ELECTRONIC COMPANION

for

Optimal Restocking Fees and Information Provision in an Integrated Demand-Supply Model of Product Returns

A.1. Modeling conditions leading to $c \geq s$ (footnote 2)

To derive the conditions under which it makes sense to assume that $c \geq s$, we engage here in a logical deduction process, as follows: we first set up the profitability of what we view to be the *most liberal* scenario about the firm's ability to process returns; then, logically, our model's assumption that returns are salvageable for value *s* (e.g., by selling them off to a third-party liquidator) should generate no greater profitability. By comparing the profits of the most liberal scenario to our model's assumption, we then obtain our condition on the relative values of *c* and *s*.

First we set up the profitability of the *most liberal* scenario. Note that by our model's symmetry, the number of consumers returning product A (and wishing to exchange it for product B) is the same as the number of consumers returning product B (and wishing to exchange it for product A). Thus, our "most liberal" scenario, which creates an upper bound on profits, is that each of the returned units of product A is seamlessly transferred to consumers wanting a unit of product A (because they have just returned a unit of product B), and conversely, that each of the returned units of product B is seamlessly transferred to consumers wanting a unit of product B (because they have just returned a unit of product A). This would leave no returned units "left over" that the firm would have to salvage in any other way. Without loss of generality, we can examine one such pair of consumers. The profit from this most liberal scenario is:

$$\pi_{(most\ liberal)} = 2 \cdot \begin{bmatrix} (\textbf{profit from initial sale}) + \\ (\textbf{net refund costs on return}) + \\ (\textbf{immediate resale revenue}) \end{bmatrix}, \text{ or } \begin{aligned} \pi_{(most\ liberal)} &= 2\left[(p-c)+(-p+f)+p\right] \\ &= 2\left[p-c+f\right] \end{aligned}.$$

Meanwhile, under our model's assumptions, consider that same pair of consumers: one who returns his initial purchase of product A, seeking an exchange to product B; and one who returns his initial purchase of product B, seeking an exchange to product A. The profit from this pair of consumers is:

$$\pi_{(our\ model)} = 2 * \begin{bmatrix} (\textbf{profit from initial sale}) + \\ (\textbf{net refund costs on return}) + \\ (\textbf{salvage value}) + \\ (\textbf{profit from replacement product sale}) \end{bmatrix}, \text{ or } \quad \begin{aligned} \pi_{(our\ model)} &= 2\left[(p-c)+(-p+f)+s+(p-c)\right] \\ &= 2\left[p-2c+f+s\right] \end{aligned}.$$

Now, why should the "our model" scenario necessarily be less profitable than the "most liberal" scenario above? In particular, both scenarios involve the same terms for "profit from initial sale" and "net refund costs on return" (for both scenarios, these two terms equal $2\left[(p-c)+(-p+f)\right]=2\left[f-c\right]$). However, above and beyond these common terms, the "most liberal" scenario also generates the "immediate resale revenue" term (equal to $2p$), while the "our model" scenario also generates the "salvage value" and "profit from replacement product sale" terms (equal to $2\left[s+(p-c)\right]$). Our assertion is that seamlessly swapping the returned units (as in the "most liberal" scenario) must be more profitable than taking in a return, selling it off to a liquidator (or as an "open box" unit), and manufacturing and selling an exchange-unit to the consumer who has returned a product. If this were *not* true, then the firm should be in the business of encouraging consumers to return products for the sole purpose of selling them as liquidated (or "open box") units and then sell newly-produced units as exchanges to these consumers, which is an unreasonable assumption. We therefore assume that our "most liberal" scenario is more profitable than the "our model" scenario. When this is true, it means that: $2p > 2\left[s+(p-c)\right]$, or simply that $c > s$.

A.2. Examples of consumer segmentation consistent with Figure 1

The example of the purchase of a tablet PC is provided in the paper's text. Here, we provide two other illustrative examples of markets consistent with the segmentation dimensions described in Figure 1:

- Contact lens purchase: Contact lenses exist in hard and soft varieties; both correct a myopic person's vision, but are horizontally differentiated (soft lenses differ from hard ones in size and flexibility) that make them differentially attractive to different consumers.[1] Some consumers simply may not be able to tolerate lenses of either type (the $1-\alpha$ subset of the population with $u_\theta = 0$), while other consumers value contact lenses (the α subset with $u = u_H$), but are unaware before purchase of their preference for hard versus soft lenses (i.e., do not know the value of $|x_j - \theta|$). Some consumers know whether or not contacts are a viable option for vision correction (falling in the γ subset of the population), while other consumers will only learn after trying a type of lens whether or not they can tolerate lenses in their eyes (members of the $1-\gamma$ subset).

- Baby clothes: Consider some expectant parents who have bought a dress for their baby even though they are uncertain about the gender: If the baby turns out to be a boy, the parents will have zero utility from the product. If it is a girl, then the only uncertainty is whether or not the item is the right size for the newborn. Hence, these expectant parents are part of the $(1-\gamma)$ portion of the consumer marketplace, where α= ½ assuming they are equally likely to have a girl or a boy (and that they do not know the baby's impending birthweight and hence clothing size, so that $|x_j - \theta|$ is unknown). Meanwhile, parents who have discovered through ultrasound that their baby is a girl still do not know the baby's birthweight (hence, $|x_j - \theta|$ is still unknown), but do know they have positive value for a dress, and are therefore in Segment 1 (part of the $\alpha\gamma$ proportion of the market).

A.3. Derivation of the condition under which consumers' product return and exchange behavior can be described by Tables 2.a and 2.b

In this Electronic Companion document, for given values of firm decisions (*p* and *f*), we identify when a consumer in segment 1 or segment 3 with a positive consumption utility ($u_\theta = u_H$) who returns his/her initial purchase will prefer to exchange it, rather than return the good and leave the market. More specifically, below we show that this returns and exchange behavior is feasible if and only if

[1] One appropriately considers the adoption of the lenses over time, rather than the purchase of a single pair of either type of lenses (because hard lenses are bought once and are a durable good, while soft lenses are disposable). Ultimately, sellers' optimal pricing decisions reflect this difference in total quantities purchased so that the two technologies are realistically substitutable.

$4(u_H - p) + 2(f + h) - d \geq 0$. The derivations in this Electronic Companion are later used in A.4. to calculate the expected ex-ante consumer utility and the utility maximizing return and exchange quantities of different consumer segments.

Consistent with current practice, we allow consumers to engage in one of three possible consumption behaviors. First, the consumer could **buy a product and keep** it (i.e., not return it). Such a consumer located at θ, who purchases and keeps a product priced at p and located at x_j, receives a utility of

$$U_\theta = u_H - p - d\,|\,x_j - \theta\,| \;.$$

A second possibility is that the consumer purchases a product, then **returns it and exchanges it** for a more preferred product after initial purchase. This consumer, located at θ, who purchases a product priced at p and located at x_j, receives total utility of

$$U_\theta = u_H - p - f - h - \underset{j \in \{A,B\}}{Min}\{d\,|\,x_j - \theta\,|\}\,.$$

Finally, a consumer may purchase a product, then **return it and not exchange it (i.e., leave the market).** The consumer located at θ who purchases a product priced at p and located at x_j receives total utility of

$$U_\theta = -f - h\,.$$

In what follows, we will model the returns behavior of a consumer who initially bought product A. Note that since a consumer is equally likely to buy either product A or product B initially, the derivation below does not depend on consideration of product A.

Let θ_1 denote the consumer between 0 and $\frac{1}{2}$ who is indifferent between keeping product A after purchase and exchanging it for product B. Buying and keeping product A yields a utility equal to $u_H - p - d\,|\,\theta_1 - 0\,|$. Returning the product and buying product B yields utility equal to $u_H - p - f - h - d\,|\,\frac{1}{2} - \theta_1\,|$. Therefore, one can easily show that the marginal consumer is located at

$\theta_1 = \dfrac{2(f+h)+d}{4d}$ such that $u_H - p - d\theta_1 = u_H - p - f - h - d(\frac{1}{2} - \theta_1)$. All consumers with $\theta \in [0, \theta_1]$ who initially buy product A earn greater utility from keeping it than from exchanging it.

Let θ_2 denote the consumer between 0 and $\frac{1}{2}$ who is indifferent between keeping product A and returning it to opt out of the market (i.e., not replacing product A with product B). For this consumer, the utility of buying and keeping the product equals the utility of buying and returning product A for a refund, $u_H - d|\theta_2 - 0| - p = -f - h$. Therefore, $\theta_2 = \dfrac{u_H - p + f + h}{d}$. All consumers with $\theta \in [0, \theta_2]$ have greater value from keeping the product than from returning it.

Let θ_3 denote the consumer between 0 and $\frac{1}{2}$ who has returned product A and is indifferent between purchasing product B and opting out of the market.. For this consumer, $u_H - p - f - h - d|\frac{1}{2} - \theta_3| = -f - h$. Therefore, $\theta_3 = \dfrac{d - 2(u_H - p)}{2d}$. All consumers with $\theta \in [\theta_3, \frac{1}{2}]$ who initially purchase product A enjoy greater utility from exchanging product A for product B than from opting out of the market.

By comparing the values of $\theta_1, \theta_2,$ and θ_3, we can order them $\theta_2 \geq \theta_1 \geq \theta_3$ if $4(u_H - p) + 2(f+h) - d \geq 0$. Otherwise, the inequalities are reversed and $\theta_2 < \theta_1 < \theta_3$. The definitions of $\theta_1, \theta_2,$ and θ_3 imply several choice rules. For consumers with:

- $1/2 > \theta > \theta_1$, returning the initial purchase and buying the other product (RB) $\succ$ keeping purchase (K),
- $1/2 > \theta > \theta_2$, returning the initial purchase and owning nothing (R) $\succ$ keeping purchase (K),
- $1/2 > \theta > \theta_3$, returning the initial purchase and buying the other product (RB) $\succ$ returning and owning nothing (R).

When $4(u_H - p) + 2(f + h) - d < 0$ (which implies $\theta_2 < \theta_1 < \theta_3$), we have:

- for $0 < \theta < \theta_2$, **K** $\succ$ R$\succ$RB ;
- for $\theta_2 < \theta < \theta_1$, **R** $\succ$ K$\succ$RB;
- for $\theta_1 < \theta < \theta_3$, **R** $\succ$ RB$\succ$K;
- for $1/2 > \theta > \theta_3$, **RB** $\succ$ R$\succ$K .

Thus, if $4(u_H - p) + 2(f + h) - d < 0$, keeping (**K**) is the dominant choice for $0 < \theta < \theta_2$; returning and owning nothing (**R**) is the dominant choice for $\theta_2 < \theta < \theta_3$; and returning the initial purchase to buy the other product (**RB**) is the dominant choice for $\theta > \theta_3$. Therefore, if $4(u_H - p) + 2(f + h) - d < 0$, there are consumers in segment 1 and 3 whose θ value lies in (θ_2, θ_3) will return the initially purchased product for a refund and opt out of the market. By symmetry, the same analysis follows for consumers for whom $1/2 < \theta < 1$.

When $4(u_H - p) + 2(f + h) - d \geq 0$ (which implies $\theta_2 > \theta_1 > \theta_3$), we have:

- for $0 < \theta < \theta_3$, **K** $\succ$ R$\succ$RB;
- for $\theta_3 < \theta < \theta_1$, **K** $\succ$ RB$\succ$R;
- for $\theta_1 < \theta < \theta_2$, **RB** $\succ$ K$\succ$R;
- for $1/2 > \theta > \theta_2$, **RB** $\succ$ R$\succ$K.

Thus, if $4(u_H - p) + 2(f + h) - d \geq 0$ keeping (**K**) is the dominant choice for $0 < \theta < \theta_1$, and returning the initial purchase to buy the other product (**RB**) is the dominant choice for $\theta_1 < \theta < \frac{1}{2}$. There does <u>not</u> exist a group of consumers who would prefer to leave the market without owning a product rather than keeping or exchanging the initial purchase. In other words, consumers who value the product will keep or exchange even with the available option to return a product for a refund with no exchange. By symmetry, the same analysis follows for consumers for whom $1/2 < \theta < 1$.

The next section of the Electronic Companion uses the conditions derived above to calculate the expected ex-ante purchasing utility of consumers as well as the expected number of returns and exchanges for segments 1, 3 and 4.

A.4. Derivation of Table 2.a and Table 2.b

In this section, we derive expected *ex ante* consumer utilities and the expected number of returns and exchanges for Segment-1, -3, and -4 consumers.

i) First we consider Segment- 1 consumers (consumers who are informed about their consumption utility). Below we derive $q_{eY}(f;d,\gamma,\alpha)=\gamma\,\alpha(\frac{1}{2}-\frac{f+h}{d})$, $q_{kY}(f;d,\gamma,\alpha)=\gamma\,\alpha(\frac{1}{2}+\frac{f+h}{d})$ and

$$E_Y(U)=u_H-p+\frac{(f+h)^2}{2d}-\frac{(f+h)}{2}-\frac{d}{8}.$$

<u>Proof:</u> Note that Segment-1 consumers constitute a fraction $\gamma\,\alpha$ of the market, and Segment-1 consumers' taste parameters are uniformly distributed on the unit circle. The consumer's lack of information implies that the *a priori* expected utility of buying product A is the same as the a *priori* expected utility of buying product B, and thus the consumer initially randomly chooses which product to buy once s/he decides to own a product. Therefore, we first look at the ½ of these consumers (that is, a proportion $\frac{\gamma\,\alpha}{2}$ of the total population of consumers) who bought product A located at 0. Furthermore, for Segment-1 consumers, we assume that $4(u_H-p)+2(f+h)-d\geq 0$**;** therefore, ex-post, each Segment-1 consumer will either keep his initial purchase or return it for an exchange. From A.3., we know that consumers with $\theta\leq\theta_1\equiv\frac{2(f+h)+d}{4d}$ will keep their initial purchase and consumers with

$\theta_1 < \theta \le \frac{1}{2}$ will return product A in exchange for product B. Following the same analysis, consumers with $1-\theta_1 < \theta \le 1$ will keep product A, and consumers with $\frac{1}{2} \le \theta < 1-\theta_1$ will return product A in exchange for product B. Therefore, with probability $\frac{2(f+h)+d}{2d} = \frac{1}{2} + \frac{f+h}{d}$, a consumer in Segment 1 who purchases product A will keep it. With probability $(\frac{1}{2} - \frac{f+h}{d})$, a consumer in Segment 1 who purchases product A will exchange it for product B.

We follow the same analysis for the $\frac{\gamma\alpha}{2}$ proportion of the total population that is in Segment 1 and buys product B initially. With probability $\frac{2(f+h)+d}{2d} = \frac{1}{2} + \frac{f+h}{d}$, a consumer in Segment 1 who purchases product B will keep it. With probability $(\frac{1}{2} - \frac{f+h}{d})$, a consumer in Segment 1 who purchases product B will exchange it for product A.

We may then write the expected proportion of the total population that is in Segment 1 and keeps the initially purchased product as $q_{kY}(f; d, \gamma, \alpha) = \gamma\alpha(\frac{1}{2} + \frac{f+h}{d})$. The expected proportion of the total population that is in Segment 1 and exchanges the initially purchased product is given by $q_{eY}(f; d, \gamma, \alpha) = \gamma\alpha(\frac{1}{2} - \frac{f+h}{d})$.

