to consumers by the unauthorized direct seller. If N authorized retailers divert q_t units then the equilibrium payoffs are in Lemma 2 with N substituted for S .

The question becomes, would any player want to unilaterally deviate. Consider an authorized retailer who chooses not to divert when the $N-1$ retailers do divert. The payoffs in this case are also in Lemma 2 where I substitute $N-1$ for S . Comparing payoffs, an authorized retailer would prefer to divert when $N-1$ retailers

also divert if $w_d \geq \underline{w}_d(q_t) \equiv w + c + \frac{t(2+(1-2N)q_t)}{N^2(1+\alpha)}$. The unauthorized direct seller would prefer to

buy Nq_t units rather than $(N-1)q_t$ units if $w_d \leq \overline{w}_d(q_t) \equiv w - \mu + \frac{\alpha t(3+\alpha) - (-1+2N)t(1+3\alpha)q_t}{2N\alpha(1+\alpha)}$.

Therefore, if $\underline{w}_d(q_t) \leq w_d \leq \overline{w}_d(q_t)$, no retailer would prefer to unilaterally deviate by not diverting and the unauthorized seller would not want to deviate by not buying from the N th authorized retailer. The

range $\underline{w}_d(q_t) \leq w_d \leq \overline{w}_d(q_t)$ exists if and only if $q_t \leq \frac{\alpha(t(3N-4+N\alpha)-2N^2(1+\alpha)(c+\mu))}{(2N-1)t(N-2\alpha+3N\alpha)}$. This

upper bound on per-retailer diverted quantity is positive if and only if $\mu < \frac{t(3N-4+N\alpha)}{2N^2(1+\alpha)} - c$.

Therefore, for any $\mu < \frac{t(3N-4+N\alpha)}{2N^2(1+\alpha)} - c$, there exists a mutual agreement between the Nth diverting

retailer and the unauthorized direct seller $\{q_t \leq \frac{\alpha(t(3N-4+N\alpha)-2N^2(1+\alpha)(c+\mu))}{(2N-1)t(N-2\alpha+3N\alpha)},$

$\underline{w}_d(q_t) \leq w_d \leq \overline{w}_d(q_t)\}$ such that both parties prefer trade than to not trade.

The question becomes is there any other stable equilibrium in which a subset $S < N$ authorized sellers divert q_t units to the unauthorized seller. If no authorized retailers divert then the payoffs are t/N^2 . If one authorized retailer diverts q_t units, the payoffs are described in Lemma 2 with $S=1$. Comparing payoffs, both the unauthorized direct seller and the single diverting retailer can earn greater payoffs than

if there were no diversion if $w+c + \frac{t(\alpha+2q_t-q_t^2)}{N^2(1+\alpha)q_t} < w_d < w-\mu + \frac{\alpha t(3+\alpha)-(t+3t\alpha)q_t}{2N\alpha(1+\alpha)}$. Thus, if

no authorized retailers were diverting, the unauthorized seller could come to a mutually beneficial agreement with an authorized retailer. Consider any $1 \leq S < N$ retailers diverting. A retailer would be

willing to become the S+1 diverting retailer if $w_d \geq \underline{w}^S \equiv w+c + \frac{t(2-(1+2S)q_t)}{N^2(1+\alpha)}$ and the unauthorized

direct seller would want to buy from the S+1 retailers (instead of S retailers) if

$w_d \leq \overline{w}^S \equiv (w-\mu) + \frac{\alpha t(3+\alpha)-(1+2S)t(1+3\alpha)q_t}{2N\alpha(1+\alpha)}$. The range $\underline{w}^S \leq w_d \leq \overline{w}^S$ exists for any

$q_t \leq \frac{\alpha(t(3N-4+N\alpha)-2N^2(1+\alpha)(c+\mu))}{(1+2S)t(N-2\alpha+3N\alpha)}$ which is greater than

$\frac{\alpha(t(3N-4+N\alpha)-2N^2(1+\alpha)(c+\mu))}{(2N-1)t(N-2\alpha+3N\alpha)}$ for any $S < N$. Therefore, if S retailers divert q_t units, an

agreement can be made for S+1 retailers to divert q_t units. Therefore, N retailers will each divert

$$q_t^* \leq \frac{\alpha(t(3N-4+N\alpha)-2N^2(1+\alpha)(c+\mu))}{(2N-1)t(N-2\alpha+3N\alpha)} \text{ units at a diversion price } w_d^* \text{ if } \underline{w}(q_t^*) < w_d^* < \overline{w}_d(q_t^*).$$

Q.E.D.

Proof of Lemma 3:

As shown in Proposition 1, if $0 < S < N$ firms are willing to divert a quantity q_t in equilibrium, then N firms will divert q_t . As such, I look at the unauthorized direct seller's payoff from N retailers diverting (see Lemmas 1 and 2 with N substituted for S). As shown in the proof of Proposition 1, the unauthorized direct seller must offer $w_d \geq \underline{w} \equiv w + c + \frac{t(2+(1-2N)q_t)}{N^2(1+\alpha)}$ so that no retailer would prefer not to divert.

Thus, I have the following constrained optimization problem:

$$\begin{aligned} \max_{q_t, w_d, \lambda_2} L_2(q_t, w_d, \lambda_2) &= (p_d^*(q_t) - w_d)Nq_t + \lambda_2(w_d - \underline{w}) \\ \text{s.t. } \lambda_2(w_d - \underline{w}) &= 0 \\ \lambda_2 &\geq 0 \end{aligned}$$

There are two possibilities to consider: 1. q_t is chosen such that the quantity constraint in the pricing game will bind and $p_d^*(q_t)$ is hence defined in Lemma 2 with $S=N$; 2. q_t is chosen such that the quantity constraint in the pricing game will not bind and $p_d^*(q_t)$ is hence defined in Lemma 1 (where it is reported as p^{**}) and $q_t = q_d(p_d^{**}, p^{**})/N$.

