ON-LINE SUPPLEMENT

for

Optimal Reverse Channel Structure for Consumer Product Returns

A.1. Proof of Equation 1 (See Section 2)

$$
\begin{aligned}
&\phi_{kj}(f_j, p_j, p_{-j}; d, \alpha) = \alpha(\frac{1}{2} + \frac{f_j - p_j + p_{-j}}{2d}) \\
&\phi_{ej}(f_j, p_j, p_{-j}; d, \alpha) = \alpha(\frac{1}{2} - \frac{f_j - p_j + p_{-j}}{2d}) \\
&\phi_r = 1 - \alpha \\
&E_{ij}(utility) = \alpha(u_i - \frac{p_j + p_{-j} + f_j}{2} - \frac{d}{4} + \frac{(f_j - p_j + p_{-j})^2}{4d}) - (1-\alpha) f_j
\end{aligned}
\qquad \textbf{(1)}
$$

Proof:

Consider two horizontally differentiated products located at 0 and 1 on a Hotelling unit line. Let the product located at 0 be denoted by j=0 and the product located at 1 be denoted by j=1. Suppose the consumer located at θ_i purchases $x_0 = 0$. Upon purchasing the product and learning the value of product fit $|x_j - \theta_i|$, with probability α the consumer will get utility equal to $u_i - p_0 - d\theta_i$ from keeping this product and utility equal to $-f_0 + u_i - p_1 - d(1-\theta_i)$ from exchanging it for $x_1 = 1$. Comparing these two utilities, consumers who purchase x_0, will prefer to keep it if $\theta_i < \frac{1}{2} + \frac{f_0 - p_0 + p_1}{2d}$ and will prefer to exchange it for $x_1 = 1$ if $\theta_i > \frac{1}{2} + \frac{f_0 - p_0 + p_1}{2d}$. With probability (1- α), the consumer will discover the product category offers zero utility and make a return. With probability α, the consumer will have positive utility from owning a product in the category. Because $\theta_i \sim U[0,1]$, the probability that someone who buys product $x_0 = 0$ decides to keep it is equal to $\alpha(\frac{1}{2} + \frac{f_0 - p_0 + p_1}{2d})$ and the probability that someone who buys product $x_0 = 0$ decides to exchange it is equal to $\alpha(\frac{1}{2} - \frac{f_0 - p_0 + p_1}{2d})$. For a given consumer *i* who purchases $x_0 = 0$ initially, we may write the probability that the consumer keeps this purchase, $\phi_{k0}(f_0, p_0, p_1; d, \alpha) = \alpha(\frac{1}{2} + \frac{f_0 - p_0 + p_1}{2d})$, the probability that the consumer exchanges this purchase for product B $\phi_{e0}(f_0, p_0, p_1; d, \alpha) = \alpha(\frac{1}{2} - \frac{f_0 - p_0 + p_1}{2d})$.

We derive the expected utility of purchasing $x_0 = 0$ for a consumer with reservation utility u_i. With probability $\phi_{k0}(f_0, p_0, p_1; d, \alpha) = \alpha(\frac{1}{2} + \frac{f_0 - p_0 + p_1}{2d})$, the consumer will keep the *j=0* product. The unit will be kept if and only if $|0 - \theta_i| < \frac{1}{2} + \frac{f_0 - p_0 + p_1}{2d}$. Therefore, the average utility derived from keeping the unit will be equal to $u_i - p_0 - \frac{d}{2}(\frac{1}{2} + \frac{f_0 - p_0 + p_1}{2d})$. The unit will be exchanged for product *j=1* if and only if $\theta_i > \frac{1}{2} + \frac{f_0 - p_0 + p_1}{2d}$, which implies $|1 - \theta_i| < \frac{1}{2} - \frac{f_0 - p_0 + p_1}{2d}$. Therefore, the average utility derived from exchanging a unit of product *j=0* for a unit of product *j=1* will be $u_i - p_1 - f_0 - \frac{d}{2}(\frac{1}{2} - \frac{f_0 - p_0 + p_1}{2d})$. With probability *(1-α),* the consumer will realize after purchase that neither product is a match. In this case, the consumer will return the initial purchase at a penalty f_0 and not make a subsequent purchase. We may now write the expected utility *ex ante* of purchasing $x_0 = 0$ for consumer *i* as:

E_{i0}(utility)=

$$\phi_{k0}(f_0, p_0, p_1; d)(u_i - p_0 - \frac{d}{2}(\frac{1}{2} + \frac{f_0 - p_0 + p_1}{2d})) + \phi_{e0}(f_0, f_1, p_0, p_1; d)(u_i - p_1 - f_0 - \frac{d}{2}(\frac{1}{2} - \frac{f_0 - p_0 + p_1}{2d})) - (1-\alpha) f_0$$

which simplifies to $E_{i0}(utility) = \alpha(u_i - \frac{p_0 + p_1 + f_0}{2} - \frac{d}{4} + \frac{(f_0 - p_0 + p_1)^2}{4d}) - (1-\alpha) f_0$.

Suppose the consumer located at θ_i purchases $x_1 = 1$. Upon purchasing the product and learning the value of product fit $|x_j - \theta_i|$, with probability α the consumer will get utility equal to $u_i - p_1 - d(1-\theta_i)$ from keeping this product and utility equal to $-f_1 + u_i - p_0 - d\theta_i$ from exchanging it for $x_0 = 0$. Comparing these two utilities, consumers who purchase $x_1 = 1$, will prefer to keep it if $\theta_i > \frac{1}{2} - \frac{f_1 - p_1 + p_0}{2d}$ and will prefer to exchange it for $x_0 = 0$ if $\theta_i < \frac{1}{2} - \frac{f_1 - p_1 + p_0}{2d}$. With probability α, the consumer will have positive utility from owning a product in the category. Because $\theta_i \sim U[0,1]$, the probability that someone who buys product $x_1 = 1$ decides to keep it is equal to $\alpha(\frac{1}{2} + \frac{f_1 - p_1 + p_0}{2d})$ and the probability that someone who buys product $x_1 = 1$ decides to exchange it is equal to $\alpha(\frac{1}{2} - \frac{f_1 - p_1 + p_0}{2d})$. For a given consumer *i* who purchases product *j=1* initially, we may write the probability that the consumer keeps this purchase as $\phi_{k1}(f_1, p_0, p_1; d, \alpha) = \alpha(\frac{1}{2} + \frac{f_1 - p_1 + p_0}{2d})$, and the probability that the consumer exchanges this purchase for product 0 as $\phi_{e1}(f_1, p_0, p_1; d, \alpha) = \alpha(\frac{1}{2} - \frac{f_1 - p_1 + p_0}{2d})$.

We derive the expected utility of making a purchase for a consumer with reservation utility u_i. With probability $\phi_{k1}(f_1, p_0, p_1; d, \alpha) = \alpha(\frac{1}{2} + \frac{f_1 - p_1 + p_0}{2d})$, the consumer will keep the unit *1*. The unit *1* will be kept if and only if $|1-\theta_i| < \frac{1}{2} + \frac{f_1 - p_1 + p_0}{2d}$. Therefore, the average utility derived from keeping the unit will be equal to $u_i - p_1 - \frac{d}{2}(\frac{1}{2} + \frac{f_1 - p_1 + p_0}{2d})$. The unit will be exchanged for product *0* if and only if $\theta_i < \frac{1}{2} + \frac{f_1 - p_1 + p_0}{2d}$, which implies

$|0-\theta_i|<\frac{1}{2}-\frac{f_1-p_1+p_0}{2d}$. Therefore, the average utility derived from exchanging a unit of product *1* for a unit of product *0* will be $u_i-p_0-f_1-\frac{d}{2}(\frac{1}{2}-\frac{f_1-p_1+p_0}{2d})$. With probability *(1-α),* the consumer will realize after purchase that neither product is a match. In this case, the consumer will return the initial purchase at a penalty f_1 and not make a subsequent purchase. We may now write the expected utility *ex ante* of making a purchase for consumer *i* as:

E_{i1}(utility)=

$$\phi_{k1}(f_1,p_0,p_1;d,\alpha)(u_i-p_1-\frac{d}{2}(\frac{1}{2}+\frac{f_1-p_1+p_0}{2d}))+\phi_{e1}(f_1,p_0,p_1;d,\alpha)(u_i-p_0-f_1-\frac{d}{2}(\frac{1}{2}-\frac{f_1-p_1+p_0}{2d}))-(1-\alpha)f_1$$

which simplifies to *E_{i1}(utility)*= $\alpha(u_i-\frac{p_1+p_0+f_1}{2}-\frac{d}{4}+\frac{(f_1-p_1+p_0)^2}{4d})-(1-\alpha)f_1$.

We may therefore generalize that for a given consumer *i* who initially purchases product *j*, the probability that the consumer exchanges this purchase for the product located at x_{-j}, the probability that the consumer keeps this purchase, the probability that the consumer returns this purchase without exchange, and the expected utility *ex ante* of making this purchase (*E_j(utility)*) can be written as follows:

$$\phi_{kj}(f_j,p_j,p_{-j};d,\alpha)=\alpha(\frac{1}{2}+\frac{f_j-p_j+p_{-j}}{2d})$$
$$\phi_{ej}(f_j,p_j,p_{-j};d,\alpha)=\alpha(\frac{1}{2}-\frac{f_j-p_j+p_{-j}}{2d})$$
$$\phi_r=1-\alpha$$
$$E_{ij}(utility)=\alpha(u_i-\frac{p_j+p_{-j}+f_j}{2}-\frac{d}{4}+\frac{(f_j-p_j+p_{-j})^2}{4d})-(1-\alpha)f_j$$

Q.E.D.

A.2. Proof of Equation 2 (See Section 2)

Initial sales of product *j* are

$$q_{bj}(f_j, p_j, f_{-j}, p_{-j}; d, \overline{u}, \alpha) = \frac{1}{2}(\overline{u} - \frac{p_j + p_{-j}}{2} - \frac{d}{4} - \frac{(2-\alpha)f_j}{2\alpha} + \frac{(f_j - p_j + p_{-j})^2}{4d} \quad \text{if}$$

$$\alpha(f_j + f_{-j})(f_j - f_k - 2p_j + 2p_{-j}) - 2d(f_j - f_{-j})(2-\alpha) = 0$$

$q_{bj}(f_j, p_j, f_{-j}, p_{-j}; d, \overline{u}, \alpha) = 0$ if $\alpha(f_j + f_{-j})(f_j - f_k - 2p_j + 2p_{-j}) - 2d(f_j - f_{-j})(2-\alpha) < 0$

and $q_{bj}(f_j, p_j, f_{-j}, p_{-j}; d, \overline{u}, \alpha) = \overline{u} - \frac{p_j + p_{-j}}{2} - \frac{d}{4} - \frac{(2-\alpha)f_j}{2\alpha} + \frac{(f_j - p_j + p_{-j})^2}{4d}$ if

$\alpha(f_j + f_{-j})(f_j - f_k - 2p_j + 2p_{-j}) - 2d(f_j - f_{-j})(2-\alpha) > 0$.

Proof:

Consumers are differentiated in their value of u_i which takes values between 0 and $\overline{u}$ with an equal density (normalized to 1) at each u_i and is independent of θ_i. Note from equation (1) in the paper's text that the difference between expected utility from buying product *j=0* and the expected utility from buying product *j=1* is invariant with respect to u_i. Therefore, if product 0 is preferred *ex ante* over product 1 by one consumer, it is preferred by all consumers (and vice versa). Also, notice that the expected utility from buying product *j* is increasing in u_i. Therefore, if a consumer located at $u_i = \hat{u}_j \equiv u_i \ s.t. \ E_{ij}(utility) = 0$ has non-negative expected utility from purchasing product *j*, then all consumers with $u_i > \hat{u}_j$ will have positive expected utility from purchasing product *j*. The consumer with $u_i \ s.t. \ E_{ij}(utility) = 0$ is located at $\hat{u}_j = \frac{p_j + p_{-j}}{2} + \frac{d}{4} + \frac{(2-\alpha)f_j}{2\alpha} - \frac{(f_j - p_j + p_{-j})^2}{4d}$. To derive the quantity sold of product *j*, there are three possibilities to consider. First, if $E_{ij}(utility) = E_{i-j}(utility)$, which occurs if and only if $\alpha(f_j + f_{-j})(f_j - f_k - 2p_j + 2p_{-j}) - 2d(f_j - f_{-j})(2-\alpha) = 0$, then consumers are indifferent between purchasing product 0 and purchasing product 1 initially. All consumers with $u_i \in [\hat{u}_j, \overline{u}]$ will buy one of the two products, randomly choosing product *j* with probability 1/2. Thus

$$q_{bj}(f_j, p_j, f_{-j}, p_{-j}; d, \overline{u}, \alpha) = \frac{\overline{u} - \hat{u}_j}{2} = \frac{1}{2}(\overline{u} - \frac{p_j + p_{-j}}{2} - \frac{d}{4} - \frac{(2-\alpha)f_j}{2\alpha} + \frac{(f_j - p_j + p_{-j})^2}{4d}).$$

Second, if $E_{ij}(utility) < E_{i-j}(utility)$, which occurs if and only if $\alpha(f_j + f_{-j})(f_j - f_k - 2p_j + 2p_{-j}) - 2d(f_j - f_{-j})(2-\alpha) < 0$, then no consumer would prefer *ex ante* purchasing product *j* over purchasing the product located at x_{-j}. Therefore, the sales quantity for product *j* is $q_{bj}(f_j, p_j, f_{-j}, p_{-j}; d, \overline{u}, \alpha) = 0$.

Third, if $E_{ij}(utility) > E_{i-j}(utility)$, which occurs if and only if $\alpha(f_j + f_{-j})(f_j - f_k - 2p_j + 2p_{-j}) - 2d(f_j - f_{-j})(2-\alpha) > 0$, then all consumers would prefer *ex ante* purchasing product *j* over purchasing the product located at x_{-j}. All consumers with $u_i \in [\hat{u}_j, \overline{u}]$ will buy product *j*. Therefore,

$$q_{bj}(f_j, p_j, f_{-j}, p_{-j}; d, \overline{u}, \alpha) = \overline{u} - \hat{u}_j = \overline{u} - \frac{p_j + p_{-j}}{2} - \frac{d}{4} - \frac{(2-\alpha)f_j}{2\alpha} + \frac{(f_j - p_j + p_{-j})^2}{4d}.$$

Q.E.D.

A.3. Condition for all consumers with positive utility (a proportion α of the population) to prefer to keep or exchange rather than return and make no other purchase

In this Appendix, for given values of retailer decisions (p_j and f_j), we identify when all consumers who have positive utility from owning one of the two products (the proportion α of the population) and who purchase product *j* initially will prefer to keep or exchange it, rather than return the good and leave the market. More specifically, we find that this returns and exchange behavior by the α consumers is feasible if and only if $\frac{4df_j - \alpha(d^2 + (f_j - p_j + p_{-j})^2)}{2d\alpha} \geq 0$.

We first show that this returns and exchange behavior is feasible for a consumer located at u_i if $2u_i - p_0 - p_1 + f_0 - d \geq 0$, and then show that this implies the condition $\frac{4df_j - \alpha(d^2 + (f_j - p_j + p_{-j})^2)}{2d\alpha} \geq 0$. The derivations in this Appendix are later used to determine parameter values for which our assumption that all units sold are kept or exchanged holds even if consumers were given the option to return without subsequent purchase.

Consider consumers who can engage in one of three possible consumption behaviors. First, the consumer could **buy a product and keep** it (i.e., not return it). Such a consumer located at θ_i, who purchases and keeps a product priced at p and located at x_j, receives a utility of $u_i - p_j - d\,|\,x_j - \theta_i\,|$.

A second possibility is that the consumer purchases a product, then **returns and exchanges it** for a more preferred product after initial purchase. This consumer, located at θ_i, who initially purchases a product priced at p_j and located at x_j, receives total utility of

$u_i - p_{-j} - f_j - \underset{x_j \in \{0,1\}}{Min}\{d\,|\,x_j - \theta_i\,|\}$.

Finally, a consumer may purchase a product, then **return it and not exchange it (i.e., leave the market).** The consumer located at θ_i who purchases a product priced at p_j and located at x_j receives total utility of

$-f_j$.

Let θ_1 denote the consumer between 0 and 1 who is indifferent between keeping the product located at x_j after purchase and exchanging it for the product located at x_{-j}. Buying and keeping the product located at x_j yields a utility equal to $u_i - p_j - d\,|\,\theta_1 - x_j\,|$. Returning the product and buying the product located at x_{-j} yields utility equal to $u_i - p_{-j} - f_j - d\,|\,x_{-j} - \theta_1\,|$.

We examine consumers who initially buy product *j=0*. The analysis generalizes to consumers who buy product *j=1* initially. Of consumers who bought product *j=0* initially, the marginal consumer indifferent between keeping the initial purchase and exchanging it is located at $\theta_1 = \frac{f_0 - p_o + p_1 + d}{2d}$ such that $u_i - p_0 - d\theta_1 = u_i - p_1 - f_0 - d(1-\theta_1)$. All consumers with $\theta \in [0, \theta_1]$ who initially buy the product located at x_0 =0 earn greater utility from keeping it than from exchanging it.

Let θ_2 denote the consumer between 0 and 1 who is indifferent between keeping the product located at x_0=0 and returning it to opt out of the market (i.e., not replacing the product located at x_0=0). For this consumer, the utility of buying and keeping the product equals the utility of buying and returning the product located at x_0=0 for a refund, $u_i - d\,|\,\theta_2 - 0\,| - p_0 = -f_0$. Therefore, $\theta_2 = \frac{u_i - p_0 + f_0}{d}$. All consumers with $\theta \in [0, \theta_2]$ have greater value from keeping the product than from returning it.

Let θ_3 denote the consumer between 0 and 1 who has returned the product located at x_0=0 and is indifferent between purchasing the product located at x_1=1 and opting out of the market. For this consumer, $u_i - p_1 - f_0 - d\,|\,1-\theta_3\,| = -f_0$. Therefore, $\theta_3 = \frac{d - (u_i - p_1)}{d}$. All consumers with $\theta \in [\theta_3, 1]$ who initially purchase the product located at x_0=0 enjoy greater utility from exchanging the product located at x_0=0 for the product located at x_1=1 than from opting out of the market.

By comparing the values of θ_1, θ_2, and θ_3, we can order them $\theta_2 \geq \theta_1 \geq \theta_3$ if $2u_i - p_0 - p_1 + f_0 - d \geq 0$. Otherwise, the inequalities are reversed and $\theta_2 < \theta_1 < \theta_3$. The definitions of θ_1, θ_2, and θ_3 imply several choice rules. For consumers with:

- $1 > \theta > \theta_1$, returning the initial purchase and buying the other product (RX) $\succ$ keeping purchase (K),
- $1 > \theta > \theta_2$, returning the initial purchase and owning nothing (R) $\succ$ keeping purchase (K),
- $1 > \theta > \theta_3$, returning the initial purchase and buying the other product (RX) $\succ$ returning and owning nothing (R).

When $2u_i - p_0 - p_1 + f_0 - d < 0$ **(which implies** $\theta_2 < \theta_1 < \theta_3$**), we have:**

- for $0 < \theta < \theta_2$, $\text{K} \succ \text{R} \succ \text{RX}$;
- for $\theta_2 < \theta < \theta_1$, $\text{R} \succ \text{K} \succ \text{RX}$;
- for $\theta_1 < \theta < \theta_3$, $\text{R} \succ \text{RX} \succ \text{K}$;
- for $1 > \theta > \theta_3$, $\text{RX} \succ \text{R} \succ \text{K}$.

