DOES IT PAY TO SHROUD IN-APP PURCHASE PRICES?

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ABSTRACT

App developers commonly sell their apps at relatively low prices and subsequently earn substantial revenue from in-app purchases. Although some consumers may do their research about in-app prices before deciding whether to buy the app, others only discover the in-app prices later in the purchase process. This paper presents an analytical model predicting the profit and welfare implications of hidden prices of the in-app purchases. Whereas practitioners posit that hidden pricing benefits firms at the disadvantage of consumers, the consensus from the academic literature is that any gains from hidden prices are negated by competition in available prices. This paper has three main contributions. First, it resolves discrepancy between theory and practice by finding a profit improvement effect of hidden prices under circumstances for which prior literature predicts profit irrelevance. In this regard, the model identifies a new mechanism driving the profit-improvement result. Second, it finds when app developers can be made better off by platforms disclosing in-app purchase prices. Third, it shows how platform decisions intended to improve pricing transparency may actually diminish consumer welfare. The findings have implications for app developers considering hidden in-app purchases as well as platforms such as Google, Apple, and Amazon who can restrict this practice.

Keywords: Game Theory, In-App Purchase, Shrouded Prices, Pricing
1. Introduction

Nowadays it is a common practice for developers on mobile platforms to first offer their apps at a low price to consumers, and subsequently charge consumers more for optional functionalities or content through in-app purchases. For example, as of April 2016, mobile game “Minecraft: Pocket Edition” is listed for $6.99 in both Apple App Store and Google Play Store.¹ Within the game, consumers can purchase new skins for game characters at $0.99 per 20-skin. A popular fitness app “7 Minute Workout Challenge” is listed for $2.99, and in the app users can purchase additional workout packs for $0.99 each. In-app purchases account for a significant portion of total app revenues on mobile platforms – market research firm Statista projects that, by 2017, roughly half of global app revenue will come from in-app purchases.²

Consumer advocacy groups and policy makers, however, often raise concerns that in-app purchases lack transparency. Triggered by “a large number of complaints in EU countries concerning in-app purchases,” the European Commission in 2014 cautioned the leading mobile platforms that “games advertised as ‘free’ should not mislead consumers about the true costs involved.”³ While as of 2016 Apple App Store requires developers to publish the (range of) cost of in-app purchases, this cost information is positioned at the very end of the app’s long description page that is reachable only after scrolling. In Google Play Store, in addition to scrolling, consumers also need to click a “read more” link to reach the cost information of in-app purchases. The practice of hiding the price of in-app purchase from consumers is not all accidental -- the CEO of one app developer, Montessorium, admits that they “buried the in-app purchases too deep”.⁴

It is widely believed in the popular press that such hidden prices of in-app purchases can result in a profit gain for developers and platforms. Chuck Jones at Forbes magazine states that “in-app purchase has become the greatest revenue driver for apps” (Jones 2013). However, the academic literature theorizes

¹ Prices checked at Apple App Store and Google Play on April 16, 2016.
that, in most cases, shrouding prices of optional add-ons (in-app purchases are one example) will have no effect on firm profitability (Lal and Matutes 1994; Verboven 1999; Gabaix and Laibson 2006). This paper intends to fill in the gap between theory and practice by identifying new conditions under which shrouded in-app purchases can benefit competing app developers.\(^5\)

Our study further asks, from the perspective of mobile platforms and app developers, how much price shrouding regarding in-app purchases is optimal. While the common wisdom would suggest that the more price shrouding, the more consumers can be taken advantage of, anecdotal evidence suggests that it is not always a monotonic relationship between price shrouding and firm profit. In the aforementioned app case of Montessorium, for example, the adoption of in-app purchases did not bring the revenue growth the business hoped for, and the CEO concluded that “it’s better to be more upfront with the purchase for the sake of transparency.”

On the consumer side, policy makers have expressed a view that consumers will be better off if the prices of in-app purchases are more transparent. The Office of Fair Trading in the United Kingdom calls for price information to be provided “clearly, accurately and prominently up-front, before the consumer … agrees to make a purchase”.\(^6\) The European Commission has actively engaged with leading mobile platforms on how to provide “clearer explanations about the costs involved in apps.”\(^7\) This paper examines whether having more consumers informed about the prices of in-app purchases actually benefits consumers.

In summary, the objective of this paper is to address the following research questions.

1. Do app developers benefit from a platform shrouding in-app purchase prices?

\(^5\) The literature on freemium pricing, e.g. Cheng et al. (2014) and Niculescu and Wu (2014), shows that a firm can profit from offering the base version for free and charging for the premium version if consumers learn the value of the premium version through the usage of the base version. Our paper does not depend on this learning effect. Furthermore, the focus of our paper is the strategic implications of the shrouded in-app purchase prices. The freemium literature does not consider price shrouding.


2. Does having more consumers uninformed about the prices of in-app purchases always lead to higher firm and platform profits? Relatedly, what should be a mobile platform’s optimal disclosure policy regarding in-app purchases?

3. Does more transparency regarding the prices of in-app purchases always lead to higher consumer welfare?

With regard to the relationship between shrouding of in-app purchase prices and firm profitability, previous literature either suggests that shrouding does not affect profitability (Lal and Matutes 1994; Verboven 1999; Gabaix and Laibson 2006), or requires price-sensitivity heterogeneity among consumers (Ellison 2005) or quality heterogeneity among competing firms (Shulman and Geng 2013) for shrouding to affect profitability. In addressing the first research question, this paper identifies a new competition-softening effect by which shrouding in-app purchase prices can increase profitability. Specifically, this research shows that consumers who do not observe the in-app purchase prices will rationally expect them to be negatively correlated with the observed base prices. Consequently, a firm’s price cut on the app price is less effective in attracting consumers, which softens competition and boosts firm profitability.

In addressing the second research question, the research finds that having more consumers uninformed about in-app purchase prices before making their firm choice is not always beneficial to app developers or platforms. This is because the competition-softening effect mentioned earlier holds only if the proportion of uninformed consumers is not too large. As a consequence, a move toward greater, but not complete, transparency in in-app purchase prices benefits app developers. The research further shows that a mobile platform should adopt a disclosure policy regarding the in-app purchase prices that is neither too transparent nor too obscure.

In addressing the third research question, the research demonstrates that initiatives to keep consumers better informed of in-app purchase prices may not have the desired outcome. In fact, consumers may be worse off when in-app purchase prices are transparent to a greater number of consumers.

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8 Uninformed consumers do not observe in-app purchase prices and may not even observe that they exist. However, rationality requires they form rational expectations. In contrast, informed consumers observe in-app purchase prices.
existence of informed and uninformed rational consumers is managerially relevant because even when firms shroud in-app purchase prices, a subset of consumers, but not all, have sufficiently low search costs to obtain this information. An implication is that consumers may be disadvantaged by recent changes to platform disclosure policies that make it such that some, but not all, consumers find the in-app purchase price information.

The rest of the paper sequentially discusses relevant literature, describes the model, and presents the results and concluding remarks.