We now calculate the expected utility of making an initial purchase for consumers in Segment 1. From above, we know that if a consumer in Segment 1 makes an initial purchase, the probability that it will be kept is given by $(\frac{1}{2} + \frac{f+h}{d})$. If the initial purchase is kept, the utility is $u_H - p - d\,|x_j - \theta|$. The value of $|x_j - \theta|$ is between 0 and θ_1. Because θ is uniformly distributed, the average utility of keeping the product is $u_H - p - \frac{d\theta_1}{2}$. If a consumer in Segment 1 makes an initial purchase, the probability of exchanging it is equal to $(\frac{1}{2} - \frac{f+h}{d})$. If the initial purchase is exchanged, utility is equal to $u_H - p - f - h - d\,|\,x_j - \theta\,|$. For consumers who exchange the product, $|\,x_j - \theta\,|$ is between 0 and $(\frac{1}{2} - \theta_1)$. The average utility of exchanging the product is $u_H - p - f - h - \frac{d}{2}(\frac{1}{2} - \theta_1)$. Therefore, the expected ex-ante utility of a Segment-1 consumer is given by: $(u_H - p - \frac{d\theta_1}{2})(\frac{1}{2} + \frac{f+h}{d}) + (u_H - p - f - h - \frac{d}{2}(\frac{1}{2} - \theta_1))(\frac{1}{2} - \frac{f+h}{d})$. Now, recall that $\theta_1 = \frac{2f+d}{4d}$. Then, the expected ex-ante utility of a Segment-1 consumer is $u_H - p - \frac{f+h}{2} + \frac{(f+h)^2}{2d} - \frac{d}{8}$. *QED*

ii) Secondly, we derive the expected ex-ante consumer utilities and the expected number of returns and exchanges for consumers who are uninformed about their consumption utility. These consumers constitute Segments 3 and 4. Specifically, below we derive:

$$q_{eZ}(f;d,\gamma,\alpha) = (1-\gamma)\alpha(\frac{1}{2} - \frac{f+h}{d}), \quad q_{kZ}(f;d,\gamma,\alpha) = (1-\gamma)\alpha(\frac{1}{2} + \frac{f+h}{d}),$$

$$q_{rZ}(f;d,\gamma,\alpha) = (1-\alpha)(1-\gamma) \text{ and } E_Z(U) = \alpha(u_H - p + \frac{(f+h)^2}{2d} - \frac{(f+h)}{2} - \frac{d}{8}) - (1-\alpha)(f+h).$$

Proof: From Figure 1, we know that a proportion $(1-\gamma)$ of the total population does not know whether or not they have positive consumption for the product. With probability $(1-\alpha)$, these consumers do not value the product, $u_\theta = 0$ and thus, they will be in Segment 4. If $0 - p < -f - h$, i.e., if the utility from keeping a purchase is less than the utility from returning it, then Segment-4 consumers will return the initial purchase without making a subsequent purchase. Consequently, $q_{rZ}(f;d,\gamma,\alpha) = (1-\alpha)(1-\gamma)$.

With probability α, an uninformed consumer has value for the product (a proportion $\alpha(1-\gamma)$ of the total consumer population falls into this group). Note that these consumers, who are shown to be in

Segment 3 *ex post*, have the same utilities from each action as the consumers in Segment 1. Therefore, we follow the same analysis as shown in the proof of (i). If a consumer has value for the product and initially makes a purchase, there is a probability of $\frac{2(f+h)+d}{2d}$ that the consumer will keep it and with a probability of $(\frac{1}{2}-\frac{f+h}{d})$, the consumer will exchange it. Therefore, the expected proportion of the total population who falls into Segment 3 and keeps the initial purchase is equal to $q_{kZ}(f;d,\gamma,\alpha)=(1-\gamma)\alpha(\frac{1}{2}+\frac{f+h}{d})$ and the expected proportion of the total population falling into Segment 3 and exchanging the initial purchase is equal to $q_{eZ}(f;d,\gamma,\alpha)=(1-\gamma)\alpha(\frac{1}{2}-\frac{f+h}{d})$.

If the consumer is one of the α consumers who value the product, then the consumer's expected utility of making an initial purchase is $u_H-p-\frac{f+h}{2}+\frac{(f+h)^2}{2d}-\frac{d}{8}$, as shown in the proof of (i). However, at the time of purchase, an uninformed consumer does not know whether s/he will have consumption utility for the product. With probability α, this consumer will have an expected utility from purchase equaling to $u_H-p-\frac{f+h}{2}+\frac{(f+h)^2}{2d}-\frac{d}{8}$ and with probability $(1-\alpha)$ the expect utility from purchase will equal $-f-h$. Therefore, at the time of purchase, the expected utility of making an initial purchase for a consumer who is uninformed about product's consumption value is given by:

$$E_Z(U)=\alpha(u_H-p+\frac{(f+h)^2}{2d}-\frac{(f+h)}{2}-\frac{d}{8})-(1-\alpha)(f+h).\ Q.E.D.$$

A.5. Proof of Proposition 1a [Separating Equilibrium Optimal Seller Strategies]

In this section, for the separating equilibrium, we derive the optimal price and restocking fee decisions of the seller. More specifically, below we present the proof of Proposition 1a.

Proposition 1a: *When the seller chooses to set a price and restocking fee that separates the market and sells only to consumers who know the products will be valued, the equilibrium price, restocking fee, seller profit, and expected number of exchanges will be given by Table 3:*

Table 3: Equilibrium Values in the Separating Case

Parameter Values	**Retail Price,** p^* **and Restocking Fee,** f^*	**Profit,** π^*	**Exchanges** q_{eY}^*
Case 1, *sep*: $d < d_{1,sep}$	$p^* = u_H - \frac{d}{4}$ $f^* \geq \frac{d-2h}{2}$	$\pi^* = \alpha\gamma(u_H - c - \frac{d}{4})$	0
Case 2, *sep*: $d_{1,sep} \leq d \leq d_{2,sep}$	$p^* = u_H - \frac{d}{8} - \frac{c-s+h}{2} + \frac{(c-s+h)^2}{2d}$ $f^* = c - s$	$\pi^* = \alpha\gamma(u_H - c - \frac{d}{8} - \frac{c-s+h}{2} + \frac{(c-s+h)^2}{2d})$	$\alpha\gamma(\frac{1}{2} - \frac{c-s+h}{d})$
Case 3, *sep*: $d \geq d_{2,sep}$	$p^* = u_H - \frac{d(\sqrt{3}-1)}{4}$ $f^* = d\left(1 - \frac{\sqrt{3}}{2}\right) - h$	$\pi^* = \alpha\gamma(u_H - c - \frac{d(2-\sqrt{3})}{2} - \frac{(c+h-s)(\sqrt{3}-1)}{2})$	$\frac{\alpha\gamma(-1+\sqrt{3})}{2}$

Where $d_{1,sep} \equiv 2(c-s+h)$ *and* $d_{2,sep} \equiv (4+2\sqrt{3})(c-s+h)$.

<u>Proof:</u>

For each consumer who keeps their initial purchase, the firm produces a unit at a marginal cost *c* and sells it at the retail price *p*. For each consumer who exchanges their initial purchase, the firm earns the unit margin *(p-c)* on the initial sale, pays out a refund of *(p-f)* for the return, salvages the good at a value *s*, and then sells the consumer's preferred product for a unit margin of *(p-c)*. As discussed in A.3.

and A.4., Segment-1 consumers either keep their initial purchase or exchange it for the other product. More specifically, $4(u_H - p) + 2(f + h) - d \geq 0$ holds.

Under the separating equilibrium, the firm maximizes profits such that consumers in Segment 1 make an initial purchase ($E_Y(U) \geq 0$), the market quantities are non-negative, and the all consumers in Segment 1 eventually keep one of the products. Thus, the seller chooses price and restocking fee to maximize profits $\pi_{sep} = (p-c)q_{kY}(f;d,\gamma,\alpha) + (2(p-c) + f - p + s)q_{eY}(f;d,\gamma,\alpha)$ subject to the constraints $E_Y(U) \geq 0$, $q_{kY}(f;d,\gamma,\alpha) \geq 0$, $q_{eY}(f;d,\gamma,\alpha) \geq 0$ and $4(u_H - p) + 2(f + h) - d \geq 0$. Because profit is strictly increasing in p ($\frac{\partial \pi_{sep}}{\partial p} = \alpha\gamma > 0$), the seller charges the highest price possible. That is, the seller will raise the price until the constraint on price binds. There are two possibilities for which the constraint will be binding at the optimal seller strategy: either $E_Y(U) = 0$ or $4(u_H - p) + 2(f + h) - d = 0$. We first fix price such that $E_Y(U) = 0$ and solve for the optimal restocking fee. Then, we fix price such that $4(u_H - p) + 2(f + h) - d = 0$, solve for the optimal restocking fee. In this case, we show that any additional potential solutions are not feasible.

When price is set such that $E_Y(U) = 0$, the seller solves the following constrained optimization problem with Lagrangian muliplIers λ_2, λ_3:

$$\begin{aligned}
\max_f L(f,\lambda_2,\lambda_3;c,s,u,d,\gamma,\alpha) = {} & (p-c)q_{kY}(f;d,\gamma,\alpha) + (2(p-c) + f - p + s)q_{eY}(f;d,\gamma,\alpha) + \\
& \lambda_2(d - 2f - 2h) + \lambda_3(4(u_H - p) + 2(f + h) - d) \\
& \textit{subject to} \\
& E_Y(U) = 0 \\
& \lambda_2 q_{eY} = 0 \\
& \lambda_3(4(u_H - p) + 2(f + h) - d) = 0 \\
& \lambda_2 \geq 0, \lambda_3 \geq 0.
\end{aligned}$$

The Kuhn-Tucker conditions will be both necessary and sufficient if the objective function is concave and the constraints are quasi-concave (Arrow and Enthoven 1961).

Lemma 1: In the separating equilibrium, the objective function of the seller π_{sep} is concave.

Proof of Lemma 1: When p solves $E_Y(U)=0$, the second derivative with respect to f is $\frac{\partial^2 \pi_{sep}}{\partial f^2} = -\frac{\alpha\gamma}{d} < 0$. Therefore, the seller's objective function under a separating equilibrium is concave in *f*.□

Lemma 2: The constraints $\lambda_2 \, q_{eY} = 0$ and $\lambda_3(4(u_H - p) + 2(f+h) - d) = 0$ are quasi-concave.

Proof of Lemma 2: The constraint is linear in *f*, and therefore is quasi-concave. When *p* is such that $E_Y(U)=0$, the second derivative with respect to f of $(4(u_H - p) + 2(f+h) - d)$ is $-\frac{4\lambda_3}{d} < 0$. Therefore both constraints are quasi-concave. Thus, the Kuhn-Tucker conditions are both necessary and sufficient for a maximum.

Thus, we use the Kuhn-Tucker conditions to characterize the optimal solution to the seller's profit maximization problem:

$$\frac{\partial L(f,\lambda_2,\lambda_3;c,s,u,d,\gamma,\alpha)}{\partial f} = \frac{\alpha\gamma(c-f-s-\lambda_2) - 4(f+h-d)\lambda_3}{d} = 0$$

$$\lambda_2 \, q_{eY} = 0$$

$$\lambda_3\left(\frac{8d(f+h) - 4(f+h)^2 - d^2}{2d}\right) = 0$$

$$\lambda_2 \geq 0;\ \lambda_3 \geq 0 .$$

We analyze four solutions that satisfy the Kuhn-Tucker conditions:

(A1) $p = u_H - \frac{d}{4},\ f = \frac{d}{2} - h,\ \lambda_1 = 1,\ \lambda_2 = c - s + h - \frac{d}{2},\ \lambda_3 = 0 .$

At this solution, $q_{eY} = 0$. λ_2 in this case is positive if and only if $d < 2(c - s + h)$.

(A2) $p = u_H - \frac{(c-s+h)}{2} + \frac{(c-s+h)^2}{2d} - \frac{d}{8},\ f = c - s,\ \lambda_2 = 0,\ \lambda_3 = 0$.

At this solution, $q_{eY} = (\frac{1}{2} - \frac{c+h-s}{d})\alpha\gamma \geq 0$ if and only if $d \geq 2(c-s+h)$. The full market condition, $4(u_H - p) + 2(f+h) - d \geq 0$ is satisfied for $d \leq (4+2\sqrt{3})(c-s+h)$. Therefore, the conditions are satisfied for $d \in [2(c-s+h),(4+2\sqrt{3})(c-s+h)]$.

(A3) $p = u_H - \frac{d(\sqrt{3}-1)}{4},\ f = d(1-\frac{\sqrt{3}}{2}) - h,$

$\lambda_2 = 0,\ \lambda_3 = \frac{\alpha\gamma((2\sqrt{3}-3)d - 2(c-s+h)\sqrt{3})}{12d}$.

At this solution, $q_{eY} = \frac{\alpha\gamma}{2}(\sqrt{3}-1) \geq 0$. The condition $\lambda_3 \geq 0$ is satisfied if and only if

$d \geq (4+2\sqrt{3})(c-s+h)$.

(A4) $p = \frac{1}{4}(1+\sqrt{3})d + u_H,\ f = \frac{1}{2}(2+\sqrt{3})d - h,\ \lambda_2 = 0,\ \lambda_3 = \frac{2\sqrt{3}(c-s+h)-(3+2\sqrt{3})d}{12d}$.

This solution has $p = \frac{1}{4}(1+\sqrt{3})d + u > u$, which implies that the price is set at a level greater than the consumer's valuation for the product. Thus, this solution is not feasible.□

Now we solve for any potential solutions when $4(u_H - p) + 2(f+h) - d = 0$. We will show that **(A3)** is again a solution to this problem, but no other potential solution satisfies the constraints and thus solutions A1, A2, and A3 are uniquely profit maximizing.

The seller's optimization problem with Lagrangian multipliers becomes:

$$\max_f L(f,\lambda_1,\lambda_2;c,s,u,d) = (p-c)q_{kY}(f;d,\gamma,\alpha) + (2(p-c)+f-p+s)q_{eY}(f;d,\gamma,\alpha) +$$
$$\lambda_1 E_Y(U) + \lambda_2(d-2f-2h)$$
$$\textit{subject to}:$$
$$4(u_H - p) + 2(f+h) - d = 0$$

$$\lambda_1 E_Y(U) = 0$$

$$\lambda_2\, q_{eY} = 0$$

$$\lambda_2 \geq 0, \lambda_3 \geq 0.$$

The Kuhn-Tucker conditions are

$$\frac{\partial L(f,\lambda_1,\lambda_2;c,s,u,d,\gamma,\alpha)}{\partial f} = \frac{\alpha\gamma(c+d-2f-h-s-\lambda_2)+\lambda_1(f+h-d)}{d} = 0$$

$$\lambda_1\left(\frac{d^2+4(f+h)^2-8d(f+h)}{8d}\right) = 0$$

$$\lambda_2\, q_{eY} = 0$$

$$\lambda_1 \geq 0;\ \lambda_2 \geq 0.$$

We analyze the four solutions that potentially satisfy the Kuhn-Tucker conditions: first, the price and restocking fee combination from (A3), which was shown above to be optimal for $d \geq (4+2\sqrt{3})(c-s+h)$; second, the price and restocking combination from (A4) which was shown above to be infeasible; third,

(A5) $p = \frac{c-s+h}{4}$, $f = \frac{c-s+d-h}{2}$ which yields $q_{eY} = \frac{\alpha * \gamma(s-c-h)}{2d} < 0$ (because $c>s$),

which is an infeasible solution because we rule out negative quantities; and fourth,

(A6) $p = u_H$, $f = \frac{d}{2} - h$, which yields expected utility=$-\frac{d}{4} < 0$. Consumers will not make the initial purchase with negative expected utility, making this also an infeasible solution.

Consequently, from the analysis above, we see that in a separating equilibrium, for $d \le 2(c-s)$ **((A1) above)**, the optimal solutions are $f = \frac{d}{2} - h$ and $p = u_H - \frac{d}{4}$. In this region, for any $f \ge \frac{d}{2} - h$ all consumers keep their initial purchase and expected utility is unchanged. Thus, any $f \ge \frac{d-2h}{2}$ will generate expected profit of $\pi^* = \alpha\gamma(u_H - c - \frac{d}{4})$ with zero exchanges and will be an equilibrium when $d \le 2(c-s+h)$ as presented in **case 1,*sep* of Proposition 1a**. In a separating equilibrium, for $2(c-s+h) \le d \le (4+2\sqrt{3})(c-s+h)$ **((A2) above)**, the optimal solution is $f = c-s$ and $p = u_H - \frac{(c-s+h)}{2} + \frac{(c-s+h)^2}{d} - \frac{d}{8}$, generating an expected profit of $\pi^* = \alpha\gamma(u_H - c - \frac{d}{8} - \frac{c-s+h}{2} + \frac{(c-s+h)^2}{2d})$ with expected total exchanges of $q^*_{eY} = \alpha\gamma(\frac{1}{2} - \frac{c-s+h}{d})$ as presented in **case 2,*sep* of Proposition 1a.** For $d \ge (4+2\sqrt{3})(c-s+h)$ **((A3) above)**, the optimal solutions are $f = d(1-\frac{\sqrt{3}}{2}) - h$ and $p = u_H - \frac{d(\sqrt{3}-1)}{4}$, generating an expected profit of $\pi^* = \alpha\gamma(u_H - c - \frac{d(2-\sqrt{3})}{2} - \frac{(c+h-s)(\sqrt{3}-1)}{2})$ with expected total exchanges of $q^*_{eY} \frac{\alpha\gamma(-1+\sqrt{3})}{2}$ as presented in **case 3,*sep* of Proposition 1a.** *Q.E.D.*

A.6. Proof of Proposition 1b [Pooling Equilibrium Optimal Seller Strategies]

In this section, for the pooling equilibrium, we derive the optimal price and restocking fee decisions of the seller. More specifically, below we present the proof of Proposition 1b.