Thus, if $2u_i - p_0 - p_1 + f_0 - d < 0$, keeping (K) is the dominant choice for $0 < \theta < \theta_2$; returning and owning nothing (R) is the dominant choice for $\theta_2 < \theta < \theta_3$; and returning the initial purchase to buy the other product (RX) is the dominant choice for $\theta > \theta_3$. Therefore, if $2u_i - p_0 - p_1 + f_0 - d < 0$, there are consumers whose θ value lies in (θ_2, θ_3) who will return the initially purchased product for a refund and opt out of the market.

When $2u_i - p_0 - p_1 + f_0 - d \geq 0$ **(which implies** $\theta_2 > \theta_1 > \theta_3$**), we have:**

- for $0 < \theta < \theta_3$, $\text{K} \succ \text{R} \succ \text{RX}$;

- for $\theta_3 < \theta < \theta_1$, K $\succ$ RX$\succ$R;
- for $\theta_1 < \theta < \theta_2$, RX $\succ$ K$\succ$R;
- for $1 > \theta > \theta_2$, RX $\succ$ R$\succ$K.

Therefore, if $2u_i - p_0 - p_1 + f_0 - d \geq 0$ keeping (K) is the dominant choice for $0 < \theta < \theta_1$, and returning the initial purchase to buy the other product (RX) is the dominant choice for $\theta_1 < \theta < 1$. There does not exist a group of consumers who would prefer to leave the market without owning a product rather than keeping or exchanging the initial purchase.

Because the condition $2u_i - p_0 - p_1 + f_0 - d \geq 0$ is less strict for higher u_i, we may say that if it holds for the lowest u_i such that the consumer is indifferent between buying and not buying initially, then it will hold for all higher u_i values. Generalizing, all consumers who purchase product *j* initially and value the product category will keep or exchange product *j* if and only if $2u_i - p_j - p_{-j} + f_j - d \geq 0$. From A.2., this marginal consumer is located at $\hat{u}_j = \frac{p_j + p_{-j}}{2} + \frac{d}{4} + \frac{(2-\alpha)f_j}{2\alpha} - \frac{(f_j - p_j + p_{-j})^2}{4d}$. Thus if $\frac{4df_j - \alpha(d^2 + (f_j - p_j + p_{-j})^2)}{2d\alpha} \geq 0$, all consumers who purchase product *j* initially and value the product category will keep or exchange even with the available option to return a product for a refund with no exchange.

A.4. Proof of Entries in Table 2 (See Section 3)

We first solve for the equilibrium decisions of a vertically integrated system with net salvage value max{*s*, *s$_r$*}. We will then show that the same outcome can be achieved in a decentralized channel in which the manufacturer uses a fixed fee, wholesale price equal to marginal cost and a refund rate equal to the net salvage value to coordinate the channel.

The vertically integrated system's objective function can be written as

$$\max_{p_0, f_0, p_B, f_B} \pi^{VI} = q_{b0}(f_0, f_1, p_0, p_1; d, \bar{u}, \alpha) \cdot$$
$$((p_0 - c) + (-p_0 + f_0 + \max\{s, s_r\})(\phi_{e0}(f_0, p_0, p_1; d, \alpha) + \phi_r) + (p_1 - c) \cdot \phi_{e0}(f_0, p_1, p_0; d, \alpha))$$
$$+ q_{b1}(f_0, f_1, p_0, p_1; d, \bar{u}, \alpha) \cdot$$
$$((p_1 - c) + (-p_1 + f_1 + \max\{s, s_r\})(\phi_{e1}(f_1, p_0, p_1; d, \alpha) + \phi_r) + (p_0 - c) \cdot \phi_{e1}(f_1, p_1, p_0; d, \alpha)).$$

We will first solve for the symmetric equilibrium in which $E_{ij}(utility) = E_{i-j}(utility)$. As shown in Appendix A.1, this implies that $q_{bj}(f_j, p_j, f_{-j}, p_{-j}; d, \bar{u}, \alpha) = \frac{1}{2}(\bar{u} - \frac{p_j + p_{-j}}{2} - \frac{d}{4} - \frac{(2-\alpha) f_j}{2\alpha} + \frac{(f_j - p_j + p_{-j})^2}{4d}$ for each product *j*. We will show in Appendix A.5 that the vertically integrated firm cannot earn greater profit by choosing asymmetric prices or return penalties. The first order conditions for each product *j* are:

$$\frac{\partial \pi}{\partial p_j} = \frac{1}{16d^2} \left(\begin{array}{l} -2\alpha d^3 + 2d^2 \left[-4(f_j + \max\{s, s_r\}) + \alpha(f_j - 2p_j + 2\max\{s, s_r\} + 4\bar{u}) - (4 - 3\alpha) f_{-j} - 6\alpha p_{-j} \right] \\ +\alpha(f_j - 2p_j - f_{-j} + 2p_{-j}) \left\{ \begin{array}{l} 4(f_j^2 - f_j p_j + p_j^2) + 3 f_j \max\{s, s_r\} + 4 f_{-j}^2 \\ + f_{-j} \left[\begin{array}{l} 4(f_j + p_j) \\ +3\max\{s, s_r\} - 4p_{-j} \end{array} \right] + 4p_{-j}(f_j - 2p_j + p_{-j}) \end{array} \right\} \\ +c \left\{ 4d^2(2+\alpha) - 2d \left[\begin{array}{l} f_j(\alpha - 4) + (4+\alpha) f_{-j} \\ +2(2+\alpha)(p_j - p_{-j}) \end{array} \right] - 3\alpha(f_j + f_{-j})(f_j - 2p_j - f_{-j} + 2p_{-j}) \right\} \\ d \left\{ \begin{array}{l} 3f_j^2(-4 + 3\alpha) - 3(\alpha - 4) f_{-j}^2 \\ +2f_j \left[-4\max\{s, s_r\} + p_j(6 - 9\alpha) + 3\max\{s, s_r\}\alpha + 4\alpha\bar{u} + 3p_{-j}(\alpha - 2) \right] \\ +2f_{-j} \left[4\max\{s, s_r\} - \alpha(\max\{s, s_r\} + 4\bar{u}) + 3p_j(2+\alpha) + 3p_{-j}(\alpha - 2) \right] \\ +2(p_j - p_{-j}) \left[4\max\{s, s_r\} + 9\alpha p_j - 2\alpha(\max\{s, s_r\} + 4\bar{u}) + 3\alpha p_{-j} \right] \end{array} \right\} \end{array} \right) = 0$$

$$\frac{\partial \pi}{\partial f_j} = \frac{1}{16\alpha d^2} \left(\begin{array}{l} -2d \left\{ \begin{array}{l} \left[4d - 2\alpha(d + f_j - p_j + p_{-j}) \right] \bullet \\ \left[-c + p_j(f_j - p_j + s)(1 - \alpha) + \frac{\alpha(d - f_j + p_j - p_{-j})(f_j - c - p_j + \max\{s, s_r\} + p_{-j})}{2d} \right] \end{array} \right\} \\ + \left\{ \begin{array}{l} \left[\alpha(d - c + 2f_j - 2p_j + \max\{s, s_r\}) - 2d + 2\alpha p_{-j} \right] \bullet \\ \left\{ \alpha d^2 - \alpha(f_j - p_j + p_{-j})^2 + d\left[2f_j(2 - \alpha) + 2\alpha(p_j - 2\bar{u} + p_{-j}) \right] \right\} \end{array} \right\} \end{array} \right).$$

There are three symmetric solutions to the first order conditions:

I) $p = \frac{\overline{u}}{2} + \frac{(c - \max\{s, s_r\})^2}{8d} + \frac{2(c + \max\{s, s_r\}) - d}{8} - \frac{(1-\alpha)(c - \max\{s, s_r\})}{2\alpha}$

$f = c - \max\{s, s_r\}$

Defining $X_{VI} \equiv \alpha^2((c - \max\{s, s_r\})^2 - 2d(c + \max\{s, s_r\} - 2\overline{u}) - d^2) - 4d\alpha(c - \max\{s, s_r\})$, we have

II)

$$p = \left[\begin{array}{l} 2\overline{u} + \frac{(c - \max\{s, s_r\})^2 - d(d + 2\max\{s, s_r\})}{2d} - \frac{\sqrt{X_{VI}}}{\alpha^2} \\ + \frac{4d(\max\{s, s_r\} - c) + (c + d - \max\{s, s_r\})\sqrt{X_{VI}}}{2d\alpha} \end{array} \right]$$

$f = c - \max\{s, s_r\} - \sqrt{X_{VI}}$

III)

$$p = \left[\begin{array}{l} 2\overline{u} + \frac{(c - \max\{s, s_r\})^2 - d(d + 2\max\{s, s_r\})}{2d} + \frac{\sqrt{X_{VI}}}{\alpha^2} \\ + \frac{4d(\max\{s, s_r\} - c) - (c + d - \max\{s, s_r\})\sqrt{X_{VI}}}{2d\alpha} \end{array} \right]$$

$f = c - \max\{s, s_r\} + \sqrt{X_{VI}}$.

The initial sales quantity q_b when evaluated at solutions I, II and III is

$q_b = \frac{\overline{u}}{2} + \frac{(c - \max\{s, s_r\})^2}{8d} - \frac{2(3c - \max\{s, s_r\}) + d}{8} - \frac{(1-\alpha)(c - \max\{s, s_r\})}{2\alpha}$, $q_b = 0$, and $q_b = 0$

respectively. We therefore rule out potential solutions II and III. We verify below that the second order conditions are satisfied.

$$\frac{\partial\partial\pi}{\partial\partial p_j}=-\frac{\alpha\overline{u}}{d}-\frac{1}{8d^2}\begin{pmatrix}4cd-6df_j-4d\max\{s,s_r\}\\+\alpha(2cd+2d^2-3cf_j+9df_j+6f_j^2-18dp_j-12f_jp_j+12p_j^2+2d\max\{s,s_r\}+3f_j\max\{s,s_r\})\\+6\alpha f_{-j}^2+6\alpha p_{-j}(d+2f_j-4p_j+2p_{-j})\\-3f_{-j}(2d+\alpha(c+d-4p_j-\max\{s,s_r\})+4p_{-j})\end{pmatrix}$$

$$\frac{\partial\partial\pi}{\partial\partial f_j}=-\frac{\alpha\overline{u}}{2d}-\frac{1-\alpha}{\alpha}-\frac{\alpha}{8}$$

$$+\frac{3\alpha(f_j-p_j)(c-2f_j+2p_j-\max\{s,s_r\})+d(-6c+6(3f_j-2p_j+\max\{s,s_r\})+\alpha(c-9f_j+9p_j-3\max\{s,s_r\}))}{8d^2}$$

$$-\frac{3p_{-j}(-4d+\alpha(d-c+\max\{s,s_r\}+4(f_j-p_j)+2p_{-j}))}{8d^2}$$

We will show that both of these second derivatives are negative when evaluated at solution I. The second derivative with respect to price when evaluated at solution I is $\frac{\partial\partial\pi}{\partial\partial p_j}|_{solution\,I}=-\frac{1}{4d}(\alpha\overline{u}+\frac{9\alpha(c-\max\{s,s_r\})^2}{4d}-\frac{4(c-\max\{s,s_r\})+2\alpha(c+\max\{s,s_r\})-7d\alpha}{4})$.

This is negative for any $\overline{u}$ such that the equilibrium total quantity (i.e. $q_b^{VI}(.)\equiv q_{b0}^{VI}(.)+q_{b1}^{VI}(.)$) $q_b^{VI}=\frac{\overline{u}}{2}+\frac{(c-\max\{s,s_r\})^2}{8d}-\frac{2(3c-\max\{s,s_r\})+d}{8}-\frac{(1-\alpha)(c-\max\{s,s_r\})}{2\alpha}>0$. The second derivative with respect to return penalty when evaluated at solution I is $\frac{\partial\partial\pi}{\partial\partial f_j}|_{solution\,I}=-\frac{\alpha\overline{u}}{8d}+\frac{c(18-7\alpha)-9s(2-\alpha)}{16d}-\frac{9\alpha(c-s)^2}{32d^2}-\frac{1-\alpha}{\alpha}-\frac{7\alpha}{32}$. This is negative for any $\overline{u}$ such that $q_b^{VI}=\frac{\overline{u}}{2}+\frac{(c-\max\{s,s_r\})^2}{8d}-\frac{2(3c-\max\{s,s_r\})+d}{8}-\frac{(1-\alpha)(c-\max\{s,s_r\})}{2\alpha}>0$. The value of the discriminant $D\equiv\frac{\partial\partial\pi}{\partial\partial p_j}\cdot\frac{\partial\partial\pi}{\partial\partial f_j}-\frac{\partial\partial\pi}{\partial p_j\partial f_j}\cdot\frac{\partial\partial\pi}{\partial f_j\partial p_j}$ at solution I is:

$$D=\overline{u}\left(\frac{2d\alpha^2\overline{u}+9\alpha^2(c-\max\{s,s_r\})^2+d^2(32-16\alpha+7\alpha^2)-2d\alpha\left[\max\{s,s_r\}(\alpha-10)+c(10+\alpha)\right]}{128d^3}\right)$$

$$-\frac{c-s}{4d\alpha}+\frac{29(c-\max\{s,s_r\})^2-32cd+12d^2+16ds}{64d^2}$$

$$-\frac{\alpha\begin{bmatrix}45c^3+28d^3+21d^2\max\{s,s_r\}-18d(\max\{s,s_r\})^2-45(\max\{s,s_r\})^3\\-c^2(38d+135\max\{s,s_r\})+c\left[56d\max\{s,s_r\}-37d^2+135(\max\{s,s_r\})^2\right]\end{bmatrix}}{128d^3}$$

$$+\frac{\alpha^2\left[9(c-\max\{s,s_r\})^2+14cd+7d^2-18d\max\{s,s_r\}\right]\left[9(c-\max\{s,s_r\})^2+7d^2+14ds-18cd\right]}{1024d^4}.$$

The discriminant is equal to $\frac{(d-c+s)^2(d(2-\alpha)-\alpha(c-s))^2}{16d^4}>0$ at the minimum value of $\overline{u}$ such that $q_b^{VI}=\frac{\overline{u}}{2}+\frac{(c-\max\{s,s_r\})^2}{8d}-\frac{2(3c-\max\{s,s_r\})+d}{8}-\frac{(1-\alpha)(c-\max\{s,s_r\})}{2\alpha}>0$. The derivative with respect to $\overline{u}$ is positive for all $\overline{u}$ in this range and d such that exchange probability is positive (i.e. $d>c-\max\{s,s_r\}$). Because $\lim_{f_j\to+/-\infty}\pi=-\infty$, $\lim_{p_j\to+/-\infty}\pi=-\infty$, the only symmetric local maximum of profit (solution I) is also a global maximum. Therefore,

$$p^{VI}=\frac{\overline{u}}{2}+\frac{(c-\max\{s,s_r\})^2}{8d}+\frac{2(c+\max\{s,s_r\})-d}{8}-\frac{(1-\alpha)(c-\max\{s,s_r\})}{2\alpha}$$

$f^{VI}=c-\max\{s,s_r\}$. Substituting this solution into the demand/return equations and the profit expression yields $q_b^{VI}=\frac{\overline{u}}{2}+\frac{(c-\max\{s,s_r\})^2}{8d}-\frac{2(3c-\max\{s,s_r\})+d}{8}-\frac{(1-\alpha)(c-\max\{s,s_r\})}{2\alpha}$,

$$\phi_e^{VI}=\frac{1}{2}-\frac{(c-\max\{s,s_r\})}{2d}$$

and $\pi_{mfgr}^{VI}=\frac{(\alpha(c-\max\{s,s_r\})^2-(4+2\alpha)cd+2d((2-\alpha)\max\{s,s_r\}+2\alpha\overline{u})-\alpha d^2)^2}{64d^2}$.

It is trivial to show that fixing the choice set to be symmetric for each product in the profit expression (rather than after the first order conditions) will produce the same symmetric equilibrium result. We now show that this outcome can be achieved in a decentralized channel in which the manufacturer charges a fixed fee, wholesale price equal to marginal cost, and a refund rate equal to the net salvage value. To simplify the presentation of analysis, we impose symmetry on the problem without loss of generality. We will then show that asymmetry in choice variables across products will not improve profit for either the manufacturer or the retailer.

A retailer's objective function can be written as:

$$\max_{p,f} \pi_{ret} = \begin{bmatrix} q_b(f,p;d,\bar{u},\alpha)\bullet \\ \left[(p-w)+(-p+f+\max\{s_r,r\})\left[\phi_e(f;d,\alpha)+\phi_r(\alpha)\right]+(p-w)\phi_e(f;d,\alpha)\right]-T \end{bmatrix},$$

where T is the fixed fee, $q_b(f,p;d,\bar{u},\alpha)=\bar{u}-p-\frac{d}{4}-\frac{(2-\alpha)f}{2\alpha}+\frac{f^2}{4d}$, $\phi_r=(1-\alpha)$ and $\phi_e(f;d,\alpha)=\alpha(\frac{1}{2}-\frac{f}{2d})$.[1]

The first order conditions are

$$\frac{\partial\pi_{ret}}{\partial p}=\alpha(\bar{u}-(p-w)+\frac{(d-f)(w-\max\{s_r,r\}-f)}{2d})-(1-\alpha)(2f+\max\{s_r,r\}-w)$$
$$+\frac{\alpha}{4}(-d+\frac{f^2}{d}-2(f+2p))=0$$

$$\frac{\partial\pi_{ret}}{\partial f}=\frac{(\alpha(d^2-f^2)+2d(f(2-\alpha)+4\alpha(p-\bar{u})))(\alpha(2f-w+\max\{s_r,r\})-(2-\alpha)d)}{8\alpha d^2}$$
$$+\frac{(2d-\alpha d-\alpha f)((2d-\alpha d-\alpha f)(f+\max\{s_r,r\}-w)-2d\alpha w+2\alpha dp)}{4\alpha d^2}=0$$

There are three possible solutions that satisfy the first order conditions.

I) $p(w,r)=\frac{\bar{u}}{2}+\frac{(w-\max\{s_r,r\})^2}{8d}+\frac{2(w+\max\{s_r,r\})-d}{8}-\frac{(1-\alpha)(w-\max\{s_r,r\})}{2\alpha}$

$f(w,r)=w-\max\{s_r,r\}$

[1] These expressions are adapted from equations (1) and (2) from the text by setting $p_j=p$ and $f_j=f$ for each product j.