2. Literature

This research adds to the information systems literature on digital goods pricing (e.g., Gupta et al. 1997, Dewan et al. 2003, Sundararajan 2004, Choudhary 2009). In particular, our paper is related to the research stream on versioning of digital goods because the combined app and in-app content can be considered a more valuable “product” than just the app alone. Dewan et al. (2003) show that versioning of digital goods may benefit a firm in both monopoly and duopoly settings. Bhargava and Choudhary (2008) consider monopolistic versioning of digital goods, and offer a general condition on when versioning is optimal. While we can view an app without in-app purchases and with in-app purchases as two different versions of the app, the research differs from the prior work in that we focus on information shrouding. To our knowledge, this is the first paper that shows the shrouding of in-app purchase prices can result in profit gains for competing app firms.

This research is also related to the literature on freemium pricing (e.g., Cheng et al. 2014, Niculescu and Wu 2014). Cheng et al. (2014) compares feature-limited, time-locked and their hybrid versions of freemium. They find that the hybrid version dominates the other two. Niculescu and Wu (2014) compare feature-limited and uniform seeding versions of freemium. They find that the optimal choice depends on factors including prior bias, cross-module synergy, word-of-mouth effect and the number of game periods. Quality learning is a core dynamic of the freemium literature – the free stage offers consumers a chance to better learn the quality of the full product. Our paper does not depend on this learning effect.
Furthermore, the focus of our paper is the strategic implications of information shrouding regarding the in-app purchase prices.

A growing literature studies the antecedents and implications of shrouded add-on prices (whereas in-app purchases are one example of add-ons) in competitive markets (e.g., Ellison 2005; Gabaix and Laibson 2006). We focus our review of this literature on how prior work answers the following two questions: whether add-on pricing benefits competing firms; and whether such benefit, if it exists, increases in the number of uninformed consumers (i.e., consumers who do not observe in-app purchase prices when they make app purchases). On the profitability of add-on pricing, one known finding that is robust to most model variations is the profit-irrelevance result: firm profits earned on hidden add-on prices are competed away due to equivalently diminished profits earned on base prices (Lal and Matutes 1994; Verboven 1999; Gabaix and Laibson 2006). This profit-irrelevance result collides with the belief in popular press that firms benefit from shrouding prices (e.g., Sullivan 2007, p4). Furthermore, this profit-irrelevance result is also at odds with some recent empirical findings: Chetty, Looney, and Kroft (2009) and Ellison and Ellison (2009) provide evidence that withholding information of add-on prices can benefit firms; Brown, Hossain, and Morgan (2010) finds that given add-on prices are hidden, firm profits increase in these fees.

Two recent papers seek to explain when the profit-irrelevance result can be overturned. Ellison (2005) finds add-on pricing improves firm profitability only if there is a significant and negative correlation between consumer price sensitivity and add-on valuation, which creates an adverse selection problem softening price competition. Shulman and Geng (2013) demonstrate that vertical differentiation between sellers in both the quality of the add-on and the quality of the base product, when coupled with consumer bounded rationality can lead competing firms to profit from add-on pricing. Our research does not impose correlation between consumer price sensitivity and add-on valuation, or quality asymmetry, or bounded rationality. Figure 1 summarizes the state of the add-on pricing literature with regard to predicting the profitability of shrouded add-on prices.
Figure 1: Summary of Add-on Pricing Literature’s Models and Predictions of Profit Effect

Only one prior paper, Gabaix and Laibson (2006), touches the question of whether a larger number of uninformed consumers will benefit competing firms more. Other papers in this literature either assume that the number of uninformed consumers is a constant (Lal and Matutes 1994; Verboven 1999; Ellison 2005), or consider a varying number of uninformed consumers yet are mute on its profit implications (Shulman and Geng 2013). Gabaix and Laibson (2006) confirm the profit-irrelevance result with a varying number of uninformed consumers, and thus suggest that firm profits are not affected by the number of uninformed consumers. To our knowledge, our paper is the first to find that firm profits can change in the number of uninformed consumers.

This research has several notable distinctions from the bundling literature (see for example Nalebuff 2004; Ghosh and Balachander 2007; Chao and Derdenger 2013). First, the consumer’s in-app purchase decision requires prior purchase of the app. Second, we allow for in-app purchase prices to be unobserved by some consumers whereas prices in the bundling literature are common knowledge. These two features create the recipe for a novel mechanism driving the profitability of in-app purchase pricing that is absent in the bundling literature. This also allows for the consideration of how the presence of consumers who are uninformed about in-app purchase prices affects consumer welfare and profitability.
3. The Model

We consider a linear city model similar to the ones in Ellison (2005) and Shulman and Geng (2013). Two app developer firms, 1 and 2, engage in horizontal competition on a linear city with unit measure. Firm 1 (2) occupies location 0 (1). Each firm offers an app and has content available for in-app purchase. Each firm \( j, \ j = 1, 2 \), chooses its app price \( p_b \) and its in-app purchase price \( p_a \). Though apps commonly have multiple in-app purchase options and prices, we treat \( p_a \) as a summary measure of a segment’s in-app purchase payments in the interest of parsimony. Firms pay an exogenous proportion of revenue \( \gamma \) to the app platform, but otherwise have zero marginal costs.

The app platform chooses the degree to which consumers have access to information about in-app prices. For example, Apple App Store and Google Play differ in their disclosure policies that affect the ease of access to in-app purchase information (see Screenshots in Appendix A). Prior to the app firms choosing their prices, the platform chooses its in-app purchase disclosure policy. We summarize this policy with the degree of difficulty in finding the in-app purchase price captured by \( \beta \). The platform sets \( \beta \) to maximize combined revenue from the app firms.

On the consumer side, we allow for three types of consumer heterogeneity. The first source of heterogeneity is derived from the observation that some consumers value the functionalities or contents available for in-app purchases while others do not. We refer to the former (latter) as app-only (in-app) consumers. For example, a majority of users of gaming apps do not purchase any in-app content.\(^9\) We normalize consumer population to 1 and assume there are \( 1 - \alpha \) app-only consumers where \( 0 < \alpha < 1 \). While we acknowledge that in some cases the value for the in-app purchases is driven by differences in the marginal utility of income (as is required for the profit improvement result in Ellison 2005), the valuation of in-app purchase options also depends on personal relevancy, availability of alternatives, and the consumption situation which may not always map onto the marginal utility of income.

The second source of consumer heterogeneity is driven by the common observation that consumers are heterogeneous in their search for and attentiveness to information. For instance, Brynjolfsson, Dick, and Smith (2004) find consumers are heterogeneous in their propensity to scroll to lower screens on an internet shopbot. Early work on eye pattern movements (see Rayner 1998 for a review) shows individuals vary widely in their eye-movement patterns and more recently Van der Lans, Pieters, and Wedel (2008) found that individuals differ in the effectiveness of individuals’ visual search strategies. We model this heterogeneity by parsimoniously assuming consumer’s willingness to scroll to find in-app purchase information, \( \phi \), is uniformly distributed between zero and one across all consumers. Given the platform’s choice of \( \beta \), consumers will choose to locate the in-app purchase price information if and only if \( \phi > \beta \).