1. Direct Seller Quantity is Constrained in Pricing Game:

Profit is concave in q_t (i.e., $\partial^2 \pi_d / \partial^2 q_t = -\frac{t(N^3 + 3N^3\alpha)}{N^2(\alpha + \alpha^2)} < 0$) and strictly decreasing in w_d . The

Kuhn-Tucker conditions (i.e., $\partial \pi_d / \partial q_t = 0$, $\partial \pi_d / \partial w_d = 0$, $\lambda_2 \geq 0$, $\lambda_2(w_d - \underline{w}) = 0$) are satisfied at

$$w_d^* = w + c + \frac{4t\alpha + Nt(-5 + \alpha)\alpha + 4N^3\alpha(1 + \alpha)(c + \mu) - 2N^2(t(-2 - 3\alpha + \alpha^2) + \alpha(1 + \alpha)(c + \mu))}{2N^2(1 + \alpha)(2\alpha - 4N\alpha + N^2(1 + 3\alpha))}$$

$q_t^* = \frac{\alpha(t(3N-4+N\alpha)-2N^2(1+\alpha)(c+\mu))}{2t(2\alpha-4N\alpha+N^2(1+3\alpha))}$, $\lambda_2^* = N$. This leads to a direct seller profit of

$$\pi_d^* = \frac{\alpha(2cN^2(1+\alpha)-t(-4+N(3+\alpha))+2N^2(1+\alpha)\mu)^2}{8Nt(1+\alpha)(2\alpha-4N\alpha+N^2(1+3\alpha))}.$$

2. Direct Seller Quantity is Unconstrained in the Pricing Game

The unauthorized direct seller's profit is presented in Lemma 1. The equilibrium pricing leads to a total

quantity diverted equal to $q_d^{**}(w_d) = \frac{\alpha(t(3+\alpha)+2N(1+\alpha)(w-\mu)-2N(1+\alpha)w_d)}{2(t+2t\alpha)}$. Thus each

retailer will divert $q_t^{**} = q_d^{**} / N$. The unauthorized direct seller's profit is decreasing in w_d for any

$q_d^{**}(w_d) > 0$. Therefore, the optimal w_d is the lowest value that will induce diversion of q_t^{**} from the N

retailers (i.e., w_d such that $w_d - \underline{w}_d(q_t^{**}(w_d)) = 0$). The unique solution is

$$w_d^{**} = w + c + \frac{t\alpha(3+\alpha) + 4N^2\alpha(1+\alpha)(c+\mu) - 2N(1+\alpha)(t(-2+\alpha) + \alpha(c+\mu))}{2N(1+\alpha)(\alpha - 2N\alpha + N^2(1+2\alpha))}$$

$$q_t^{**} = \frac{\alpha(t(3N-4+N\alpha)-2N^2(1+\alpha)(c+\mu))}{2t(\alpha-2N\alpha+N^2(1+2\alpha))}$$

leading to direct seller profit equal to $\pi_d^{**} = \frac{N\alpha(2cN^2(1+\alpha)-t(-4+N(3+\alpha))+2N^2(1+\alpha)\mu)^2}{8t(\alpha-2N\alpha+N^2(1+2\alpha))^2}$.

Comparing Quantity-Constrained Profit to Unconstrained Profit

Comparing payoffs above, the difference between constrained profit and unconstrained profit is equal to

$$\pi_d^* - \pi_d^{**} = \frac{(-1+N)^4\alpha^3(4t+N(2cN(1+\alpha)-t(3+\alpha)+2N(1+\alpha)\mu))^2}{8Nt(1+\alpha)(N^2+(2+N(-4+3N))\alpha)(\alpha+N(N+2(-1+N)\alpha))^2} > 0. \quad \text{Therefore, the}$$

unauthorized direct seller will maximize profit by offering to buy

$q_t^* = \frac{\alpha(t(3N-4+N\alpha)-2N^2(1+\alpha)(c+\mu))}{2t(2\alpha-4N\alpha+N^2(1+3\alpha))}$ from each retailer at a rate of

$$w_d^* = w + c + \frac{4t\alpha + Nt(-5+\alpha)\alpha + 4N^3\alpha(1+\alpha)(c+\mu) - 2N^2(t(-2-3\alpha+\alpha^2) + \alpha(1+\alpha)(c+\mu))}{2N^2(1+\alpha)(2\alpha-4N\alpha+N^2(1+3\alpha))}.$$

Retailers earn profit equal to

$$\pi_r^* = \frac{(t(-1+\alpha)\alpha + 2N^2\alpha(1+\alpha)(c+\mu) - N(t(-2-3\alpha+\alpha^2) + 2\alpha(1+\alpha)(c+\mu)))^2}{4t(1+\alpha)(2\alpha - 4N\alpha + N^2(1+3\alpha))^2}$$

For completeness, I must verify the conditions for which the retailers will not choose to focus only on the $1-\alpha$ consumers who do not consider the unauthorized direct seller, thereby forsaking α consumers.