Defining $X \equiv \alpha^2((w-\max\{s_r,r\})^2 - 2d(w+\max\{s_r,r\}-2\overline{u}) - d^2) - 4d\alpha(w-\max\{s_r,r\})$, we have

$$\text{II)}\ \ p(w,r) = \begin{bmatrix} 2\overline{u} + \dfrac{(w-\max\{s_r,r\})^2 - d(d+2\max\{s_r,r\})}{2d} \\ -\dfrac{\sqrt{X}}{\alpha^2} + \dfrac{4d(\max\{s_r,r\}-w) + (w+d-\max\{s_r,r\})\sqrt{X}}{2d\alpha} \end{bmatrix}$$

$$f = w - r - \sqrt{X}$$

$$\text{III)}\ \ p(w,r) = \begin{bmatrix} 2\overline{u} + \dfrac{(w-\max\{s_r,r\})^2 - d(d+2\max\{s_r,r\})}{2d} \\ +\dfrac{\sqrt{X}}{\alpha^2} + \dfrac{4d(\max\{s_r,r\}-w) - (w+d-\max\{s_r,r\})\sqrt{X}}{2d\alpha} \end{bmatrix}$$

$$f(w,r) = w - \max\{s_r,r\} + \sqrt{X}\ .$$

The initial sales quantity q_b when evaluated at solutions I, II and III is

$$q_b = \frac{\overline{u}}{2} - \frac{w-\max\{s_r,r\}}{2\alpha} - \frac{d^2 + 2d(w+\max\{s_r,r\}) - (w-\max\{s_r,r\})^2}{8d},\quad q_b = 0,\quad \text{and}\quad q_b = 0$$

respectively. We therefore rule out potential solutions II and III. Therefore, the retailer's reaction functions are

$$p(w,r) = \frac{\overline{u}}{2} + \frac{(w-\max\{s_r,r\})^2}{8d} + \frac{2(w+\max\{s_r,r\}) - d}{8} - \frac{(1-\alpha)(w-\max\{r,s_r\})}{2\alpha}$$

$$f(w,r) = w - \max\{s_r,r\}\ .$$

We now examine two cases separately (when $r>s_r$ and when $r<s_r$) and indentify which will hold in equilibrium.

Assuming the manufacturer accepts returns and offers $r>s_r$

The manufacturer's objective is:

$$\max_{w,r,T} \pi_{mfgr} = T + q_b(f(w,r), p(w,r); d, \overline{u})((w-c)+(w-c-(r-s))\phi_e(f(w,r);d) - \phi_r(r-s))$$
$$s.t.\, \pi_{ret} \geq 0$$

Using a Lagrangian multiplier λ on the constraint, we have the following Kuhn-Tucker conditions:

$$\frac{\partial \pi_{mfgr}}{\partial w} = \frac{(2d+\alpha(d+r-w))(2cd+\alpha(c(d+r-w)+(r-w-d)(r-s-w))-2d(w+s-r))}{8\alpha d^2}$$
$$+\frac{(\alpha(c+2r-s-2w)+d(2+\alpha))(\alpha(w-r)^2-d^2\alpha-2d(w(2+\alpha)-r(2-\alpha)-2\alpha\overline{u}))}{16\alpha d^2}$$
$$+\lambda\frac{(2d+\alpha(r+d-w))(\alpha(d^2-(w-r)^2+2d(w+r)-2\overline{u})+4d(w-r))}{16\alpha d^2}=0$$

$$\frac{\partial \pi_{mfgr}}{\partial r} = \frac{(-2d+\alpha(d-r+w))(2cd+\alpha(c(d+r-w)+(r-w-d)(r-s-w))-2d(w+s-r))}{8\alpha d^2}$$
$$+\frac{(d(2-\alpha)+\alpha(c+2r-s-2w))(\alpha(w-r)^2-d^2\alpha-2d(w(2+\alpha)-r(2-\alpha)-2\alpha\overline{u}))}{16\alpha d^2}$$
$$+\lambda\frac{(\alpha(d+w-r)-2d)(\alpha(d^2-(w-r)^2+2d(w+r)-2\overline{u})+4d(w-r))}{16\alpha d^2}$$

$$\frac{\partial \pi_{mfgr}}{\partial T} = 1-\lambda$$

$$\lambda \pi_{ret} = 0, \lambda > 0$$

There are three potential solutions that satisfy these conditions. The first is:

1. $w=c,\ r=s,\ \lambda=1,\ T=\dfrac{(\alpha(c-s)^2-(4+2\alpha)cd+2d((2-\alpha)s+2\alpha\overline{u})-\alpha d^2)^2}{64d^2}$.

Defining $Y_M = \left[\alpha(c-s)^2 - d^2\alpha - 2d\left[c(2+\alpha)-s(2-\alpha)-2\alpha\overline{u}\right]\right]$, the second and third potential solutions are:

2. $T=0,\ \lambda=1,\ r = s - \dfrac{Y_M}{2d\alpha} + \dfrac{\sqrt{-\alpha^3 Y_M(\alpha(c-s)-d(2+\alpha))^2}}{2d\alpha^3}$,

$$w = c - \frac{Y_M}{2d} - \frac{(d(2-\alpha)-\alpha(c-s))\sqrt{-\alpha^3 Y_M(\alpha(c-s)-d(2+\alpha))^2}}{2d\alpha^3(\alpha(c-s)-d(2+\alpha))}$$

3. $T=0,\ \lambda=1,\ r=s-\frac{Y_M}{2d\alpha}-\frac{\sqrt{-\alpha^3 Y_M(\alpha(c-s)-d(2+\alpha))^2}}{2d\alpha^3}$,

$$w=c-\frac{Y_M}{2d}+\frac{(d(2-\alpha)-\alpha(c-s))\sqrt{-\alpha^3 Y_M(\alpha(c-s)-d(2+\alpha))^2}}{2d\alpha^3(\alpha(c-s)-d(2+\alpha))}.$$

The retailer's reaction to possible solutions 2 and 3 leads to $q_b=0$ and zero profit for both the retailer and the manufacturer. Therefore, the first possibility is the only solution for which profit and q_b are potentially positive. As such, assuming the manufacturer accepts returns and offers $r>s_r$ (which holds true in equilibrium for $s>s_r$) the equilibrium is described by

$$p^{VI}=\frac{\bar{u}}{2}+\frac{(c-s)^2}{8d}+\frac{2(c+s)-d}{8}-\frac{(1-\alpha)(c-s)}{2\alpha}$$

$$f^{VI}=c-s,$$

$$q_b^{VI}=\frac{\bar{u}}{2}+\frac{(c-s)^2}{8d}-\frac{2(3c-s)+d}{8}-\frac{(1-\alpha)(c-s)}{2\alpha},$$

$$\phi_e^{VI}=\alpha\left(\frac{1}{2}-\frac{(c-s)}{2d}\right)$$

and $\pi_{mfgr}^{VI}=\frac{(\alpha(c-s)^2-(4+2\alpha)cd+2d((2-\alpha)s+2\alpha\bar{u})-\alpha d^2)^2}{64d^2}$ which is equivalent to a vertically integrated system when $s>s_r$.

If the manufacturer does not accept returns ($r<s_r$):

The manufacturer's objective is

$$\max_{w,T} T+q_b(f(w,r),p(w,r);d,\bar{u})((w-c)+(w-c)\phi_e(f(w,r);d))$$
$$s.t.\ \pi_{ret}\geq 0$$

Defining $Y_R\equiv\alpha((w-s_r)^2-2d(w+s_r-2\bar{u})-d^2)-4d(w-s_r)$, the Kuhn-Tucker conditions (with a Lagrangian multiplier λ) are

$$\frac{\partial \pi_{mfgr}}{\partial w} = \frac{1}{16\alpha d^2}(2(c-w)(\alpha(s_r-w)+d(2+\alpha))^2+(\alpha(c+s_r-2w)+d(2+\alpha))Y_R)$$
$$+\lambda\frac{(2d+\alpha(s_r+d-w))(\alpha(d^2-(w-s_r)^2+2d(w+s_r)-2\overline{u})+4d(w-s_r))}{16\alpha d^2}=0$$

$$\frac{\partial \pi_{mfgr}}{\partial T} = 1-\lambda=0,\quad \lambda\pi_{mfgr}\geq 0,\quad \lambda>0.$$

There are three possible solutions:

A. $w=c,\ \lambda=1,\ T=\frac{(\alpha(c-s_r)^2-(4+2\alpha)cd+2d((2-\alpha)s_r+2\alpha\overline{u})-\alpha d^2)^2}{64d^2}$

B. $w=\frac{3\alpha s_r+3d\alpha(2+\alpha)-\sqrt{6d\alpha^2(2\alpha^2(s_r-\overline{u})+d(2+2\alpha+\alpha^2))}}{3\alpha^2},\ \lambda=1$
$T=\frac{(2\alpha^2(s_r-\overline{u})+d(2+2\alpha+\alpha^2))^2}{36\alpha^3}$

C. $w=\frac{3\alpha s_r+3d\alpha(2+\alpha)+\sqrt{6d\alpha^2(2\alpha^2(s_r-\overline{u})+d(2+2\alpha+\alpha^2))}}{3\alpha^2},\ \lambda=1$
$T=\frac{(2\alpha^2(s_r-\overline{u})+d(2+2\alpha+\alpha^2))^2}{36\alpha^3}$

For possibilities B and C, the quantity sold simplifies to $-\frac{2\alpha^2(s_r-\overline{u})+d(2+2\alpha+\alpha^2)}{6\alpha^2}$ which is positive if and only if $2\alpha^2(s_r-\overline{u})+d(2+2\alpha+\alpha^2)<0$. However, if $2\alpha^2(s_r-\overline{u})+d(2+2\alpha+\alpha^2)$ <0, then the wholesale price in possible solutions B and C will be imaginary. Thus we rule out these possibilities because either quantities will be non-positive or wholesale price will be imaginary. The quantity for solution A simplifies to $q_b^{VI}=\frac{\overline{u}}{2}+\frac{(c-s_r)^2}{8d}-\frac{2(3c-s_r)+d}{8}-\frac{(1-\alpha)(c-s_r)}{2\alpha}$ which is positive for sufficiently high $\overline{u}$.

The resulting equilibrium is:

$$p^{VI}=\frac{\overline{u}}{2}+\frac{(c-s_r)^2}{8d}+\frac{2(c+s_r)-d}{8}-\frac{(1-\alpha)(c-s_r)}{2\alpha}$$

$f^{VI} = c - s_r$,

$$q_b^{VI} = \frac{\overline{u}}{2} + \frac{(c-s_r)^2}{8d} - \frac{2(3c-s_r)+d}{8} - \frac{(1-\alpha)(c-s_r)}{2\alpha},$$

$$\phi_e^{VI} = \alpha\left(\frac{1}{2} - \frac{(c-s_r)}{2d}\right)$$

and $\pi_{mfgr}^{VI} = \dfrac{(\alpha(c-s_r)^2 - (4+2\alpha)cd + 2d((2-\alpha)s_r + 2\alpha\overline{u}) - \alpha d^2)^2}{64d^2}$.

We compare manufacturer profit when the manufacturer salvages returned units to when the retailer salvages returned units. At $s=s_r$, the two profits are equal.

Claim: The manufacturer's profit when the manufacturer salvages returns is increasing in s for all parameters such that exchanges and quantity are non-negative.

Proof of Claim: $\dfrac{\partial \pi_{mfgr}^{VI}}{\partial s} = \dfrac{d(2-\alpha)-\alpha(c-s)}{2d}\left(\dfrac{\overline{u}}{2} + \dfrac{(c-s)^2}{8d} - \dfrac{2(3c-s)+d}{8} - \dfrac{(1-\alpha)(c-s)}{2\alpha}\right)$.

Exchanges are non-negative if and only if $d>c-s$ which implies $\dfrac{d(2-\alpha)-\alpha(c-s)}{2d}>0$. Quantity is non-negative if and only if $\dfrac{\overline{u}}{2} + \dfrac{(c-s)^2}{8d} - \dfrac{2(3c-s)+d}{8} - \dfrac{(1-\alpha)(c-s)}{2\alpha} > 0$. Therefore, the manufacturer's profit when the manufacturer salvage returns is increasing in s for all parameters such that exchanges and quantity are non-negative.

Since the manufacturer's profit when the retailer salvages returns is invariant with respect to s, the manufacturer's profit when the manufacturer salvages returns is increasing with respect to s and the two profits are equal when $s=s_r$, it can be concluded that the manufacturer earns greater profit from taking back returns if and only if $s>s_r$.

Thus, when the manufacturer may offer a fixed fee in addition to a wholesale price and refund rate, the equilibrium can be described as:

Term	Equilibrium Value
Retail Price	$p^{VI}=\frac{\bar{u}}{2}+\frac{(c-\max\{s,s_r\})^2}{8d}+\frac{2(c+\max\{s,s_r\})-d}{8}-\frac{(1-\alpha)(c-\max\{s,s_r\})}{2\alpha}$
Return Penalty	$f^{VI}=c-\max\{s,s_r\}$
Quantity Sold Initially	$q_b^{VI}=\frac{\bar{u}}{2}+\frac{(c-\max\{s,s_r\})^2}{8d}-\frac{2(3c-\max\{s,s_r\})+d}{8}-\frac{(1-\alpha)(c-\max\{s,s_r\})}{2\alpha}$
Exchange Probability	$\phi_e^{VI}=\alpha(\frac{1}{2}-\frac{(c-\max\{s,s_r\})}{2d})$
Channel Profit	$\pi^{VI}=\frac{(\alpha(c-\max\{s,s_r\})^2-(4+2\alpha)cd+2d((2-\alpha)\max\{s,s_r\}+2\alpha\bar{u})-\alpha d^2)^2}{64d^2}$
Wholesale Price	$w^{VI}=c$
Refund Rate	$r^{VI}=s$
Fixed Fee	$T^{VI}=\frac{(\alpha(c-\max\{s,s_r\})^2-(4+2\alpha)cd+2d((2-\alpha)\max\{s,s_r\}+2\alpha\bar{u})-\alpha d^2)^2}{64d^2}$

This is equivalent to a vertically integrated system in which the greatest net salvage value for returned units $\max\{s,s_r\}$ is attainable.

Q.E.D.

A.5. Proof that Asymmetric Prices and Restocking Fees Yield Equivalent Profit to Symmetric Equilibrium of Vertically-Integrated System.

In this Appendix, we show that allowing for asymmetric equilibria in a vertically-integrated channel system will produce the same profit, total number of returns, total number of exchanges, and total number of initial sales as the symmetric equilibrium.

Note from equation (1) in the paper's text that the difference between expected utility from buying product *j=0* and the expected utility from buying product *j=1* is invariant with respect to u_i. Therefore, if product 0 is preferred *ex ante* over product 1 by one consumer, it is preferred for all consumers (and vice versa). Also, notice that the expected utility from buying product *j* is increasing in u_i. Therefore, if a consumer located at $u_i=\hat{u}_j$ has non-negative expected utility from purchasing product *j*, then all consumers with $u_i > \hat{u}_j$ will have positive expected utility from purchasing the product.

Suppose that there is a $\hat{u}$ for which prices and restocking fees are chosen such that all consumers with $u_i < \hat{u}$ do not buy either product initially, and all consumers with $u_i \in [\hat{u}, \overline{u}]$ do initially buy one of the two products. We will prove that all resulting asymmetric equilibria will produce the same profit, total number of returns, total number of exchanges, and total number of initial sales as the symmetric equilibrium. This implies two things: 1) for any population size that it is optimal to serve when restricted to symmetric choices, the profit from the symmetric choices is NOT dominated by the profit produced from asymmetric choices; 2) for any population size that it is optimal to serve when making asymmetric choices, the profit from symmetric choices is NOT dominated by the profit produced from asymmetric choices. Thus, the symmetric equilibrium established in Appendix A.4. which also identifies the optimal population to serve initially, cannot be improved upon with asymmetric choices.

The form of the profit expression will depend on whether the expected utility of buying product 0 is equal or unequal to that of buying product 1. Three possibilities exist: (1) $E_{j=0}(utility \mid u_i = \hat{u}) = E_{j=1}(utility \mid u_i = \hat{u}) \geq 0$; (2) $E_{j=0}(utility \mid u_i = \hat{u}) > E_{j=1}(utility \mid u_i = \hat{u}) \geq 0$; or (3) $E_{j=0}(utility \mid u_i = \hat{u}) \geq 0$ and $\mathrm{E}_{j=1}(utility \mid u_i = \hat{u}) < 0$. We next explore each of these possibilities.

Possibility 1: Product 0 offers expected utility equal to the expected utility offered by product 1.

If Product 0 offers the same expected utility as product 1, then consumers are indifferent between buying 0 and buying 1. Therefore consumers for whom $u_i \in [\hat{u}, \overline{u}]$ will randomly choose between initial purchase of product 0 or 1, resulting in half the market (of total size $\overline{u} - \hat{u}$) buying product 0 and the other half buying product 1. Assuming that $s > s_r$, the profit can be written as the sum of profits from initially-bought products that are kept (subscript *k*), those that are exchanged (subscript *e*), and those that are returned without exchange (subscript *r*), summed across the two product offerings (see equation (1) in the paper for definitions of the ϕ expressions):

$$\pi = \frac{(\bar{u}-\hat{u})}{2}((p_0-c)\phi_{k0} + (p_1-c+f_0-c+s)\phi_{e0} + (f_0-c+s)\phi_{r0}$$
$$+(p_1-c)\phi_{k1} + (p_0-c+f_1-c+s)\phi_{e1} + (f_1-c+s)\phi_{r1})$$

The seller's objective is to maximize profit subject to the constraint that consumers with $u_i = \hat{u}$ have non-negative expected utility from making an initial purchase. We also have the constraint that expected utility from product 0 is equal to that of product 1. The constrained optimization problem can be described with Lagrangian multipliers λ_1 and λ_2:

$$\max_{p_0,p_1,f_0,f_1,\lambda_1,\lambda_2} L = \pi + \lambda_1(E_{j=0}(utility \mid u_i=\hat{u}) - E_{j=1}(utility \mid u_i=\hat{u})) + \lambda_2 E_{j=0}(utility \mid u_i=\hat{u})$$
$$s.t.\ E_{j=0}(utility \mid u_i=\hat{u}) - E_{j=1}(utility \mid u_i=\hat{u}) = 0$$
$$\lambda_2 E_{j=0}(utility \mid u_i=\hat{u}) = 0$$
$$\lambda_1 \ge 0,\ \lambda_2 \ge 0$$

The Kuhn-Tucker conditions are

$$\frac{dL}{dp_0} = (\bar{u}-\hat{u})\frac{\alpha(d+f_0-f_1-2(p_0-p_1)+\lambda_1(f_1+f_0)-\lambda_2(d+f_0+p_0-p_1))}{2d} = 0,$$
$$\frac{dL}{dp_1} = (\bar{u}-\hat{u})\frac{\alpha(d+f_1-f_0-2(p_1-p_0)-\lambda_1(f_1+f_0)-\lambda_2(d-f_0+p_0-p_1))}{2d} = 0,$$
$$\frac{dL}{df_0} = (\bar{u}-\hat{u})\frac{\alpha(c-s+2(p_0-f_0-p_1)(1+\lambda_1-\lambda_2))}{4d} + \frac{(2-\alpha)(1+2\lambda_1-2\lambda_2)}{4} = 0,$$
$$\frac{dL}{df_1} = (\bar{u}-\hat{u})\frac{(2-\alpha)(1-2\lambda_1)}{4} + \frac{\alpha(c-s+2(p_1-f_1-p_0)(1-\lambda_1))}{4d} = 0$$
$$\frac{\partial L}{\partial \lambda_1} = \frac{(2-\alpha)(f_1-f_0)}{2} + \frac{\alpha(f_1+f_0)(f_1-f_0-2(p_0-p_1))}{4d} = 0$$
$$\lambda_2(\alpha(\hat{u} - \frac{p_0+p_1+f_0}{2} - \frac{d}{4} + \frac{(f_0-p_0+p_1)^2}{4d}) - (1-\alpha)f_0) = 0$$
$$\lambda_1 \ge 0,\ \lambda_2 \ge 0$$

The unique solution to the Kuhn-Tucker conditions is:

$$p_0 = p_1 = \hat{u} + \frac{(c-s)^2}{4d} - \frac{(c-s)(2-\alpha)}{2\alpha} - \frac{d}{4}$$

$$f_0 = f_1 = c - s$$

$$\lambda_1 = (\bar{u} - \hat{u})/2,\ \lambda_2 = (\bar{u} - \hat{u})\ .$$

The equilibrium results in the following outcome.