Note that in the pooling equilibrium (contrary to the separating equilibrium), in addition to consumers who keep or exchange their initial purchase, we have also consumers who opt out of the market after the product's consumption utility is revealed and the product is returned to the seller. These consumers are in Segment 4 and constitute a fraction $(1-\alpha)(1-\gamma)$ of the market.

Proposition 1b: *When the seller chooses to set a price and restocking fee that pools the market and sells to all consumers have the possibility of valuing the products, the equilibrium price, restocking fee, seller profit, and expected number of exchanges will be given by Table 4:*

Table 4: Equilibrium Values in the Pooling Case

Parameter Values	Retail Price, p^* and Restocking Fee, f^*	Profit, π^*	Exchanges, $(q_{eY}^* + q_{eZ}^*)$
Case 1, *pool*: $d \le d_{1,pool}$	$p^* = u_H - \frac{d(2-\alpha)}{4\alpha}$ $f^* = \frac{d-2h}{2}$	$\alpha u_H - \frac{d}{4}(\alpha + 2\gamma(1-\alpha)) + (s-h)(1-\alpha)(1-\gamma) - c(1-\gamma(1-\alpha))$	0
Case 2,*pool*: $d_{1,pool} \ge d \ge d_{2,pool}$	$p^* = u_H - \frac{d}{8} - \frac{c-s+h-\frac{d\gamma(1-\alpha)}{\alpha}}{2} + \frac{(c-s+h-\frac{d\gamma(1-\alpha)}{\alpha})^2}{2d}) - \frac{(1-\alpha)}{\alpha}(c-s+h-\frac{d\gamma(1-\alpha)}{\alpha})$ $f^* = c-s-\frac{d\gamma(1-\alpha)}{\alpha}$	$\alpha u_H + \frac{d}{8\alpha}(4\gamma^2(1-\alpha)^2 - \alpha^2) + \frac{\alpha(s-h-c)^2}{2d} + (s-h)(1-\frac{\alpha}{2}) - c(1+\frac{\alpha}{2})$	$\frac{\alpha}{2} - \frac{\alpha(c-s+h)}{d} + \gamma(1-\alpha)$
Case 3, *pool*: $d \ge d_{2,pool}$	$p^* = u_H - \frac{d(\sqrt{4-\alpha^2}-2+\alpha)}{4\alpha}$ $f^* = \frac{d(2-\sqrt{(4-\alpha^2)})}{2\alpha} - h$	$\alpha u_H - \frac{d(1+\gamma(1-\alpha))}{\alpha}(1-\frac{\sqrt{4-\alpha^2}}{2}) - (c+h-s)(\frac{\sqrt{4-\alpha^2}}{2} - \gamma(1-\alpha)) - \frac{\alpha(c-h+s)}{2}$	$\frac{(\sqrt{4-\alpha^2}-(2-\alpha))}{2}$

Where $d_{1,pool} \equiv \frac{2\alpha(c-s+h)}{\alpha+2\gamma(1-\alpha)}$ *and* $d_{2,pool} \equiv \frac{2\alpha(c+h-s)}{2(1+\gamma-\alpha\gamma)-\sqrt{4-\alpha^2}}$.

Proof:

The seller earns the unit margin *(p-c)* on all units initially sold; a fraction $(1-\gamma(1-\alpha))$ of the market initially buys (which includes everyone in Segments 1, 3, or 4). For each unit exchanged, the seller pays out a refund of *(p-f)* for the return, salvages the good at a value *s,* and then sells the consumer's preferred product for a unit margin of *(p-c).* For each consumer who returns their purchase and opts out of the market, the firm pays out a refund of (*p-f*) for the return and salvages the good at a value s. All consumers who have value for the products will keep their initial purchase or exchange it if $4(u_H - p) + 2(f + h) - d \geq 0$ (see A.1. for derivation of this condition).

The firm maximizes profits such that consumers in Segments, 1, 3 and 4 make an initial purchase ($E_Y(U) \geq 0, E_Z(U) \geq 0$). Market quantities are non-negative and all consumers in Segments 1 and 3 eventually keep one of the products. As in the proof of Proposition 1a, one can show that the profit-maximizing p is set such that $E_Z(U) = 0$. We will then show that when p is set such that $4(u_H - p) + 2(f + h) - d = 0$, the additional potential solutions are infeasible. Also note that $E_Y(U) \geq E_Z(U)$; and when $E_Z(U) = 0$ (which implies that $E_Y(U) \geq 0$), we have the following constrained optimization problem with Lagrangian mulipliers λ_2, λ_3:

$$
\begin{aligned}
\max_f \; & L(f, \lambda_2, \lambda_3; c, s, u, d, \gamma, \alpha) = (p - c)(1 - \gamma(1 - \alpha)) + (f + s - p)q_{rY}(f; d, \gamma, \alpha) \\
& + (f + s - p + p - c)(q_{eZ}(f; d, \gamma, \alpha) + q_{eY}(f; d, \gamma, \alpha)) \\
& + \lambda_2(2f + 2h - d) + \lambda_3(4(u_H - p) + 2(f + h) - d)) \\
& \textit{subject to} \\
& \qquad E_Z(U) = 0 \\
& \qquad \lambda_2(2f + 2h - d) = 0 \\
& \qquad \lambda_3(4(u_H - p) + 2(f + h) - d) = 0
\end{aligned}
$$

The Kuhn-Tucker conditions will be both necessary and sufficient if the objective function is concave and the constraints are quasi-concave (Arrow and Enthoven 1961).

Lemma 3: The seller's objective function under a pooling equilibrium is concave.

Proof of Lemma 3: When p solves $E_Z(U) = 0$, the second derivative with respect to f is $\frac{\partial^2 \pi_{pool}}{\partial f^2} = -\frac{\alpha}{d} < 0$. Therefore, the objective function is concave in *f*.□

Lemma 4: The constraints $\lambda_2(2f+2h-d)=0$ and $\lambda_3(4(u_H-p)+2(f+h)-d)=0$ are quasi-concave.

Proof of Lemma 4: The constraint $\lambda_2(2f+2h-d)=0$ is linear in *f*, and therefore is quasi-concave. When *p* is such that $E_Z(U)=0$, the second derivative with respect to f of $4(u_H-p)+2(f+h)-d$ is $-\frac{4\lambda_3}{d}<0$. Therefore both constraints are quasi-concave. Thus, the Kuhn-Tucker conditions are both necessary and sufficient for a maximum.

Next, we list the necessary and sufficient the Kuhn-Tucker conditions:

$$\frac{\partial L(f,\lambda_2,\lambda_3;c,s,u,d,\gamma,\alpha)}{\partial f}=\frac{4\lambda_3(d-\alpha(f+h))-\alpha d\gamma(1-\alpha)+\alpha^2(f-c+s+\lambda_2(1-\gamma))}{d\alpha}=0$$

$$\lambda_2\, q_{eY}=0$$

$$\lambda_3(\frac{8d(f+h)-\alpha d^2-4\alpha(f+h)^2}{2d\alpha})=0$$

$$\lambda_2\geq 0;\ \lambda_3\geq 0\,.$$

We analyze the four solutions that satisfy the Kuhn-Tucker conditions:

(B1) $p=u_H-\frac{d(2-\alpha)}{4\alpha}$, $f=\frac{d}{2}-h$, $\lambda_2=\frac{2\alpha(c+h-s)-d(\alpha+\gamma-\alpha\gamma)}{2\alpha(1-\gamma)}$, $\lambda_3=0$. This implies that $q_{eZ}=0$. The condition that $\lambda_2\geq 0$ is satisfied if and only if $d\leq\frac{2\alpha(c-s+h)}{\alpha+2\gamma(1-\alpha)}$.

(B2) $p=\alpha(u_H-\frac{d}{8}-\frac{c-s-\frac{d\gamma(1-\alpha)}{\alpha}+h}{2}+\frac{(c-s-\frac{d\gamma(1-\alpha)}{\alpha}+h)^2}{2d})-(1-\alpha)(c-s-\frac{d\gamma(1-\alpha)}{\alpha}-h)$

$f=c-s-\frac{d\gamma(1-\alpha)}{\alpha}$, $\lambda_2=0,\lambda_3=0$. This implies $q_{eZ}=\frac{\alpha(1-\gamma)(d-2(c+h-s-\frac{d\gamma(1-\alpha)}{\alpha}))}{2d}\geq 0$ if and only if $d\geq\frac{2\alpha(c-s+h)}{\alpha+2\gamma(1-\alpha)}$. This solution satisfies the constraint $4(u_H-p)+2(f+h)-d\geq 0$, if and only if $d \le \frac{2\alpha(c-s+h)}{2(1+\gamma-\alpha\gamma)-\sqrt{4-\alpha^2}}$. Therefore the solution is feasible if and only if $d \in [\frac{2\alpha(c-s+h)}{\alpha+2\gamma(1-\alpha)}, \frac{2\alpha(c+h-s)}{2(1+\gamma-\alpha\gamma)-\sqrt{4-\alpha^2}}]$.

(B3) $p = u_H - \frac{d(\sqrt{4-\alpha^2}-2+\alpha)}{4\alpha}$, $f = \frac{d(2-\sqrt{(4-\alpha^2)})}{2\alpha} - h$, $\lambda_2 = 0$, $\lambda_3 = \frac{\alpha}{4}(-1+\frac{2d(1+\gamma-\gamma\alpha)-2\alpha(c+h-s)}{d\sqrt{4-\alpha^2}})$. This implies $q_{eZ} = \frac{(1-\gamma)(\sqrt{4-\alpha^2}-(2-\alpha))}{2} \ge 0$. (To check that $q_{eZ} \ge 0$, we examine the numerator. $(1-\gamma)(\sqrt{4-\alpha^2}-(2-\alpha)) \ge 0$ if $\sqrt{4-\alpha^2} \ge (2-\alpha)$. Using the positive root of $\sqrt{4-\alpha^2}$, this is satisfied if $4-\alpha^2 \ge (2-\alpha)^2$, implying $2\alpha(2-\alpha) \ge 0$. This holds by the fact that $\alpha \le 1$.)

The value of λ_3 is non-negative if and only if $d \ge \frac{2\alpha(c+h-s)}{2(1+\gamma-\alpha\gamma)-\sqrt{4-\alpha^2}}$. Therefore, **(B3)** satisfies the Kuhn-Tucker conditions if and only if $d \ge \frac{2\alpha(c+h-s)}{2(1+\gamma-\alpha\gamma)-\sqrt{4-\alpha^2}}$.

(B4) $p = u_H + \frac{d(2-\alpha+\sqrt{4-\alpha^2})}{4\alpha}$, $f = \frac{d(2+\sqrt{4-\alpha^2})}{2\alpha} - h$, $\lambda_2 = 0$, $\lambda_3 = \frac{\alpha}{4}(-1+\frac{2\alpha(c+h-s)-2d(1+\gamma-\alpha\gamma)}{d\sqrt{4-\alpha^2}})$. This solution has $p > u_H$ for positive roots of $\sqrt{4-\alpha^2}$. We rule out this potential solution because $p > u_H$ and this implies that the price of the good is greater than the maximum consumer's valuation for it. □

When price is set such that $4(u_H - p) + 2(f+h) - d$ =0, the seller faces the following constrained optimization problem with Lagrangian multipliers λ_1, λ_2 :

$$\max_f \; L(f,\lambda_1,\lambda_2;c,s,u,d,\gamma,\alpha) = (p-c)(1-\gamma(1-\alpha)) + (f+s-p)q_{rY}(f;d,\gamma,\alpha)$$
$$+(f+s-p+p-c)(q_{eZ}(f;d,\gamma,\alpha)+q_{eY}(f;d,\gamma,\alpha))$$
$$+\lambda_1 E_Z(U)+\lambda_2(2f+2h-d)$$

subject to

$$4(u_H-p)+2(f+h)-d=0$$
$$\lambda_1 E_Z(U)=0$$
$$\lambda_2 q_{eY}=0$$

The Kuhn-Tucker conditions are:

$$\frac{\partial L(f,\lambda_1,\lambda_2;c,s,u,d,\gamma,\alpha)}{\partial f} = \frac{d(1-\gamma+\alpha\gamma-\lambda_1)+\alpha(c-h-s-f(2-\lambda_1)+h\lambda_1-(1-\gamma)\lambda_2)}{d}=0$$

$$\lambda_1\left(\frac{\alpha d^2+4\alpha(f+h)^2-8d(f+h)}{8d}\right)=0$$

$$\lambda_2\alpha\gamma\left(\frac{1}{2}-\frac{f+h}{d}\right)=0$$

$$\lambda_1\ge 0;\ \lambda_2\ge 0.$$

We analyze the four solutions that potentially satisfy the Kuhn-Tucker conditions:

The first two solutions correspond to the price and restocking fee combination from **(B3)**, which was shown above to be optimal for $d \ge \dfrac{2\alpha(c+h-s)}{2(1+\gamma-\alpha\gamma)-\sqrt{4-\alpha^2}}$ and the price and restocking combination from **(B4)** which was shown above to be infeasible. The two remaining potential solutions are:

(B5) $p=\dfrac{\alpha(c+h-s+4u_H)+d(1-\alpha)(1-\gamma)}{4\alpha}$, $f=\dfrac{d+\alpha(c-h-s)-d\gamma(1-\alpha)}{2\alpha}$

$\lambda_1=0,\lambda_2=0$. This solution implies $q_{eZ}=-\dfrac{(1-\gamma)(\alpha(c+h-s)+d(1-\gamma)(1-\alpha))}{2d}<0$ (by the fact that $c>s,\gamma\le 1,\alpha\le 1$), which is an infeasible solution because we rule out negative quantities.

(B6) $p = u_H$, $f = \frac{d-2h}{2}$, $\lambda_1 = 0,\ \lambda_2 = d(-1+\frac{1}{\alpha}) + \frac{c+h-s}{1-\gamma}$, which yields expected utility$= -\frac{d(2-\alpha)}{4} < 0$. Consumers will not make the initial purchase with negative expected utility, making this an infeasible solution.

Consequently, in a pooling equilibrium, the seller will optimally choose solution **(B1)** iff $d \le \frac{2\alpha(c-s+h)}{\alpha+2\gamma(1-\alpha)}$ **(Case 1, *pool*)**, leading to expected profit of $\pi^* = \alpha u_H - \frac{d}{4}(\alpha + 2\gamma(1-\alpha)) + (s-h)(1-\alpha)(1-\gamma) - c(1-\gamma(1-\alpha))$ with zero exchanges. The solution **(B2)** is optimal when $d \in [\frac{2\alpha(c-s+h)}{\alpha+2\gamma(1-\alpha)}, \frac{2\alpha(c+h-s)}{2(1+\gamma-\alpha\gamma)-\sqrt{4-\alpha^2}}]$ **(Case 2, *pool*)**, leading to expected profit of $\pi^* = \alpha u_H + \frac{d}{8\alpha}(4\gamma^2(1-\alpha)^2 - \alpha^2) + \frac{\alpha(s-h-c)^2}{2d} + (s-h)(1-\frac{\alpha}{2}) - c(1+\frac{\alpha}{2})$ with expected total exchanges equal to $\frac{\alpha}{2} - \frac{\alpha(c-s+h)}{d} + \gamma(1-\alpha)$. Lastly, the solution **(B3)** will be optimal when $d \ge \frac{2\alpha(c+h-s)}{2(1+\gamma-\alpha\gamma)-\sqrt{4-\alpha^2}}$ **(Case 3, *pool*)** leading to expected profit of $\pi^* = \alpha u_H - \frac{d(1+\gamma(1-\alpha))}{\alpha}(1-\sqrt{1-\frac{\alpha^2}{4}}) - (c+h-s)(\sqrt{1-\frac{\alpha^2}{4}} - \gamma(1-\alpha)) - \frac{\alpha(c-h+s)}{2}$ with expected total exchanges equal to $\frac{(\sqrt{4-\alpha^2}-(2-\alpha))}{2}$. *Q.E.D.*

A.7. Derivation of Table EC.1 [Comparative Statics of the Separating & Pooling Equilibria]

The comparative static effects of the parameters *c, d, h* and *s* are summarized in Table EC.1 as follows.