In other words, would a retailer prefer to maximize $\pi^i = (1-\alpha)(p-w)(\frac{p^* + (t/N) - p}{t})$? By solving

the first order conditions and substituting into the profit expression, the maximum profit a retailer earns

by this deviation is

$$\pi^{i,*} = -\frac{1}{4N^2(1+\alpha)^2} [t(-1+\alpha)(2+\alpha)^2 + N^2t(-1+\alpha)q_i^2 + 2Nq_i(2t + 2Nw - t\alpha + 4Nw\alpha - t\alpha^2 + 2Nw\alpha^2 + 2cN(1+\alpha)^2 - 2N(1+\alpha)^2w_d)]$$

By algebra, $\pi_r^* - \pi^{i,*} > 0$ if

$$\mu \geq \underline{\mu} \equiv -\frac{1}{2N^6\alpha(1+\alpha)^2(3+5\alpha)} [2cN^6\alpha(1+\alpha)^2(3+5\alpha) + 4N^3t\alpha(2+7\alpha+6\alpha^2+\alpha^3) - 4N^4t\alpha(1+6\alpha+7\alpha^2+2\alpha^3) + N^5t(4+17\alpha+27\alpha^2+15\alpha^3+\alpha^4) - 4\sqrt{-N^6t^2(-1+\alpha)(1+\alpha)^5(2\alpha-4N\alpha+N^2(1+3\alpha))^2}] \quad Q.E.D.$$

Proof of Proposition 2

By Lemma 3, profit from constraining quantity by buying

$$q_i^* = \frac{\alpha(t(3N-4+N\alpha) - 2N^2(1+\alpha)(c+\mu))}{2t(2\alpha-4N\alpha+N^2(1+3\alpha))}$$

is greater than profit from unconstrained quantity of

$$q_i^{**} = \frac{\alpha(t(3N-4+N\alpha) - 2N^2(1+\alpha)(c+\mu))}{2t(\alpha-2N\alpha+N^2(1+2\alpha))} \text{ and } q_i^* < q_i^{**}. \quad Q.E.D.$$

Proof of Propositions 3 and 4

Pprofit with diversion is increasing in c :

$$\partial \pi_r^* / \partial c = \frac{(N-1)N\alpha(Nt(3\alpha-\alpha^2) + t\alpha^2 + (2N-1)t + t(1-\alpha) + 2N(N-1)\alpha(1+\alpha)(c+\mu))}{t(2\alpha-4N\alpha+N^2(1+3\alpha))^2} > 0.$$

The maximum value of c such that diverted quantity is non-negative is equal to

$$\hat{c} \equiv \frac{-4t + 3Nt + Nt\alpha - 2N^2\mu - 2N^2\alpha\mu}{2N^2(1+\alpha)}. \text{ Evaluated at } c = \hat{c}, \pi_r^* - t/N^2 = -(t\alpha)/N^2(1+\alpha) < 0.$$

Therefore, for all $c < \hat{c}$, $\pi_r^* - t/N^2 < 0$. *Q.E.D.*

Proof of Proposition 5

I will first solve for the demand system. I will then solve for the equilibrium prices in the third stage given the diversion agreements (or lack thereof) from the previous stage. I will substitute these prices into the unauthorized direct seller's objective function to find its equilibrium diversion quantities from each retailer and the diversion price.

Consumer utility from buying from retailer 1 is equal to $u + \delta - t|0 - \theta_i| - p_1$. Consumer utility from buying from retailer 2 is equal to $u - t|(1/2) - \theta_i| - p_2$. Consumer utility from buying from the direct seller is equal to $u - \mu - p_d$. First consider the interval between 0 and $1/2$. The consumer who is

indifferent between buying from retailer 1 and retailer 2 is located at $\theta' = \frac{t + 2\delta - 2p_1 + 2p_2}{4t}$. The

consumer who is indifferent between buying from retailer 1 and buying from the unauthorized direct

seller is located at $\theta^1 = \frac{\delta + \mu - p_1 + p_d}{t}$. The consumer who is indifferent between buying from retailer

2 and buying from the unauthorized direct seller is located at $\theta^2 = \frac{1}{2} - \frac{\mu - p_2 + p_d}{t}$. Consider prices

such that the unauthorized direct seller competes with each retailer (i.e., $\theta^1 < \theta' < \theta^2$). I will verify that

this condition holds at the equilibrium prices. The proportion α of consumers who consider the

unauthorized direct seller will buy from retailer 1 if $\theta_i \leq \theta^1$, from the unauthorized direct seller if

$\theta^1 < \theta_i < \theta^2$, and from retailer 2 if $\theta_i \geq \theta^2$. The proportion $(1 - \alpha)$ of consumers who do not consider

the unauthorized direct seller will buy from retailer 1 if $\theta_i \leq \theta'$ and will buy from retailer 2 if $\theta_i > \theta'$.

The interval 0 to $\frac{1}{2}$ is identical to the remaining interval and therefore demand for retailer 1, demand for retailer 2 and demand for the direct seller can be written as

$$q_1 = 2(1-\alpha)\theta' + 2\alpha\theta^1 = [(1-\alpha)(t+2\delta-2p_1+2p_2)+4\alpha(\delta+\mu-p_1+p_d)]/2t$$

$$q_2 = 2(1-\alpha)(1/2-\theta') + 2\alpha(1/2-\theta^2) = [(1-\alpha)(t-2\delta+2p_1-2p_2)+4\alpha(\mu-p_2+p_d)]/2t \quad (1)$$

$$q_d = 2\alpha(\theta^2 - \theta^1) = \alpha(t-2\delta-4\mu+2p_1+2p_2-4p_d)/t.$$

Outcome when no diversion occurs:

When no diversion occurs, retailer 1 earns profit $\pi_1 = (p_1 - w - c_v)q_1$ and retailer 2 earns profit $\pi_2 = (p_2 - w)q_2$ where q_j for each retailer ($j=1,2$) is defined in equation 1 simplified with $\alpha = 0$.