Total quantity sold initially: $(\bar{u} - \hat{u})$

Total quantity returned (without subsequent exchange): $\frac{(\bar{u}-\hat{u})}{2}\phi_{r0} + \frac{(\bar{u}-\hat{u})}{2}\phi_{r1} = (1-\alpha)(\bar{u}-\hat{u})$

Total quantity of exchanges:

$$\frac{(\bar{u}-\hat{u})}{2}\phi_{e0} + \frac{(\bar{u}-\hat{u})}{2}\phi_{e1} = \frac{\alpha(\bar{u}-\hat{u})(d-c+s)}{4d} + \frac{\alpha(\bar{u}-\hat{u})(d-c+s)}{4d} = \alpha(\bar{u}-\hat{u})(\frac{1}{2} - \frac{c-s}{2d})$$

Manufacturer profit: $\alpha(\bar{u}-\hat{u})(\hat{u} - c + \frac{(c-s)^2}{4d} - \frac{(c-s)(2-\alpha)}{2\alpha} - \frac{d}{4})$.

Possibility 2: $E_{j=0}(utility \mid u_i = \hat{u}) > E_{j=1}(utility \mid u_i = \hat{u}) \geq 0$.

If Product 0 offers greater expected utility than product 1, all consumers who buy initially (consumers with $u_i \in [\hat{u}, \bar{u}]$) will initially buy product 0. In this case, the seller's profit can be written as

$$\pi = (\bar{u} - \hat{u})((p_0 - c)\phi_{k0} + (p_1 - c + f_0 - c + s)\phi_{e0} + (f_0 - c + s)\phi_{r0})\ .$$

The seller's objective is to maximize profit subject to the constraint that consumers with $u_i = \hat{u}$ have $E_{j=0}(utility \mid u_i = \hat{u}) > E_{j=1}(utility \mid u_i = \hat{u}) \geq 0$. The constrained optimization problem can be described with Lagrangian multipliers λ_1 and λ_2:

$$\max_{p_0, p_1, f_0, f_1, \lambda_1, \lambda_2} L = \pi + \lambda_1 (E_{j=0}(utility \mid u_i = \hat{u}) - E_{j=1}(utility \mid u_i = \hat{u})) + \lambda_2 E_{j=1}(utility \mid u_i = \hat{u})$$

$$s.t.\ \lambda_1 (E_{j=0}(utility \mid u_i = \hat{u}) - E_{j=1}(utility \mid u_i = \hat{u})) = 0$$

$$\lambda_2 E_{j=1}(utility \mid u_i = \hat{u}) = 0$$

$$\lambda_1 = 0,\ \lambda_2 \geq 0$$

The Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial p_0}=(\overline{u}-\hat{u})\frac{\alpha(d-c+s+2(p_1-p_0+f_0)-(f_0+f_1)\lambda_1-\lambda_2(f_1+p_0-p_1-d))}{2d}=0$$
$$\frac{\partial L}{\partial p_1}=(\overline{u}-\hat{u})\frac{\alpha(c-s+d+2(p_0-p_1-f_0)+\lambda_1(f_0+f_1)-\lambda_2(d+f_1+p_0-p_1))}{2d}=0$$
$$\frac{\partial L}{\partial f_0}=(\overline{u}-\hat{u})(\frac{(2-\alpha)(1-\lambda_1)}{2}+\frac{\alpha(c-s-(2-\lambda_1)(p_1-p_0+f_0))}{2d})=0$$
$$\frac{\partial L}{\partial f_1}=(\overline{u}-\hat{u})\frac{(\alpha(d+f_1+p_0-p_1)-2d)(\lambda_2-\lambda_1)}{2d}=0$$
$$\lambda_1(\frac{(2-\alpha)(f_1-f_0)}{2}+\frac{\alpha(f_1+f_0)(f_1-f_0-2(p_0-p_1))}{4d})=0$$
$$\lambda_2(\alpha(\hat{u}-\frac{p_1+p_0+f_1}{2}-\frac{d}{4}+\frac{(f_1-p_1+p_0)^2}{4d})-(1-\alpha)f_1)=0$$
$$\lambda_1=0,\ \lambda_2\geq 0$$

There are no solutions that satisfy λ_1=0 and all of the first-order conditions jointly; therefore, the firm will never choose to set prices and restocking fees so that $E_{j=0}(utility\,|\,u_i=\hat{u})>E_{j=1}(utility\,|\,u_i=\hat{u})\geq 0$. In other words, the firm will not choose to sell only product 0 and to give the marginal consumer strictly positive utility. We therefore rule out Possibility 2.

Possibility 3: $E_{j=0}(utility\,|\,u_i=\hat{u})\geq 0$ and $E_{j=1}(utility\,|\,u_i=\hat{u})<0$.

We now turn our attention to potential equilibria in which the seller sets prices and restocking fees such that $E_{j=0}(utility\,|\,u_i=\hat{u})\geq 0$ and $E_{j=1}(utility\,|\,u_i=\hat{u})<0$. The objective function becomes:

$$\max_{p_0,p_1,f_0,f_1,\lambda_2} L=\pi+\lambda_2 E_{j=0}(utility\,|\,u_i=\hat{u})$$
$$s.t.\ \lambda_2(E_{j=0}(utility\,|\,u_i=\hat{u}))=0$$
$$\lambda_2\geq 0$$
$$E_{j=1}(utility\,|\,u_i=\hat{u})<0$$

The Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial p_0} = (\bar{u} - \hat{u}) \frac{\alpha(d + 2(f_0 - p_0 + p_1) + s - c - \lambda_2(d + f_0 - p_0 + p_1))}{2d} = 0$$
$$\frac{\partial L}{\partial p_1} = (\bar{u} - \hat{u}) \frac{\alpha(d - 2(f_0 - p_0 + p_1) - s + c - \lambda_2(d - f_0 + p_0 - p_1))}{2d} = 0$$
$$\frac{\partial L}{\partial f_0} = (\bar{u} - \hat{u})(\frac{(2-\alpha)(1-\lambda_2)}{2} + \frac{\alpha(c - s - (2-\lambda_2)(p_1 - p_0 + f_0))}{2d}) = 0$$
$$\lambda_2(\alpha(\hat{u} - \frac{p_1 + p_0 + f_0}{2} - \frac{d}{4} + \frac{(f_0 - p_0 + p_1)^2}{4d}) - (1-\alpha) f_0) = 0$$
$$\lambda_2 \geq 0$$

The Kuhn-Tucker conditions are satisfied at any price and restocking fee combination such that

$p_0 = \hat{u} + \frac{(c-s)^2}{4d} - \frac{(c-s)}{2} - \frac{f_0(1-\alpha)}{\alpha} - \frac{d}{4}$, $p_1 = \hat{u} + \frac{(c-s)^2}{4d} + \frac{(c-s)}{2} - \frac{f_0}{\alpha} - \frac{d}{4}$ and

$E_{j=1}(utility \mid u_i = \hat{u}) = \frac{(2-\alpha)(f_0 - f_1)}{2} + \frac{\alpha(f_1 + f_0)(f_1 + f_0 - 2(c-s))}{4d} < 0$ (which implies $\lambda_2 = \bar{u} - \hat{u}$).

The equilibrium outcome in terms of profit, total returns, total sales, and total exchanges will be the same as in the symmetric solution where $E_{j=0}(utility \mid u_i = \hat{u}) = E_{j=1}(utility \mid u_i = \hat{u})$. Specifically,

Total quantity sold initially: $(\bar{u} - \hat{u})$

Total quantity returned (without subsequent exchange): $(\bar{u} - \hat{u})\phi_{r0} + 0 \bullet \phi_{r1} = (1-\alpha)(\bar{u} - \hat{u})$

Total quantity of exchanges: $(\bar{u} - \hat{u})\phi_{e0} + 0 \bullet \phi_{e1} = \alpha(\bar{u} - \hat{u})(\frac{1}{2} - \frac{c-s}{2d})$

Manufacturer profit: $\alpha(\bar{u} - \hat{u})(\hat{u} - c + \frac{(c-s)^2}{4d} - \frac{(c-s)(2-\alpha)}{2\alpha} - \frac{d}{4})$.

Moreover, if a consumer initially purchases product 0, the out-of-pocket marginal expense to return it and buy product 1 is also the same as in the symmetric case, namely: $p_1 - (p_0 - f_0) = (c - s)$.

Summarizing the insights from analyzing Possibilities 1, 2, and 3, we have: (a) Possibility 2 cannot exist; (b) Possibility 1 produces symmetric solution values; and (c) Possibility 3 produces the same profit, total returns, total sales, and total exchanges as in the symmetric solution. Thus, allowing asymmetry in the vertically-integrated channel structure results in the same outcomes as assuming symmetry.

In particular, given that the vertically-integrated channel structure can choose the most efficient salvage technology, we can replace s with $\max\{s, s_r\}$ in the above expressions for the asymmetric vertically-integrated case. From Appendix A.4., we know that $q_b = \overline{u} - \hat{u}$ implies at the optimum, $\hat{u} = \frac{\overline{u}}{2} - \frac{(c - \max\{s, s_r\})^2}{8d} + \frac{2(3c - \max\{s, s_r\}) + d}{8} + \frac{(1-\alpha)(c - \max\{s, s_r\})}{2\alpha}$.

Substituting this expression into the above solution values in Possibilities 1 and 3 shows that a symmetric solution and an asymmetric solution both yield the same values for total quantity sold, total quantity returned, total exchanges, and total channel profit, as follows:

Total quantity sold: $q_b^{VI} = \frac{\overline{u}}{2} + \frac{(c - \max\{s, s_r\})^2}{8d} - \frac{2(3c - \max\{s, s_r\}) + d}{8} - \frac{(1-\alpha)(c - \max\{s, s_r\})}{2\alpha}$

Total quantity returned: $(1-\alpha)q_b^{VI}$

Total exchanges: $\alpha q_b^{VI}(\frac{1}{2} - \frac{c-s}{2d})$

Channel profit: $\pi_{mfgr}^{VI} = \frac{(\alpha(c - \max\{s, s_r\})^2 - (4 + 2\alpha)cd + 2d((2-\alpha)\max\{s, s_r\} + 2\alpha\overline{u}) - \alpha d^2)^2}{64d^2}$

Q.E.D.

A.6. Proof of Proposition 1

PROPOSITION 1: *Consider a manufacturer who can charge a fixed fee, a per-unit wholesale price to the retailer, and a per-unit refund rate. If and only if $s > s_r$, then the manufacturer earns greater profit from the reverse channel structure in which the manufacturer salvages returns than the reverse channel structure in which the retailer salvages returns. The equilibrium is described in Table 2.*

From Appendix A.4., the manufacturer's profit from accepting and salvaging returns is greater than the manufacturer's profit from the retailer salvaging returns if and only if $s>s_r$.

Q.E.D.

A.7. Parameter values for which all consumers who buy initially prefer to keep or exchange, rather than return without subsequent purchase in the VI system

From A.3., we know that all consumers who initially buy a product will prefer to keep or exchange it rather than return it without subsequent purchase if $\frac{4df_j - \alpha(d^2 + (f_j - p_j + p_{-j})^2)}{2d\alpha} \geq 0$.

Substituting the equilibrium values from Table 2, this implies that $d \leq \frac{(2+\sqrt{4-\alpha^2})(c-s)}{\alpha}$ is sufficient to ensure that consumers who buy initially will prefer to keep or exchange their initial purchase even if they have the option to return without subsequent purchase.

A.8. Proof of Retailer's Reaction Functions (See Equation 3)

To simplify the presentation of analysis and without loss of generality, we first examine the equilibrium when symmetry is imposed. We will then show that neither the manufacturer nor the retailer can improve profit by making asymmetric choices. Given a wholesale price and a refund that is symmetric for each product, the retailer has the following objective function:

$$\max_{p,f} q_b(f,p;d,\bar{u})((p-w)+(-p+f+\max\{s_r,r\})(\phi_e(f;d)+\phi_r)+(p-w)\phi_e(f;d)) \text{ , where}$$

$$q_b(f,p;d,\bar{u},\alpha)=\bar{u}-p-\frac{d}{4}-\frac{(2-\alpha)f}{2\alpha}+\frac{f^2}{4d} \text{ , } \phi_r=(1-\alpha) \text{ and } \phi_e(f;d,\alpha)=\alpha(\frac{1}{2}-\frac{f}{2d}).$$[2]

The first order conditions are

$$\frac{\partial\pi_{ret}}{\partial p}=\alpha(\bar{u}-(p-w)+\frac{(d-f)(w-\max\{s_r,r\}-f)}{2d})-(1-\alpha)(2f+\max\{s_r,r\}-w)$$
$$+\frac{\alpha}{4}(-d+\frac{f^2}{d}-2(f+2p))=0$$

$$\frac{\partial\pi_{ret}}{\partial f}=\frac{(\alpha(d^2-f^2)+2d(f(2-\alpha)+4\alpha(p-\bar{u})))(\alpha(2f-w+\max\{s_r,r\})-(2-\alpha)d)}{8\alpha d^2}$$
$$+\frac{(2d-\alpha d-\alpha f)((2d-\alpha d-\alpha f)(f+\max\{s_r,r\}-w)-2d\alpha w+2\alpha dp)}{4\alpha d^2}=0$$

There are three possible solutions that satisfy the first order conditions.

I) $p(w,r)=\frac{\bar{u}}{2}+\frac{(w-\max\{s_r,r\})^2}{8d}+\frac{2(w+\max\{s_r,r\})-d}{8}-\frac{(1-\alpha)(w-\max\{s_r,r\})}{2\alpha}$

$$f(w,r)=w-\max\{s_r,r\}$$

Defining $X\equiv\alpha^2((w-\max\{s_r,r\})^2-2d(w+\max\{s_r,r\}-2\bar{u})-d^2)-4d\alpha(w-\max\{s_r,r\})$, we have

$$\text{II) } p(w,r)=\begin{bmatrix} 2\bar{u}+\dfrac{(w-\max\{s_r,r\})^2-d(d+2\max\{s_r,r\})}{2d} \\ -\dfrac{\sqrt{X}}{\alpha^2}+\dfrac{4d(\max\{s_r,r\}-w)+(w+d-\max\{s_r,r\})\sqrt{X}}{2d\alpha} \end{bmatrix}$$

$$f=w-r-\sqrt{X}$$

[2] These expressions are adapted from equations (1) and (2) from the text by setting $p_j=p$ and $f_j=f$ for each product j.

III) $$p(w,r)=\begin{bmatrix} 2\bar{u}+\dfrac{(w-\max\{s_r,r\})^2-d(d+2\max\{s_r,r\})}{2d} \\ +\dfrac{\sqrt{X}}{\alpha^2}+\dfrac{4d(\max\{s_r,r\}-w)-(w+d-\max\{s_r,r\})\sqrt{X}}{2d\alpha} \end{bmatrix}$$

$$f(w,r)=w-\max\{s_r,r\}+\sqrt{X}\ .$$

The initial sales quantity q_b when evaluated at solutions I, II and III is

$$q_b=\frac{\bar{u}}{2}-\frac{w-\max\{s_r,r\}}{2\alpha}-\frac{d^2+2d(w+\max\{s_r,r\})-(w-\max\{s_r,r\})^2}{8d},\quad q_b=0,\quad \text{and}\quad q_b=0$$

respectively. We therefore rule out potential solutions II and III. Therefore, the retailer's reaction functions are

$$p(w,r)=\frac{\bar{u}}{2}+\frac{(w-\max\{s_r,r\})^2}{8d}+\frac{2(w+\max\{s_r,r\})-d}{8}-\frac{(1-\alpha)(w-\max\{r,s_r\})}{2\alpha}$$

$$f(w,r)=w-\max\{s_r,r\}$$

Q.E.D.

A.9. Proof of Entries in Table 3 (See Section 3)

The manufacturer's objective function takes into account the retailer's reaction (from Appendix A.8.) and can be described as $\max\limits_{w,r,z} q_b(f(w,r),p(w,r);d,\bar{u})((w-c)+(w-c-I_z(r-s))\phi_e(f(w,r);d)-I_z\,\phi_r(r-s))$ where I_z is an indicator variable equal to one if the manufacturer accepts and salvages returned units from the retailer (CRM reverse channel structure) and equal to zero otherwise (CR reverse channel structure). We solve the model recursively, first identifying the wholesale price and refund for each reverse channel structure. We will then identify when each reverse channel structure will give the manufacturer greater profit. The model cannot simply be solved by finding the optimal *w,* and *r* when the manufacturer accepts returns and identifying parameters for which $r^*<s_r$.

Such a solution method operates under the assumption that the value of the returned unit is s and the refund rate r factors into the retailer's reaction function. However, when return responsibility is allocated to the retailer, the returned units are valued at s_r (though by the retailer) and the wholesale price is the only element in retailer's decision. The wholesale price in each reverse channel structure will be set differently because of where units are salvaged, at what value they are salvaged, and the retailer's decisions based on these facts. Thus, we solve for the equilibrium wholesale price and refund rate in each reverse channel structure separately.

First, we examine when the retailer salvages returned units.

Defining $Y_R \equiv \alpha((w-s_r)^2 - 2d(w+s_r-2\overline{u}) - d^2) - 4d(w-s_r)$, the first order condition is

$$\frac{\partial \pi_{mfgr}}{\partial w} = \frac{1}{16\alpha d^2}(2(c-w)(\alpha(s_r-w)+d(2+\alpha))^2 + (\alpha(c+s_r-2w)+d(2+\alpha))Y_R) = 0.$$

Defining $A \equiv 44d^2 + 4\alpha d(-3c+11d+3s_r) + \alpha^2(3(c-s_r)^2 + 19d^2 + 38ds_r - 6cd - 32\overline{u})$

and

$B \equiv -9\alpha^3(-2d + a(c-d-s_r))(-12d^2 - 4ad(c+3d-s_r) + a^2((c-s)^2 - 7d(d+2s_r) - 2cd + 16\overline{u}))$,

the first order condition is satisfied at:

$$\text{I)}\ \ w = \frac{1}{4}(c+3(s_r+d+\frac{2d}{\alpha})) - \frac{3^{2/3}}{12}(\frac{A}{12(B+\sqrt{B^2-3\alpha^6A^3})^{1/3}} + \frac{(B+\sqrt{B^2-3\alpha^6A^3})^{1/3}}{3^{1/3}\alpha^2}$$

$$\text{II)}\ \ w = \frac{1}{4}(c+3(s_r+d+\frac{2d}{\alpha})) + \frac{3^{2/3}(1+i\sqrt{3})A}{24(B+\sqrt{B^2-3\alpha^6A^3})^{1/3}} + \frac{3^{1/3}(1-i\sqrt{3})(B+\sqrt{B^2-3\alpha^6A^3})^{1/3}}{24\alpha^2}$$

$$\text{III)}\ \ w = \frac{1}{4}(c+3(s_r+d+\frac{2d}{\alpha})) + \frac{3^{2/3}(1-i\sqrt{3})A}{24(B+\sqrt{B^2-3\alpha^6A^3})^{1/3}} + \frac{3^{1/3}(1+i\sqrt{3})(B+\sqrt{B^2-3\alpha^6A^3})^{1/3}}{24\alpha^2}$$

We restrict our attention to real solutions and therefore we may discard solutions II and III.