We refer to consumers who ultimately observe in-app purchase price information as *informed consumers*. Given the uniform distribution of \( \phi \), there will be \( (1 - \beta) \) informed consumers. We refer to the \( \beta \) proportion of consumers who ultimately do not observe in-app purchase price information as *uninformed consumers*. The more challenging the app platform makes it to find in-app pricing on its platform, the more uninformed consumers there will be. Note in a model of rational consumers, “there is no room in the scheme for unanticipated consequences” (Simon 1955, p. 103). Thus, our model of uninformed consumers applies equivalently to situations in which the uninformed consumers are not even informed that the in-app items carry a separate charge as well as when the uninformed consumers are informed that there is an in-app price but are not informed of its value. The equivalence comes from the fact that rationality requires that uninformed consumers form accurate expectations of in-app purchase prices in equilibrium.

The third consumer source of heterogeneity is driven by the observation that consumers have horizontal preferences between app firms. Within each consumer segment, consumers are also differentiated by a stylized taste parameter, \( \theta \), that is uniformly distributed on \([0,1]\) and represents a consumer’s location on the linear city. Consumer \( i \) at location \( \theta_i \) suffers a fit cost of \( t \theta_i \left( t(1 - \theta_i) \right) \) if she purchases from firm 1 (2).
Note that, in reality, it is possible that consumers may become informed about the in-app purchase price via other means such as word-of-mouth. Though we abstract from this possibility in our model, the qualitative results remain unchanged. Our findings merely rely on the fact that a platform’s disclosure policy affects the number of people who find the in-app purchase information, but does not need to be the sole source of information.

We solve for the Subgame Perfect Nash Equilibrium in which the expectations of uninformed consumers are confirmed to be rational. Let \( \hat{p}_{j\mu}(p_{1b}, p_{2b}) \) denote uninformed consumers' expectation of firm \( j \)'s in-app purchase price upon observing both firms' app prices, \( j=1,2 \). All consumers have unit demand of the app with reservation value \( v_b \), and in-app consumers have unit demand of the in-app purchase item with reservation value \( v_a \). Consistent with Ellison (2005), Gabaix and Laibson (2006) and Shulman and Geng (2013), we assume \( v_b \) is large enough for full market coverage to avoid having key findings obscured by technical discussions of the magnitude of \( v_b \).

We focus attention on situations in which there exists a pure strategy equilibrium in the pricing game. As such, we assume \( t > v_a / 2 \), i.e., the level of horizontal differentiation is sufficiently large relative to the value of the in-app purchase item. This assumption rules out the possibility that a firm abandons all informed consumers in pursuit of a maximum margin on the in-app purchase of uninformed consumers.

Our timeline, depicted in Figure 2, assumes that the platform leads in its choice of in-app purchase price disclosure policy, app firms simultaneously choose app prices then simultaneously choose in-app purchase prices before consumers choose which app to buy and then choose whether to make the in-app purchase. The sequential choice of advertised app prices and in-app purchase prices is consistent with convention established in the add-on pricing literature (e.g., Lal and Matutes 1994; Ellison 2005) and thus allows for isolation of the factors driving our unique results. Moreover, the unique results are subsequently shown to hold in an alternative timeline involving simultaneous price-setting.

As shown in Figure 2, we assume that uninformed consumers have to purchase the app before observing the in-app purchase prices. This fits the scenario in which a consumer discovers the prices of
desired functionality or contents of the in-app purchase items as they are viewed within the app. We further assume that, once a consumer has purchased the app from a firm, she is stuck with this firm in that she can only pick between making the in-app purchase from the same firm or not to purchase it.

Figure 2. Timeline of the Game

4. Results

We first derive outcomes for any given Stage 0 platform choice of $\beta$. We subsequently identify the equilibrium $\beta^*$. Using backward induction, we first characterize consumer choices in Stages 3 and 4 conditional on observed app prices and uninformed consumer expectations. We then discuss rational beliefs of uninformed consumers in Stage 3 and firm in-app prices in Stage 2. Finally we discuss equilibrium app pricing, firm profits and consumer surplus.

The behaviors of each of the three consumer segments must first be analyzed: $(1 - \alpha)$ app-only consumers, $\alpha(1 - \beta)$ informed consumers and $\alpha\beta$ uninformed consumers. The last two segments together are referred to as in-app consumers. In Stage 4 and conditional on Stage 3 purchase of the app, in-app consumers make an in-app purchase from the same firm if the price is no more than $v_a$. Hereafter we consider only $p_{ja} \leq v_a$ to rule out the uninteresting and never optimal case where a firm charges a too high in-app purchase price and gets no sales on the in-app content.
In Stage 3, informed consumers observe in-app prices $p_{la}$ and $p_{2a}$ prior to making their app purchase decisions. The location of the marginal informed consumer that is indifferent between the two firms, $\theta_j$, satisfies $v_b + v_a - t\theta_j - p_{lb} - p_{la} = v_b + v_a - t(1 - \theta_j) - p_{2b} - p_{2a}$. Therefore, $\theta_j = 1/2 + (p_{2b} + p_{2a} - p_{lb} - p_{la})/(2t)$. Uninformed consumers do not observe in-app prices prior to making the decision regarding which app to buy; instead, they form rational expectations of these in-app purchases, $\hat{p}_{la}(p_{lb}, p_{2b})$ and $\hat{p}_{2a}(p_{lb}, p_{2b})$. Accordingly, the marginal uninformed consumer is located at $\theta_U = 1/2 + (p_{2b} + \hat{p}_{2a}(p_{lb}, p_{2b}) - p_{lb} - \hat{p}_{la}(p_{lb}, p_{2b}))/(2t)$. Lastly, the marginal app-only consumer is located at $\theta_a = 1/2 + (p_{2b} - p_{lb})/(2t)$. Demand of each firm's app, $D_{jb}$, and in-app purchases, $D_{ja}$, are then

$$D_{jb} = \frac{1}{2} - \eta(j) \frac{(p_{lb} - p_{2b}) + \alpha(1 - \beta)(p_{la} - p_{2a}) + \beta(p_{lb} - p_{2b}) - \hat{p}_{1a}(p_{lb}, p_{2b}) - \hat{p}_{2a}(p_{lb}, p_{2b}))}{2t}$$

and

$$D_{ja} = \frac{\alpha}{2} - \eta(j) \frac{(p_{lb} - p_{2b}) + (1 - \beta)(p_{la} - p_{2a}) + \beta(p_{lb} - p_{2b}) - \hat{p}_{1a}(p_{lb}, p_{2b}) - \hat{p}_{2a}(p_{lb}, p_{2b}))}{2t},$$

where $\eta(j) = 1$ if $j = 1$, and $\eta(j) = -1$ if $j = 2$. Therefore, for any given app prices and in-app prices, firm profits are

$$\pi_j = (1 - \gamma)(p_{lb}D_{jb} + p_{ja}D_{ja})$$

for $j = 1, 2$. We next analyze firm in-app purchase pricing, which will affect consumers’ rational beliefs.