Resulting prices are $p_1^{nd} = w + \frac{1}{6}(3t + 2\delta + 4c_v)$ and $p_2^{nd} = w + \frac{1}{6}(3t - 2\delta + 2c_v)$. Resulting payoffs

$$\text{are } \pi_1^{nd} = \frac{(3t + 2\delta - 2c_v)^2}{36t} \text{ and } \pi_2^{nd} = \frac{(3t - 2\delta + 2c_v)^2}{36t} \text{ with the direct seller earning zero profit.}$$

Outcome when diversion occurs:

When each retailer diverts $q_{jt} \geq 0$ at a diversion price w_d , retailer 1 earns profit $\pi_1 = (p_1 - w - c_v)q_1 + (w_d - w)q_{1t}$, retailer 2 earns profit $\pi_2 = (p_2 - w)q_2 + (w_d - w)q_{2t}$, and the unauthorized direct seller earns $\pi_d = (p_d - w_d - w) \cdot \min\{q_d, q_{1t} + q_{2t}\}$ where q_j and q_d are defined in equation 1 and q_{jt} is determined in advance of the pricing game. The supply constraint means that the unauthorized direct seller's objective function can be rewritten as the following Lagrangian:

$$\max_{p_d, \lambda} L(p_d, \lambda) = (p_d - w_d)q_d + \lambda(q_{2t} + q_{1t} - q_d)$$

$$s.t. \lambda(q_{2t} + q_{1t} - q_d) = 0, \lambda \geq 0$$

I solve each firm's first order conditions simultaneously. If

$$\lambda^*(w_d, q_{1t}, q_{2t}) = \frac{1}{4\alpha(1+\alpha)}(\alpha(t(3+\alpha) + 2(1+\alpha)(2w - \delta - 2\mu)) + 2\alpha(1+\alpha)c_v - 2t(1+2\alpha)(q_t + q_{2t}) - 4\alpha(1+\alpha)w_d) > 0$$

then the prices as functions of diversion quantities and diversion price are:

$$p_d^*(w_d, q_{1t}, q_{2t}) = [\alpha(t(3+\alpha) + 2(1+\alpha)(2w - \delta - 2\mu)) + 2\alpha(1+\alpha)c_v - t(1+3\alpha)(q_{1t} + q_{2t})] / 4\alpha(1+\alpha)$$

$$p_1^*(w_d, q_{1t}, q_{2t}) = \frac{t(3+\alpha) + 2(1+\alpha)(w(3+\alpha) + \delta) + 2(1+\alpha)(2+\alpha)c_v - t(3+\alpha)(q_{1t} + q_{2t})}{2(1+\alpha)(3+\alpha)}$$

$$p_2^*(w_d, q_{1t}, q_{2t}) = \frac{(3+\alpha)(t + 2w(1+\alpha)) - 2(1+\alpha)\delta + 2(1+\alpha)c_v - t(3+\alpha)(q_{1t} + q_{2t})}{2(1+\alpha)(3+\alpha)}$$

Payoffs for each firm are:

$$\pi_1^*(w_d, q_{1t}, q_{2t}) = \frac{(-3t - t\alpha - 2\delta - 2\alpha\delta + 2(1+\alpha)c_v + t(3+\alpha)q_{1t} + 3tq_{2t} + t\alpha q_{2t})^2}{4t(1+\alpha)(3+\alpha)^2} + q_{1t}(w_d - w)$$

$$\pi_2^*(w_d, q_{1t}, q_{2t}) = \frac{(-3t - t\alpha + 2\delta + 2\alpha\delta - 2(1+\alpha)c_v + t(3+\alpha)q_{1t} + 3tq_{2t} + t\alpha q_{2t})^2}{4t(1+\alpha)(3+\alpha)^2} + q_{2t}(w_d - w)$$

$$\pi_d^*(w_d, q_{1t}, q_{2t}) = \frac{(q_{1t} + q_{2t})}{4\alpha(1+\alpha)} (\alpha(t(3+\alpha) + 2(1+\alpha)(2w + c_v - \delta - 2\mu)) - t(1+3\alpha)(q_{1t} + q_{2t}) - 4\alpha(1+\alpha)w_d)$$

Following the logic from the proof of Lemma 3, I focus on the outcome in which $\lambda^*(w_d, q_{1t}, q_{2t}) > 0$ at the equilibrium diversion quantities and diversion price.

To divert, or not to divert, that is the question:

Now consider each retailer's decision in stage 2. If retailer 1 is diverting $q_{1t} > 0$ units, then retailer 2 will accept the offer to divert q_{2t} units if and only if $\pi_2^*(w_d, q_{1t}, q_{2t}) > \pi_2^*(w_d, q_{1t}, q_{2t} | q_{2t} = 0)$. If retailer 2 is diverting $q_{2t} > 0$, then retailer 1 will accept the offer to divert q_{1t} if and only if $\pi_1^*(w_d, q_{1t}, q_{2t}) > \pi_1^*(w_d, q_{1t}, q_{2t} | q_{1t} = 0)$. If retailer 1 is not diverting, then retailer 2 will accept the offer to divert q_{2t} if and only if $\pi_2^*(w_d, q_{1t}, q_{2t} | q_{1t} = 0) > \pi_2^{nd}$. If retailer 2 is not diverting, then retailer 1 will accept the offer to divert q_{1t} units if and only if $\pi_1^*(w_d, q_{1t}, q_{2t} | q_{2t} = 0) > \pi_1^{nd}$.

Equilibrium offer by unauthorized direct seller:

In stage 1, there are four possibilities to consider. The unauthorized direct seller may offer to buy: $\{q_{1t} = 0, q_{2t} = 0\}$, $\{q_{1t} > 0, q_{2t} > 0\}$, $\{q_{1t} > 0, q_{2t} = 0\}$ or $\{q_{1t} = 0, q_{2t} > 0\}$. I identify the optimal

diversion quantities and diversion price for each of these possibilities and compare profit to determine the equilibrium.