Therefore, the manufacturer's wholesale price when returns responsibility is allocated to the retailer ($r^* = 0$) will be:

$$w^* = \frac{1}{4}(c + 3(s_r + d + \frac{2d}{\alpha})) - \frac{3^{2/3}}{12}(\frac{A}{12(B + \sqrt{B^2 - 3\alpha^6 A^3})^{1/3}} + \frac{(B + \sqrt{B^2 - 3\alpha^6 A^3})^{1/3}}{3^{1/3}\alpha^2}.$$

Secondly we solve for when the manufacturer accepts returns

If the manufacturer chooses to take back returns (*I_z=1*), the manufacturer's objective function is given by

$\max_{w,r} q_b(f(w,r), p(w,r); d, \bar{u})((w-c) + (w-c-r+s))\phi_e(f(w,r); d))$ where the retailer's reaction functions are defined in Appendix A.6. The first order conditions for the manufacturer are

$$\frac{\partial \pi_{mfgr}}{\partial w} = \frac{(2d + \alpha(d + r - w))(2cd + \alpha(c(d + r - w) + (r - w - d)(r - s - w)) - 2d(w + s - r))}{8\alpha d^2}$$
$$+ \frac{(\alpha(c + 2r - s - 2w) + d(2 + \alpha))(\alpha(w - r)^2 - d^2\alpha - 2d(w(2 + \alpha) - r(2 - \alpha) - 2\alpha\bar{u}))}{16\alpha d^2} = 0$$
$$\frac{\partial \pi_{mfgr}}{\partial r} = \frac{(-2d + \alpha(d - r + w))(2cd + \alpha(c(d + r - w) + (r - w - d)(r - s - w)) - 2d(w + s - r))}{8\alpha d^2}$$
$$+ \frac{(d(2 - \alpha) + \alpha(c + 2r - s - 2w))(\alpha(w - r)^2 - d^2\alpha - 2d(w(2 + \alpha) - r(2 - \alpha) - 2\alpha\bar{u}))}{16\alpha d^2}$$

There are three possible solutions that satisfy the first order conditions for the manufacturer.

A) $w^* = c + \frac{\bar{u}}{2} + \frac{(c-s)^2}{8d} - \frac{2(3c-s)+d}{8} - \frac{(1-\alpha)(c-s)}{2\alpha}$,

$$r^* = s + \frac{\bar{u}}{2} + \frac{(c-s)^2}{8d} - \frac{2(3c-s)+d}{8} - \frac{(1-\alpha)(c-s)}{2\alpha}$$

Defining $Y_M = (\alpha(c-s)^2 - d^2\alpha - 2d(c(2+\alpha) - s(2-\alpha) - 2\alpha\bar{u}))$

B) $r = s - \frac{Y_M}{2d\alpha} + \frac{\sqrt{-\alpha^3 Y_M(\alpha(c-s) - d(2+\alpha))^2}}{2d\alpha^3}$,

$$w = c - \frac{Y_M}{2d} - \frac{(d(2-\alpha) - \alpha(c-s))\sqrt{-\alpha^3 Y_M(\alpha(c-s) - d(2+\alpha))^2}}{2d\alpha^3(\alpha(c-s) - d(2+\alpha))}$$

C) $r = s - \frac{Y_M}{2d\alpha} - \frac{\sqrt{-\alpha^3 Y_M(\alpha(c-s)-d(2+\alpha))^2}}{2d\alpha^3}$,

$$w = c - \frac{Y_M}{2d} + \frac{(d(2-\alpha)-\alpha(c-s))\sqrt{-\alpha^3 Y_M(\alpha(c-s)-d(2+\alpha))^2}}{2d\alpha^3(\alpha(c-s)-d(2+\alpha))}.$$

The retailer's reaction to possible solutions B and C leads to $q_b = 0$ and zero profit for both the retailer and the manufacturer. Therefore, A is the only solution for which profit and q_b are potentially positive. Defining q_b^{CRM} as the equilibrium quantity sold initially when the wholesale and refund rate are given by solution A with the manufacturer accepting returns: $q_b^{CRM} = \frac{\bar{u}}{4} + \frac{(c-s)^2}{16d} - \frac{c-s}{4\alpha} - \frac{2(c+s)+d}{16}$. We verify that the solution to the first order conditions also satisfies the second order conditions for any parameters such that $q_b^{CRM} > 0$.

Evaluated at the *w, r* of solution A,

$\frac{\partial\partial\pi_{mfgr}}{\partial\partial w}|_{solutionA} = -\frac{1}{2d}(\frac{\alpha\bar{u}}{4} + \frac{9\alpha(c-s)^2}{16d} + \frac{2d(1+\alpha)}{\alpha} - \frac{9(c-s)}{4} - \frac{\alpha(18c-7(d+2s))}{16})$, which is less than zero for all $\bar{u} > \frac{9(c-s)}{\alpha} + \frac{\alpha(18c-7(d+2s))}{4\alpha} - \frac{8d(1+\alpha)}{\alpha^2} - \frac{9(c-s)^2}{4d}$. This inequality holds true for all $\bar{u}$ such that $q_b^{CRM} = \frac{\bar{u}}{4} + \frac{(c-s)^2}{16d} - \frac{c-s}{4\alpha} - \frac{2(c+s)+d}{16} > 0$ (i.e., $q_b^{CRM} > 0$ holds when $\bar{u} > \frac{c-s}{\alpha} + \frac{2(c+s)+d}{4} - \frac{(c-s)^2}{4d}$), by the fact that $\frac{c-s}{\alpha} + \frac{2(c+s)+d}{4} - \frac{(c-s)^2}{4d} > \frac{9(c-s)}{\alpha} + \frac{\alpha(18c-7(d+2s))}{4\alpha} - \frac{8d(1+\alpha)}{\alpha^2} - \frac{9(c-s)^2}{4d}$. The latter inequality holds because

$$\frac{c-s}{\alpha} + \frac{2(c+s)+d}{4} - \frac{(c-s)^2}{4d} - (\frac{9(c-s)}{\alpha} + \frac{\alpha(18c-7(d+2s))}{4\alpha} - \frac{8d(1+\alpha)}{\alpha^2} - \frac{9(c-s)^2}{4d})$$

is equal to $\frac{2(d(2+\alpha)-(c-s))^2}{d\alpha^2}>0$.

$\frac{\partial\partial\pi_{mfgr}}{\partial\partial r}|_{solutionA}=-\frac{1}{2d}(\frac{\alpha\bar{u}}{4}+\frac{2d(1-\alpha)}{\alpha}+\frac{9\alpha(c-s)^2}{16d}-\frac{9(c-s)}{4}+\frac{\alpha(14c-18s+7d)}{16})$, which is less than zero for all $\bar{u}>\frac{9(c-s)}{\alpha}-\frac{8d(1-\alpha)}{\alpha^2}-\frac{9\alpha(c-s)^2}{4\alpha d}-\frac{(14c-18s+7d)}{4}$. This is true for all $\bar{u}$ such $q_b^{CRM}=\frac{\bar{u}}{4}+\frac{(c-s)^2}{16d}-\frac{c-s}{4\alpha}-\frac{2(c+s)+d}{16}>0$

(i.e., $q_b^{CRM}>0$ holds when $\bar{u}>\frac{c-s}{\alpha}+\frac{2(c+s)+d}{4}-\frac{(c-s)^2}{4d}$), by the fact that $\frac{c-s}{\alpha}+\frac{2(c+s)+d}{4}-\frac{(c-s)^2}{4d}>\frac{9(c-s)}{\alpha}-\frac{8d(1-\alpha)}{\alpha^2}-\frac{9\alpha(c-s)^2}{4\alpha d}-\frac{(14c-18s+7d)}{4}$. The latter inequality holds because

$\frac{c-s}{\alpha}+\frac{2(c+s)+d}{4}-\frac{(c-s)^2}{4d}-(\frac{9(c-s)}{\alpha}-\frac{8d(1-\alpha)}{\alpha^2}-\frac{9\alpha(c-s)^2}{4\alpha d}-\frac{(14c-18s+7d)}{4})$ is equal to $\frac{2(d(2-\alpha)-\alpha(c-s))^2}{d\alpha^2}>0$. The determinant of the Hessian

$|H_{mfgr}|=\frac{\alpha^2}{2d}(\frac{\bar{u}}{4}+\frac{(c-s)^2}{16d}-\frac{c-s}{4\alpha}-\frac{2(c+s)+d}{16})>0$ if and only if $\bar{u}$ is such that $q_b^{CRM}=\frac{\bar{u}}{4}+\frac{(c-s)^2}{16d}-\frac{c-s}{4\alpha}-\frac{2(c+s)+d}{16}>0$. Because $\lim_{w\to+/-\infty}\pi_{mfgr}=-\infty$ and $\lim_{r\to+/-\infty}\pi_{mfgr}=-\infty$, the only local maximum of manufacturer's profit (solution A) is also a global maximum.

In conclusion

Making simplifications using the retailer's reaction functions, we may characterize the equilibrium as in Table 3 from the paper:

Table 3. Equilibrium under a {Wholesale Price, Refund Rate} Wholesale Contract

Term	**Equilibrium Value †**
Retail Price	$p^* = \frac{\bar{u}}{2} + \frac{(w^* - \max\{r^*, s_r\})^2}{8d} + \frac{2(w^* + \max\{r^*, s_r\}) - d}{8} - \frac{(1-\alpha)(w^* - \max\{r^*, s_r\})}{2\alpha}$
Return Penalty	$f^* = w^* - \max\{r^*, s_r\}$
Quantity Sold Initially	$q_b^* = \frac{\bar{u}}{2} - \frac{w^* - \max\{r^*, s_r\}}{2\alpha} - \frac{d^2 + 2d(w^* + \max\{r^*, s_r\}) - (w^* - \max\{r^*, s_r\})^2}{8d}$
Exchange Probability	$\phi_e^* = \alpha(\frac{1}{2} - \frac{(w^* - \max\{r^*, s_r\})}{2d})$
Retailer Profit	$\pi_{ret}^* = \frac{(d^2\alpha - \alpha(w^* - \max\{r^*, s_r\})^2 + 2d((2+\alpha)w^* - (2-\alpha)\max\{r^*, s_r\} - 2\alpha\bar{u}))^2}{64\alpha d^2}$
CRM Manufacturer Profit (*I_z=1*)	$\pi_{mfgr}^{CRM} = \frac{(\alpha(c-s)^2 + d(2s(2-\alpha) + \alpha(4\bar{u} - d) - 2c(2+\alpha)))^2}{128\alpha d^2}$
CRM Wholesale Price (I_z=1)	$w^{*,CRM} = c + \frac{\bar{u}}{2} + \frac{(c-s)^2}{8d} - \frac{2(3c-s)+d}{8} - \frac{(1-\alpha)(c-s)}{2\alpha}$
CRM Refund Rate (I_z=1)	$r^{*,CRM} = s + \frac{\bar{u}}{2} + \frac{(c-s)^2}{8d} - \frac{2(3c-s)+d}{8} - \frac{(1-\alpha)(c-s)}{2\alpha}$
CR Manufacturer Profit (*I_z=0*)	$\pi_{mfgr}^{CR} = \frac{(w^*-c)(\alpha(s_r - w^*) + d(2+\alpha))(\alpha(w^* - \max\{r^*, s_r\})^2 - \alpha d^2)}{16\alpha d^2} + \frac{(w^*-c)(\alpha(s_r - w^*) + (2+\alpha)d)((2-\alpha)s_r + 2\alpha\bar{u} - (2+\alpha)w^*)}{8\alpha d}$
CR Wholesale Price (*I_z=0*) ††	$w^{*,CR} = \frac{1}{4}(c + 3(s_r + d + \frac{2d}{\alpha})) - \frac{3^{2/3}}{12}(\frac{A}{12(B + \sqrt{B^2 - 3\alpha^6 A^3})^{1/3}} + \frac{(B + \sqrt{B^2 - 3\alpha^6 A^3})^{1/3}}{3^{1/3}\alpha^2}$
CR Refund Rate (*I_z=0*)	$r^{*,CR}=0$

† In the equations for retail price, return penalty, quantity sold initially, exchange probability, and retailer profit, $w^* = w^{*,CRM}$ and $r^* = r^{*,CRM}$ when the channel structure is CRM;

$w^* = w^{*,CR}$ and $r^* = r^{*,CR}$ when the channel structure is CR. In the equation for CR manufacturer profit above, $w^* = w^{*,CR}$ and $r^* = r^{*,CR}$.

†† $A \equiv 44d^2 + 4\alpha d(-3c + 11d + 3s_r) + \alpha^2(3(c - s_r)^2 + 19d^2 + 38ds_r - 6cd - 32\overline{u})$ and
$B \equiv -9\alpha^3(-2d + a(c - d - s_r))(-12d^2 - 4ad(c + 3d - s_r) + a^2((c - s)^2 - 7d(d + 2s_r) - 2cd + 16\overline{u}))$.
Q.E.D.

A.10. Proof that Asymmetric Prices and Restocking Fees Yield Equivalent Profit to Symmetric Equilibrium in a Decentralized Channel.

Suppose that there is a $\hat{u}$ such that prices and restocking fees are chosen such that all consumers with $u_i < \hat{u}$ do not buy either product initially and all consumers with $u_i \in [\hat{u}, \overline{u}]$ buy one of the two products. We will prove that for any $\hat{u}$, asymmetric firm choices will not increase profit for either firm relative to the symmetric equilibrium. This implies two things: 1) for any population size that it is optimal to serve when restricted to symmetric choices, the profit from the symmetric choices is NOT dominated by the profit produced from asymmetric choices; 2) for any population size that it is optimal to serve when making asymmetric choices, the profit from symmetric choices is NOT dominated by profit produced from asymmetric choices. The form of the profit expression will depend on whether the expected utility of buying product 0 is equal or unequal to that of buying product 1. Three possibilities exist: (1) $E_{j=0}(utility \mid u_i = \hat{u}) = E_{j=1}(utility \mid u_i = \hat{u}) \geq 0$; (2) $E_{j=0}(utility \mid u_i = \hat{u}) > E_{j=1}(utility \mid u_i = \hat{u}) \geq 0$; or (3) $E_{j=0}(utility \mid u_i = \hat{u}) \geq 0$ and $\mathrm{E}_{j=1}(utility \mid u_i = \hat{u}) < 0$. We explore each of these possibilities solving recursively.

Possibility 1: Product 0 offers expected utility equal to the expected utility offered by product 1.

If Product 0 offers the same expected utility as product 1, then consumers are indifferent between buying 0 and buying 1. Therefore consumers for whom $u_i \in [\hat{u}, \overline{u}]$ will randomly choose between products resulting in half the market (of total size $\overline{u}-\hat{u}$) buying product 0 and the other half buying product 1. We solve the model recursively, starting with the retailer's problem. The retailer's profit can be written as the sum of profits from initially-bought products that are kept (subscript *k*), those that are exchanged (subscript *e*), and those that are returned without exchange (subscript *r*), summed across the two product offerings (see equation (1) in the paper for definitions of the ϕ expressions):

$$\pi_{ret} = \frac{(\overline{u}-\hat{u})}{2}((p_0 - w_0)\phi_{k0} + (p_1 - w_1 + f_0 - w_0 + \max\{r_0, s_r\})\phi_{e0} + (f_0 - w_0 + \max\{r_0, s_r\})\phi_{r0} \\ +(p_1 - w_1)\phi_{k1} + (p_0 - w_0 + f_1 - w_1 + \max\{r_1, s_r\})\phi_{e1} + (f_1 - w_1 + \max\{r_1, s_r\})\phi_{r1})$$

The retailer's objective is to maximize profit subject to the constraint that consumers with $u_i = \hat{u}$ have non-negative expected utility from making an initial purchase. We also have the constraint that expected utility from product 0 is equal to that of product 1. The constrained optimization problem can be described with Lagrangian multipliers λ_1 and λ_2:

$$\max_{p_0, p_1, f_0, f_1, \lambda_1, \lambda_2} L^{ret} = \pi_{ret} + \lambda_1(E_{j=0}(utility \mid u_i = \hat{u}) - E_{j=1}(utility \mid u_i = \hat{u})) + \lambda_2 E_{j=0}(utility \mid u_i = \hat{u})$$
$$s.t.\ E_{j=0}(utility \mid u_i = \hat{u}) - E_{j=1}(utility \mid u_i = \hat{u}) = 0$$
$$\lambda_2 E_{j=0}(utility \mid u_i = \hat{u}) = 0$$
$$\lambda_1 \geq 0,\ \lambda_2 \geq 0$$

The Kuhn-Tucker conditions are

$$\frac{\partial L^{ret}}{\partial p_0} = \frac{\alpha(2d - 2f_1(1-\lambda_1) - 4p_0 + 4p_1 + \max\{r_0, s_r\} - \max\{r_1, s_r\} + w_0 - w_1 + 2f_0(1+\lambda_1-\lambda_2) + 2\lambda_2(p_0 - p_1 - d)}{4d} = 0$$

$$\frac{\partial L^{ret}}{\partial p_1} = \frac{\alpha(2d + 2f_1(1-\lambda_1) + 4p_0 - 4p_1 - \max\{r_0, s_r\} + \max\{r_1, s_r\} - w_0 + w_1 - 2f_0(1+\lambda_1-\lambda_2) - 2\lambda_2(p_0 - p_1 - d)}{4d} = 0$$

$$\frac{\partial L^{ret}}{\partial f_0} = \frac{\alpha(w_1 - \max\{r_0, s_r\} + 2(p_0 - p_1 - f_0)(1+\lambda_1-\lambda_2)) + d(2-\alpha)(1+2\lambda_1-2\lambda_2)}{4d} = 0$$

$$\frac{\partial L^{ret}}{\partial f_1} = \frac{d(2-\alpha)(1-2\lambda_1) + \alpha(w_0 - \max\{r_1, s_r\} + 2(p_1 - f_1 - p_0)(1-\lambda_1))}{4d} = 0$$

$$E_{j=0}(utility \mid u_i = \hat{u}) - E_{j=1}(utility \mid u_i = \hat{u}) = 0$$
$$\lambda_2 E_{j=0}(utility \mid u_i = \hat{u}) = 0,\ \lambda_1 \geq 0,\ \lambda_2 \geq 0$$

The unique solution to these conditions is

$$p_0 = \hat{u} - \frac{d}{4} - \frac{w_1 - \max\{r_0, s_r\} + (1-\alpha)(w_0 - \max\{r_1, s_r\})}{2\alpha} + \frac{(1-\alpha)(w_0 - \max\{r_1, s_r\})^2 + (\max\{r_0, s_r\} - w_1)^2}{4d(2-\alpha)}$$
$$p_1 = \hat{u} - \frac{d}{4} - \frac{w_0 - \max\{r_1, s_r\} + (1-\alpha)(w_1 - \max\{r_0, s_r\})}{2\alpha} + \frac{(w_0 - \max\{r_1, s_r\})^2 + (1-\alpha)(\max\{r_0, s_r\} - w_1)^2}{4d(2-\alpha)}$$
$$f_0 = \frac{(\max\{r_0, s_r\} + \max\{r_1, s_r\} - w_0 - w_1)(\alpha(\max\{r_0, s_r\} - \max\{r_1, s_r\} + w_0 - w_1) - 2d(2-\alpha))}{4d(2-\alpha)}$$
$$f_1 = \frac{(w_0 + w_1 - \max\{r_0, s_r\} - \max\{r_1, s_r\})(2d(2-\alpha) + \alpha(\max\{r_0, s_r\} - \max\{r_1, s_r\} + w_0 - w_1))}{4d(2-\alpha)}$$

We first examine when the manufacturer accepts returns (CRM reverse channel structure).