4.1 Rational Beliefs

Given any app prices $p_{lb}$ and $p_{2b}$, we solve for the in-app purchase prices of firms $(p_{la}^*, p_{2a}^*, \hat{p}_{la}^*, \hat{p}_{2a}^* \mid p_{lb}, p_{2b})$ such that:

- firms optimize in-app purchase prices given uninformed consumer beliefs, i.e.,

$$p_{ja}^* = \arg \max_{p_{ja}} \pi_j(p_{ja}, p_{ja}^*, \hat{p}_{1a}, \hat{p}_{2a} \mid p_{lb}, p_{2b}) \text{ for } j = 1, 2,$$

- and firms anticipate that the Stage 3 beliefs of uninformed consumers will be consistent with equilibrium firm strategy, i.e.,

$$\hat{p}_{ja} = p_{ja}^* \text{ for any } (p_{lb}, p_{2b}) \text{ and } j = 1, 2.$$
To avoid repetition in describing actions, payoffs, and beliefs regarding each firm, we let "\(-j\)" indicate the firm other than firm \(j\). We only need to consider beliefs of uninformed consumers because the belief of app-only consumers is not relevant to their actions, and for informed consumers they will directly observe in-app purchase prices before making app purchase decisions. Solutions to (2) for \(j = 1, 2\) can be derived by using (1) and solving the first-order conditions. The following lemma describes the rational beliefs.

**Lemma 1** Given \(p_{1b}, p_{2b}, \hat{p}_{1a}(p_{1b}, p_{2b}), \hat{p}_{2a}(p_{1b}, p_{2b})\), and assume that \(\omega_j \leq \omega_{-j}\).\(^{10}\) Optimal in-app purchase prices are

\[
(p^*_j, p^*_{-j}) = \begin{cases} 
(\omega_j, \omega_{-j}) & \text{if } \omega_j < v_a, \\
(\mu_j, v_a) & \text{if } \mu_j < v_a \leq \omega_{-j}, \\
(v_a, v_a) & \text{if } \mu_j \geq v_a.
\end{cases}
\]

where \(\omega_j = \frac{t}{1-\beta} - p_{jb} + \frac{\beta}{3(1-\beta)}(p_{-jb} + \hat{p}_{-ja}(p_{1b}, p_{2b}) - p_{jb} - \hat{p}_{ja}(p_{1b}, p_{2b}))\) and

\[
\mu_j = -p_{jb} + \frac{1}{2}[(p_{-jb} + v_a) + \frac{t}{1-\beta} + \frac{\beta}{1-\beta}(p_{-jb} + \hat{p}_{-ja}(p_{1b}, p_{2b}) - p_{jb} - \hat{p}_{ja}(p_{1b}, p_{2b})]].
\]

All proofs are in the Appendix. From (3) and (4) we can then solve for the rational belief for uninformed consumers given any app prices \(p_{1b}\) and \(p_{2b}\).

**Proposition 1** Given \(p_{1b}, p_{2b}\), and assume that \(p_{jb} \geq p_{-jb}\). The unique rational expectations of in-app purchase prices for uninformed consumers are the following:

(i). \((\hat{p}_{ja}, \hat{p}_{-ja}) = (\frac{t}{1-\beta} - p_{jb}, \frac{t}{1-\beta} - p_{-jb})\) if \(p_{-jb} > \frac{t}{1-\beta} - v_a\).

(ii). \((\hat{p}_{ja}, \hat{p}_{-ja}) = (\frac{t + p_{-jb} + v_a}{2 - \beta} - p_{jb}, v_a)\) if \(p_{-jb} \leq \frac{t}{1-\beta} - v_a\) and \(p_{jb} > \frac{t + p_{-jb} - v_a(1-\beta)}{2 - \beta}\).

(iii). \((\hat{p}_{ja}, \hat{p}_{-ja}) = (v_a, v_a)\) if \(p_{jb} \leq \frac{t + p_{-jb} - v_a(1-\beta)}{2 - \beta}\).

\(^{10}\)This assumption does not affect the generality of the lemma: if \(\omega_j > \omega_{-j}\), simply switch notations \(j\) and \(-j\). The same is true about the assumption subsequently in Proposition 1.
The result in part (iii) of Proposition 1 is consistent with prior works including Lal and Matutes (1994), Verboven (1999), Ellison (2005), and Gabaix and Laibson (2006). When both firms charge low enough app prices, uninformed (but rational) consumers expect firm $j$ to charge the maximum in-app purchase prices (i.e., the consumer’s reservation value on its in-app content) and consumers take this into account in their purchase decisions.

Part (i) of Proposition 1, in contrast, contributes a new insight to the literature: when both firms’ app prices are higher than $t/(1-\beta)-v_a$, instead of expecting a maximal possible in-app purchase price, uninformed consumers rationally expect firm $j$’s in-app purchase price to be an inner solution that *linearly decreases in its app price*. This negative correlation between the observed app price and the in-app purchase price subsequently softens the competition between the app firms: a firm’s price cut on the app price is now less effective in attracting uninformed consumers than that without in-app purchase, as the uninformed consumers expects a lesser cut on the total price (i.e., app price plus in-app purchase price) she pays.

Notice that the existence of enough informed consumers is instrumental to this new finding: for any given $p_{jb}$, $p_{jb} > t/(1-\beta)-v_a$ will not hold if $1-\beta$ is close enough to zero. Therefore, our assumption that consumers are heterogeneous in their willingness to scroll to find information resulting in both informed and uninformed consumers coexisting plays a pivotal role in driving this new finding.

The intuition is best seen (though not limited to) when the vast majority in-app consumers are informed, i.e. $\beta$ is very close to 0. In this case and if we assume $\max\{\omega_j, \omega_{-j}\} < v_a$ is true -- which indeed holds for any given Stage 1 app prices and any consumer belief as long as $\beta$ is small enough, firm $j$ chooses in-app purchase price $p_{j\sigma}^* = \omega_j$. Therefore

$$\frac{\partial p_{j\sigma}^*}{\partial p_{jb}} = -1 + \frac{\beta}{3(1-\beta)} \frac{\partial (p_{-j\sigma} + \hat{p}_{-j\sigma}(p_{1b},p_{2b}) - p_{j\sigma} - \hat{p}_{j\sigma}(p_{1b},p_{2b}))}{\partial p_{jb}}.$$

As long as all prices and expectations are finite, the above derivative is negative if $\beta$ is small enough. In other words, the existence of a large number of informed consumers incentivizes a firm to
charge an in-app purchase price inversely related to its app price. The lower the app price, the higher the in-app purchase price regardless of the beliefs of uninformed consumers. This in turn implies that uninformed consumers’ rational expectation needs to take app prices into account rather than ignoring them. Though the intuition is best seen when $\beta$ is small, part (i) of Proposition 1 shows that this intuition applies for a wide range of values for $\beta$ as long as $p_{jb} > t / (1 - \beta) - v_a$ holds for both $j = 1, 2$.

The result regarding $\hat{p}_{jb}(p_{1b}, p_{2b})$ in part (ii) of Proposition 1 is analogous to part (i) with one difference. As now uninformed consumers observe a very low $p_{-jb}$, they rationally expect firm $-j$ to charge the maximum in-app purchase price $v_a$. The fact that firm $-j$’s in-app purchase price is now binding at $v_a$ influences firm $j$’s optimization problem, and as a response firm $j$ sets the in-app purchase price at a level lower than $t / (1 - \beta) - p_{jb}$ though the price still decreases in $p_{jb}$.

We next study firm profits and consumer surplus by solving for Stage 1 app pricing game.

4.2 Firm Profits and Platform Disclosure Policy

In Stage 1, the app firms simultaneously choose app prices to maximize profit. The equilibrium prices and profits are presented in the following Proposition. For notational convenience, denote $\beta_1 \triangleq \frac{v_a}{t + v_a}$.