If the unauthorized direct seller chooses to buy diverted units from each retailer, then the unauthorized direct seller chooses q_{1t}, q_{2t}, w_d to maximize profit subject to the constraint that the retailers accept the offers. The unauthorized direct seller has the following constrained optimization problem:

$$\max_{\lambda_{d1}, \lambda_{d2}, q_{1t}, q_{2t}, w_d} \pi_d^*(q_{1t}, q_{2t}, w_d) + \lambda_{d1}[\pi_1^*(w_d, q_{1t}, q_{2t}) - \pi_1^*(w_d, q_{1t}, q_{2t} | q_{1t} = 0)] \\ + \lambda_{d2}[\pi_2^*(w_d, q_{1t}, q_{2t}) - \pi_2^*(w_d, q_{1t}, q_{2t} | q_{2t} = 0)]$$

$$s.t. \lambda_{d1}[\pi_1^*(w_d, q_{1t}, q_{2t}) - \pi_1^*(w_d, q_{1t}, q_{2t} | q_{1t} = 0)] = 0 \\ \lambda_{d2}[\pi_2^*(w_d, q_{1t}, q_{2t}) - \pi_2^*(w_d, q_{1t}, q_{2t} | q_{2t} = 0)] = 0 \\ \lambda_{d1} \geq 0, \lambda_{d2} \geq 0$$

The unique solution satisfying the Kuhn Tucker conditions such that $q_{1t} > 0, q_{2t} > 0$ is

$$q_{1t}^* = (1 + \alpha)(t\alpha(3 + \alpha) - 2((8 + 15\alpha + \alpha^2)(\delta - c_v) + 2\alpha(3 + \alpha)\mu)) / 2t(6 + 11\alpha + 3\alpha^2)$$

$$q_{2t}^* = (1 + \alpha)(t\alpha(3 + \alpha) + 2((8 + 9\alpha - \alpha^2)(\delta - c_v) - 2\alpha(3 + \alpha)\mu)) / 2t(6 + 11\alpha + 3\alpha^2)$$

$$w_d^* = [t(8 + 9\alpha - 3\alpha^2) + 2(1 + \alpha)(8w + 12w\alpha + 3\alpha\delta + 6\alpha\mu) - 6\alpha(1 + \alpha)c_v] / 8(2 + 5\alpha + 3\alpha^2)$$

$$\lambda_{1d}^* = \alpha(3 + \alpha)(t - 2(\delta + 2\mu) + 2c_v) / [t\alpha(3 + \alpha) - 2((8 + 15\alpha + \alpha^2)(\delta - c_v) + 2\alpha(3 + \alpha)\mu)]$$

$$\lambda_{2d}^* = \frac{\alpha(3 + \alpha)(t - 2(\delta + 2\mu) + 2c_v)}{t\alpha(3 + \alpha) + 2((8 + 9\alpha - \alpha^2)(\delta - c_v) - 2\alpha(3 + \alpha)\mu)}. \text{ The unauthorized direct seller earns profit}$$

equal to $\frac{\alpha(1 + \alpha)(t - 2(\delta + 2\mu) + 2c_v)^2}{8t(2 + 3\alpha)}$. It should be noted that $q_{1t}^* > 0$ if and only if

$$\delta < \delta^m \equiv c_v + \frac{\alpha(3 + \alpha)(t - 4\mu)}{2(8 + 15\alpha + \alpha^2)}.$$

To verify that this critical point is a maximum rather than a minimum, I take the derivative of profit through this point. Let \vec{q} denote the vector of choice variables at the critical point. The partial derivatives

$$\text{through this vector are } \frac{\partial \pi_d(\vec{q})}{\partial q_{2t}} = \frac{\partial \pi_d(\vec{q})}{\partial q_{1t}} = \frac{3t}{8(1 + \alpha)} \frac{\partial \pi_d(\vec{q})}{\partial w_d} = -\frac{3\alpha(t - 2(\delta + 2\mu) + 2c_v)}{8(2 + 3\alpha)} \text{ which are}$$

negative for all parameters that lead to positive sales by the unauthorized direct seller. Thus increasing any of the choice variables will reduce the unauthorized direct seller's profit, yet decreasing any of the choice variables will violate the constraint that retailer 1 accepts the offer. The vector \vec{q} is the profit maximizing decision given the unauthorized seller buys from each retailer since \vec{q} is the only critical point in the feasible set, local deviations cannot improve profit, and $q_{1t} = q_{2t} = 0$ results in zero profit.

If the unauthorized direct seller chooses to buy diverted units from retailer 1 only, then the unauthorized direct seller chooses q_{1t}, w_d to maximize profit subject to the constraint that retailer 1 accepts the offer to be the only retailer that diverts. The unauthorized direct seller has the following constrained optimization problem:

$$\begin{aligned} \max_{\lambda_{d1}, q_{1t}, w_d} & \pi_d^*(q_{1t}, q_{2t}, w_d \mid q_{2t} = 0) + \lambda_{d1}[\pi_1^*(w_d, q_{1t}, q_{2t} \mid q_{2t} = 0) - \pi_1^{nd}] \\ \text{s.t. } & \lambda_{d1}[\pi_1^*(w_d, q_{1t}, q_{2t} \mid q_{2t} = 0) - \pi_1^{nd}] = 0 \\ & \lambda_{d1} \geq 0 \end{aligned}$$