The manufacturer takes the retailer's reaction into account in maximizing its profit

$$\pi_{mfgr}^{CRM} = \frac{(\bar{u} - \hat{u})}{2}((w_0 - c)\phi_{k0} + (w_1 - c + w_0 - c - r_0 + s)\phi_{e0} + (w_0 - c - r_0 + s)\phi_{r0}$$
$$+(w_1 - c)\phi_{k1} + (w_0 - c + w_1 - c - r_1 + s)\phi_{e1} + (w_1 - c - r_1 + s)\phi_{r1})$$

where the expressions $\phi_{k0}, \phi_{k1}, \phi_{e0}, \phi_{e1}, \phi_{r0}, and\ \phi_{r1}$ from equation (1) in the text are evaluated at the optimal prices and restocking fees as functions of wholesale prices and refunds as described above. The manufacturer has the constraint that the contract offered to the retailer must provide non-negative profit. Thus, the manufacturer's constrained optimization problem is

$$\max_{w_0,\, w_1,\, r_0, r_1} L^{mfgr} = \pi_{mfgr}^{CRM} + \lambda_3 \pi_{ret}$$
$$s.t.\ \lambda_3\, \pi_{ret} = 0,\ \lambda_3 \geq 0$$

The Kuhn-Tucker conditions are

$$\frac{\partial L^{mfgr}}{\partial w_0} = \frac{(\bar{u} - \hat{u})}{2}(1 - \lambda_3 + \frac{\alpha((c - s - 2w_0 + 2r_1 + d) + \lambda_3(w_0 - r_1 - d))}{2d})$$

$$\frac{\partial L^{mfgr}}{\partial w_1} = \frac{(\bar{u} - \hat{u})}{2}(1 - \lambda_3 + \frac{\alpha((c - s - 2w_1 + 2r_0 + d) + \lambda_3(w_1 - r_0 - d))}{2d})$$

$$\frac{\partial L^{mfgr}}{\partial r_0} = \frac{(\bar{u}-\hat{u})}{2}((1-\alpha)(\lambda_3-1) - \frac{\alpha((c-s-2w_1+2r_0+d)+\lambda_3(w_1-r_0-d))}{2d})$$

$$\frac{\partial L^{mfgr}}{\partial r_0} = \frac{(\bar{u}-\hat{u})}{2}((1-\alpha)(\lambda_3-1) - \frac{\alpha((c-s-2w_0+2r_1+d)+\lambda_3(w_0-r_1-d))}{2d})$$

$$\lambda_3\,\pi_{ret} = 0,\ \lambda_3 \geq 0$$

These conditions are satisfied at

$$r_1 = w_0 - c + s$$
$$w_1 = 2\hat{u} - w_0 + \frac{(c-s)^2}{2d} - \frac{d}{2} - \frac{(2-\alpha)(c-s)}{\alpha}$$
$$r_0 = 2\hat{u} - w_0 + \frac{(c-s)^2}{2d} - \frac{d}{2} - \frac{2(c-s)}{\alpha}$$
$$\lambda_3 = 1$$

Plugging these wholesale prices and refund rates into the retailer's reaction function, the resulting prices and restocking fees will be

$$p_0 = p_1 = \hat{u} + \frac{(c-s)^2}{4d} - \frac{(c-s)(2-\alpha)}{2\alpha} - \frac{d}{4}.$$
$$f_0 = f_1 = c - s$$

This equilibrium results in the following outcome:

Total quantity sold initially: $(\bar{u}-\hat{u})$

Total quantity returned (without subsequent exchange): $\frac{(\bar{u}-\hat{u})\phi_{r0}}{2} + \frac{(\bar{u}-\hat{u})\phi_{r1}}{2} = (1-\alpha)(\bar{u}-\hat{u})$

Total quantity of exchanges:

$$\frac{(\bar{u}-\hat{u})}{2}\phi_{e0} + \frac{(\bar{u}-\hat{u})}{2}\phi_{e1} = \frac{\alpha(\bar{u}-\hat{u})(d-c+s)}{4d} + \frac{\alpha(\bar{u}-\hat{u})(d-c+s)}{4d} = \frac{\alpha(\bar{u}-\hat{u})(d-c+s)}{2d}$$

Manufacturer profit: $\alpha(\bar{u}-\hat{u})(\hat{u} - c + \frac{(c-s)^2}{4d} - \frac{(c-s)(2-\alpha)}{2\alpha} - \frac{d}{4})$

We now examine when the manufacturer does not accept returns (CR reverse channel structure).

The manufacturer earns profit equal to

$$\pi_{mfgr}^{CR} = \frac{(\bar{u}-\hat{u})}{2}((w_0-c)\phi_{k0} + (w_1-c+w_0-c)\phi_{e0} + (w_0-c)\phi_{r0} \\ +(w_1-c)\phi_{k1} + (w_0-c+w_1-c)\phi_{e1} + (w_1-c)\phi_{r1})$$

where the expressions $\phi_{k0}, \phi_{k1}, \phi_{e0}, \phi_{e1}, \phi_{r0},$ *and* ϕ_{r1} from equation (1) in the text are evaluated at the optimal prices and restocking fees as functions of wholesale prices and refunds as described above. The manufacturer has the constraint that the contract offered to the retailer must provide non-negative profit. Thus, the manufacturer's constrained optimization problem is:

$$\max_{w_0,\, w_1} L^{mfgr} = \pi_{mfgr}^{CR} + \lambda_3 \pi_{ret}$$
$$s.t.\ \lambda_3\, \pi_{ret} = 0,\ \lambda_3 \geq 0$$

The Kuhn-Tucker conditions are

$$\frac{\partial L^{mfgr}}{\partial w_0} = \frac{(\bar{u}-\hat{u})}{2}(1-\lambda_3 + \frac{\alpha((c-2w_0+2s_r+d)+\lambda_3(w_0-s_r-d))}{2d})$$

$$\frac{\partial L^{mfgr}}{\partial w_1} = \frac{(\bar{u}-\hat{u})}{2}(1-\lambda_3 + \frac{\alpha((c-2w_1+s_r+d)+\lambda_3(w_1-s_r-d))}{2d}),\ \ \lambda_3\, \pi_{ret} = 0,\ \lambda_3 \geq 0$$

The three potential solutions are each symmetric:

$$w_0 = w_1 = \frac{c+d+s_r}{2},\ \lambda_3 = 0$$

$$w_0 = w_1 = \frac{(2d+\alpha(-c+d+s_r))(2d+\alpha(d+s_r)) \pm \sqrt{2d(2d+\alpha(-c+d+s_r))^2(d(2+2\alpha+\alpha^2)-2\alpha^2(\hat{u}-s_r))}}{\alpha(2d+\alpha(-c+d+s_r))},$$

$$\lambda_3 = \frac{4d^2(2+2\alpha+\alpha^2)-8d\alpha^2(\hat{u}-s_r) \pm \sqrt{2d(2d+\alpha(-c+d+s_r))^2(d(2+2\alpha+\alpha^2)-2\alpha^2(\hat{u}-s_r))}}{2d(d(2+2\alpha+\alpha^2)-2\alpha^2(\hat{u}-s_r))}.$$

Possibility 2: $E_{j=0}(utility \mid u_i = \hat{u}) > E_{j=1}(utility \mid u_i = \hat{u}) \geq 0$.

If Product 0 offers greater expected utility than product 1, all consumers who buy initially (consumers with $u_i \in [\hat{u}, \bar{u}]$) will initially buy product 0. In which case, the seller's profit can be written as

$$\pi_{ret} = (\bar{u} - \hat{u})((p_0 - w_0)\phi_{k0} + (p_1 - w_1 + f_0 - w_0 + \max\{r_0, s_r\})\phi_{e0} + (f_0 - w_0 + \max\{r_0, s_r\})\phi_{r0}).$$

The retailer's objective is to maximize profit subject to the constraints that consumers with $u_i = \hat{u}$ have $E_{j=0}(utility \mid u_i = \hat{u}) > E_{j=1}(utility \mid u_i = \hat{u}) \geq 0$. The constrained optimization problem can be described with Lagrangian multipliers λ_1 and λ_2:

$$\begin{aligned}
&\max_{p_0, p_1, f_0, f_1, \lambda_1, \lambda_2} L^{ret} = \pi_{ret} + \lambda_1(E_{j=0}(utility \mid u_i = \hat{u}) - E_{j=1}(utility \mid u_i = \hat{u})) + \lambda_2 E_{j=1}(utility \mid u_i = \hat{u}) \\
&\quad s.t.\ \lambda_1(E_{j=0}(utility \mid u_i = \hat{u}) - E_{j=1}(utility \mid u_i = \hat{u})) = 0 \\
&\quad \lambda_2 E_{j=1}(utility \mid u_i = \hat{u}) = 0 \\
&\quad \lambda_1 = 0,\ \lambda_2 \geq 0
\end{aligned}$$

The Kuhn-Tucker conditions are

$$\begin{aligned}
&\frac{\partial L^{ret}}{\partial p_0} = (\bar{u} - \hat{u})\frac{\alpha(d - w_1 + \max\{r_0, s_r\} + 2(p_1 - p_0 + f_0) - (f_0 + f_1)\lambda_1 - \lambda_2(f_1 + p_0 - p_1 - d))}{2d} = 0 \\
&\frac{\partial L^{ret}}{\partial p_1} = (\bar{u} - \hat{u})\frac{\alpha(w_1 - \max\{r_0, s_r\} + d + 2(p_0 - p_1 - f_0) + \lambda_1(f_0 + f_1) - \lambda 2(d + f_1 + p_0 - p_1))}{2d} = 0 \\
&\frac{\partial L^{ret}}{\partial f_0} = (\bar{u} - \hat{u})(\frac{(2-\alpha)(1-\lambda_1)}{2} + \frac{\alpha(w_1 - \max\{r_0, s_r\} - (2-\lambda_1)(p_1 - p_0 + f_0))}{2d}) \\
&\frac{\partial L^{ret}}{\partial f_1} = (\bar{u} - \hat{u})\frac{(\alpha(d + f_1 + p_0 - p_1) - 2d)(\lambda_2 - \lambda_1)}{2d} = 0 \\
&\lambda_1(\frac{(2-\alpha)(f_1 - f_0)}{2} + \frac{\alpha(f_1 + f_0)(f_1 - f_0 - 2(p_0 - p_1))}{4d}) = 0 \\
&\lambda_2(\alpha(\hat{u} - \frac{p_1 + p_0 + f_1}{2} - \frac{d}{4} + \frac{(f_1 - p_1 + p_0)^2}{4d}) - (1-\alpha)f_1) = 0 \\
&\lambda_1 = 0,\ \lambda_2 \geq 0
\end{aligned}$$

There are no solutions that satisfy λ_1=0 and all of the first-order conditions jointly; therefore, the retailer will never choose to set prices and restocking fees so that $E_{j=0}(utility \mid u_i = \hat{u}) > E_{j=1}(utility \mid u_i = \hat{u}) \geq 0$. In other words, the retailer will not choose to sell only product 0 and to give the marginal consumer strictly positive utility. We therefore rule out Possibility 2.

Possibility 3: $E_{j=0}(utility \mid u_i = \hat{u}) \geq 0$ and $E_{j=1}(utility \mid u_i = \hat{u}) < 0$.

The objective function with the Lagrangian multiplier on the expected utility from buying product 0 initially becomes

$$\max_{p_0, p_1, f_0, f_1, \lambda_1, \lambda_2} L^{ret} = \pi_{ret} + \lambda_2 E_{j=0}(utility \mid u_i = \hat{u})$$
$$s.t.\ \lambda_2 (E_{j=0}(utility \mid u_i = \hat{u})) = 0$$
$$\lambda_2 \geq 0,\ E_{j=1}(utility \mid u_i = \hat{u}) < 0$$

The Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial p_0} = \frac{\alpha((\bar{u} - \hat{u})(d + 2(f_0 - p_0 + p_1) + \max\{r_0, s_r\} - w_1) - \lambda_2(d + f_0 - p_0 + p_1))}{2d} = 0$$
$$\frac{\partial L}{\partial p_1} = (\bar{u} - \hat{u})\frac{\alpha((\bar{u} - \hat{u})(d - 2(f_0 - p_0 + p_1) - \max\{r_0, s_r\} + w_1) - \lambda_2(d - f_0 + p_0 - p_1))}{2d} = 0$$
$$\frac{\partial L}{\partial f_0} = \lambda_2(\frac{\alpha(d + f_0 - p_0 + p_1)}{2d} - 1) + (\bar{u} - \hat{u})(1 + \frac{\alpha(-d - 2(f_0 - p_0 + p_1) - \max\{r_0, s_r\} + w_1)}{2d} = 0$$

The Kuhn-Tucker conditions are satisfied (and profit is the same) at any price and restocking fee combination such that

$$p_0 = \hat{u} + \frac{(w_1 - \max\{r_0, s_r\})^2}{4d} - \frac{(w_1 - \max\{r_0, s_r\})}{2} - \frac{f_0(1-\alpha)}{\alpha} - \frac{d}{4},$$

$$p_1 = \hat{u} + \frac{(w_1 - \max\{r_0, s_r\})^2}{4d} + \frac{(w_1 - \max\{r_0, s_r\})}{2} - \frac{f_0}{\alpha} - \frac{d}{4}$$ and

$$E_{j=1}(utility \mid u_i = \hat{u}) = \frac{(2-\alpha)(f_0 - f_1)}{2} + \frac{\alpha(f_1 + f_0)(f_1 + f_0 - 2(w_1 - \max\{r_0, s_r\}))}{4d} < 0$$ (with $\lambda_2 = \bar{u} - \hat{u}$).

We first examine when the manufacturer accepts returns (CRM reverse channel structure).

The manufacturer takes the retailer's reaction into account in maximizing its profit

$$\pi_{mfgr}^{CRM} = (\bar{u} - \hat{u})((w_0 - c)\phi_{k0} + (w_1 - c + w_0 - r_0 - c + s)\phi_{e0} + (w_0 - c - r_0 + s)\phi_{r0}),$$

where the expressions $\phi_{k0}, \phi_{e0}, and\ \phi_{r0}$ from equation (1) in the text are evaluated at the optimal prices and restocking fees as functions of wholesale prices and refunds as described above. The manufacturer has the constraint that the contract offered to the retailer must provide non-negative profit. Thus, the manufacturer's constrained optimization problem is

$$\max_{w_0, w_1, r_0, r_1} L^{mfgr} = \pi_{mfgr}^{CRM} + \lambda_3 \pi^{ret}$$
$$s.t.\ \lambda_3\, \pi_{ret} = 0,\ \lambda_3 \geq 0$$

The Kuhn-Tucker Conditions are

$$\frac{\partial L^{mfgr}}{\partial w_0} = (\overline{u} - \hat{u})(1 - \lambda_3) = 0$$
$$\frac{\partial L^{mfgr}}{\partial w_1} = \frac{\alpha(\overline{u} - \hat{u})(c + d + 2r_0 - s - 2w_1 - \lambda_3(d + r_0 - w_B))}{2d} = 0$$
$$\frac{\partial L^{mfgr}}{\partial r_0} = \frac{\overline{u} - \hat{u}(\alpha((2 - \lambda_3)(w_1 - r_0) - c + s) - d(2 - \alpha)(1 - \lambda_3))}{2d} = 0$$

These conditions are satisfied at

$$w_0 = r_0 + \alpha(\hat{u} + \frac{(c-s)^2}{4d} - \frac{c-s}{2} - \frac{d}{4} - r_0)$$

$$w_1 = r_0 + c - s\ ,\ \lambda_3 = 1\,.$$

Plugging these wholesale prices and refund rates into the retailer's reaction function, the resulting prices and restocking fees will be any price and restocking fee combination such that

$$p_0 = \hat{u} + \frac{(c-s)^2}{4d} - \frac{(c-s)}{2} - \frac{f_0(1-\alpha)}{\alpha} - \frac{d}{4}\,,\ p_1 = \hat{u} + \frac{(c-s)^2}{4d} + \frac{(c-s)}{2} - \frac{f_0}{\alpha} - \frac{d}{4}\ \text{and}$$

$$E_{j=1}(utility\,|\,u_i = \hat{u}) = \frac{(2-\alpha)(f_0 - f_1)}{2} + \frac{\alpha(f_1 + f_0)(f_1 + f_0 - 2(c-s))}{4d} < 0\,.$$

The equilibrium results in the following outcome:

Total quantity sold initially: $(\overline{u} - \hat{u})$

Total quantity returned (without subsequent exchange): $(\overline{u} - \hat{u})\phi_{r0} + 0 \bullet \phi_{r1} = (1-\alpha)(\overline{u} - \hat{u})$

Total quantity of exchanges: $(\bar{u}-\hat{u})\phi_{e0}+0\bullet\phi_{e1}=\dfrac{\alpha(\bar{u}-\hat{u})(d-c+s)}{2d}$

Manufacturer profit: $\alpha(\bar{u}-\hat{u})(\hat{u}-c+\dfrac{(c-s)^2}{4d}-\dfrac{(c-s)(2-\alpha)}{2\alpha}-\dfrac{d}{4})$

Moreover, if a consumer initially purchases product 0, the out-of-pocket marginal expense to return it and buy product 1 is also the same as in the symmetric case, namely: $p_1-\left(p_0-f_0\right)=(c-s)$.

We now examine when the manufacturer does not accept returns (CR reverse channel structure).