Table EC.1: Comparative Statics for the Separating and Pooling Equilibria

		Marginal Cost		Disutility of Mismatch		Hassle Cost		Salvage Value	
Parameter Values	Returns Behavior	$\frac{\partial f^*}{\partial c}$	$\frac{\partial p^*}{\partial c}$	$\frac{\partial f^*}{\partial d}$	$\frac{\partial p^*}{\partial d}$	$\frac{\partial f^*}{\partial h}$	$\frac{\partial p^*}{\partial h}$	$\frac{\partial f^*}{\partial s}$	$\frac{\partial p^*}{\partial s}$
Case 1 *Separating: for* $d \le d_{1,sep}$ *Pooling: for* $d \le d_{1,pool}$	*No Returns*	**0**	**0**	↑	↓	↓	**0**	**0**	**0**
Case 2 *Separating: for* $d_{1,sep} \le d \le d_{2,sep}$ *Pooling: for* $d_{1,pool} \le d \le d_{2,pool}$	*Positive Returns (Incremental Profits from Returns=0)*	↑	↓	*****	******	**0**	↓	↓	↑
Case 3 *Separating: for* $d \ge d_{2,sep}$ *Pooling: for* $d \ge d_{2,pool}$	*Positive Returns (Incremental Profits from Returns > 0)*	**0**	**0**	↑	↓	↓	**0**	**0**	**0**

Notes: entries denote the sign of the comparative statics result. For example, the restocking fee from case 1 changes positively with the disutility of mismatch (*d*).
(*) The comparative-static effect of *d* on f^* is zero for the separating equilibrium, and negative for the pooling equilibrium.
(**) The comparative-static effect of *d* on p^* is negative in the separating equilibrium, and negative in the pooling equilibrium for α high enough and γ low enough; and positive in the pooling equilibrium for α low enough and γ high enough (see below for details).

<u>Cases 1 (Low *d* values)</u>

Separating equilibrium

As shown in the proof of Proposition 1a, all consumers keep the product when $d \le d_{1,sep} = 2(c - s + h)$ and the optimal price and restocking fee are given by $p^* = u_H - \frac{d}{4}$ and $f^* \ge \frac{d}{2} - h$.

One can easily show that $\frac{\partial f^*}{\partial c} = 0$ and $\frac{\partial p^*}{\partial c} = 0$; $\frac{\partial f^*}{\partial d} = \frac{1}{2} > 0$ and $\frac{\partial p^*}{\partial d} = -\frac{1}{4} < 0$; $\frac{\partial f^*}{\partial h} = -1 < 0$ and $\frac{\partial p^*}{\partial h} = 0$; $\frac{\partial f^*}{\partial s} = 0$ and $\frac{\partial p^*}{\partial s} = 0$.

Pooling equilibrium

As shown in the proof of Proposition 1b, all consumers keep the product when $d \le d_{1,pool} = \frac{2\alpha(c-s+h)}{\alpha + 2\gamma(1-\alpha)}$ and the optimal price and restocking fee are given by $p^* = u_H - \frac{d(2-\alpha)}{4\alpha}$ and $f^* \ge \frac{d}{2} - h$.

One can easily show that $\frac{\partial f^*}{\partial c} = 0$ and $\frac{\partial p^*}{\partial c} = 0$; $\frac{\partial f^*}{\partial d} = \frac{1}{2} > 0$ and $\frac{\partial p^*}{\partial d} = -\frac{(2-\alpha)}{4\alpha} < 0$, where $\alpha \le 1$; $\frac{\partial f^*}{\partial h} = -1 < 0$ and $\frac{\partial p^*}{\partial h} = 0$; $\frac{\partial f^*}{\partial s} = 0$ and $\frac{\partial p^*}{\partial s} = 0$.

<u>Cases 2 (Medium *d* values)</u>

Separating equilibrium

As shown in the proof of Proposition 1a, when $d_{1,sep} \le d \le d_{2,sep}$ (in case 2, *sep*) the firm charges $p^* = u_H - \frac{(c-s+h)}{2} + \frac{(c-s+h)^2}{2d} - \frac{d}{8}$ and $f^* = c - s$.

One can easily show that $\frac{\partial f^*}{\partial c} = 1 > 0$ and $\frac{\partial f^*}{\partial s} = -1$. Meanwhile $\frac{\partial f^*}{\partial d} = 0$ and $\frac{\partial f^*}{\partial h} = 0$.

Below, we examine how the optimal price changes as *c*, *d*, *h* and *s* increase.

$$\frac{\partial p^*}{\partial c} = -\frac{1}{2} + \frac{c-s+h}{d} < 0 \text{ because } d > 2(c-s+h).$$

$$\frac{\partial p^*}{\partial d} = -\frac{d^2 + 4(c-s+h)^2}{8d^2} < 0.$$

$$\frac{\partial p^*}{\partial s} = \frac{1}{2} - \frac{c-s+h}{d} = -\frac{\partial p^*}{dc} > 0.$$

$$\frac{\partial p^*}{\partial h} = -\frac{1}{2} + \frac{c-s+h}{d} = \frac{\partial p^*}{dc} < 0 \text{ (because } d > 2(c-s+h)\text{)}.$$

Pooling equilibrium

As shown in the proof of Proposition 1b, the firm charges

$$p^* = \alpha(u_H - \frac{d}{8} - \frac{c-s-\frac{d\gamma(1-\alpha)}{\alpha}+h}{2} + \frac{(c-s-\frac{d\gamma(1-\alpha)}{\alpha}+h)^2}{2d}) - (1-\alpha)(c-s-\frac{d\gamma(1-\alpha)}{\alpha}-h)$$

and $f^* = c-s-\frac{d\gamma(1-\alpha)}{\alpha}$.

One can easily show that $\frac{\partial f^*}{\partial c} = 1 > 0$, $\frac{\partial f^*}{\partial s} = -1$ $\frac{\partial f^*}{\partial d} = -\frac{\gamma(1-\alpha)}{\alpha} < 0$ and $\frac{\partial f^*}{\partial h} = 0$. Notice that in the pooling equilibrium's case 2, the restocking fee is decreasing in *d*.

Below we examine how the optimal price changes with *c*, *d*, *h* and *s*.

$\frac{\partial p^*}{\partial c} = \frac{2\alpha(c+h-s) - d(2+2\gamma-\alpha(1+2\gamma))}{2d\alpha} < 0$. Evaluated at the lower bound on $d = \frac{2\alpha(c-s+h)}{\alpha+2\gamma(1-\alpha)}$, this derivative is equal to $\frac{-1+\alpha}{\alpha} < 0$ where $\alpha \le 1$. Because the numerator is decreasing in *d*, it is also negative for all $d \ge \frac{2\alpha(c-s+h)}{\alpha+2\gamma(1-\alpha)}$. Furthermore,

$$\frac{\partial p^*}{\partial d} = \frac{(2-\alpha)(1-\alpha)\gamma + \gamma^2(1-\alpha)^2}{2\alpha^2} - \frac{d^2+4(c-s+h)^2}{8d^2}.$$

This derivative can be shown to be negative for low enough γ, or high enough α, throughout the entire range of *d* for Case2,*pool*. For example, when $d = d_{1,pool}$, the derivative is negative for $\gamma < \frac{\alpha^2}{4(1-\alpha)^2}$, and when $d = d_{2,pool}$, the derivative is negative for $\gamma < \frac{2-\sqrt{4-\alpha^2}}{(1-\alpha)\left(\sqrt{4-\alpha^2}-\alpha\right)}$.

$$\frac{\partial p^*}{\partial s} = -\frac{2\alpha(c+h-s)-d(2+2\gamma-\alpha(1+2\gamma))}{2d\alpha} = -\frac{\partial p^*}{\partial c} > 0 \text{, and}$$

$$\frac{\partial p^*}{\partial h} = \frac{2\alpha(c+h-s)-d(2+2\gamma-\alpha(1+2\gamma))}{2d\alpha} = \frac{\partial p^*}{\partial c} < 0 .$$

Case 3 (high *d* values):

Separating equilibrium

As shown in the proof of Proposition 1a, the firm's optimal restocking fee and price are given by

$$f^* = d(1-\frac{\sqrt{3}}{2})-h \text{ and } p^* = u_H - \frac{d(\sqrt{3}-1)}{4} \text{ when } d > (4+2\sqrt{3}) .$$

One can easily show that $\frac{\partial f^*}{\partial c} = 0$ and $\frac{\partial p^*}{\partial c} = 0$; $\frac{\partial f^*}{\partial d} = (1-\frac{\sqrt{3}}{2}) > 0$ and $\frac{\partial p^*}{\partial d} = -\frac{(\sqrt{3}-1)}{4} < 0$; $\frac{\partial f^*}{\partial h} = -1 < 0$ and $\frac{\partial p^*}{\partial h} = 0$; $\frac{\partial f^*}{\partial s} = 0$ and $\frac{\partial p^*}{\partial s} = 0$.

Pooling equilibrium

As shown in the proof of Proposition 1b, the firm charges $f^* = \frac{d(2-\sqrt{(4-\alpha^2)})}{2\alpha} - h$ and

$$p^* = u_H - \frac{d(\sqrt{4-\alpha^2}-2+\alpha)}{4\alpha} \text{ when } d \geq d_{2,pool} \equiv \frac{2\alpha(c+h-s)}{2(1+\gamma-\alpha\gamma)-\sqrt{4-\alpha^2}} .$$

From these expressions, it follows that $\frac{\partial f^*}{\partial c}$ = 0 and $\frac{\partial p^*}{\partial c}$ = 0 ; $\frac{\partial f^*}{\partial d}$ = $\frac{(2-\sqrt{(4-\alpha^2)})}{2\alpha}$ > 0 and $\frac{\partial p^*}{\partial d}$ = $-\frac{(\sqrt{4-\alpha^2}-2+\alpha)}{4\alpha}$ < 0 ; $\frac{\partial f^*}{\partial h}$ = -1 < 0 and $\frac{\partial p^*}{\partial h}$ = 0 ; $\frac{\partial f^*}{\partial s}$ = 0 and $\frac{\partial p^*}{\partial s}$ = 0.

Q.E.D.

Note that only the consumer parameters, *d* and *h*, influence f^* and p^* in Cases 1 and 3; this is because demand-side factors (not supply-side ones) constrain the solution in both of these Cases. Specifically, a higher disutility of product mismatch (*d*) leads to a lower equilibrium price, in order to compensate for the possibility of a poorly-fitting product that the consumer nevertheless might keep. Meanwhile, parameter changes that make the likelihood of a return higher (namely, higher *d* or lower *h*) both lead to an increase in f^*.

In Case 2, the intermediate range of *d*-values, these demand-side constraints are not binding, and thus the seller balances demand and supply-side factors in setting the optimal price and restocking fee. The optimal restocking fee increases, and the optimal price decreases to compensate, as returns are more costly for the seller to handle (i.e., as either *c* increases or *s* decreases). Meanwhile, the optimal price generally declines as consumer costs of a product misfit rise (i.e., with an increase in either *d* or *h*), and these consumer-based parameters have either a zero impact, or (in the case of $\partial f^* / \partial d$ in the pooling equilibrium only) a negative impact, on the optimal restocking fee. The exception to this occurs for low α and high γ values; then, the seller faces a market heavily weighted toward zero-utility consumers, and toward consumers who are unaware of whether their consumption utility is positive or zero for the category. To achieve a pooling equilibrium in this type of market, the seller optimally sets f^* very low and, to compensate, is able to increase price while just inducing all consumers to buy *a priori*. These effects reflect the incentives discussed above, which include first a need to balance the total cost to the consumer of buying in the category (which includes initial purchase price and the expected cost of a return); and second, the impetus to optimally control returns volumes through manipulation of the restocking fee.

A.8. Proof of Proposition 2

In this section, we prove the condition on *u* that makes the profits of the separating equilibrium dominate the profits of the pooling equilibrium. To this end, we first show how the critical *d* values ($d_{1,sep}$, $d_{2,sep}$, $d_{1,pool}$, and $d_{2,pool}$) rank in the α - γ space. This helps us identify the appropriate profit comparisons among the three cases of the separating and the three cases of the pooling equilibrium. Next, we perform the profit comparisons to derive Proposition 2.

Proposition 2: *There exists a critical reservation value $\bar{u}$ such that $u_H < \bar{u}$ implies that the seller will optimally set the price and restocking fee to skim the market and only sell to the consumers in segment 1 (consumers who know that they will have value from owning one of the products, i.e. $u_\theta = u_H > 0$). For $u_H \geq \bar{u}$, the seller will optimally set the price and restocking fee such that consumers in segments 1, 3, and 4 purchase initially (all consumers who potentially value the product), purposefully selling to consumers in segment 4 who will return the product and opt out of the market (because $u_\theta = 0$).*

Proof:

Because there are three cases in the pooling equilibrium and three cases in the separating equilibrium, the appropriate comparisons must be determined. In Figure EC.1, we have three regions of the parameter space that identify the correct ordering of $d_{1,sep}$, $d_{2,sep}$, $d_{1,pool}$ and $d_{2,pool}$.

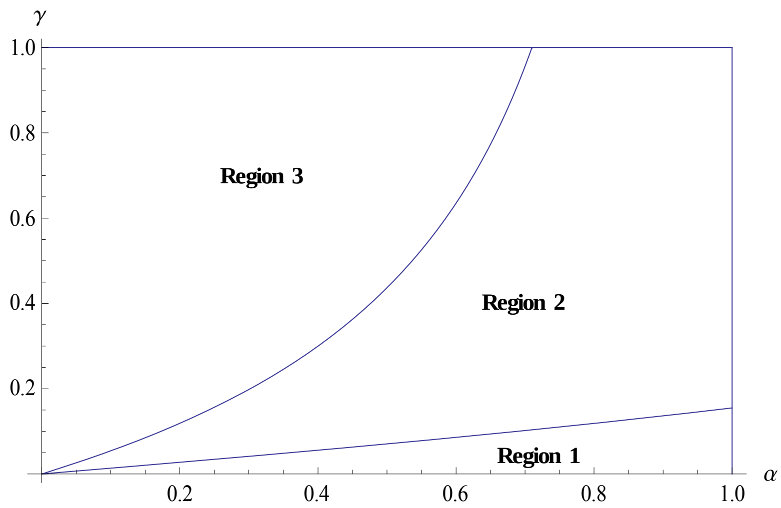

Figure EC.1: Comparison of Critical *d* Values from Separating and Pooling Equilibria
Note: In Region 1, $d_{1,pool} < d_{1,sep} < d_{2,sep} < d_{2,pool}$ **; in Region 2,** $d_{1,pool} < d_{1,sep} < d_{2,pool} < d_{2,sep}$ **; and in Region 3,** $d_{1,pool} < d_{2,pool} < d_{1,sep} < d_{2,sep}$ **.**

In Table EC.2 below, we identify each of the possible comparisons of profits.

Table EC.2: Feasible Profit Comparisons Between Pooling and Separating Equilibrium Cases in Regions 1, 2 and 3

			Pooling Equilibrium	
		Case 1P	Case 2P	Case 3P
Separating Equilibrium	Case 1S	Regions 1, 2, 3 Criteria in R1, R2 & R3: $d \le d_{1,pool}$	Regions 1, 2, 3 Criteria in R1 & R2: $d_{1,pool} \le d \le d_{1,sep}$ in R3: $d_{1,pool} \le d \le d_{2,pool}$	Region 3 Criteria in R3: $d_{2,pool} \le d \le d_{1,sep}$
	Case 2S	NOT FEASIBLE	Regions 1, 2 Criteria in R1: $d_{1,sep} \le d \le d_{2,sep}$ in R2: $d_{1,sep} \le d \le d_{2,pool}$	Regions 2, 3 Criteria in R2: $d_{2,pool} \le d \le d_{2,sep}$ in R3: $d_{1,sep} \le d \le d_{2,sep}$
	Case 3S	NOT FEASIBLE	Region 1 Criteria in R1: $d_{2,sep} \le d \le d_{2,pool}$	Regions 1, 2, 3 Criteria in R1 : $d \ge d_{2,pool}$ in R2&R3: $d \ge d_{2,sep}$

Note: Entries in Table EC.2 should be read as follows: Case 1P should be compared with Case 1S in Regions 1, 2, and 3 of Figure EC.1, under the criterion that $d \le d_{1,pool}$. Meanwhile, Case 2P should be compared with Case 1S in Regions 1, 2, and 3 of Figure EC.1, under the criterion (for Regions 1 and 2 only) that $d_{1,pool} \le d \le d_{1,sep}$ and (in Region 3) that $d_{1,pool} \le d \le d_{2,pool}$.