The unique solution satisfying the Kuhn-Tucker conditions such that $q_{1t} > 0, q_{2t} = 0$ is as follows:

$$q_{1t}^* = \frac{\alpha(1+\alpha)(t(3+\alpha) - 2(5+\alpha)(\delta - c_v) - 4(3+\alpha)\mu)}{2t(3+\alpha)(1+2\alpha)}, \lambda_{d1}^* = 1,$$

$$\begin{aligned} w_d^* = w + & \frac{1}{(72(1+\alpha)^2(3+\alpha)(1+2\alpha)(t(3+\alpha) - 2(5+\alpha)\delta - 4(3+\alpha)\mu + 2(5+\alpha)c_v))} \\ & [-9t^2(3+\alpha)^2(-8+\alpha(-27+(-22+\alpha)\alpha)) \\ & +12t(1+\alpha)(3+\alpha)((\delta - c_v)(-1+\alpha)(20+\alpha(37+3\alpha)) + 6(-6-11\alpha+\alpha^3)\mu) \\ & +4(1+\alpha)(-\delta - c_v)^2(192+\alpha(845+\alpha(815+\alpha(155+9\alpha)))) - 36(\delta - c_v)(1+\alpha)(3+\alpha)(2+\alpha(9+\alpha))\mu \\ & -36\alpha(1+\alpha)(3+\alpha)^2\mu^2] \end{aligned}$$

It should be noted that $q_{1t}^* > 0$ if and only if $\delta < \delta' \equiv c_v + \frac{(3+\alpha)(t-4\mu)}{2(5+\alpha)}$.

This critical point is a maximum rather than a minimum because, at the choice vector satisfying the Kuhn-Tucker conditions, profit is decreasing in any deviations for which the constraint is satisfied.

If the unauthorized direct seller chooses to buy diverted units from retailer 2 only, then the unauthorized direct seller chooses q_{2t}, w_d to maximize profit subject to the constraint that retailer 2 accepts the offer to become the only retailer that diverts. The unauthorized direct seller has the following constrained optimization problem

$$\begin{aligned} \max_{\lambda_{d1}, q_{2t}, w_d} & \pi_d^*(q_{1t}, q_{2t}, w_d \mid q_{1t} = 0) + \lambda_{d2} [\pi_2^*(w_d, q_{1t}, q_{2t} \mid q_{1t} = 0) - \pi_2^{nd}] \\ \text{s.t. } & \lambda_{d2} [\pi_2^*(w_d, q_{1t}, q_{2t} \mid q_{1t} = 0) - \pi_2^{nd}] = 0 \\ & \lambda_{d2} \geq 0 \end{aligned}$$

The unique solution satisfying the Kuhn-Tucker conditions such that $q_{1t} = 0, q_{2t} > 0$ is as follows:

$$q_{2t}^* = \frac{\alpha(1+\alpha)(t(3+\alpha) - 2(\delta + \alpha\delta + 6\mu + 2\alpha\mu) + 2(1+\alpha)c_v)}{2t(3+7\alpha+2\alpha^2)}$$

$$\begin{aligned} w_d^* = & \frac{1}{72(1+\alpha)^2(3+\alpha)(1+2\alpha)(t(3+\alpha) - 2(x(1+\alpha) + 2(3+\alpha)\mu))} \cdot \\ & [-9t^2(3+\alpha)^2(-8-27\alpha-22\alpha^2+\alpha^3) \\ & + 12t(3+4\alpha+\alpha^2)((\delta-c_v)(-16-49\alpha-34\alpha^2+3\alpha^3) + 6(-6-11\alpha+\alpha^3)\mu) \\ & - 4(1+\alpha)((\delta-c_v)^2(-24-91\alpha-121\alpha^2-61\alpha^3+9\alpha^4) + 36(\delta-c_v)(-6-17\alpha-11\alpha^2+\alpha^3+\alpha^4)\mu \\ & + 36\alpha(1+\alpha)(3+\alpha)^2\mu^2)] \end{aligned}$$

It should be noted that $q_{2t}^* > 0$ if and only if $\delta < \delta'' \equiv c_v + \frac{(3+\alpha)(t-4\mu)}{2(1+\alpha)}$.

This critical point is a maximum rather than a minimum because, at the choice vector satisfying the Kuhn-Tucker conditions, profit is decreasing in any deviations for which the constraint is satisfied.

Now consider whether the unauthorized direct seller will buy only from retailer 1, buy only from retailer 2, or buy from both. Each case has a maximum delta such that quantities are non-negative. By algebra, $\delta''' < \delta' < \delta''$. The profit from buying only from retailer 1 minus the profit from buying from

retailer 2 is $\frac{\alpha(\delta-c_v)(-t(5+7\alpha)+6(1+\alpha)(\delta+2\mu-c_v))}{3t(3+\alpha)(1+2\alpha)}$ which is negative for all $\delta < \delta'$ by fact that

$t > 4\mu$. The profit from buying from both retailer 1 and retailer 2 is greater than the profit from buying only from retailer 2 for all $c_v < \delta < \delta'''$. Therefore, in the range that buying from both retailers results in

positive quantity from each retailer (i.e., $\delta < \delta'''$), the unauthorized direct seller will buy from both retailers. In the range $\delta''' < \delta < \delta''$, the unauthorized direct seller will buy only from retailer 2. There is no range such that the unauthorized direct seller buys only from firm 1. In each case $q_{1t}^* < q_{2t}^*$ for any $\delta > c_v$.

It remains to be shown that the equilibrium satisfies the inequalities presumed in the derivation of demand. Specifically, $\theta^1 < \theta' < \theta^2$. At the equilibrium price vector when both retailers divert, the

difference $\theta^2 - \theta' = \frac{(1+\alpha)(t-2(\delta+2\mu)+2c_v)}{4t(2+3\alpha)}$ and the difference

$\theta' - \theta^1 = \frac{(1+\alpha)(t-2(\delta+2\mu)+2c_v)}{4t(2+3\alpha)}$ are each positive for any positive diverted quantity. When only

retailer 2 diverts, these differences are equal to $\frac{(1+\alpha)(t(3+\alpha)-2(\delta+\alpha\delta+6\mu+2\alpha\mu)+2(1+\alpha)c_v)}{8t(3+\alpha)(1+2\alpha)}$

which are positive for any positive diverted quantity.