The manufacturer takes the retailer's reaction into account in maximizing its profit

$$\pi_{mfgr}^{CR}=(\bar{u}-\hat{u})((w_0-c)\phi_{k0}+(w_1-c+w_0-c)\phi_{e0}+(w_0-c)\phi_{r0})$$

where the expressions $\phi_{k0}, \phi_{e0}, and\ \phi_{r0}$ from equation (1) in the text are evaluated at the optimal prices and restocking fees as functions of wholesale prices and refunds as described above. The manufacturer has the constraint that the contract offered to the retailer must provide non-negative profit. Thus, the manufacturer's constrained optimization problem is

$$\max_{w_0,\,w_1} L^{mfgr}=\pi_{mfgr}^{CR}+\lambda_3\pi^{ret}$$
$$s.t.\ \lambda_3\,\pi^{ret}=0,\ \lambda_3\geq 0$$

The Kuhn-Tucker Conditions are

$$\frac{\partial L^{mfgr}}{\partial w_0}=(\bar{u}-\hat{u})(1-\lambda_3)=0$$
$$\frac{\partial L^{mfgr}}{\partial w_1}=\frac{\alpha(\bar{u}-\hat{u})(c+d+s_r-2w_1-\lambda_3(d+s_r-w_B))}{2d}=0$$
$$\lambda_3\,\pi_{ret}=0,\ \lambda_3\geq 0$$

The solution to these conditions is

$$w_0 = \frac{2d(\alpha(2\hat{u}-c)+s_r(2-\alpha))-\alpha(d^2-(c-s_r)^2)}{4d}$$
$$w_1 = c$$
$$\lambda_3 = \bar{u}-\hat{u}$$

The above wholesale contract would be the manufacturer's optimal contract if the retailer were in fact to react by setting prices and restocking fees according to Possibility 3 (i.e., such that $E_{j=0}(utility \mid u_i = \hat{u}) \geq 0,\ and\ E_{j=1}(utility \mid u_i = \hat{u}) < 0$). However, the retailer gets to choose its reaction to the wholesale contract. Plugging this wholesale contract into the retailer's possible reaction from Possibility 3 ($E_{j=0}(utility \mid u_i = \hat{u}) \geq 0,\ and\ E_{j=1}(utility \mid u_i = \hat{u}) < 0$) and into the retailer's reaction from Possibility 1 ($E_{j=0}(utility \mid u_i = \hat{u}) = E_{j=1}(utility \mid u_i = \hat{u})$), the retailer in fact earns greater profit from reacting with the best response function from Possibility 1 (in which $E_{j=0}(utility \mid u_i = \hat{u}) = E_{j=1}(utility \mid u_i = \hat{u})$) for any value of $\hat{u}$ that yields non-negative *manufacturer profit* in Possibility 3 ($\hat{u} \geq \frac{2cd(2+\alpha)+\alpha d^2-\alpha(c-s_r)^2-2ds_r(2-\alpha)}{4d\alpha}$). Thus, the retailer's actual reaction to the manufacturer contract from Possibility 3 (asymmetric choices set assuming $E_{j=0}(utility \mid u_i = \hat{u}) \geq 0,\ and\ E_{j=1}(utility \mid u_i = \hat{u}) < 0$) will be to offer prices and restocking fees according to Possibility 1 (expected utility from each product is equivalent). The manufacturer would recognize the retailer's actual reaction. As shown above, when the retailer responds as in Possibility 1, the manufacturer optimally offers symmetric wholesale prices. Therefore, in the CR reverse channel structure, there will only be a symmetric equilibrium.

Conclusion

Summarizing the insights from analyzing Possibilities 1, 2, and 3 for the CRM reverse channel structure, we have: (a) Possibility 2 cannot exist; (b) Possibility 1 produces symmetric solution values; and (c) Possibility 3 produces the same profit, total returns, total sales, and total exchanges as in the symmetric solution. Thus, allowing asymmetry in a decentralized channel in which the manufacturer offers a per-unit wholesale price and refund rate results in the same outcomes as assuming symmetry.

When prices and restocking fees are chosen such that all consumers with $u_i < \hat{u}$ do not buy either product initially and all consumers with $u_i \in [\hat{u}, \overline{u}]$ buy one of the two products, asymmetric prices, restocking fees, wholesale prices, or refund rates will not affect manufacturer profit, total quantity of exchanges, or the total quantity sold. Therefore, for any given $\hat{u}$, asymmetric wholesale prices do not improve profit or change the number of product returns. To increase the number of consumers who buy initially, the prices and restocking fees may be lowered, but there is no additional benefit on sales or profit to charge asymmetric prices (retail or wholesale). Thus we have shown that the asymmetric wholesale prices (and consequently asymmetric retail prices) will not increase profit or change the number of exchanges for any number of initial sales that the manufacturer would like to induce. *Q.E.D.*

A.11. Proof of Observation 1

OBSERVATION 1: *Consider a situation in which the manufacturer charges the retailer a per-unit wholesale price but not a fixed fee. If* $s > s_r$, *then the return penalty, f, charged to consumers is greater when the retailer salvages returned units than when the manufacturer accepts and salvages returned units from the retailer. There exists a critical salvage value for the manufacturer,* $\tilde{s} < s_r$, *such that* $\tilde{s} < s < s_r$ *also implies the return penalty, f, charged to consumers is greater when the retailer salvages returned units than when the manufacturer accepts and salvages returned units from the retailer. If* $s \le \tilde{s}$, *then the return penalty, f,* *charged to consumers is lower when the retailer salvages returned units than when the manufacturer accepts and salvages returned units from the retailer.*

Proof:

We first prove that for $s = s_r$, the return penalty, *f,* charged to consumers is greater when the retailer salvages returned units than when the manufacturer accepts and salvages returned units from the retailer. From Table 3, we know that $f^* = w^* - \max\{s_r, r\}$. When the manufacturer accepts returns responsibility, this simplifies to $f^* = c - s$. When the manufacturer does not accept returns responsibility, then $f^* = w^* - s_r$ and it must be that $w^* > c$ for the manufacturer to earn positive profit. Thus, when $s = s_r$, the return penalty is greater when the retailer is responsible for salving returned units than when the manufacturer accepts this responsibility.

The return penalty charged to consumers when the manufacturer salvages returns is decreasing in *s*. The return penalty charged to consumers when the retailer salvages returns is invariant with respect to *s.* We define $f^{CRM}(s)$ as the equilibrium return penalty charged to consumers when the manufacturer accepts returns responsibility (as a function of *s*) and $f^{CR}(s)$ as the equilibrium return penalty charged to consumers when the retailer salvages returns (as a function of *s).* Because $f^{CRM}(s|_{s=s_r}) < f^{CR}(s|_{s=s_{sr}})$, $\frac{\partial f^{CRM}(s)}{\partial s} < 0$ and $\frac{\partial f^{CR}(s)}{\partial s} = 0$, there exists an $\tilde{s} < s_r$ such that the functions $f^{CRM}(s) = f^{CR}(s)$ at $s = \tilde{s}$. If $s > \tilde{s}$ then $f^{CRM}(s) < f^{CR}(s)$ and if $s \le \tilde{s}$, then $f^{CRM}(s) \ge f^{CR}(s)$.

Q.E.D.

A.12. Proof of Proposition 2

PROPOSITION 2: *Consider the situation in which the manufacturer charges the retailer a per-unit wholesale price but not a fixed fee. If $s > s_r$, then the manufacturer accepts and salvages returned units from the retailer. There exists a critical salvage value for the manufacturer, $\bar{s} < s_r$, such that $\bar{s} < s < s_r$ also implies the manufacturer will accept and salvage returned units from the retailer. If $s \le \bar{s}$, then the manufacturer chooses to allocate responsibility for salvaging returned units to the retailer.*

We first show that the manufacturer earns greater profit from accepting and salvaging returned units from the retailer when $s = s_r$. From Tables 3, we know that $\pi_{mfgr}^{CR} = \frac{(w^* - c)(\alpha(s_r - w^*) + (2+\alpha)d)(\alpha(w^* - s_r)^2 + 2d((2-\alpha)s_r + 2\alpha\bar{u} - (2+\alpha)w^*) - \alpha d^2)}{16\alpha d^2}$ where w^* is defined in Table 3 and $\pi_{mfgr}^{CRM} = \frac{(\alpha(c-s)^2 + d(2s(2-\alpha) + \alpha(4\bar{u} - d) - 2c(2+\alpha)))^2}{128 d^2}$. We will show that at $s = s_r$, $\pi_{mfgr}^{CR} - \pi_{mfgr}^{CRM} < 0$ for all $w > c$.

We will do this in two parts.

Part 1: We will show that for $d > \frac{\alpha(w^{*,CR} - s_r)}{2+\alpha}$, any value of $\bar{u}$ will make $\pi_{mfgr}^{CR} - \pi_{mfgr}^{CRM} < 0$.

Part 2: We will show that for $d \le \frac{\alpha(w^{*,CR} - s_r)}{2+\alpha}$, the value of $\pi_{mfgr}^{CR} - \pi_{mfgr}^{CRM} < 0$ for $\bar{u}$ greater than the minimum $\bar{u}$ for which quantity is positive in the CR model.

<u>We begin with part 1</u> $d > \frac{\alpha(w^{*,CR} - s_r)}{2+\alpha}$. The second derivative of $(\pi_{mfgr}^{CR} - \pi_{mfgr}^{CRM})$ with respect to $\bar{u}$ is equal to $-\frac{\alpha}{4}$. Therefore, the expression is concave in $\bar{u}$ and has a unique maximum.

Evaluated at the expression maximizing value of $\overline{u}$, the maximum value that $(\pi_{mfgr}^{CR} - \pi_{mfgr}^{CRM})$ can take is $\frac{(w^{*,CR} - c)^3(\alpha(w^{*,CR} - s_r) - d(2+\alpha))}{16d^2}$. This maximum value is negative by fact that $d > \frac{\alpha(w^{*,CR} - s_r)}{2+\alpha}$. Since the expression $(\pi_{mfgr}^{CR} - \pi_{mfgr}^{CRM})$ is negative at its maximum value over $\overline{u}$, it is negative for all values of $\overline{u}$.

<u>We now show part 2, $d \le \frac{\alpha(w^{*,CR} - s_r)}{2+\alpha}$</u>. From Table 3, the quantity of consumers who buy initially (heretofore defined as q_b^{CR}) is equal to $q_b^{CR} = \frac{\overline{u}}{2} - \frac{w^{*,CR} - s_r}{2\alpha} - \frac{d^2 + 2d(w^{*,CR} + s_r) - (w^{*,CR} - s_r)^2}{8d}$. This is positive if and only if $\overline{u} > \overline{u}^{CR} \equiv \frac{w^{*,CR} - s_r}{\alpha} + \frac{d^2 + 2d(w^{*,CR} + s_r) - (w^{*,CR} - s_r)^2}{4d}$, where we define $\overline{u}^{CR}$ as the value of $\overline{u}$ such that q_b^{CR}=0.

We can show that $\overline{u} > \overline{u}^{R}$ implies that $(\pi_{mfgr}^{CR} - \pi_{mfgr}^{CRM}) < 0$ for $d \le \frac{\alpha(w^{*,CR} - s_r)}{2+\alpha}$. At $\overline{u} = \overline{u}^{CR}$, the expression $(\pi_{mfgr}^{CR} - \pi_{mfgr}^{CRM}) = -\frac{(w^{*,CR} - c)^2(\alpha(w^{*,CR} + c - 2s_r) - 2d(2+\alpha))^2}{128\alpha d^2}$, which is negative. The expression $\frac{\partial(\pi_{mfgr}^{CR} - \pi_{mfgr}^{CRM})}{\partial \overline{u}}$ evaluated at $\overline{u} = \overline{u}^{CR}$ is equal to $\frac{\partial(\pi_{mfgr}^{CR} - \pi_{mfgr}^{CRM})}{\partial \overline{u}} = \frac{(w^{*,CR} - c)(\alpha(c + 2s_r - 3w^{*,CR}) + 2d(2+\alpha))}{16d}$. This derivative is negative for all $d < \frac{\alpha(2(w^{*,CR} - s_r) + w^{*,CR} - c)}{2(2+\alpha)}$.

Because $\frac{\alpha(2(w^{*,CR}-s_r)+w^{*}-c)}{2(2+\alpha)}=\frac{\alpha(w^{*,CR}-s_r)}{2+\alpha}+\frac{\alpha(w^{*,CR}-c)}{2(2+\alpha)}>\frac{\alpha(w^{*,CR}-s_r)}{2+\alpha}$, the derivative is also negative for all $d\leq\frac{\alpha(w^{*,CR}-s_r)}{2+\alpha}$. By fact that $\frac{\partial\partial(\pi_{mfgr}^{CR}-\pi_{mfgr}^{CRM})}{\partial\overline{u}\partial\overline{u}}=-\frac{1}{4}$, the derivative $\frac{\partial(\pi_{mfgr}^{CR}-\pi_{mfgr}^{CRM})}{\partial\overline{u}}<0$ for all $\overline{u}>\overline{u}^{CR}$.

Because $(\pi_{mfgr}^{CR}-\pi_{mfgr}^{CRM})<0$ at $\overline{u}=\overline{u}^{CR}$ and $\frac{\partial(\pi_{mfgr}^{CR}-\pi_{mfgr}^{CRM})}{\partial\overline{u}}<0$ for all $\overline{u}>\overline{u}^{CR}$, the expression $(\pi_{mfgr}^{CR}-\pi_{mfgr}^{CRM})<0$ for all $\overline{u}>\overline{u}^{CR}$.

Therefore, $\pi_{mfgr}^{CRM}>\pi_{mfgr}^{CR}$ at $s=s_r$.

We have thus far proven that if $s=s_r$, then $\pi_{mfgr}^{CRM}>\pi_{mfgr}^{CR}$. We now show that $\frac{\partial\pi_{mfgr}^{CRM}}{\partial s}>0$ and $\frac{\partial\pi_{mfgr}^{CR}}{\partial s}=0$.

$\frac{\partial\pi_{mfgr}^{CRM}}{\partial s}=\frac{d(2-\alpha)-\alpha(c-s)}{2d}(\frac{\overline{u}}{4}+\frac{(c-s)^2}{16d}-\frac{c-s}{4\alpha}-\frac{2(c+s)+d}{16})>0$ for all $\overline{u}$ such that $q_b^M=\frac{\overline{u}}{4}+\frac{(c-s)^2}{16d}-\frac{c-s}{4\alpha}-\frac{2(c+s)+d}{16}$>0 and all d such that the equilibrium exchange rate when the manufacturer accepts returns (heretofore defined as ϕ_e^{CRM}) $\phi_e^{CRM}=\alpha(\frac{1}{2}-\frac{(c-s)}{2d})$>0.

Therefore, when $\pi_{mfgr}^{CRM}\geq 0$ and $\pi_{mfgr}^{CR}\geq 0$, there exists an $\overline{s}<s_r$ such that the functions $\pi_{mfgr}^{CRM}(s)=\pi_{mfgr}^{CR}(s)$ at $s=\overline{s}$. If $\overline{s}<s$ then $\pi_{mfgr}^{CRM}>\pi_{mfgr}^{CR}$ and $s\leq\overline{s}$, then $\pi_{mfgr}^{CRM}\leq\pi_{mfgr}^{CR}$.

Q.E.D.

A.13. Proof of Proposition 3

PROPOSITION 3**:** *Consider a situation in which the manufacturer charges the retailer a per-unit wholesale price but not a fixed fee. If the manufacturer's salvage value is greater than the retailer's salvage value ($s \geq s_r$) the manufacturer can replicate the return penalty charged to consumers in a vertically-integrated channel by accepting product returns from the retailer. The retail price charged to consumers will be distorted upward and manufacturer profit will be distorted downward from a vertically-integrated channel. Otherwise, when $s < s_r$, the return penalty charged to consumers will be greater in a decentralized channel than in a vertically integrated system.*

Proof:

In the vertically integrated system, the return penalty charged to consumers will be equal to $f^{VI} = c - \max\{s, s_r\}$ and the retail price will be $p^{VI} = \frac{\bar{u}}{2} + \frac{(c - \max\{s, s_r\})^2}{8d} + \frac{2(c + \max\{s, s_r\}) - d}{8} - \frac{(1-\alpha)(c - \max\{s, s_r\})}{2\alpha}$. When the manufacturer accepts returns (CRM reverse channel structure), then the price and consumer return penalty from Table 3 simplify to $f^{CRM} = c - s$ and $p^{CRM} = \frac{3}{2}(\frac{\bar{u}}{2} + \frac{(c-s)^2}{8d} + \frac{2(c-s)-d}{8} - \frac{(c-s)}{2\alpha} + \frac{c}{6})$.

If $s > s_r$:

In this case max*{s,s$_r$}*=*s*. Clearly, the return penalty when the manufacturer accepts returns is the equivalent to the fully-coordinated solution. The price difference is equal to $p^{CRM} - p^{VI} = \frac{\bar{u}}{4} + \frac{(c-s)^2}{16d} - \frac{c-s}{4\alpha} - \frac{2(c+s)+d}{16}$ which is greater than zero for all parameters such that initial quantity sold $q_b^{CRM} = \frac{\bar{u}}{4} + \frac{(c-s)^2}{16d} - \frac{c-s}{4\alpha} - \frac{2(c+s)+d}{16}$ is non-negative.

If $s < s_r$ *:*

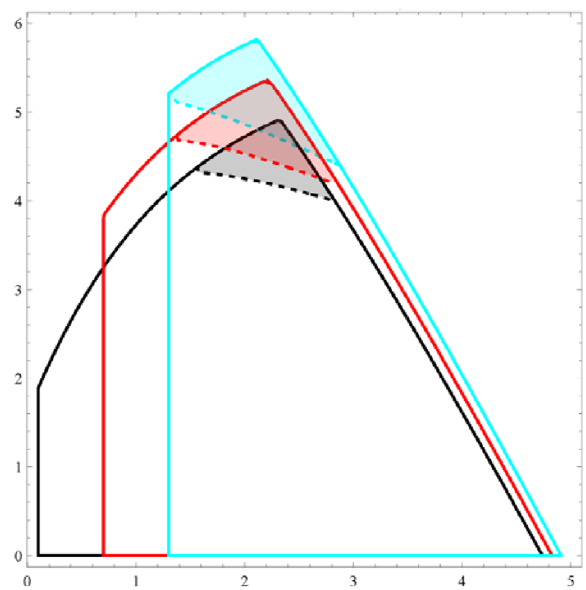
Consider $\bar{s} < s < s_r$. From Proposition 2, in this region the manufacturer will accept returns from the retailer. However, $f^{VI} = c - s_r$ which is less than $f^{CRM} = c - s$ by fact that $s < s_r$.

If $\bar{s} < s < s_r$, then the manufacturer does not accept returns. In which case, $f^{CR} = w^{*,CR} - s_r$ which is greater than $f^{VI} = c - s_r$ for any $w^* > c$, a condition which is necessary for the manufacturer to earn non-negative profit when not accepting returns (when in the CR reverse channel structure). *Q.E.D.*

A.14. Examples of When Manufacturer Should Accept Returns

In this section we examine how the model's parameters affect whether or not the manufacturer optimally takes product returns from the retailer. While the analytical solutions for the manufacturer's profit in the CRM channel structure as well as the CR channel structure are presented above, the complexity of these analytical solutions makes examining the comparative statics infeasible without the use of numerical analysis. Figures 14.3 and 14.4 illustrate parametric situations when a manufacturer would earn greater profit from salvaging returned units itself than from allocating this responsibility to the retailer. We have chosen two graphs that represent small numbers and larger numbers respectively. In each graph, we see for a broad range of examples the impact of *d* and s_r on the optimal reverse channel structure. In the graph below, we also see the impact of marginal cost on the optimal reverse channel structure.

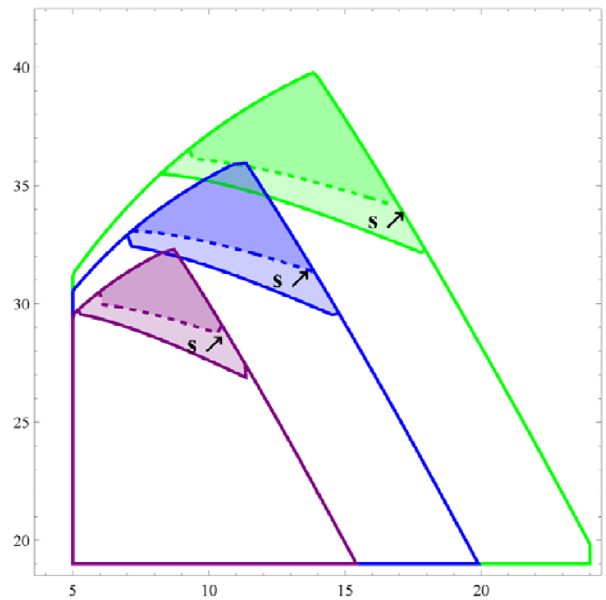
Figure 14.3 How The Optimal Reverse Channel Structure Varies with *c, d,* and s_r
(α=1, *s*=1.5, $\bar{u}=15$, Black: $c=1.6$, Red: $c=2.2$, Cyan $c=2.8$, Solid Line: *s*=15, Dashed Line: *s*=19)
(Horizontal Axis=*d*, Vertical Axis=s_r)



Shaded regions denote parameter space for which $\pi_{mfgr}^{CR} > \pi_{mfgr}^{CRM}$. Outlined regions represent the feasible parameter space. Parameter values outside the outlined regions violate rational conditions on f^{CR} and w^{CR}.