Table EC.3 presents the values of u for which the profits from separating (as found in Proposition 1a) equal the profits from pooling (as found in Proposition 1b) for each given range of *d:*

Table EC.3: Values of $\bar{u}$ Such That the Pooling Equilibrium Generates Profits Equivalent to the Separating Equilibrium

-- When Comparing Pooling Equilibrium: ---------------------------

---------------------- With Separating Equilibrium: ----------------------

	Case 1P	Case 2P	Case 3P
Case 1S	$\frac{1}{\alpha(1-\gamma)}\cdot\left\{\begin{array}{l}\frac{d(\alpha+\gamma(2-3\alpha))}{4}\\ -(s-h)(1-\alpha)(1-\gamma)\\ +c(1-\gamma)\end{array}\right\}$	$\left\{\begin{array}{l}\frac{d(\alpha^2-4\gamma^2(1-\alpha)^2)}{8\alpha^2(1-\gamma)}\\ -\frac{(s-h)^2}{2d(1-\gamma)}-\frac{(s-h)((2-\alpha)d-2c\alpha)}{2\alpha(1-\gamma)d}\\ -\frac{c^2}{2d(1-\gamma)}+\frac{c(2(2+\alpha)-4\gamma\alpha)-\gamma d\alpha}{4\alpha(1-\gamma)}\end{array}\right\}$	$\left\{\begin{array}{l}\frac{d((1+(1-\alpha)\gamma)(1-\sqrt{1-\frac{\alpha^2}{4}})-\frac{\alpha^2\gamma}{4})}{\alpha^2(1-\gamma)}\\ -\frac{(s-h)(\sqrt{1-\frac{\alpha^2}{4}}-\gamma(1-\alpha)-\frac{\alpha}{2})}{\alpha(1-\gamma)}\\ +\frac{c(\sqrt{1-\frac{\alpha^2}{4}}-\gamma+\frac{\alpha}{2})}{\alpha(1-\gamma)}\end{array}\right\}$
Case 2S	NOT FEASIBLE	$\frac{1}{\alpha(1-\gamma)}\left\{\begin{array}{l}\frac{d(\alpha^2(1-\gamma)-4\gamma^2(1-\alpha)^2)}{8\alpha}\\ +\frac{(h-s)(d(2-\alpha(1+\gamma))-2c\alpha(1-\gamma))}{2d}\\ -\frac{(c^2+(h-s)^2)\alpha(1-\gamma)}{2d}\\ +\frac{c(2+\alpha(1-3\gamma))}{2}\end{array}\right\}$	$\frac{1}{\alpha(1-\gamma)}\left\{\begin{array}{l}\frac{d((8-(-8+\alpha(8+\alpha))\gamma)-4\sqrt{4-\alpha^2}(1+\gamma-\alpha\gamma))}{8\alpha}\\ +\frac{(h-s)(\sqrt{4-\alpha^2}-2\gamma-\alpha(1-\gamma))}{2}\\ -\frac{(c+h-s)^2\alpha\gamma}{2d}\\ +\frac{c(\alpha-\gamma(2+\alpha)+\sqrt{4-\alpha^2})}{2}\end{array}\right\}$
Case 3S	NOT FEASIBLE	$\frac{1}{\alpha(1-\gamma)}\left\{\begin{array}{l}\frac{d(\alpha^2-4(2-\sqrt{3})\alpha^2\gamma-4(1-\alpha)^2\gamma^2)}{8\alpha}\\ -\frac{(s-h)(d(1-\frac{\alpha}{2}(1+\gamma(\sqrt{3}-1))-c\alpha)}{d}\\ -\frac{\alpha(s-h)^2}{2d}\\ +\frac{c((2+\alpha)-\alpha\gamma(1+\sqrt{3}))}{2}-\frac{\alpha c^2}{2d}\end{array}\right\}$	$\frac{1}{\alpha(1-\gamma)}\left\{\begin{array}{l}\frac{d((2+(2+\alpha(-2+(-2+\sqrt{3})\alpha))\gamma)-\sqrt{4-\alpha^2}(1+\gamma-\alpha\gamma))}{2\alpha}\\ -\frac{(h-s)(-\sqrt{4-\alpha^2}+(-3+\sqrt{3})\alpha\gamma+\alpha+2\gamma)}{2}\\ -\frac{c(\alpha(-1+\gamma(\sqrt{3}-1))-\sqrt{4-\alpha^2}+2\gamma))}{2}\end{array}\right\}$

In a separating equilibrium (as proven in Proposition 1a), the seller's price and restocking fee skim the market, selling only to $\alpha\gamma$ consumers initially. In a pooling equilibrium (as proven in Proposition 1b), the seller's price and restocking fee pool the market, selling to a $1-\gamma(1-\alpha)$ consumers initially. In Table EC.3, we have defined the values of u for which the seller is indifferent between inducing a separating equilibrium and inducing a pooling equilibrium, $\bar{u}(c,s,d,h,\alpha,\gamma)$.

In all cases, changes in the value of u_H change the profit from the pooling equilibrium at the rate α. Changes in the value of u_H change the profit from the separating equilibrium at the rate $\alpha\gamma$. Because $\gamma \le 1$, the seller will earn greater profits from a separating equilibrium if and only if $u_H < \bar{u}(c,s,d,h,\alpha,\gamma)$, where $\bar{u}(c,s,d,h,\alpha,\gamma)$ is uniquely defined for a given set of parameter values as described in Table EC.3. Therefore, the seller will optimally set the price and the restocking fee to skim the market (induce a separating equilibrium) if and only if $u_H < \bar{u}(c,s,d,h,\alpha,\gamma)$. The seller will optimally set the price and restocking fee to sell to all consumers who potentially value the products (induce a pooling equilibrium) if and only if $u_H \ge \bar{u}(c,s,d,h,\alpha,\gamma)$.

Q.E.D.

A.9. Derivation of the Value of Information (Table 5)

From section 5, we know that with informed consumers, the quantity of goods purchased equals $q_p(p;u,d,\alpha) = \min\{\frac{4\alpha(u_H - p)}{d}, \alpha\}$. The seller solves: $\max_p (p-c)q_p(p;u,d,\alpha)$. Given the assumption that the seller serves the entire market ($u_H > c + d/2$), the optimal price is then given by: $p^* = u_H - \frac{d}{4}$. Thus, the profits from informed consumers are $\pi^* = \alpha(u_H - c - \frac{d}{4})$. The value of information is the additional profit a seller earns by serving informed consumers relative to serving uninformed consumers.

The profit of the seller in a separating equilibrium and a pooling equilibrium are found in Propositions 1a and 1b respectively. The values of Table 5 are straightforwardly derived by subtracting the profit values given in the Propositions 1a and 1b from the profits that the firm attains by serving consumers who are informed about u_θ and $|x_j - \theta|$. □

A.10. Proof of Proposition 3a

Proposition 3a: *When it is costless to inform consumers about their preferences and value for each product (*u_θ *and* $\left|x_j - \theta\right|$*) and* u_H *is such that the uninformed separating equilibrium dominates the uninformed pooling equilibrium, then the value of information (VOI) is positive or negative according to the conditions below:*

	Case 1,*sep* $d < d_{1,sep}$	Case 2,*sep* $d_{1,sep} \le d < d_{2,sep}$	Case 3,*sep* $d \ge d_{2,sep}$
If $d_{2,sep} \ge \hat{d}_{sep}$:	*VOI>0 Always*	*VOI>0 for* $d_{1,sep} \le d < \hat{d}_{sep}$ *VOI<0 for* $\hat{d}_{sep} \le d < d_{2,sep}$	*VOI<0 Always*
If $d_{2,sep} < \hat{d}_{sep}$:	*VOI>0 Always*	*VOI>0 Always*	*VOI>0 for* $d_{2,sep} \le d < \tilde{d}_{sep}$ *VOI<0 for* $d \ge \tilde{d}_{sep}$

where $d_{1,sep} \equiv 2(c - s + h)$, $d_{2,sep} \equiv (4 + 2\sqrt{3})(c - s + h)$, $\tilde{d}_{sep} \equiv \frac{4(1-\gamma)(u_H - c) - \gamma(2 - 2\sqrt{3})(c - s + h)}{1 - 4\gamma + 2\gamma\sqrt{3}}$ and

$\hat{d}_{sep} \equiv \frac{2}{2-\gamma}(2(1-\gamma)(u_H - c) + \gamma(c - s + h) + \sqrt{(2(1-\gamma)(u_H - c) + \gamma(c - s + h))^2 - \gamma(2-\gamma)(c - s + h)^2}$.

Proof:

The value of information is listed in Table 5. We look at the three cases across columns.

For $d < d_{1,sep}$, the choices described by Case 1, *sep* will be optimal when consumers are uninformed and the value of information is $\alpha(1-\gamma)(u_H - c - \frac{d}{4}) = \alpha(1-\gamma)\pi_{INFORMED}$, which is positive if and only if the informed profits (maximum profits that the seller attains if he plans to operate in the market, assumed to ensure the existence of the seller in the market) is positive. Therefore, the VOI is always positive for $d < d_{1,sep}$. □

For $d_{1,sep} \le d < d_{2,sep}$, the choices described by Case 2, *sep* will be optimal when consumers are uninformed and the value of information will be

$\alpha(1-\gamma)(u_H - c - \frac{d}{4}) - \alpha\gamma(\frac{d}{8} - \frac{c-s+h}{2} + \frac{(c-s+h)^2}{2d})$. This expression is equal to zero at $\hat{d}_{sep} = \frac{2}{2-\gamma}(2(1-\gamma)(u_H - c) + \gamma(c-s+h) + \sqrt{(2(1-\gamma)(u_H-c)+\gamma(c-s+h))^2 - \gamma(2-\gamma)(c-s+h)^2}$ and $\breve{d}_{sep} = \frac{2}{2-\gamma}(2(1-\gamma)(u_H - c) + \gamma(c-s+h) - \sqrt{(2(1-\gamma)(u_H-c)+\gamma(c-s+h))^2 - \gamma(2-\gamma)(c-s+h)^2}$. Notice that for positive *d*, the VOI is quadratic in *d*. The derivative with respect to *d* is $\frac{\partial VOI}{\partial d} = \frac{\alpha(4\gamma(c-s+h)^2 - (2-\gamma)d^2)}{8d^2}$, which is negative for all $d \ge d_{1,sep}$. Also, at $d = d_{1,sep}$, the VOI in Case 2, *sep* is equal to the VOI of Case 1, *sep,* which we know is positive from above. Because the VOI is positive at $d = d_{1,sep}$ and decreasing in *d* for $d \ge d_{1,sep}$, we conclude that only the larger root $\hat{d}_{sep}$ is within the range for Case 2, *sep.* Moreover, if $d_{2,sep} \ge \hat{d}_{sep}$, then VOI is positive for $d_{1,sep} < d \le \hat{d}_{sep}$ and negative for $\hat{d}_{sep} \le d < d_{2,sep}$. If $d_{2,sep} < \hat{d}_{sep}$, then VOI is positive for all $d_{1,sep} \le d < d_{2,sep}$. □

For $d \ge d_{2,sep}$, the choices described by Case 3, *sep* will be optimal when consumers are uninformed. The value of information is $\alpha(1-\gamma)(u_H - c - \frac{d}{4}) + \alpha\gamma(\frac{d}{4}(3-2\sqrt{3}) + \frac{(c+h-s)(\sqrt{3}-1)}{2})$. If $d_{2,sep} \ge \hat{d}_{sep}$, then from the above proof, the VOI for Case 2, *sep* is negative at $d = d_{2,sep} \ge \hat{d}_{sep}$. Since VOI for Case 3, *sep* at $d_{2,sep}$ equals VOI for Case 2,sep at $d_{2,sep}$ and the derivative of VOI for Case 3, *sep* with respect to *d* is $\frac{\partial VOI}{\partial d} = -\frac{\alpha(1-\gamma(4-2\sqrt{3}))}{4} < 0$ by fact that $\gamma \le 1$, then if $d_{2,sep} \ge \hat{d}_{sep}$ VOI for Case 3, *sep* will be negative for $d \ge d_{2,sep}$.

If $d_{2,sep} < \hat{d}_{sep}$, then the VOI at $d = d_{2,sep}$ is positive. The VOI in Case 3, *sep* is equal to zero at $d = \tilde{d}_{sep} = \frac{4(1-\gamma)(u_H - c) - \gamma(2 - 2\sqrt{3})(c - s + h)}{1 - 4\gamma + 2\gamma\sqrt{3}}$. Because the derivative of VOI for Case 3, *sep* with respect to d is negative, VOI is positive for $d < \tilde{d}_{sep}$ and the VOI is negative for $d > \tilde{d}_{sep}$ and positive otherwise.□ *Q.E.D.*

A.11. Price Comparisons between Informed Consumer Model and the Separating Equilibrium

In this section, we show that the price with informed consumers is lower than the price with uninformed consumers in a separating equilibrium when $d > d_{1,sep}$.

To this end, we consider three cases. Below, we show the price difference between the informed consumer model and the separating equilibrium cases.

Parameters	Informed Price - Uninformed Price
Case 1, *sep* { $d < d_{1,sep}$ }	0
Case 2, *sep* { $d_{1,sep} \le d < d_{2,sep}$ }	$-\frac{d}{8} + \frac{c-s+h}{2} - \frac{(c-s+h)^2}{2d}$
Case 3, *sep* { $d \ge d_{2,sep}$ }	$\frac{d(\sqrt{3}-2)}{4}$

These difference in prices are negative for $d > d_{1,sep}$, implying that pricing to the "marginal consumer" in the informed case will lead to a lower retail price than pricing to the "average consumer" in the uninformed case when $d > d_{1,sep}$.

Q.E.D.

A.12. Proof of Proposition 3b

Proposition 3b*: When it is costless to inform consumers about their preferences and value for each product (u_θ and $\left| x_j - \theta \right|$) and u_H is such that the uninformed pooling equilibrium dominates the uninformed separating equilibrium, then the value of information (VOI) is positive or negative according to the conditions below:*

	Case 1,*pool* $d < d_{1,pool}$	Case 2,*pool* $d_{1,pool} \le d < d_{2,pool}$	Case 3,*pool* $d \ge d_{2,pool}$
If $d_{2,pool} \ge \hat{d}_{pool}$:	*VOI>0 Always*	*VOI>0 for* $d_{1,pool} \le d < \hat{d}_{pool}$ *VOI<0 for* $\hat{d}_{pool} < d < d_{2,pool}$	*VOI<0 Always*
If $d_{2,sep} < \hat{d}_{pool}$:	*VOI>0 Always*	*VOI>0 Always*	*VOI>0 for* $d_{2,pool} \le d < \tilde{d}_{pool}$ *VOI<0 for* $d > \tilde{d}_{pool}$

where $d_{1,pool} = \frac{2\alpha(c+h-s)}{\alpha+2\gamma(1-\alpha)}$, $d_{2,pool} = \frac{2\alpha(c+h-s)}{\alpha+2\gamma(1-\alpha)-\sqrt{4-\alpha^2}+(2-\alpha)}$, $\hat{d}_{pool} = \frac{2\alpha\left(2-\alpha+2\sqrt{1-\alpha-\gamma^2(1-\alpha)^2}\right)(c+h-s)}{\alpha^2+4\gamma^2(1-\alpha)^2}$ *and*

$\tilde{d}_{pool} = \frac{2\alpha\left(\sqrt{4-\alpha^2}-\alpha-2\gamma(1-\alpha)\right)(c+h-s)}{\alpha^2+2\left(2-\sqrt{4-\alpha^2}\right)(\alpha\gamma-1-\gamma)}$.

Proof: The value of information is listed in Table 5. We look at the three cases across the columns above.