Q.E.D.

Proof of Proposition 6

When both firms divert, Retailer 1 earns profit equal to

$$\pi_1^* = \frac{(t(-12-19\alpha-2\alpha^2+\alpha^3)-2(1+\alpha)((-4-3\alpha+\alpha^2)\delta+2\alpha(3+\alpha)\mu)+2(-4+\alpha)(1+\alpha)^2c_v)^2}{16t(1+\alpha)(3+\alpha)^2(2+3\alpha)^2}$$

and Retailer 2 earns profit equal to

$$\pi_2^* = \frac{(t(-12-19\alpha-2\alpha^2+\alpha^3)-2(1+\alpha)((4+9\alpha+\alpha^2)\delta+2\alpha(3+\alpha)\mu)+2(4+13\alpha+10\alpha^2+\alpha^3)c_v)^2}{16t(1+\alpha)(3+\alpha)^2(2+3\alpha)^2}$$

The difference $\pi_2^* - \pi_1^* = \frac{(\delta-c_v)(t(4+5\alpha-\alpha^2)+2\alpha(1+\alpha)(\delta-c_v+2\mu))}{t(3+\alpha)(2+3\alpha)} > 0$ for all $\delta > c_v$. The

total quantity sold is $(q_{1t}^* + q_1^*) = \frac{t(3+\alpha)-6(1+\alpha)(\delta-c_v)}{2t(3+\alpha)}$ for retailer 1 and

$(q_{2t}^* + q_2^*) = \frac{t(3+\alpha)+6(1+\alpha)(\delta-c_v)}{2t(3+\alpha)}$ for retailer 2. *Q.E.D.*

APPENDIX B. IMPACT ON MARKET EXPANDING EFFORT

Carlton and Chevalier (2001) hypothesize that manufacturers that rely strongly upon physical sales effort may limit the availability of their products on the internet to control the free-rider problem. For example, manufacturers of salon products cite the importance of salons in helping consumers understand the value of the product.¹³ In this appendix, I demonstrate that online free-riding of market expanding effort can lead to fewer manufacturer sales and thus be an impetus for the manufacturer to not authorize the direct seller. To this end, I model a specific form of effort, though the finding is robust to several modifications. The purpose of this model is not to definitively prove why manufacturers, in some cases, do not authorize online direct sellers. Rather, it demonstrates a context that leads a manufacturer to authorize only service-providing brick-and-mortar retailers. Moreover, it shows why retailer diversion to the unauthorized direct seller can be a concern for the manufacturer. It should be noted that this appendix abstracts from the elements that can lead free-riding to soften competition and increase retailer profit (See Shin 2007).

I first model a manufacturer's decision of whether or not to authorize the direct seller when there is no diversion. Recall that retailers are distributed at even intervals around a Salop circle of mass 1. In this companion model, I assume that there are $\sum_{j=1}^N e_j$ identical circles where e_j is the market-expanding effort by retailer j . Retailers incur convex costs of effort equal to βe_j^2 . In summary, whereas the main model assumes fixed market size equal to 1, this extension allows the market size to be equal to the sum of retailer efforts with the same distribution of consumer preferences for firms. The findings persist if instead a retailer's effort only expands its home market defined to be the arc of length $1/N$ surrounding the retailer's location.

The market expansion can be in the form of either new consumers from the same location or additional purchases by existing consumers. The effort includes sales assistance, events, flyers, or advertisements that boost the awareness or value of the brand for the exposed consumers. Consider the lead-in example.

¹³ From Palmer (2004): "Consumers don't know if a product is going to give them the results they want without the help from stylists."

Retailers who host in-store gaming events will afford its participants greater value out of purchasing the trading cards used in the game. Moreover, the in-store gaming can build greater awareness and lead participants to encourage new customers to try the product. Consider also the value of salons in terms of helping customers use products best suited to the customer's hair type, thereby increasing the value and the demand of the hair product. While the retailer benefits from the increased brand value or customer base, some consumers may choose to buy from the online seller (if there is one). In stage 0, the manufacturer decides whether or not to authorize the direct seller. In stage 1, the retailers simultaneously choose effort. In stage 2, the retailers and direct seller simultaneously choose prices.

Now consider demand for retailer A's product when choosing price p_A . The market shares are as derived in the proof of Lemma 1 with the market size of $e_A + (N-1)e_r$. As such, the resulting demand for retailer A is $q_A^E = (e_A + (N-1)e_r) \left[\frac{(1-\alpha)}{t} (p_r + \frac{t}{N} - p_A) + 2\alpha \left(\frac{p_d - p_A + \mu}{t} \right) \right]$ and the demand for the direct seller is $q_d^E = (e_r + (N-1)e_A) \left[\alpha \left(\frac{1}{N} - 2 \frac{p_d - p_A + \mu}{t} \right) + \alpha(N-1) \left(\frac{1}{N} - 2 \frac{p_d - p_r + \mu}{t} \right) \right]$ where the superscript E represents that these quantities are derived in this market-expanding effort extension.