**Proposition 2** In the hidden in-app pricing game:

(i). If $\beta < \beta_1$, in equilibrium, firms set in-app purchase prices $p_{1b}^* = p_{2b}^* = \frac{\beta t}{1 - \beta}$ and app prices $p_{1b}^* = p_{2b}^* = t$. App firm profits are $\pi_1^* = \pi_2^* = (1 - \gamma)(1 + \frac{\alpha \beta}{1 - \beta})\frac{t}{2}$ and the platform earns $\pi_p = \gamma t(1 + \frac{\alpha \beta}{1 - \beta})$.

(ii). If $\beta > \beta_1$, in equilibrium, firms set in-app purchase prices $p_{1b}^* = p_{2b}^* = v_a$. If $t > \alpha v_a$, then app prices are $p_{1b}^* = p_{2b}^* = t - \alpha v_a$, app firm profits are $\pi_1^* = \pi_2^* = (1 - \gamma)\frac{t}{2}$ and the platform earns

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We use the Pareto-Dominance refinement for equilibrium selection when multiple equilibria arises.
\[ \pi_p = \gamma t \]. If \( t < \alpha \nu_u \), then app prices are zero, app firm profits are \((1 - \gamma)\alpha \nu_u / 2\) and the platform earns \( \pi_p = \alpha \gamma \nu_u \).

Proposition 2 highlights how the app platform’s disclosure policy affects the pricing decisions of competing app firms. Previous research on unadvertised prices (e.g., Lal and Matutes 1994) suggests that the platform’s disclosure policy should be irrelevant to firm profits. However, we find the heterogeneity in consumers’ willingness to scroll to find in-app prices combined with the heterogeneity in consumer’s demand for the in-app functions or contents can overturn this profit irrelevance result. In fact, if the platform changes its disclosure policy regarding the prominence of in-app purchase prices, the pricing and app firm profits are in fact affected. Note that the full-information game can be computed by setting \( \beta = 0 \), which leads to the standard profitability of \( \pi_1 = \pi_2 = (1 - \gamma)t / 2 \). Interestingly, Proposition 2 part (i) shows that increasing \( \beta \) from zero in an effort to create more uninformed consumers can increase firm profitability.

The finding thus identifies a novel mechanism for why hidden in-app purchases can increase profit. In the case of \( \beta < \beta_1 \), firms will pick app prices in Stage 1 such that the rational expectation of uninformed consumers will be a decreasing function of the app price according to Part (i) of the rational belief system specified in Proposition 1. Consequently, uninformed consumers, in making the app purchase decision, are less price-sensitive with respect to the app prices than the informed consumers. Firms then face less competitive pressure in pricing apps as compared to a standard pricing game, and thus avoid losses from loss-leader pricing. On the other hand, in Stage 2 firms can still gain from the information disadvantage of uninformed consumers by setting profitable in-app prices, as shown in Part (i) of Proposition 2. Part (i) of Proposition 2 thus shows that the existence of enough informed consumers enables firms to avoid the perils of loss-leader pricing while reaping the benefit of high margins on in-app purchases.

We now turn our attention to the app platform’s optimal choice of \( \beta \). The platform chooses \( \beta \) to maximize \( \pi_p = \gamma \sum_{j=1}^{2} (p_j^*D_{ja} + p_j^*D_{jb}) \). We present the results in the following proposition.
Proposition 3 The platform will choose $\beta^* = v_a / (t + v_a)$ and earn profit $\pi_p = \gamma(t + \alpha v_a)$ while app firms earn $\pi_1 = \pi_2 = (1 - \gamma)(t + \alpha v_a) / 2$.

Proposition 3 makes several contributions to the literature and to practice. First, hidden in-app purchase prices actually boost app firm profit. Second, this happens as the platform chooses an intermediate level of prominence given to in-app purchase prices in equilibrium. As a consequence, the decision by platforms such as Apple and Google to display in-app purchase price information actually increases its profit and the profit of its app developers. However, the platforms have buried the information to some degree (see Appendix A for examples), thereby creating uninformed consumers from the population who have a lower propensity to scroll as described in Byrnjolfsson, Dick, and Smith (2004). The heterogeneity in consumers’ discovery of in-app purchase information is key to the result. If all in-app consumers are informed, or if they are all uninformed, the profit-irrelevance of hidden pricing result holds. Previous papers on rational consumers have studied both extremes yet not the middle; this model thus highlights the importance of explicitly accounting for heterogeneity in the consumers’ propensity to scroll and the platform’s ability make the in-app purchase price information less prominent.

To understand proposition 3, consider what happens when the in-app purchase price is hidden from too many consumers. As a consequence, all consumers rationally expect in-app purchase prices to be set at their maximum. Thus, if the platform chooses a policy such that there are too many uninformed consumers, it can actually intensify competition among app developers and reduce their revenue that they share with the app platform. It is only when the price information is transparent to a reasonable amount of informed consumers (i.e., $0 < \beta < \beta_i$) that the common intuition of profit increasing in $\beta$ holds.

4.3 Consumer Welfare

Initially, platforms did not require disclosure of in-app purchase prices. In our model this would lead to $\beta=1$ uninformed consumers. Even if some consumers obtained in-app purchase price information via word-of-mouth (a possibility abstracted from in our model), the number of uninformed consumers would be relatively high. Intuition would suggest that the recent move toward more transparency in in-app
purchase prices would increase consumer welfare. We identify the effect of greater transparency in the following Proposition. For notational convenience, denote \( \hat{\beta} \triangleq \begin{cases} \frac{0}{0} & \text{if } t > \alpha v_a \\ \frac{\alpha v_a - t}{\alpha v_a - (1 - \alpha) t} & \text{if } t < \alpha v_a \end{cases} \) and
\[
\hat{\beta} \triangleq \begin{cases} \frac{(1 - \alpha) v_a}{t + (1 - \alpha) v_a} & \text{if } t > \alpha v_a \\ \frac{1 - t}{v_a} & \text{if } t < \alpha v_a \end{cases} .
\]

**Proposition 4** Total consumer welfare is greater when a large proportion of the market is uninformed about in-app prices (i.e., \( \beta > \beta_i \)) than when a small proportion of the market is uninformed about in-app prices (i.e., \( \hat{\beta} < \beta < \beta_i \)). App-only consumer welfare is greater for all \( \beta > \beta_i \) than \( 0 < \beta < \beta_i \). In-app consumer welfare is greater for all \( \beta > \beta_i \) than \( \hat{\beta} < \beta < \beta_i \).

Proposition 4 suggests that recent efforts by app platforms to require more transparency of in-app purchases can actually reduce consumer welfare. The result arises because a greater number of informed consumers actually prevent an app developer from charging a monopoly in-app purchase price. The counterintuitive result of Proposition 4 stems from the fact that the uninformed consumers’ rational expectations regarding the relationship between the app price and the in-app purchase price diminish competitive intensity in app prices. This has a strictly negative impact on welfare for app-only consumers, though the effect on welfare for in-app consumers depends on the effect relative to the savings on the in-app purchases. Proposition 4 thus demonstrates again that considering the extremes of the \( \beta \) range results in a different set of predictions than considering the interior.