In the following graph, we are able to see the impact of $\bar{u}$ and s on the optimal reverse channel structure over a large set of parameters.

Figure 14.4 How The Optimal Reverse Channel Structure Changes with $\bar{u}$, *s*, *d*, and s_r
(α=1, *c*=20, Green: $\bar{u}=100$, Blue: $\bar{u}=85$, Purple $\bar{u}=70$, Solid Line: *s*=15, Dashed Line: *s*=19)
(Horizontal Axis=*d*, Vertical Axis=s_r



Shaded regions denote parameter space for which $\pi_{mfgr}^{CR} > \pi_{mfgr}^{CRM}$. Outlined regions represent the feasible parameter space. Parameter values outside the outlined regions violate rational conditions on f^{CR} and w^{CR}.

Consistent with our analytical results above, these examples show that the manufacturer prefers to salvage the returned units except when the retailer has a significant salvage advantage (high s_r) and consumers place a higher value on getting the right product (high *d*). A higher value of s_r makes each returned unit more valuable to the retailer and a higher value of *d* creates a greater number of returns (making the retailer's salvage advantage more valuable). In Figure 14.3, we see the effect of the marginal cost of production, *c*, while Figure 14.4 shows the effects of the manufacturer's salvage value for returned units, *s*, and the highest value a consumer may obtain from owning a unit in the product category, $\bar{u}$.

The minimum values of *d* and s_r necessary for the manufacturer to prefer that the retailer salvages returns are increasing as *c* increases (as shown in Figure 14.3). While an increase in *c* decreases manufacturer profits whether or not the manufacturer accepts returns, the magnitude of the effect is greater when the retailer salvages returns. This comes as a result of the distortion in f^{CR}. In either channel structure, the manufacturer increases wholesale price with an increase in the marginal cost of production. However, in the CRM channel structure, the manufacturer also raises the refund for returned units, which dampens the effect on the restocking fee and the resulting equilibrium quantities. So although $\frac{\partial w_{CR}}{\partial c} > \frac{\partial w_{CRM}}{\partial c} > 0$, this leads to $|\frac{\partial q_{CRM}}{\partial c}| < |\frac{\partial q_{CR}}{\partial c}|$, which has a stronger net effect on manufacturer profits in the CR structure than in the CRM structure. Thus an increase in *c* implies that higher values of sr and *d* are necessary in order for $\pi_{mfgr}^{CR} > \pi_{mfgr}^{CRM}$.

The minimum values of *d* and *sr* necessary for the manufacturer to prefer that the retailer salvages returns are also increasing as *s* or $\bar{u}$ increases. An increase in *s* has a positive effect on manufacturer profit in the CRM structure and no effect in the CR structure. An increase in $\bar{u}$ has a positive effect on manufacturer profit regardless of who salvages returned units. However, it has a greater impact on manufacturer profit in a CRM structure than a CR structure, again because of the distortion in restocking fee that occurs in the CR structure. Although equilibrium sales quantity increases with $\bar{u}$ at a faster rate in the CR structure than in the CRM structure, the reverse is true for the equilibrium wholesale price. Overall, manufacturer profit increases with $\bar{u}$ at a faster rate in the CRM structure than the CR structure. Thus, for higher $\bar{u}$, the retailer's salvage advantage (and/or the consumer disutility of mismatch) must be greater for the manufacturer to earn greater profit when the retailer salvages returned units.[3]

[3] It should be noted that larger $\bar{u}$ or *s* expands the feasible parameter space and larger *c* shrinks the feasible set. A larger $\bar{u}$ means an expanded market and thus a greater set of $\{sr,d\}$ that satisfy the constraints of the model. For larger *c* or smaller *s*, there is greater incentive to shut down returns entirely (a scenario which could not address the issue of optimal reverse channel structure).

A.15. Single-Product Model

In this Appendix, we generate the analogues to Observation 1 and Proposition 2 in a single-product model.

For a product located at $x=0$, a consumer with reservation utility u_i chooses to keep his initial purchase if $u_i-p-d\theta_i>-f$, that is, if $\theta_i<(u_i-p+f)/d$.[4] Ex ante, consumer i's expected utility of making an initial purchase is then equal to

$$E(u_i)=(\frac{u_i-p+f}{d})(u_i-p-\frac{d}{2}(\frac{u_i-p+f}{d}))+(1-\frac{u_i-p+f}{d})(-f)=\frac{(f-p+u_i)^2-2df}{2d}.$$

Consumers for whom $E(u_i)\geq 0$ (that is, for whom $u_i\in[p-f+\sqrt{2df},\bar{u}]$) buy initially. Thus, the quantity sold is $\bar{u}-p+f-\sqrt{2df}$. For each consumer, the probability of a return is equal to $(1-(u_i-p+f)/d)$. Integrating over all u_i such that an initial purchase is made gives the total expected number of returns as: $\int_{p-f+\sqrt{2df}}^{\bar{u}}(1-\frac{u_i-p+f}{d})\,du_i=\bar{u}+2f-\sqrt{2df}-p-\frac{(\bar{u}-p+f)^2}{2d}$.

There are several points to note about the demand function and the return function. The quantity sold is decreasing in price. The quantity sold is decreasing in the restocking fee, due to the fact that the derivative with respect to f is $1-\frac{\sqrt{d}}{\sqrt{2f}}$, which is negative if and only if the consumer located at $u_i=p-f+\sqrt{2df}$ has a positive probability of making a return. The expected total number of returns is decreasing in price (the derivative of expected total returns with respect to p is $-(1-\frac{\bar{u}-p+f}{d})$, which is negative if and only if the probability that the

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[4] If the product is located at $x_j=1/2$, the probability that it is kept doubles. In such a case, a consumer with reservation utility u_i would keep the purchase if $u_i-p-d|1/2-\theta|>-f$, leading to an expected utility equal to $\frac{(f-p+u_i)^2-df}{d}$. The subsequent analysis for the product located $x_j=0$ follows for $x_j=1/2$ as well but with a different scale.

consumer with the highest $u_i = \bar{u}$ has a positive probability of making a return). The expected total number of returns is decreasing in the restocking fee (its derivative with respect to *f* is $(2 - \frac{\bar{u} - p + f}{d} - \frac{\sqrt{d}}{\sqrt{2f}}))$. For sales to be positive it must be that $\bar{u} > \sqrt{2df} - f + p$. Evaluated at this minimum bound on $\bar{u}$, the derivative of expected total number of returns with respect to the restocking fee is equal to $\frac{1}{(\sqrt{f}(\sqrt{2d} - \sqrt{f}) + p)}(2 - \frac{(d+2f)}{\sqrt{2df}}) < 0$ for all values of *f* such that the consumer located at $u_i = p - f + \sqrt{2df}$ has a positive probability of making a return (by fact that $(2 - \frac{(d+2f)}{\sqrt{2df}})$ is equal to zero at *f=d/2* and increasing in *f* through this point). Thus, the derivative of the total number of returns with respect to *f* is $(2 - \frac{\bar{u} - p + f}{d} - \frac{\sqrt{d}}{\sqrt{2f}})$, which is negative at the minimum bound $\bar{u} = \sqrt{2df} - f + p$ and is decreasing in $\bar{u}$. Therefore this derivative is negative for all $\bar{u}$ such that sales are positive.

Letting *Qsales* represent the total initial sales and *Qreturns* represent the total number of returns (rather than returns probability), it can then be noted that:

$\frac{\partial Qsales}{\partial p} < 0, \frac{\partial Qsales}{\partial f} < 0, \frac{\partial Qreturns}{\partial p} < 0, \frac{\partial Qreturns}{\partial f} < 0,$ and

$\frac{\partial Qsales}{\partial p} \bullet \frac{\partial Qreturns}{\partial f} - \frac{\partial Qsales}{\partial f} \bullet \frac{\partial Qreturns}{\partial p} = \frac{1}{\bar{u}}(\frac{(\bar{u} - p + f) - \sqrt{2df}}{\sqrt{2df}}) > 0$ for all $\bar{u}$ such that sales are positive $\bar{u} > \sqrt{2df} - f + p$.

To obtain interpretable results, we use general linear demand and return functions that preserve the directional effects of *p* and *f*. We examine a single product setting with demand and return functions of the form:

Qsales=α-βp-δf

Qreturns= z-yp-vf

Where *Qsales* represents the total initial sales and *Qreturns* represents the total number of returns (rather than returns probability).

We assume $(v\beta - y\delta) > 0$, to mirror the result above that

$$\frac{\partial Qsales}{\partial p} \bullet \frac{\partial Qreturns}{\partial f} - \frac{\partial Qsales}{\partial f} \bullet \frac{\partial Qreturns}{\partial p} > 0.$$

A15.1. CR Reverse Channel Structure

The retailer chooses price and restocking fee to maximize

profitretCR=(p-w)Qsales+(f-p+s_r)Qreturns.

Taking first order conditions, the retailer maximizes profit with the reaction functions:

$$p(w) = \frac{s_r v(v+y-\delta)+(y+\delta)(z+w\delta)+v(z-2\alpha-w(2\beta+\delta))}{v^2+2v(y-2\beta-\delta)+(y+\delta)^2}$$

$$f(w) = \frac{yz+y\alpha+wy\beta-2z\beta+v(z-\alpha-w\beta)+2wy\delta-z\delta+\alpha\delta-w\beta\delta-s_r(v(y-2\beta)+y(y+\delta))}{v^2+2v(y-2\beta-\delta)+(y+\delta)^2}.$$

The second order conditions are satisfied if

$$\frac{\partial^2 profitretCR}{\partial^2 p} = 2(y-\beta) < 0$$

$$\frac{\partial^2 profitretCR}{\partial^2 f} = -2v < 0$$

and the determinant of the Hessian: $-(v^2+2v(y-2\beta-\delta)+(y+\delta)^2) > 0$.

The manufacturer chooses wholesale price to maximize

profitmanCR=(w-c)Qsales(p(w),f(w)).

Since it is a single-product setting, returns do not generate an additional sale for the manufacturer. Taking first order conditions, the manufacturer maximizes profit at

$$w^{CR}=\frac{1}{4(\beta+\delta)(v\beta-y\delta)}\Big(v(2\alpha\beta+2c\beta^2+y(-2\alpha+s_r(\beta-\delta))+\alpha\delta+\beta\delta(2c+s_r)+z(\beta+\delta))-$$
$$(v^2(\alpha-\beta s_r)+z\delta(\beta+\delta)+y^2(\alpha+\delta s_r)+y(-z(\beta+\delta)+\delta(\alpha+\delta s_r+2c(\beta+\delta))))\Big).$$

The second order condition for the above wholesale price to be a profit maximum is

$\dfrac{\partial^2 profitmanCR}{\partial^2 w}=\dfrac{4(\beta+\delta)(v\beta-y\delta)}{v^2+2v(y-2\beta-\delta)+(\gamma+\delta)^2}<0$ by assumption that $(v\beta-y\delta)$>0 and

$-(v^2+2v(y-2\beta-\delta)+(y+\delta)^2)>0$.

The manufacturer's equilibrium profit in the CR reverse channel structure is given by

$$profitmanCR=-\frac{1}{8(\beta+\delta)(v\beta-y\delta)(v^2+2v(y-2\beta-\delta)+(y+\delta)^2)}$$
$$\Big(v(2\alpha\beta-2c\beta^2+y(-2\alpha+s_r(\beta-\delta))+\alpha\delta+\beta\delta(-2c+s_r)+z(\beta+\delta))-$$
$$(v^2(\alpha-\beta s_r)+z\delta(\beta+\delta)+y^2(\alpha+\delta s_r)+y(-z(\beta+\delta)+\delta(\alpha+\delta s_r-2c(\beta+\delta))))\Big)^2.$$

A15.2. CRM Reverse Channel Structure

The retailer chooses price and restocking fee to maximize

profitretCRM=(p-w)Qsales+(f-p+r)Qreturns.

Taking first order conditions, the retailer maximizes profit with the reaction functions:

$$p(w,r)=\frac{rv(v+y-\delta)+(y+\delta)(z+w\delta)+v(z-2\alpha-w(2\beta+\delta))}{v^2+2v(y-2\beta-\delta)+(y+\delta)^2}$$

$$f(w,r)=\frac{yz+y\alpha+wy\beta-2z\beta+v(z-\alpha-w\beta)+2wy\delta-z\delta+\alpha\delta-w\beta\delta-r(v(y-2\beta)+y(y+\delta))}{v^2+2v(y-2\beta-\delta)+(y+\delta)^2}$$

.

The second order conditions are satisfied for the same parameter values as the CR model above.

The manufacturer chooses wholesale price and refund to maximize

profitmanCRM=(w-c)Qsales(p(w,r),f(w,r))+(s-r)Qreturns(p(w,r),f(w,r)).

Taking first order conditions, the manufacturer's profit is maximized at

$$w^{CRM} = \frac{v\alpha + cv\beta - cy\delta - z\delta}{2(v\beta - y\delta)}$$
$$r^{CRM} = \frac{v\alpha + y\alpha + sv\beta - z\beta - sy\delta - z\delta}{2(v\beta - y\delta)}.$$

The equilibrium manufacturer profit is

$$profitmanCRM = \frac{1}{2(v^2 + 2v(y - 2\beta - \delta) + (y+\delta)^2}\big(\delta((sy - z)(sv + z) + (-sv + z)\alpha) -$$
$$\beta(sv+z)^2 + sz\delta^2 - c^2(\beta+\delta)(v\beta - y\delta) + \alpha((v+y)(sv+z) - v\alpha) +$$
$$c(v^2(s\beta - \alpha) - z\delta(\beta+\delta) - y^2(\alpha + s\delta) + v(-2y\alpha + \beta(sy + z + 2\alpha + s\delta) + \delta(z + \alpha - sy)) + y(z\beta - \delta(\alpha + s\delta - z)))\big).$$

The second order conditions are satisfied if

$$\frac{\partial^2 profitmanCRM}{\partial^2 w} = \frac{4(\beta+\delta)(v\beta - y\delta)}{v^2 + 2v(y - 2\beta - \delta) + (y+\delta)^2} < 0$$
$$\frac{\partial^2 profitmanCRM}{\partial^2 r} = \frac{4v(v\beta - y\delta)}{v^2 + 2v(y - 2\beta - \delta) + (y+\delta)^2} < 0$$

and the determinant of the Hessian $-\frac{4(v\beta - y\delta)^2}{v^2 + 2v(y - 2\beta - \delta) + (y+\delta)^2} > 0$ by assumption that

$-(v^2 + 2v(y - 2\beta - \delta) + (y+\delta)^2) > 0$.

A.15.3. Comparing CRM to CR (assuming $-(v^2 + 2v(y - 2\beta - \delta) + (y+\delta)^2) > 0, (v\beta - y\delta) > 0$, and $y < \beta$)

PROPOSITION 2: *Consider the situation in which the manufacturer charges the retailer a per-unit wholesale price but not a fixed fee. If* $s > s_r$*, then the manufacturer accepts and salvages returned units from the retailer. There exists a critical salvage value for the manufacturer,* $\overline{s} < s_r$*, such that* $\overline{s} < s < s_r$ *also implies the manufacturer will accept and salvage returned units from the retailer. If* $s \le \overline{s}$*, then the manufacturer chooses to allocate responsibility for salvaging returned units to the retailer.*

Proof: For $s = s_r$, $profitmanCRM\text{-}profitmanCR = \frac{(v(\alpha - s\beta) - z(\beta+\delta) + y(\alpha + s\delta))^2}{8(\beta+\delta)(v\beta - y\delta)}$.

This is greater than zero if and only if $(v\beta - y\delta) > 0$, which we have assumed previously to be true.

$$\frac{\partial \pi_{mfgr}^{CRM}}{\partial s} = \frac{v^2(\alpha + \beta(c-2s)) - \delta(cy - z)(y+\delta) + v(-2z\beta - z\delta - \alpha\delta + c\beta\delta + y(\alpha + c\beta - c\delta + 2s\delta))}{2(v^2 + 2v(y - 2\beta - \delta) + (y+\delta)^2)},$$

(which equals the equilibrium value of $Qreturns \geq 0$) and $\frac{\partial \pi_{mfgr}^{CR}}{\partial s} = 0$. Therefore, when $\pi_{mfgr}^{CRM} \geq 0$ and $\pi_{mfgr}^{CR} \geq 0$, there exists an $\overline{s} < s_r$ such that the functions $\pi_{mfgr}^{CRM}(s) = \pi_{mfgr}^{CR}(s)$ at $s = \overline{s}$. If $\overline{s} < s$ then $\pi_{mfgr}^{CRM} > \pi_{mfgr}^{CR}$ and $s \leq \overline{s}$, then $\pi_{mfgr}^{CRM} \leq \pi_{mfgr}^{CR}$. *Q.E.D.*

Observation 1 can be proven with the additional assumption on the demand and return equations $v(2\beta - y) - y(y+\delta) > 0$.

OBSERVATION 1: *Consider a situation in which the manufacturer charges the retailer a per-unit wholesale price but not a fixed fee. If $s > s_r$, then the return penalty, f, charged to consumers is greater when the retailer salvages returned units than when the manufacturer accepts and salvages returned units from the retailer. There exists a critical salvage value for the manufacturer, $\tilde{s} < s_r$, such that $\tilde{s} < s < s_r$ also implies the return penalty, f, charged to consumers is greater when the retailer salvages returned units than when the manufacturer accepts and salvages returned units from the retailer. If $s \leq \tilde{s}$, then the return penalty, f, charged to consumers is lower when the retailer salvages returned units than when the manufacturer accepts and salvages returned units from the retailer.*

Proof:

If $s = s_r$, then $f^{CRM} - f^{CR} = \dfrac{\beta(v(-\alpha + s\beta) + z(\beta+\delta) - y(\alpha + s\delta))}{4(\beta+\delta)(v\beta - y\delta)}$. If $v(-\alpha + s\beta) + z(\beta+\delta) - y(\alpha + s\delta) < 0$, this difference is negative, implying $f^{CRM} < f^{CR}$. The derivative $\dfrac{\partial f^{CRM}}{\partial s} = \dfrac{v(2\beta - y) - y(y+\delta)}{2(v^2 + 2v(y - 2\beta - \delta) + (y+\delta)^2)} < 0$ (by the fact that the S.O.C. implies $(v^2 + 2v(y - 2\beta - \delta) + (y+\delta)^2)$<0 and under assumption $v(2\beta - y) - y(y+\delta) > 0$. The derivative of the restocking fee in the CR structure with respect to the manufacturer's salvage value is $\dfrac{\partial f^{CR}}{\partial s} = 0$. Therefore, there exists an $\tilde{s} < s_r$ such that the functions $f^{CRM}(s) = f^{CR}(s)$ at $s = \tilde{s}$. If $s > \tilde{s}$ then $f^{CRM} < f^{CR}$ and if $s \le \tilde{s}$, then $f^{CRM} \ge f^{CR}$.

Q.E.D.