The profit of retailer A is $\pi_A^E = (p_A - w)q_A^E - \beta e_A^2$ and the profit of the authorized direct seller is $\pi_d^E = (p_d - w)q_d^E$. Notice that retailer A profit is equivalent to $(\sum_{j=1}^N e_j) [(p_A - w)q_A] - \beta e_A^2$ and direct seller profit is equal to $(\sum_{j=1}^N e_j) [(p_d - w)q_d]$ where q_A and q_d are the market shares as defined in the proof of Lemma 1. From the first order conditions, it is straightforward to show that in stage 1,

$$(p_A^* - w)q_A^* = \frac{(1+\alpha)(2\alpha\mu N + 2t - \alpha t)^2}{4(1+2\alpha)^2 N^2 t} \quad \text{if} \quad \frac{t(4Z - 2\alpha Z - 3\alpha - 4)}{2\alpha N(1-2Z)} < \mu < \frac{t(3+\alpha)}{2N(1+\alpha)} \quad \text{where}$$

$$Z \equiv \sqrt{\frac{1+\alpha}{1-\alpha}}. \quad \text{The minimum bound is the condition which ensures that the retailers do not cede all}$$

consumers who would consider the direct seller. Retailer A's stage 1 objective function can be rewritten

as $\max_{e_A} e_A(p_A^* - w)q_A^* - \beta e_A^2 + (N-1)e_r(p_A^* - w)q_A^*$. Taking the first order condition and setting

$$e_A = e_r \text{ leads to } e^* = \frac{(1+\alpha)(2\alpha\mu N + 2t - \alpha t)^2}{8\beta(1+2\alpha)^2 N^2 t}.$$

Now consider the manufacturer's decision of whether or not authorize the direct seller in stage 0. Market demand will be equal to Ne^* . When the direct seller is authorized, manufacturer profit will be

$$(w-c)Ne^* = (w-c)\frac{(1+\alpha)(2\alpha\mu N + 2t - \alpha t)^2}{8\beta(1+2\alpha)^2 Nt}.$$

When there is no direct seller, the manufacturer profit will be $(w-c)Ne^* = (w-c)t/(2\beta N)$. The difference $(w-c)\left[\frac{(1+\alpha)(2\alpha\mu N + 2t - \alpha t)^2}{8\beta(1+2\alpha)^2 Nt} - \frac{t}{2\beta N}\right] < 0$

for all μ that leads to positive sales by the direct seller (i.e., $\mu < \frac{t(3+\alpha)}{2N(1+\alpha)}$). Thus the manufacturer, in

this model, will prefer to have only retailers as authorized sellers rather than have a direct seller. Notice that this result holds for any wholesale price that is constant across conditions.

I also show that the manufacturer may not authorize the direct seller even if the wholesale price is not constant across conditions. Consider the case in which the manufacturer is able to extract all profit from its intermediaries via a fixed fee. Industry profit with the direct seller authorized is equal to $N[Ne^*(p_A^* - w)q_A^* - \beta e^{*2}] + Ne^*(p_d^* - w)q_d^*$ which is equal to

$$\frac{(1+\alpha)(t(2-\alpha) + 2N\alpha\mu)^2}{64N^3 t^2 (1+2\alpha)^4 \beta} \cdot [(t^2(-(2-\alpha)^2(1+\alpha) + N(8+9\alpha+3\alpha^3)) - 4Nt\alpha(1+\alpha)(2-\alpha - N(1-3\alpha))\mu + 4N^2\alpha(1+\alpha)(N-\alpha+3N\alpha)\mu^2)]$$

Industry profit with only N authorized retailers is equal to

$$N[Ne^*(p_A^* - w)q_A^* - \beta e^{*2}] = (2N-1)t^2 / 4\beta N^3 \text{ where } e^* = t / (2\beta N^2).$$

The industry profit with an authorized direct seller minus the industry profit without a direct seller evaluated at the upper bound on μ that leads to positive direct sales (i.e., $\mu = t(3+\alpha) / 2N(1+\alpha)$) is equal to

$-\frac{(2N-1)t^2\alpha(2+\alpha)}{4N^3(1+\alpha)^2\beta} < 0$. The difference is increasing in μ through this point. Thus, locally

around $\mu < t(3+\alpha)/2N(1+\alpha)$ industry profit is lower when the direct seller is authorized than when there is no direct seller.

In this set-up, it is also easy to see why diversion by the retailers has an adverse effect on manufacturer profitability. Cross, Stephans, and Benjamin (1990) point out that unauthorized sellers can free ride on the promotion and service provide by the authorized channel members. Practitioners often cite the retailers' incentive to provide valuable service as an important factor in manufacturer decisions.¹⁴ Let effort be

simultaneously set before the stages of the game depicted in Figure 1 in the text. Retailer A profit in the effort-setting stage can be written as

$\max_{e_A} e_A [(p_A^* - w)q_A^* + (w_d^* - w)q_t^*] - \beta e_A^2 + (N-1)e_r [(p_A^* - w)q_A^* + (w_d^* - w)q_t^*]$. The equilibrium effort

by each retailer will be equal to $e^* = [(p_A^* - w)q_A^* + (w_d^* - w)q_t^*] / 2\beta$. Since Proposition 3 demonstrated

that $(p_A^* - w)q_A^* + (w_d^* - w)q_t^*$ is less than t/N^2 and both are independent of the manufacturer's wholesale price, the effort levels by the retailers and consequently the manufacturer's sales will be less when there is diversion in equilibrium than when diversion can be prevented.

Thus this Appendix has proven when an unauthorized direct seller free-rides on retailer market-expanding effort, diversion can lead to diminished manufacturer sales relative to when the free-riding online sellers are not authorized (for any linear wholesale contract). The intuition of managers and the logic behind why within-market, channel-flow diversion can be a concern for manufacturers is confirmed. While the logic holds for a variety of different models of market-expanding effort, there are other factors such as market expansion, contract negotiation costs, or scale economies that may lead a manufacturer to benefit from diversion or prefer to authorize direct sellers. However, this Appendix demonstrates that diversion can occur in a set-up in which the manufacturer does not authorize the direct seller.

¹⁴ From <http://www.divecenterbusiness.com/public/190.cfm?sd=36>, "The scuba business is as much about service as it is about equipment."