**4.4. Robustness Check: Simultaneous Pricing Decisions**

In the base model we assumed that in-app purchase prices are set subsequent to app prices. This follows the convention of Ellison (2005) and Lal and Matutes (1994). In this extension we consider an alternative scenario where app prices and in-app purchase prices are set simultaneously. While there is a
unique equilibrium under sequential pricing, in this extension we demonstrate that simultaneous pricing implies multiple equilibria and discuss when our main findings are preserved.

The uniqueness of equilibrium under sequential pricing is driven by the fact that the in-app purchase pricing is a *subgame* following app pricing. In Stage 2 of the in-app purchase pricing game (as shown in Figure 2) the sequential timing of the game requires any firm *j* to optimally set its in-app purchase price conditional on both app prices, i.e., \( p^*_{j\mu}(p_{\beta}, p_{-\beta}) \) as depicted in Lemma 1. Any off-equilibrium app price deviation by firm *j* will then trigger a corresponding change of \( p^*_{j\mu} \) as dictated by Lemma 1. In other words, under sequential pricing uninformed consumers rationally believe that the unobserved in-app purchase prices are particular functions of the observed app prices as dictated by rational expectations -- the uniqueness of this belief structure is then shown in Proposition 1.

Under simultaneous pricing, however, no such sequential relationship between app prices and in-app purchase prices exist. If uninformed consumers observe an off-equilibrium app price by a firm, there is no restriction on how these consumers construe their belief of the firm’s unobserved in-app purchase price. The multiple possible belief structures lead to multiple possible equilibria. In this extension we consider symmetric linear belief structures. Upon observing firm *j*’s app price \( p_{\beta} \), consider the following belief structure for uninformed consumers:

\[
\hat{p}_{j\mu}(p_{\beta}) = Ap_{\beta} + H,
\]

where \( A \) and \( H \) are constants, \( j = 1,2 \). To avoid having technical details distract the focus on key insights of this extension, we assume the belief structure will not lead to beliefs that the in-app purchase price exceeds \( v_a \). This is the case when the number of uninformed consumers is not too large. When the number of uninformed consumers is very large, in equilibrium in-app purchase prices will be set at the boundary value \( v_a \) similar to part (ii) of Proposition 2, and thus beliefs will be independent of the app prices.

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12 Kyle (1985) also limits the belief structure to linear functions when such a belief structure can constitute equilibria.
Given this belief structure, both firms simultaneously optimize their app prices and in-app purchase prices. Furthermore, their equilibrium in-app purchase prices need to be consistent with the belief structure (as evaluated at the equilibrium point of app prices). The result equilibrium is shown in the following proposition.

**Proposition 5** In the simultaneous pricing game there is a continuum of equilibria. For any 

\[ A \in (-1 - \frac{2\sqrt{\alpha(1-\alpha)(1-\beta)}}{\alpha\beta}, -1 + \frac{2\sqrt{\alpha(1-\alpha)(1-\beta)}}{\alpha\beta}) \],

there is an equilibrium with app prices 

\[ p_{1b}^* = p_{2b}^* = \frac{(1 - \alpha - \beta - A\alpha\beta)t}{(1 - \alpha)(1 - \beta)} \], in-app purchase prices 

\[ p_{1u}^* = p_{2u}^* = \frac{(1 + A\alpha)\beta t}{(1 - \alpha)(1 - \beta)} \], belief structure 

\[ \hat{p}_{ju} (p_{1b}) = Ap_{1b} + \frac{((1 + A)(1 + A\alpha)\beta - A(1 - \alpha)t)}{(1 - \alpha)(1 - \beta)} \] and each app firm profit 

\[ (1 - \gamma)(\frac{t}{2} - \frac{A\alpha\beta t}{2(1 - \beta)}) \].

Again recall that, for a concise discussion, we only consider cases where \( \beta < \frac{\nu_a(1-\alpha)}{t(1 + A\alpha) + \nu(1 - \alpha)} \) so that \( \hat{p}_{ju} \) above will not exceed \( \nu_a \). Proposition 5 demonstrates that, under simultaneous pricing, equilibrium prices and profits are dependent on the belief structure of uninformed consumers. Note that Proposition 1 under the sequential pricing game corresponds to the special case of \( A = -1 \) under the simultaneous pricing game.

The following corollary summarizes the profit implications of Proposition 5.

**Corollary 1** In the simultaneous pricing game, hidden in-app purchase prices improves app firm profits if uninformed consumers believe a negative correlation between a firm’s app price and its hidden in-app purchase price, i.e., \( A < 0 \). Profit-irrelevance of hidden in-app purchase prices holds if the belief structure is constant and independent of the app prices, i.e., \( A = 0 \).

Corollary 1 shows that sequential pricing decisions are not necessary for shrouded in-app purchase prices to improve app profitability relative to a complete information game in which \( \beta = 0 \). This contrasts with the standard profit irrelevancy result from the add-on pricing literature and is consistent with part (i) of Proposition 2 for any \( A < 0 \). That is, the results are preserved when in-app purchase prices
and app prices are set simultaneously if consumers believe app firms with higher in-app purchase prices try to lure customers with lower app prices. Note that an inverse relationship between the price that is posted and the price that is hidden is consistent with the empirical finding of Derdenger, Liu and Sun (2012). Moreover, behavioral research on belief formation suggests that an “item of information about a given object will thus have implications for many other beliefs about the object” (Fishbein and Ajzen 1975, p. 214). Thus, logic would suggest an inferential belief system in which the app price serves as a cue affecting the beliefs about the in-app purchase price (i.e., \( A \neq 0 \)). The qualitative results of Proposition 2 (ii) and Proposition 3 can similarly be replicated when considering the boundary condition of \( p_{ja} = v_a \).

5. Conclusion

This paper uniquely demonstrates that shrouding in-app purchase prices by symmetric firms can increase firm profit even without price-sensitivity heterogeneity among consumers or quality heterogeneity among firms. Key to this finding is the consideration of in-app purchase by both consumers who are informed about the in-app purchase prices before app purchase and consumes who are uninformed of the in-app purchase prices until after app purchase. When the platform’s disclosure policy results in a sufficient number of informed consumers, the uninformed consumers will rationally expect the in-app purchase prices to be negatively correlated with the app price. This belief system implies that uninformed consumers can be less price sensitive over app prices than informed consumers. Two dynamics then follow. With respect to app prices, price cuts turn out to be ineffective in attracting uninformed consumers (as they expect a corresponding increase in the in-app purchase prices), which dampens the price competition over app prices. Second and with respect to in-app purchase prices, firms can still take advantage of the unobservability of the in-app purchase prices by uninformed consumers and charge high in-app purchase prices. In other words, firms avoid the perils of loss-leader pricing while still reaping the benefit of high margins on in-app purchases.
One may intuit that the presence of informed consumers would help to drive down in-app purchase prices and benefit the uninformed consumers. However, the model shows how the presence of informed consumers can deteriorate uninformed consumer surplus and total consumer surplus. When the presence of informed consumers suppresses the app developers’ incentive to charge monopoly in-app purchase prices, it also creates rational expectations that a lower app price implies higher in-app purchase prices. In other words, uninformed consumers rationally expect that an attempt to lure them with a lower app price will generally be followed by an attempt to harvest their value with a higher in-app purchase price. This is the unique rational belief system in our model and it consequently diminishes firm incentive to cut app prices. One implication we then highlight is that the co-existence of informed and uninformed consumer segments can be detrimental to each segment's surplus, as compared to surplus when only one of the two segments exist. Another consequence we find is that the platform’s optimal shrouding is incomplete. In other words, the platform optimally chooses a disclosure policy that results in a subset of consumers becoming informed about in-app purchase prices.