For $d < d_{1,pool}$ *,* the choices described by Case 1, *pool* are optimal when consumers are uninformed and the value of information is $\frac{(1-\alpha)(d\gamma+2(1-\gamma)(c-s+h))}{2}$. This is always positive by fact that *c>s.*

For $d_{1,pool} \le d < d_{2,pool}$, the choices described by Case 2, *pool* are optimal when consumers are uninformed; the value of information is then $\frac{\alpha(8d(c-s+h)-\alpha(2c-2s+d+2h)^2)-4d^2\gamma^2(1-\alpha)^2}{8d\alpha}$. This expression is equal to zero at $d = \hat{d}_{pool} \equiv \frac{2\alpha\left(2-\alpha+2\sqrt{1-\alpha-\gamma^2(1-\alpha)^2}\right)(c+h-s)}{\alpha^2+4\gamma^2(1-\alpha)^2}$ and $d = \breve{d}_{pool} \equiv \frac{2\alpha\left(2-\alpha-2\sqrt{1-\alpha-\gamma^2(1-\alpha)^2}\right)(c+h-s)}{\alpha^2+4\gamma^2(1-\alpha)^2}$. Notice that the numerator for VOI is quadratic in *d.* The derivative with respect to *d* is given by $\frac{\partial VOI}{\partial d} = \frac{d^2(2\alpha-1)+\alpha^2(4(c-s+h)^2-d^2(1+4\gamma^2))}{8\alpha d^2}$. At

$d = d_{1,pool}$, this derivative is positive and equals $\frac{\gamma(1-\alpha)}{2} \geq 0$. Also note that at $d = d_{1,pool}$ VOI for Case 2, *pool* equals VOI for Case1, *pool* which is – as shown above - greater than zero. Because the second derivative with respect to *d* is negative *(*$\frac{\partial^2 VOI}{\partial d^2} = -\frac{\alpha(c+h-s)^2}{d^3} < 0$*)* for positive *d* values, the VOI expression is concave. Since the expression is positive and increasing at $d = d_{1,pool}$, and the VOI for Case 2 is a concave quadratic function, we conclude that only $\hat{d}_{pool}$ is potentially in the range for Case 2, *pool*. Moreover, if $d_{2,pool} \geq \hat{d}_{pool}$, then VOI is positive for $d_{1,pool} \leq d < \hat{d}_{pool}$ and negative for $\hat{d}_{pool} < d \leq d_{2,pool}$. If $d_{2,pool} < \hat{d}_{pool}$, then VOI is positive for all $d_{1,pool} \leq d < d_{2,pool}$. □

For $d \geq d_{2,pool}$, the choices described by Case 3, *pool* are optimal when consumers are uninformed and the value of information is then:

$$\frac{1}{4\alpha}(d(-\alpha^2 - 2\alpha\gamma(2-\sqrt{4-\alpha^2}) + 2(1+\gamma)(2-\sqrt{4-\alpha^2}))$$
$$+2\alpha(c+h-s)(\sqrt{4-\alpha^2} - 2\gamma - \alpha(1-2\gamma))).$$

If $d_{2,pool} \geq \hat{d}_{pool}$, then from the above proof, VOI for Case 2, *pool* is negative at $d = d_{2,pool} \geq \hat{d}_{pool}$. Also note that the VOI for Case 2, *pool* equals VOI for Case 3, *pool* at $d = d_{2,pool}$. The derivative of VOI for Case 3 with respect to *d* is $\frac{\partial VOI}{\partial d} = -\frac{\alpha^2 - 2(1-\alpha\gamma+\gamma)(2-\sqrt{4-\alpha^2})}{4\alpha}$. This derivative is negative if and only if $\alpha^2 > 2(1-\alpha\gamma+\gamma)(2-\sqrt{4-\alpha^2})$, which is true if and only if $\frac{\alpha^2(-(4-\alpha^2)+4\gamma^2(1-\alpha)^2)}{4(1+\gamma-\alpha\gamma)^2} < 0$. This is true because $-(4-\alpha^2)+4\gamma^2(1-\alpha)^2$ is negative for all $\{0 \leq \alpha \leq 1,\ 0 \leq \gamma \leq 1\}$. Therefore, the $\frac{\partial VOI}{\partial d} < 0$ and VOI is always negative for $d \geq d_{2,pool}$.

If $d_{2,pool} < \hat{d}_{pool}$, then VOI at $d = d_{2,pool}$ is positive. VOI for Case 3, *pool* is equal to zero at

$$\tilde{d}_{pool} = \frac{2\alpha\left(\sqrt{4-\alpha^2}-\alpha-2\gamma(1-\alpha)\right)(c+h-s)}{\alpha^2+2\left(2-\sqrt{4-\alpha^2}\right)(\alpha\gamma-1-\gamma)}$$

. Therefore, when $d_{2,pool} < \hat{d}_{pool}$,VOI for Case 3,*pool* is positive when $d_{2,pool} \le d < \tilde{d}_{pool}$ and negative when $d \ge \tilde{d}_{pool}$. □

Q.E.D.

A.13. Critical Minimum Values of $d/(c-s+h)$ Above Which VOI<0 in Pooling Equilibrium

The Table EC.4 below summarizes the critical minimum values of $d/(c-s+h)$ above which the VOI is negative relative to the pooling equilibrium, for various values of α and γ. This Table is completely general and does not rely on any specific parameterization. For any $\{\alpha,\gamma\}$ combination, there is a unique value of $[d/(c-s+h)]$ above which the value of information is negative in the pooling case, relative to the uninformed pooling equilibrium. From Proposition 3b, we know that if $\frac{d_{2,pool}}{c-s+h} \ge \frac{\hat{d}_{pool}}{c-s+h}$, then VOI is negative for $\frac{d}{c-s+h} \ge \frac{\hat{d}_{pool}}{c-s+h}$. If $\frac{d_{2,pool}}{c-s+h} < \frac{\hat{d}_{pool}}{c-s+h}$, then VOI is negative for $\frac{d}{c-s+h} > \frac{\tilde{d}_{pool}}{c-s+h}$. For each given value of α and γ, we derive each of these critical values of *d*. For an $\{\alpha,\gamma\}$ pair, if $\frac{d_{2,pool}}{c-s+h} \ge \frac{\hat{d}_{pool}}{c-s+h}$, then the numerical value of $\frac{\hat{d}_{pool}}{c-s+h}$ is reported. For other entries, the numerical value of $\frac{\tilde{d}_{pool}}{c-s+h}$ is reported.

Table EC.4 shows several interesting insights about the effect of segmentation variables on the value of information. First, higher values of α are associated with a larger range of *d* values for which the uninformed pooling equilibrium is more profitable than informing consumers. The intuition for this builds on the fact that, when α is already high, many consumers have a positive utility for the product category, and even if all consumers are uninformed, their expected utility from *a priori* purchase is already high because of each consumer's assessment of the probability that he is in the positive-*u* group. The increase in willingness to pay due to informing a population of consumers like this is accordingly relatively low. Meanwhile, the firm would still need to price "on the margin" to this population of consumers if it chooses to inform them – a significant cost, given their already high expected utility of *a priori* purchase.

Table EC. 4: Values of $\left[d/(c-s+h)\right]$ Above Which VOI is Negative In the Pooling Equilibrium

Values of γ =	**α = 0.25**	**α = 0.5**	**α = 0.75**	**α = 0.9**
0	27.8564	11.6569	6.0	3.8499
0.1	27.5303	11.459	5.97013	3.8473
0.2	27.1434	11.2121	5.88206	3.8395
0.3	26.6809	10.9351	5.75406	3.82656
0.4	26.1181	10.622	5.61733	3.80852
0.5	25.4186	10.2653	5.47207	3.78549
0.6	24.5255	9.85518	5.31746	3.75757
0.7	23.3456	9.37877	5.15257	3.7249
0.8	21.7145	8.81852	4.97634	3.68764
0.9	19.3119	8.15017	4.78756	3.64594
1.0	15.4211	7.33908	4.58484	3.6

The range of *d*-values for which the value of information is negative in Table EC.4 also increases as γ increases, for any given α value. Note that higher γ values mean that a higher proportion of the consumer population knows whether or not its *u* value is positive (although they still do not know their specific value of θ). One of the benefits of informing uninformed consumers is in making them aware of whether they have a positive valuation for a perfectly-fitting product in the category; this increases a consumer's willingness-to-pay by increasing the expected utility from purchasing (since among positive-category-utility consumers, there is now a zero weight placed on the possibility that $u_\theta = 0$). This benefit is lower, the higher is γ, because there are fewer consumers who do not already know their category valuation for the product. Hence, the value of information falls as γ rises.

A.14. Price Comparisons between Informed Consumer Model and the Pooling Equilibrium

In this section, we show that the price with informed consumers is lower than the price with uninformed consumers in a pooling equilibrium when $d > \frac{2\alpha(c+h-s)}{2(1+\gamma-\alpha\gamma)-2\sqrt{1-\alpha}}$. There are three cases to consider. Below, we show the price with informed consumers minus the price with uninformed consumers for each of the cases:

CASE	Informed Price - Uninformed Price
Case 1, *pool* $d < d_{1,pool}$	$\frac{2d(1-\alpha)}{4\alpha}$
Case 2, *pool* $d_{1,pool} \le d < d_{2,pool}$	$-\frac{d}{8}+\frac{c-s+h-\frac{d\gamma(1-\alpha)}{\alpha}}{2}-\frac{(c-s+h-\frac{d\gamma(1-\alpha)}{\alpha})^2}{2d})+\frac{(1-\alpha)}{\alpha}(c-s+h-\frac{d\gamma(1-\alpha)}{\alpha})$
Case 3, *pool* $d \ge d_{2,pool}$	$\frac{d(\sqrt{4-\alpha^2}-2)}{4\alpha}$

In Case 1, *pool*, the difference is positive and therefore the price is higher with informed consumers than with uninformed consumers. In Case 2, *pool* the difference in prices equals $\frac{(c-s+h)(1-\alpha)}{2\gamma(1-\alpha)+\alpha} \ge 0$ at the lower bound on *d*. At the upper bound on *d* for Case 2, *pool*, the difference in prices is $-\frac{(c-s+h)(2-\sqrt{4-\alpha^2})}{2(2\gamma(1-\alpha)+2-\sqrt{4-\alpha^2})} < 0$. The prices are equal to each other at $d = \frac{2\alpha(c+h-s)}{2(1+\gamma-\alpha\gamma)-2\sqrt{1-\alpha}}$. Therefore, if $\frac{2\alpha(c+h-s)}{2(1+\gamma-\alpha\gamma)-2\sqrt{1-\alpha}} < d \le d_{2,pool}$, then the difference in prices is negative. In Case 3,*pool*, the difference in prices is clearly negative. Therefore iff $d > \min\{d_{2,pool}, \frac{2\alpha(c+h-s)}{2(1+\gamma-\alpha\gamma)-2\sqrt{1-\alpha}}\}$, pricing to the "marginal consumer" with informed consumers leads to a lower price than pricing to the "average consumer" with uninformed consumers in a pooling equilibrium.

Q.E.D.

A.15. Derivation of Table 6.

To simplify the exposition of our proofs, we will first derive rows 1 and 2 of Table 6 and discuss our results for the single product case under different information structures. Next, we will present the 2-product cases under different information structures which are Rows 3, 4 and 5 of Table 6.

1. The Single Product Case

(Consumers either keep or return their initial purchase)

We first examine the equilibrium strategies for a firm selling one product (rows 1 and 2) under one and two types of uncertainties. This will help us identify the impact of different information structures on the seller's return policy.

1A. Single Product & 1 Type of Uncertainty (Column 1, rows 1 and 2)

Consider a firm selling one product located at $x_j = \frac{1}{2}$ *to consumers equally uninformed with 1 type of uncertainty about* $|x_j - \theta|$.

Consumers will keep their purchase if $u_H - d\,|\frac{1}{2} - \theta| - p > -f - h$. That is if

$\frac{1}{2} - \frac{u_H - p + f + h}{d} < \theta_i < \frac{1}{2} + \frac{u_H - p + f + h}{d}$. Let the subscript 1A denote the one product, one type of uncertainty setting. Therefore, the probability that someone keeps the good is equal to

$q_{k,1A} = \frac{2(u_h + f + h - p)}{d}$ and the probability that the good is returned is equal to $q_{r,1A} = 1 - \frac{2(u_H + f + h - p)}{d}$.

The expected utility from buying is then $E_{1A}(U) = \frac{(u_H - p + f + h)^2}{d} - f - h$. To be able to make a fair comparison to our more general model, we assume that the size of the market is equal to α which is the size of the segment of consumers who know that their utility $u_i > 0$. The seller faces the following constrained maximization problem (with Lagrangian multipliers λ_1, λ_2):

$$\max_{p,f,\lambda_1,\lambda_2} \pi_{1A} = \alpha((p-c)q_{k,1A} + (f-c+s)q_{r,1A}) + \lambda_1 E_{1A}(U) + \lambda_2 (1 - \frac{2(u_H + f + h - p)}{d}).$$
$$s.t. \lambda_2 q_{r,1A} = 0, \lambda_1 E_{1A}(U) = 0, \lambda_1 \geq 0, \lambda_2 \geq 0$$

The Kuhn-Tucker conditions are satisfied at:

(1) $p^* = u_H - \frac{d}{4},\ f^* = \frac{d}{4} - h\ \ \lambda_1 = \alpha, \lambda_2 = \frac{\alpha(2(h-s+u_H)-d)}{2}$

(2) $p^* = \frac{(u_H - s + h)^2}{d} - h + s,\ f^* = \frac{(u_H - s + h)^2}{d} - h,\ \lambda_1 = \alpha, \lambda_2 = 0.$

(1) is the equilibrium if and only if $\lambda_2 > 0$, which occurs if and only if $d < 2(h - s + u_H)$. Note that $2(h - s + u_H) > 0$ by fact that $u_H > c \geq s$. When the firm chooses $p^* = u_H - \frac{d}{4},\ f^* = \frac{d}{4} - h$, then there will be zero returns. Therefore, for any $f^* \geq \frac{d}{4} - h$, the firm will not collect the restocking fee, consumers will have the same expected utility as with $f^* = \frac{d}{4} - h$ and the firm will get the same profit. Therefore, any $f^* \geq \frac{d}{4} - h$ will also be an equilibrium.

Therefore, when $d \leq 2(h - s + u_H)$ **[case of low d values: (row 1, column 1) of Table 6],** then $p^* = u_H - \frac{d}{4},\ f^* \geq \frac{d}{4} - h$ implies $p^* - f^* \leq u_H + h - \frac{d}{2}$ and there are no returns. The refund may be less than, equal to or greater than *s*. It is less than *s* for high *f*, it can be equal to *s* (i.e. , when $d = 2(h - s + u_H)$ and $f^* = \frac{d}{4} - h$) and it can be greater than *s* (i.e. when $f^* = \frac{d}{4} - h$ and $d < 2(h - s + u_H)$).

When $d > 2(h-s+u_H)$ **[case of high d values: (row 2, column 1) of Table 6]** then the objective function is maximized at $p^* = \frac{(u_H - s+h)^2}{d} - h + s$, $f^* = \frac{(u_H - s+h)^2}{d} - h$, in which case $q_{r,1A} = 1 - \frac{2(u_H - s+h)}{d}$ and $p^* - f^* = s$.

1B. Single Product & 2 Types of Uncertainty (Column 2, rows 1 and 2)

Consider a firm selling one product located at $x_j = \frac{1}{2}$ *to consumers who are uncertain about both* u_θ *and* $|x_j - \theta_\theta|$.

Consumers get utility from the good $u_H - p - d\,|x_j - \theta|$ with probability α and have $u_i = 0$ with probability $1-\alpha$ where $|x_j - \theta|$ is unknown before purchase. Of the α consumers who get utility $u_H - p - d\,|x_j - \theta|$, the good will be kept if $\frac{1}{2} - \frac{u_H - p + f + h}{d} < \theta < \frac{1}{2} + \frac{u_H - p + f + h}{d}$. Otherwise the good will be returned. Therefore the probability that someone keeps the good is equal to

$q_{k,1B} = \frac{2\alpha(u_H + f + h - p)}{d}$ and the probability that the good is returned is equal to

$q_{r,1B} = 1 - \alpha + \alpha(1 - \frac{2(u_H + f + h - p)}{d})$. The expected utility from buying is then equal to

$E_{1B}(U) = \frac{\alpha(u_H - p + f + h)^2}{d} - (1-\alpha)(f+h)$ where the 1B subscript denotes the single product case under two types of uncertainties.

The seller has the following constrained maximization problem (with Lagrangian multipliers λ_1, λ_2):

$$\max_{p,f,\lambda_1,\lambda_2} \pi_{1B} = (p-c)q_{k,1B} + (f-c+s)q_{r,1B} + \lambda_1 E_{1B}(U) + \lambda_2 (1 - \frac{2(u_H + f + h - p)}{d})$$
$$s.t.\ \lambda_2\, q_{r,1B} = 0,\ \lambda_1 E_{1B}(U) = 0,\ \lambda_1 \ge 0,\ \lambda_2 \ge 0$$

The Kuhn-Tucker conditions are satisfied at:

(1) $p^* = u_H - \frac{d(2-\alpha)}{4}$, $f^* = \frac{\alpha d}{4} - h$ $\lambda_1 = 1,\ \lambda_2 = \frac{\alpha(2(h-s+u_H) - d)}{2}$

(2) $p^* = \frac{\alpha(u_H - s + h)^2}{d} - h + s$, $f^* = \frac{\alpha(u_H - s + h)^2}{d} - h$, $\lambda_1 = 1$, $\lambda_2 = 0$.

(1) is the equilibrium if and only if $\lambda_2 > 0$, which occurs if and only if $d < 2(h - s + u_H)$ .While no consumer with $u_\theta = u_H$ will return their purchase, there will be returns equal to $1-\alpha$ from consumers with $u_\theta = 0$. Therefore, for any $f \geq \frac{\alpha d}{4} - h$, the consumers' expected utility of purchase will be $\alpha(u_H - p - d/4) + (1-\alpha)(-f - h)$. Any price and restocking fee combination such that $u_H - \frac{\alpha d}{4} - h \geq f \geq \frac{\alpha d}{4} - h$ and $\alpha(u_H - p - d/4) + (1-\alpha)(-f - h) = 0$ (where the upper bound on f^* comes from the requirement that $f^* + h < p^*$), will yield profit equal to $\alpha u_H + (s-h)(1-\alpha) - c - d/4$ and thus will also maximize profit if $d < 2(h - s + u_H)$.

Therefore, when $d \leq 2(u_H - s + h)$ **[the case of low d values: (row 1, column 2) of Table 6],** any price and restocking fee combination such that $u_H - \frac{\alpha d}{4} - h \geq f^* \geq \frac{\alpha d}{4} - h$, and $\alpha(u_H - p^* - d/4) + (1-\alpha)(-f^* - h) = 0$ is an equilibrium. All consumers with $u_\theta = u_H$ will keep their purchase and the refund is $p^* - f^* \leq u_H + h - \frac{d}{2}$, which may be less than, equal to, or greater than the salvage value. It is less than *s* for high *f*, it can be equal to *s* (i.e when $p^* = u_H - \frac{d(2-\alpha)}{4}$, $f^* = \frac{\alpha d}{4} - h$ and $d = 2(u_H - s + h)$) and it can be greater than *s* (i.e. when $f = \frac{\alpha d}{4} - h$ and $d < 2(h - s + u_H)$).

If $d > 2(u_H - s + h)$ **[the case of high d values: (row 2, column 2) of Table 6]**, then $p^* = \frac{\alpha(u_H - s + h)^2}{d} - h + s$, $f^* = \frac{\alpha(u_H - s + h)^2}{d} - h$. This implies $q_{r,1B} = 1 - \frac{2\alpha(u_H - s + h)}{d}$ and $p^* - f^* = s$.

1C. Single Product, 2 Types of Uncertainty & 2 Consumer Segments

(Column 3, rows 1 and 2)

Consider a firm selling one product located at $x_j = \frac{1}{2}$ *to a consumer segment with 2 types of uncertainty (both* u_θ *and* $|x_j - \theta|$*) and another segment with only product fit uncertainty(*$|x_j - \theta|$*).*

Consumers get utility from the good $u_H - p - d|x_j - \theta|$ with probability α and have $u_\theta = 0$ with probability $1-\alpha$ where $|x_j - \theta|$ is unknown before purchase. A proportion γ knows the value of u_θ before purchase and a proportion $1-\gamma$ learns the value of u_θ only after purchase.

The $\gamma(1-\alpha)$ consumers who know $u_\theta = 0$ will not buy. The actions of the consumers who have one type of uncertainty and $u_\theta = u_H$ is derived in 1.A (occurring with probability $\gamma\alpha$). The actions of the consumers who have two types of uncertainty is derived in 1.B (occurring with probability $1-\gamma$). Let the subscript 1C denote the scenario with one product, two forms of uncertainty, and two segments varying in information. Therefore, the probability that someone keeps the good is equal to $q_{k,1C} = \frac{2\alpha(u_H + f + h - p)}{d}$ and the probability that the good is returned is equal to $q_{r,1C} = (1-\gamma)(1-\alpha) + \alpha(1 - \frac{2(u_H + f + h - p)}{d})$. Notice that expected utility of the consumer with two types of uncertainty is less than the expected utility of the consumer with one type of uncertainty (i.e.,) $E_{1B}(U) < E_{1A}(U)$, so the firm must make $E_B(U) \geq 0$ to get both consumer segments to buy. The 1A, 1B and 1C denote the three information structures described in 1A, 1B, and 1C respectively.

The seller's has the following constrained maximization problem (with Lagrangian multipliers λ_1, λ_2):

$$\max_{p, f, \lambda_1, \lambda_2} \pi_{1C} = (p-c)q_{k,1C} + (f - c + s)q_{r,1C} + \lambda_1 E_{1B}(U) + \lambda_2 (1 - \frac{2(u_H + f + h - p)}{d}) .$$
$$s.t. \lambda_2 q_{r,1C} = 0, \lambda_1 E_{1B}(U) = 0, \lambda_1 \geq 0, \lambda_2 \geq 0$$

The Kuhn-Tucker conditions are satisfied at:

(1) $p^* = u_H - \frac{d(2-\alpha)}{4}$, $f^* = \frac{\alpha d}{4} - h$ $\lambda_1 = 1 - \gamma + \alpha\gamma,$ $\lambda_2 = \frac{\alpha(2(h - s + u_H) - d(1 + \gamma - \alpha\gamma))}{2}$

(2) $p^* = \frac{\alpha(u_H - s + h)^2}{d(1+\gamma-\alpha\gamma)^2} - \frac{h - s - u_H\gamma(1-\alpha)}{(1+\gamma-\alpha\gamma)}$, $f^* = \frac{\alpha(u_H - s + h)^2}{d(1+\gamma-\alpha\gamma)^2} - h$, $\lambda_1 = 1-\gamma+\alpha$, $\lambda_2 = 0$

(1) is the equilibrium if and only if $\lambda_2 > 0$, which occurs if and only if $d < \frac{2(h - s + u_H)}{1+\gamma-\alpha\gamma}$. In this case, no consumer with $u_\theta = u_H$ will return their purchase, there will be returns equal to $1-\alpha$ from consumers with $u_\theta = 0$. However, only $f^* = \frac{\alpha d}{4} - h$ will maximize profit. At this point, increasing *f* by ε would require that *p* is decreased by $\frac{(1-\alpha)}{\alpha}\varepsilon$ to ensure that $E_{1B}(U) \geq 0$. The restocking fee will only be paid by the $(1-\gamma)(1-\alpha)$ while the price decrease will apply to all α consumers who keep a product. Therefore, the net profit effect of an ε increase in restocking fee is $-\gamma(1-\alpha)$. Therefore:

If $d \leq \frac{2(u_H - s + h)}{1+\gamma-\alpha\gamma}$ **(the case of low d values: (row 1, column 3 of Table 6)**, then

$p^* = u_H - \frac{d(2-\alpha)}{4}$, $f^* = \frac{d\alpha}{4} - h$. All consumers with $u_\theta = u_H$ will keep their purchase and

$p^* - f^* = u_H + h - \frac{d}{2} > s$.

If $d > \frac{2(u_H - s + h)}{1+\gamma-\alpha\gamma}$ **(the case of high d values: (row 2, column 3 of Table 6),** then

$p^* = \frac{\alpha(u_H - s + h)^2}{d(1+\gamma-\alpha\gamma)^2} - \frac{h - s - u_H\,\gamma(1-\alpha)}{(1+\gamma-\alpha\gamma)}$, $f^* = \frac{\alpha(u_H - s + h)^2}{d(1+\gamma-\alpha\gamma)^2} - h$. This implies $q_{r,1C} = 1-\gamma+\alpha - \frac{2\alpha(u_H - s + h)}{d(1+\gamma-\alpha\gamma)}$

and $p^* - f^* = \frac{s + \gamma(u_H + h)(1-\alpha)}{1+\gamma-\alpha\gamma}$. To show that $p^* - f^* > s$, we evaluate

$p^* - f^* - s = \frac{(u_H - s + h)(1-\alpha)\gamma}{(1+\gamma-\alpha\gamma)} > 0$ by fact that $u_H > c > s$.

2. Two - Product Case

(Consumers keep, return or exchange their initial purchase)

In this section we examine seller's return policies under different information structures when there are two horizontally differentiated products offered in the market. In this analysis, we will derive the entries of Table 6 in rows 3, 4 and 5.

2A. Two Products & 1 Type of Uncertainty (Column 1, rows 3, 4 and 5)

Consider a firm selling two products located at at 0 and 1/2 to consumers equally uninformed with 1 type of uncertainty about $|x_j - \theta|$.

Let the subscript 2A denote the 2-product, one uncertainty setting. Following the logic in section A.4. of this Electronic Companion, when consumers know the value of u_θ, but not the value of $|x_j - \theta|$, then:

$$q_{e2A} = \frac{1}{2} - \frac{f+h}{d}$$
$$q_{r2A=0}$$
$$q_{k2A} = \frac{1}{2} + \frac{f+h}{d}$$
$$E_{2A}(U) = u_H - p + \frac{(f+h)^2}{2d} - \frac{(f+h)}{2} - \frac{d}{8}$$

The seller's profit function π_{2A} is equal to $\frac{\pi_{sep}}{\alpha}$, and thus will be maximized at the same price and restocking fee as in Proposition 1a. As proven in section A.5., the price and restocking fee that will maximize π_D are given by

Parameter Values	Retail Price, p^* and Restocking Fee, f^*	$p^* - f^*$
Case 1: (row 1 of Table 6) $d < d_{1,A}$	$p^* = u_H - \frac{d}{4}$ $f^* \geq \frac{d-2h}{2}$	$p^* - f^* \leq u_H - \frac{3d}{4} + h$
Case 2: (row 2 of Table 6) $d_{1,A} \leq d \leq d_{2,A}$	$p^* = u_H - \frac{d}{8} - \frac{c-s+h}{2} + \frac{(c-s+h)^2}{2d}$ $f^* = c - s$	$p^* - f^* = s + (p^* - c)$
Case 3: (row 3 of Table 6) $d \geq d_{2,A}$	$p^* = u_H - \frac{d(\sqrt{3}-1)}{4}$ $f^* = d\left(1 - \frac{\sqrt{3}}{2}\right) - h$	$p^* - f^* = u_H + h - \frac{d(3-\sqrt{3})}{4}$

where $d_{1,2A} \equiv 2(c-s+h)$ *and* $d_{2,2A} \equiv (4+2\sqrt{3})(c-s+h)$.

If $d < d_{1,2A} \equiv 2(c-s+h)$ **(the case of low d values: (row 3, column 1 of Table 6)**, there exist a wide range of refunds possible in equilibrium but it is not paid to anyone because all consumers keep their purchase (Case 1). By fact that $f^* \geq \frac{d-2h}{2}$, there is a maximum value on the refund: $p^* - f^* \leq u_H - \frac{3d}{4} + h$. We now show that the value of the refund may be greater than, equal to, or less than *s*. Because $p^* - f^* \leq u_H - \frac{3d}{4} + h$, it can clearly be less than *s* for high *f*. It can be equal to *s* (i.e.

$f^* = u_H - s - \frac{d}{4}$ for values of u_H such that $u_H - s - \frac{d}{4} > \frac{d-2h}{2}$), and it can be greater than s (i.e. when $f = \frac{d-2h}{2}$ and $d < 2(c+h-s)$).

If $2(c+h-s) < d < d_{2,2A} \equiv (4+2\sqrt{3})(c-s+h)$ **(the case of mid d values: (row 4, column 1 of Table 6)**, then Case 2 is the equilibrium, $q_{e2A} = \frac{1}{2} - \frac{c-s+h}{d} > 0$ and $p^* - f^* = s + (p^* - c)$.

If $d > d_{2,2A}$ **(the case of high d values: (row 5 , column 1 of Table 6)**, then Case 3 is the equilibrium. Because $f^* = d\left(1 - \frac{\sqrt{3}}{2}\right) - h > c - s$ by fact that $d \geq d_{2,2A}$. The refund $p^* - f^* < s + p^* - c$ for all u_H. The conditions $p^* > 0$ and $p^* - f^* < s$ hold if $d(\sqrt{3}-1) < u_H < s + \frac{d(3-\sqrt{3})}{4} - h$. If $u_H > s + \frac{d(3-\sqrt{3})}{4} - h$ then $p^* - f^* > s$ □

2B. Two Products & 2 Types of Uncertainty (Column 2, rows 3, 4, and 5)

Consider a firm selling two products located at 0 and 1/2 to consumers who are uncertain about both u_θ *and* $|x_j - \theta|$.

Here we assume that all consumers have a probability α of having $u_\theta = u_H$ and $(1-\alpha)$ probability of having $u_\theta = 0$. Let the 2B subscript denote the two-product, two types of uncertainty setting. By the same logic in A.2., the expected quantities and utilities when all consumers have two types of uncertainty can be written:

$$q_{e2B}(f;d,\gamma,\alpha) = \alpha(\frac{1}{2} - \frac{f+h}{d}),\ q_{k2B}(f;d,\gamma,\alpha) = \alpha(\frac{1}{2} + \frac{f+h}{d}),\ q_{r2B}(f;d,\gamma,\alpha) = (1-\alpha)$$

and $E_{2B}(U) = \alpha(u_H - p + \frac{(f+h)^2}{2d} - \frac{(f+h)}{2} - \frac{d}{8}) - (1-\alpha)(f+h)$. We restrict attention to when market quantities are non-negative and all consumers in Segments 1 and 3 eventually keep one of the products. As in the proof of Proposition 1a, one can show that the profit-maximizing p is set such that $E_{2B}(U)=0$. We first solve for the equilibrium and then we will evaluate the restocking fee and the refund. We have the following constrained optimization problem with Lagrangian muliplers λ_2, λ_3 :

$$\max_{f} \; L(f,\lambda_2,\lambda_3;c,s,u,d,\gamma,\alpha)=(p-c)q_{k2B}+(f+s-c)q_{r2B}$$
$$+(p-c+f-p+s+p-c)(q_{e2B})$$
$$+\lambda_2 q_{e2B}+\lambda_3(4(u_H-p)+2(f+h)-d))$$

subject to

$$E_{2B}(U)=0$$
$$\lambda_2 q_{e2B}=0$$
$$\lambda_3(4(u_H-p)+2(f+h)-d)=0$$

We analyze the four solutions that satisfy the Kuhn-Tucker conditions:

(Solution 2B1) $p=u_H-\dfrac{d(2-\alpha)}{4\alpha}$, $f=\dfrac{d}{2}-h$, $\lambda_2=\dfrac{2\alpha(c+h-s)-d\alpha}{2\alpha}$, $\lambda_3=0$. This implies that $q_{e2B}=0$. The condition that $\lambda_2\geq 0$ is satisfied if and only if $d\leq 2(c-s+h)$. While no consumer with $u_\theta=u_H$ will exchange their purchase, there will be returns equal to $1-\alpha$ from consumers with $u_\theta=0$. Therefore, for any $f\geq\dfrac{\alpha d}{4}-h$, the consumers' expected utility of purchase will be $\alpha(u_H-p-d/4)+(1-\alpha)(-f-h)$. Any price and restocking fee combination such that $f\geq\dfrac{\alpha d}{4}-h$ and $\alpha(u_H-p-d/4)+(1-\alpha)(-f-h)=0$, will yield profits equal to $\alpha u_H+(s-h)(1-\alpha)-c-d/4$ and thus will maximize profit when $d\leq 2(c-s+h)$.

(Solution 2B2) $p=\alpha(u_H-\dfrac{d}{8}-\dfrac{c-s+h}{2}+\dfrac{(c-s-+h)^2}{2d})-(1-\alpha)(c-s-h)$

$f = c - s,\ \lambda_2 = 0, \lambda_3 = 0$. This implies $q_{e2B} = \dfrac{\alpha(d - 2(c + h - s))}{2d} \geq 0$ if and only if $d \geq 2(c - s + h)$.

This solution satisfies the full market coverage constraint $4(u_H - p) + 2(f + h) - d \geq 0$, if and only if $d \leq \dfrac{2\alpha(c - s + h)}{2 - \sqrt{4 - \alpha^2}}$. Therefore the solution is feasible if and only if $d \in [2(c - s + h), \dfrac{2\alpha(c + h - s)}{2 - \sqrt{4 - \alpha^2}}]$.

(Solution 2B3) $p = u_H - \dfrac{d(\sqrt{4 - \alpha^2} - 2 + \alpha)}{4\alpha}$, $f = \dfrac{d(2 - \sqrt{(4 - \alpha^2)})}{2\alpha} - h$, $\lambda_2 = 0$,

$\lambda_3 = \dfrac{\alpha}{4}(-1 + \dfrac{2d - 2\alpha(c + h - s)}{d\sqrt{4 - \alpha^2}})$. This implies $q_{e2B} = \dfrac{(\sqrt{4 - \alpha^2} - (2 - \alpha))}{2} \geq 0$. (To check that $q_{eZ} \geq 0$, we examine the numerator. $(\sqrt{4 - \alpha^2} - (2 - \alpha)) \geq 0$ if $\sqrt{4 - \alpha^2} \geq (2 - \alpha)$. Using the positive root of $\sqrt{4 - \alpha^2}$, this is satisfied if $4 - \alpha^2 \geq (2 - \alpha)^2$, implying $2\alpha(2 - \alpha) \geq 0$. This holds by the fact that $\alpha \leq 1$.)

The value of λ_3 is non-negative if and only if $d \geq \dfrac{2\alpha(c + h - s)}{2 - \sqrt{4 - \alpha^2}}$. Therefore, **(2B3)** satisfies the Kuhn-Tucker conditions if and only if $d \geq \dfrac{2\alpha(c + h - s)}{2 - \sqrt{4 - \alpha^2}}$.