In summary, the consideration of heterogeneity in consumers’ willingness to scroll to find in-app purchase price information in a rational model produces novel results that reverse established intuition and offers a novel explanation of the profitability of in-app purchases. For managers, the research finds that shrouded in-app purchase prices may improve profit under conditions prior theory would suggest a profit-irrelevance result. The research also suggests that having some consumers informed about the in-app purchase prices can actually increase profit relative to serving a market with a relatively high number of consumers who do not know the in-app purchase prices at the time of firm choice. For policy makers and consumer advocates, increasing transparency will not always improve consumer welfare. Efforts to increase awareness of in-app purchase prices will be detrimental to consumers unless there is already a substantial portion of the population aware of the prices.
APPENDIX A: Screenshots of In-App purchase disclosures

Example Apple App Store Screenshots

Screenshot upon clicking on App

Continued screenshot after full scroll
In-App Purchases

1. Bag of Tokens $0.99
2. Chest of Tokens $2.99
3. Suitcase of Tokens $1.99
Example Google Play Screenshots

Screenshot upon clicking on app

Screenshot upon first scroll
Turn your phone or tablet into a real-time flight tracker.

Features that have helped make Flightradar24 - Flight Tracker the #1 selling app in 160+ countries and the #1 Travel app in 140+ countries (United States, France, United Kingdom, Germany and more) include:

- Watch planes move in real-time on detailed maps
- Identify planes flying overhead by pointing your device at the sky (the augmented reality view requires a rear camera, accelerometer & magnetic sensor)
- Experience what the pilot of an aircraft sees in real-time and in 3D
- Tap on a plane for comprehensive flight and aircraft information such as route, estimated time of arrival, actual time of departure, aircraft type, speed, altitude, and high-resolution picture
- Easy to search for individual flights using flight number, airport, or airline
- Easy to search for individual flights using flight number, airport, or airline

HOW IT WORKS
Most aircraft are equipped with so-called ADS-B transponders that transmit positional data. Flightradar24 has a rapidly growing network of several thousand ground stations around the world to receive this data that then shows up as aircraft moving on a map in the app. In an expanding number of regions Flightradar24, with the help of multilateration, is able to calculate the positions of aircraft that don't have ADS-B transponders.

Traditional radar data is also used in the app thanks to a direct feed from the US Federal Aviation Administration (FAA).
***IMPORTANT NOTICES***
Minimum required screen resolution is 320x480px

If you are only interested in tracking air traffic in a specific region, we suggest that you check FlightRadar24.com BEFORE purchasing the app. FlightRadar24 provides unrivaled positional aircraft data coverage around the world but there are areas where we don't have coverage.

Overview of coverage as of January 2016:
* Europe: close to 100%
* North America: 100% of US and Canada via slightly delayed radar data. Real-time coverage for most of US, Canada and Mexico for ADS-B-equipped aircraft
* South America: Substantial coverage in most countries including Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador, and Venezuela.
* Asia: Substantial coverage in most major Asian countries including Japan, India, Indonesia, South Korea, Thailand, UAE, Malaysia, Taiwan and many other countries. Rapidly expanding coverage in China
* Oceania: Nearly complete coverage in Australia and New Zealand
* Africa: Substantial coverage in Africa with coverage

Screenshot after 2nd scroll on “Read More”

Screenshot after 3rd scroll on “Read More”
SCREENSHOT AFTER 4TH SCROLL ON “READ MORE”

TECHNICAL APPENDIX

**Proof of Lemma 1** For \( j = 1, 2 \), from (1) we have

\[
\frac{\hat{\partial} \pi_j}{\hat{\partial} p_{ja}} = \alpha (1 - \beta) \left[ (p_{jb} + p_{ja}) - 2(p_{jb} + p_{ja}) + \frac{t}{1 - \beta} + \frac{\beta}{1 - \beta} (p_{jb} + \hat{p}_{ja}(p_{ib}, p_{2b}) - p_{jb} - \hat{p}_{ja}(p_{ib}, p_{2b})) \right].
\]

Jointly solving \( \hat{\partial} \pi_1 / \hat{\partial} p_{ja} = 0 \) and \( \hat{\partial} \pi_2 / \hat{\partial} p_{2a} = 0 \), we get \( p_{ja} = \omega_j \) for \( j = 1, 2 \). It is straightforward to verify that the second-order conditions are negative for any in-app purchase prices.

Because the reservation value for in-app consumers is \( v_a \), the above inner solution applies only if \( \omega \leq v_a \) (which also implies \( \omega_j < v_a \) as we assume \( \omega_j \leq \omega_{-j} \)). If \( \omega_j > v_a \), there are two cases:

(i) \( \mu_j < v_a \). The optimal in-app purchase price for firm \(-j\) is \( p_{ja} = v_a \) by fact that
The optimal in-app purchase price for firm \( j \) is the solution to
\[
\frac{\partial^2 \pi_j}{\partial p_{ja} \partial p_{ja}} = \frac{\alpha(1-\beta)}{2t} > 0.
\]
which results in \( p_{ja} = \mu_j \). Notice that \( \mu_j < \omega_j \) if \( \omega_j > v_a \). In other words, the fact that firm \(-j\)'s in-app purchase price is binding at its maximum \( v_a \) (which is lower than \( \omega_{-j} \)) influences firm \( j \)'s in-app purchase price as well: instead of charging \( \omega_j \), it charges a lower price equal to \( \mu_j \).

(ii) \( \mu_j \geq v_a \). Now \( \frac{\partial \pi_j}{\partial p_{ja}} \bigg|_{p_{ja} = v_a} = 0 \) for all feasible \( p_{ja} \). Therefore, optimal in-app purchase prices for both firms are \( v_a \). Q.E.D.

**Proof of Proposition 1** Notice that the assumption \( p_{jb} \geq p_{-jb} \) does not affect the generality of this proposition: if the reverse is true, simply switch notations \( j \) and \(-j\). The proof proceeds in two steps. In Step 1 we prove that, given \( p_{jb} \geq p_{-jb} \), it must be true that \( \omega_j \leq \omega_{-j} \). In Step 2 we prove the three cases of this proposition sequentially. For notational convenience, denote \( r = \frac{t}{1-\beta} \).

**Step 1** We prove \( \omega_j \leq \omega_{-j} \) by contradiction. Assume that \( \omega_j > \omega_{-j} \), i.e., \( \omega_j > \omega_{-j} \). Rational expectation then requires
\[
(\hat{p}_{ja}, \hat{p}_{-ja}) = (\omega_j, \omega_{-j}), \quad \text{i.e.,} \quad \hat{p}_{ja} = r - p_{jb} + \frac{\beta}{3(1-\beta)}(p_{-jb} + \hat{p}_{-ja} - p_{jb} - \hat{p}_{ja}) \quad \text{and}
\]
\[
\hat{p}_{-ja} = r - p_{-jb} + \frac{\beta}{3(1-\beta)}(p_{jb} + \hat{p}_{ja} - p_{-jb} - \hat{p}_{-ja}).
\]
Solving these two equations, we get \( \hat{p}_{ja} = r - p_{jb} \) and \( \hat{p}_{-ja} = r - p_{-jb} \). Therefore \( \frac{2\beta}{3-\beta} (\hat{p}_{-ja} - \hat{p}_{ja}) = \frac{2\beta}{3-\beta} (p_{jb} - p_{-jb}) \leq p_{jb} - p_{-jb} \), a contradiction.