(Solution 2B4) $p = u_H + \dfrac{d(2 - \alpha + \sqrt{4 - \alpha^2})}{4\alpha}$, $f = \dfrac{d(2 + \sqrt{4 - \alpha^2})}{2\alpha} - h$, $\lambda_2 = 0$,

$\lambda_3 = \dfrac{\alpha}{4}(-1 + \dfrac{2\alpha(c + h - s) - 2d}{d\sqrt{4 - \alpha^2}})$. This solution has $p > u_H$ for positive roots of $\sqrt{4 - \alpha^2}$. We rule out this potential solution because $p > u_H$ implies that the price of the good is greater than the maximum consumer's valuation for it. □

We summarize these findings on price and restocking fee in the following table.

Parameter Values	Retail Price, p^* and Restocking Fee, f^*	$p^* - f^*$
Case 1,2B: (row 3 of Table 6) $d < d_{1,2B}$	$\alpha(u_H - p^* - d/4) + (1-\alpha)(-f^* - h) = 0$ $f^* \geq \frac{d-2h}{2}$	$p^* - f^* \leq u_H + h - \frac{d(2+\alpha)}{4\alpha}$
Case 2,2B: (row 4 of Table 6) $d_{1,2B} \leq d \leq d_{2,2B}$	$p^* = u_H - \frac{d}{8} - \frac{c-s+h}{2} + \frac{(c-s+h)^2}{2d}$ $f^* = c - s$	$p^* - f^* = s + (p^* - c)$
Case 3,2B: (row 5 of Table 6) $d \geq d_{2,2B}$	$p^* = u_H - \frac{d(2-\alpha-\sqrt{4-\alpha^2})}{4\alpha}$ $f^* = d\left(\frac{2-\sqrt{4-\alpha^2}}{2\alpha}\right) - h$	$p^* - f^* = u_H + h - \frac{d(2+\alpha-\sqrt{4-\alpha^2})}{4\alpha}$

Where $d_{1,2B} \equiv 2(c+h-s)$ and $d_{2,2B} \equiv \frac{2(c+h-s)}{2-\sqrt{4-\alpha^2}}$.

If $d \leq 2(c+h-s)$ **(the case of low d values: (row 3, column 2 of Table 6)**, then consumers with $u_i = u_H$ all keep their initial purchase. By fact that $\alpha(u_H - p^* - d/4) + (1-\alpha)(-f^* - h) = 0$, $p^* - f^* = u_H + h - \frac{(f+h)}{\alpha} - \frac{d}{4}$**.** By fact that $f^* \geq \frac{d-2h}{2}$, there is a maximum value on the refund: $p^* - f^* \leq u_H + h - \frac{d(2+\alpha)}{4\alpha}$. We now show that the value of the refund may be greater than, equal to, or less than *s*. Because $p^* - f^* \leq u_H + h - \frac{d(2+\alpha)}{4\alpha}$, it can clearly be less than s for high *f*. It can be equal to *s* (i.e. u_H values for which $p^* = \alpha u_H + (s-h)(1-\alpha) - \frac{\alpha d}{4}$ and $f^* = \alpha u_H + h(1-\alpha) + s\alpha - \frac{\alpha d}{4}$ satisfy the requirements on *p* and *f*), and it can be greater than *s* (i.e. when $f = \frac{d-2h}{2}$ and $d < 2(c+h-s)$).

If $2(c+h-s) \le d \le \frac{2(c+h-s)}{2-\sqrt{4-\alpha^2}}$ **(the case of mid d values: (row 4, column 2 of Table 6)**, then clearly $p^* - f^* = s + (p^* - c)$.

If $d > \frac{2(c+h-s)}{2-\sqrt{4-\alpha^2}}$ **(the case of high d values: (row 5 , column 2 of Table 6)**, then the equilibrium will be as described in Case 3,2B. In this case, $p^* > 0$ and $p^* - f^* < s$ if $\frac{d(a-2+\sqrt{4-\alpha^2})}{4\alpha} < u_H < s + \frac{d(2+\alpha-\sqrt{4-\alpha^2})}{4\alpha} - h$. If $u_H > s + \frac{d(2+\alpha-\sqrt{4-\alpha^2})}{4\alpha} - h$, then $p^* - f^* > s$.

And $f^* = d\left(\frac{2-\sqrt{4-\alpha^2}}{2\alpha}\right) - h > c - s$ by fact that $d \ge d_{2,2B}$ implies that $p^* - f^* > s + (p^* - c)$.

□

2C. Two Products, 2 Types of Uncertainty & 2 Consumer Segments

2C. *Consider a firm selling two products located at 0 and 1/2 to a consumer segment with 2 types of uncertainty (both* u_θ *and* $|x_j - \theta|$*) and another segment with only product fit uncertainty(*$|x_j - \theta|$*).*

This case corresponds to the Pooling Equilibrium of the general model we presented in section A.4. When parameter values are such that the firm will choose to target only the more informed segment (Separating Equilibrium), the results will be as described in the paper and refund will be as shown in part 2A above. Therefore, we will not regenerate the same results here. To obtain the results in rows 3-5 of column 3 in Table 6, we compare $p^* - f^*$ to $(s + p^* - c)$ by evaluating $p^* - f^* - (s + p^* - c)$ for each range of *d* .

Parameter Values	$p^* - f^* - (s + p^* - c)$
Case 1, *pool*: (row 3, col. 3 of Table 6) $d \le d_{1,pool}$	$c + h - s - d/2$
Case 2,*pool*: (row 4, col. 3 of Table 6) $d_{1,pool} \le d < d_{2,pool}$	$\frac{d\gamma(1-\alpha)}{\alpha}$
Case 3, *pool*: (row 5, col. 3 of Table 6) $d \ge d_{2,pool}$	$c + h - s - \frac{d(2-\sqrt{4-\alpha^2})}{2\alpha}$

Where $d_{1,pool} \equiv \frac{2\alpha(c-s+h)}{\alpha + 2\gamma(1-\alpha)}$ *and* $d_{2,pool} \equiv \frac{2\alpha(c+h-s)}{2(1+\gamma-\alpha\gamma) - \sqrt{4-\alpha^2}}$.

If $d \le \frac{2\alpha(c-s+h)}{\alpha + 2\gamma(1-\alpha)}$, **(the case of low d values: (row 4, column 3 of Table 6)** then $c + h - s - d/2$ is positive implying $p^* - f^* > (s + p^* - c)$

If $d_{1,pool} \le d < \frac{2\alpha(c+h-s)}{2(1+\gamma-\alpha\gamma) - \sqrt{4-\alpha^2}}$, **(the case of mid d values: (row 4, column 3 of Table 6)** then $p^* - f^* > (s + p^* - c)$.

If $\frac{2\alpha(c+h-s)}{2-\sqrt{4-\alpha^2}} > d \ge \frac{2\alpha(c+h-s)}{2(1+\gamma-\alpha\gamma) - \sqrt{4-\alpha^2}}$, **(the case of high d values: (row 4, column 3 of Table 6)** then $p^* - f^* > (s + p^* - c)$. Otherwise, $p^* - f^* \le (s + p^* - c)$. Moreover, $p^* > 0$ and $p^* - f^* < s$ if $\frac{d(\alpha - 2 + \sqrt{4-\alpha^2})}{4\alpha} < u_H < s - h + \frac{d(2+\alpha-\sqrt{4-\alpha^2})}{4\alpha}$. If $u_H > s - h + \frac{d(2+\alpha-\sqrt{4-\alpha^2})}{4\alpha}$, then $p^* - f^* > s$

Q.E.D.

A.16. Derivation of Incremental Value of Each Type of Information.

There are three information structures to compare in terms of the value of information to the seller. More specifically, these are:

Model 1: 2 types of product uncertainty: All consumers are uncertain about u_θ and $|x_j - \theta|$.

Model 2: 1 type of product uncertainty: All consumers know value of u_θ, but are uncertain about $|x_j - \theta|$.

Model 3: Full Information: All consumers know the value of both u_θ and $|x_j - \theta|$.

1. The optimal price and restocking fee for the first model is derived in part 2B of A.15. The resulting profit is:

Parameter values	Seller Profit
$d \le 2(c+h-s)$	$\alpha u_H + (s-h)(1-\alpha) - c - d/4$
$2(c+h-s) < d \le \dfrac{2(c+h-s)}{2-\sqrt{4-\alpha^2}}$	$\alpha(u_H - \dfrac{d}{8} + \dfrac{(s-h-c)^2}{2d}) + (s-h)(1-\dfrac{\alpha}{2}) - c(1+\dfrac{\alpha}{2})$
$d > \dfrac{2(c+h-s)}{2-\sqrt{4-\alpha^2}}$	$\alpha(u_H + \dfrac{-c+h-s}{2}) - \dfrac{d(2-\sqrt{4-\alpha^2})}{2\alpha} - \dfrac{(c+h-s)(\sqrt{4-\alpha^2})}{2}$

2. The optimal price and restocking fee for the second model is derived in part 2A of A.15. The resulting profit is:

Parameter values	Seller Profit
$d \le 2(c+h-s)$	$\alpha(u_H - c - \frac{d}{4})$
$2(c+h-s) < d \le (4+2\sqrt{3})(c-s+h)$	$\pi^* = \alpha(u_H - c - \frac{d}{8} - \frac{c-s+h}{2} + \frac{(c-s+h)^2}{2d})$
$d > (4+2\sqrt{3})(c-s+h)$	$\pi^* = \alpha(u_H - c - \frac{d(2-\sqrt{3})}{2} - \frac{(c+h-s)(\sqrt{3}-1)}{2})$

3. The profit from the third model is presented in Section 5 in the paper. Profit equals $\pi^* = \alpha(u_H - c - \frac{d}{4})$

Comparison of Seller Profits from model 1 to model 2

Below we show that if the seller were able to give information only about u_θ to consumers who had both types of uncertainty, this would increase profits for lower values of *d* (that is for low *d*, profit from model 2 is greater than profit from model 1 and for high *d*, profit from model 1 is greater than model 2).

We have four ranges of d to examine. We will show that for low *d*, profit from model 2 is greater than profit from model 1 and for high *d*, profit from model 1 is greater than model 2.

a) If $d \le 2(c+h-s)$, the difference (profit model 1- profit model 2) equals $-(1-\alpha)(c+h-s) < 0$.

b) If $2(c+h-s) < d \le (4+2\sqrt{3})(c-s+h)$, the difference (profit model 1- profit model 2) also equals $-(1-\alpha)(c+h-s) < 0$.

c) If $(4+2\sqrt{3})(c-s+h) < d \le \dfrac{2(c+h-s)}{2-\sqrt{4-\alpha^2}}$, the difference (profit model 1- profit model 2) equals

$\dfrac{4\alpha(c+h-s)^2}{8d}+\dfrac{\alpha d(7-4\sqrt{3})}{8}-\dfrac{(c+h-s)(2-\alpha\sqrt{3})}{2}$. Next we show that if α is such that

$\dfrac{2(c+h-s)}{2-\sqrt{4-\alpha^2}} > \dfrac{2(c+h-s)(2+2\sqrt{(1-\alpha)(1+\alpha-\alpha\sqrt{3})}-\alpha\sqrt{3})}{(7-4\sqrt{3})} > (4+2\sqrt{3})(c-s+h)$, then the expression (profit model 1- profit model 2) is negative for $d < \dfrac{2(c+h-s)(2+2\sqrt{(1-\alpha)(1+\alpha-\alpha\sqrt{3})}-\alpha\sqrt{3})}{(7-4\sqrt{3})}$ and positive otherwise.

Note that the derivative of this expression is equal to $\dfrac{\alpha((7-4\sqrt{3})d^2-4(c+h-s)^2)}{8d^2}$, which is positive for all $d > (4+2\sqrt{3})(c-s+h)$ and the expression has two roots:

$d = \dfrac{2(c+h-s)(2+2\sqrt{(1-\alpha)(1+\alpha-\alpha\sqrt{3})}-\alpha\sqrt{3})}{(7-4\sqrt{3})}$ and

$d = \dfrac{2(c+h-s)(2-2\sqrt{(1-\alpha)(1+\alpha-\alpha\sqrt{3})}-\alpha\sqrt{3})}{(7-4\sqrt{3})} < (4+2\sqrt{3})(c-s+h)$. We may ignore the second root because it is outside the relevant range. Therefore, if α is such that

$\dfrac{2(c+h-s)}{2-\sqrt{4-\alpha^2}} > \dfrac{2(c+h-s)(2+2\sqrt{(1-\alpha)(1+\alpha-\alpha\sqrt{3})}-\alpha\sqrt{3})}{(7-4\sqrt{3})} > (4+2\sqrt{3})(c-s+h)$, then the expression is negative for $d < \dfrac{2(c+h-s)(2+2\sqrt{(1-\alpha)(1+\alpha-\alpha\sqrt{3})}-\alpha\sqrt{3})}{(7-4\sqrt{3})}$ and positive otherwise.

Because the profit difference expression is negative if and only if

$d < \dfrac{2(c+h-s)(2+2\sqrt{(1-\alpha)(1+\alpha-\alpha\sqrt{3})}-\alpha\sqrt{3})}{(7-4\sqrt{3})}$, it will be positive over the range

$(4+2\sqrt{3})(c-s+h) < d \le \dfrac{2(c+h-s)}{2-\sqrt{4-\alpha^2}}$ if α is such that

$\frac{2(c+h-s)(2+2\sqrt{(1-\alpha)(1+\alpha-\alpha\sqrt{3}})-\alpha\sqrt{3})}{(7-4\sqrt{3})} < (4+2\sqrt{3})(c-s+h)$, and it will negative over this range if α is

such that $\frac{2(c+h-s)(2+2\sqrt{(1-\alpha)(1+\alpha-\alpha\sqrt{3}})-\alpha\sqrt{3})}{(7-4\sqrt{3})} > \frac{2(c+h-s)}{2-\sqrt{4-\alpha^2}}$.

d) If $d > \frac{2(c+h-s)}{2-\sqrt{4-\alpha^2}}$, the difference (profit model 1 – profit model 2) equals

$\frac{(c-s+h)(\alpha\sqrt{3}-\sqrt{4-\alpha^2})+d(-2+(2-\sqrt{3})\alpha^2+\sqrt{4-\alpha^2})}{2\alpha}$. Below we show that this expression is negative if

and only if $d < \frac{\alpha(c+h-s)(\sqrt{4-\alpha^2}-\alpha\sqrt{3})}{-2+\alpha^2(2-\sqrt{3})+\sqrt{4-\alpha^2}}$.

Note that the derivative with respect to d is $(-2+(2-\sqrt{3})\alpha^2+\sqrt{4-\alpha^2})$. We show this derivative is positive. The inequality $(-2+(2-\sqrt{3})\alpha^2+\sqrt{4-\alpha^2})>0$ holds if $\sqrt{4-\alpha^2} > 2-(2-\sqrt{3})\alpha^2$, which is true if $4-\alpha^2 > (2-(2-\sqrt{3})\alpha^2)^2$, which is true if $\alpha^2(7-4\sqrt{3})(1-\alpha^2)>0$ which is true by $\alpha<1$. The expression is equal to zero at $d = \frac{\alpha(c+h-s)(\sqrt{4-\alpha^2}-\alpha\sqrt{3})}{-2+\alpha^2(2-\sqrt{3})+\sqrt{4-\alpha^2}}$ and is therefore negative if and only if

$d < \frac{\alpha(c+h-s)(\sqrt{4-\alpha^2}-\alpha\sqrt{3})}{-2+\alpha^2(2-\sqrt{3})+\sqrt{4-\alpha^2}}$. □

Therefore, for low *d*, profit from model 2 is greater than profit from model 1 and for high *d,* profit from model 1 is greater than model 2. That is, if the seller were able to give information only about u_i to consumers who had both types of uncertainty, this would increase profits for lower values of *d* and reduce profits for higher *d* values.

Q.E.D.

Comparison of Seller Profits from model 2 to model 3

Below we show that when all consumers have information about u_θ, providing information about product fit $|x_j - \theta|$ will have a detrimental effect on seller profit (profit from model 2 is greater than profit from model 3).

Again, we have three regions of *d* to examine.

a) If $d < 2(c+h-s)$, the difference (profit model 2- profit model 3) equals to 0

b) If $2(c+h-s) < d \le (4+2\sqrt{3})(c-s+h)$, the difference (profit model 2- profit model 3) equals to

$\frac{\alpha(d-2(c+h-s))}{8d}$, which is greater than zero by fact that $d > 2(c+h-s)$.

c) If $d > (4+2\sqrt{3})(c-s+h)$, the difference (profit model 2- profit model 3) equals to

$\frac{\alpha(d(-3+2\sqrt{3})+(2-2\sqrt{3})(c+h-s)}{4}$, which is greater than zero by fact that $d > (4+2\sqrt{3})(c-s+h)$.

Therefore, profit from model 2 is greater than profit from model 3 for all *d*.

Q.E.D.

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