If \( \mu_{-j} < v_a \leq \omega_j \), from Lemma 1 we have \( (p_{ja}^*, p_{-ja}^*) = (v_a, \mu_{-j}) \). Rational expectation then requires
\[
(\hat{p}_{ja}, \hat{p}_{-ja}) = (v_a, \mu_{-j}), \quad \text{i.e.,} \quad \hat{p}_{ja} = v_a \quad \text{and} \quad \hat{p}_{-ja} = -p_{-jb} + \frac{1}{2}(p_{jb} + v_a) + r + \frac{\beta}{1-\beta} (p_{jb} + \hat{p}_{ja} - p_{-jb} - \hat{p}_{-ja})].
\]
Solving these two equations, we get \( \hat{p}_{ja} = v_a \) and \( \hat{p}_{-ja} = \frac{(1-\beta)r + p_{jb} + v_a}{2-\beta} - p_{-jb} \). Therefore
\[
\frac{2\beta}{3-\beta} (\hat{p}_{ja} - \hat{p}_{ja}) = \frac{2\beta}{3-\beta} [(1-\beta)r + p_{jb} + v_a - p_{ja} - v_a] < 0 \leq p_{jb} - p_{ja}, \text{ a contradiction. The first inequality above comes from } \\
\mu_{ja} = \frac{(1-\beta)r + p_{jb} + v_a}{2-\beta} - p_{ja} < v_a.
\]

If \( \mu_{ja} \geq v_a \), from Lemma 1 we have \((p^*_{ja}, p^*_{ja}) = (v_a, v_a)\). Rational expectation then requires \((\hat{p}_{ja}, \hat{p}_{ja}) = (v_a, v_a)\). Therefore \((\hat{p}_{ja} - \hat{p}_{ja}) = 0 \leq p_{jb} - p_{ja}, \text{ a contradiction.}\)

**Step 2** Now we have established that \( \omega_j \leq \omega_j \). We proceed to prove the three cases of this proposition sequentially. For each case, we consider all three possibilities in Lemma 1, and rule out the impossible ones.

(i) \( p_{jb} \geq p_{ja} > r - v_a \).

Notice that \( \omega_j + \omega_j = 2r - (p_{jb} + p_{ja}) < 2v_a \). Therefore the third case in Lemma 1 is impossible. We next show that the second case in Lemma 1 is impossible by contradiction. Assume that \( \mu_j < v_a \) and \( \omega_j \geq v_a \). Lemma 1 then says \((p^*_{ja}, p^*_{ja}) = (\mu_j, v_a)\). Rational expectation then requires

\[
\hat{p}_{ja} = \mu_j = p_{jb} + \frac{1}{2}[(p_{ja} + v_a) + r + \frac{\beta}{1-\beta}(p_{ja} - p_{ja} - \hat{p}_{ja})]
\]

and \( \hat{p}_{ja} = v_a \). Replacing the second equation into the first, we have \( \hat{p}_{ja} = \frac{(1-\beta)r + p_{jb} + v_a}{2-\beta} - p_{jb} \). Thus

\[
\omega_j = r - p_{ja} + \frac{\beta}{3(1-\beta)}(p_{ja} + \hat{p}_{ja} - p_{ja} - \hat{p}_{ja})
\]

\[
= \frac{6-2\beta}{3(2-\beta)}(r - p_{ja}) - \frac{\beta}{3(2-\beta)}v_a < \frac{6-2\beta}{3(2-\beta)}v_a - \frac{\beta}{3(2-\beta)}v_a = v_a, \text{ a contradiction.}
\]

Therefore, only the first case in Lemma 1 applies, where \((p^*_{ja}, p^*_{ja}) = (\omega_j, \omega_j)\). From \( \hat{p}_{ja} = \omega_j \) and \( \hat{p}_{ja} = \omega_j \) we get rational expectations as shown in case (i) of the proposition.

(ii) \( p_{ja} \leq r - v_a \) and \( p_{jb} > \frac{(1-\beta)(r - v_a) + p_{ja}}{2-\beta} \).
We first rule out the first case in Lemma 1 by contradiction. Assume that \( \omega_i > v_i \). From Lemma 1 we then have \((p_j^*, p_{-ja}^*)=(\omega, \omega_j)\). From \( \hat{p}_{ja} = \omega_j \) and \( \hat{p}_{-ja} = \omega_j \) we get rational expectations \( \hat{p}_{ja} = r - p_{jb} \) and \( \hat{p}_{-ja} = r - p_{-jb} \). Thus \( \omega_j = r - p_{-jb} + \frac{\beta}{3(1-\beta)}(p_{jb} + \hat{p}_{ja} - \hat{p}_{-ja}) = r - p_{-jb} \geq v_i \), a contradiction.

We next rule out the third case in Lemma 1 by contradiction. Assume that \( \mu_j > v_a \). From Lemma 1 we then have \((p_j^*, p_{ja}^*)=(\mu, v_a)\). Thus rational expectations are \( \hat{p}_{ja} = v_a \) and \( \hat{p}_{-ja} = v_a \). We next rule out the third case in Lemma 1 by contradiction. Assume that \( \mu_j > v_a \). From Lemma 1 we then have \((p_j^*, p_{ja}^*)=(\mu, v_a)\). Thus rational expectations are \( \hat{p}_{ja} = v_a \) and \( \hat{p}_{-ja} = v_a \). We get rational expectations as in case (ii) of the proposition.

Therefore, only the second case in Lemma 1 applies, where \((p_j^*, p_{ja}^*)=(\mu, v_a)\). Thus \( \hat{p}_{ja} = \mu_j \) and \( \hat{p}_{-ja} = v_a \). Plug the latter into the former, we get rational expectations as in case (ii) of the proposition.

\[
\text{We first rule out the first case in Lemma 1 by contradiction. Assume that } \omega_j < v_i. \text{ From Lemma 1 we then have } (p_j^*, p_{-ja}^*) = (\omega, \omega_j). \text{ From } \hat{p}_{ja} = \omega_j \text{ and } \hat{p}_{-ja} = \omega_j \text{ we get rational expectations } \hat{p}_{ja} = r - p_{jb} \text{ and } \hat{p}_{-ja} = r - p_{-jb}. \text{ Thus } \omega_j = r - p_{-jb} + \frac{\beta}{3(1-\beta)}(p_{jb} + \hat{p}_{ja} - \hat{p}_{-ja}) = r - p_{-jb} \geq v_i, \text{ a contradiction.}
\]

\[
\text{We next rule out the third case in Lemma 1 by contradiction. Assume that } \mu_j > v_a. \text{ From Lemma 1 we then have } (p_j^*, p_{ja}^*) = (\mu, v_a). \text{ Thus } \hat{p}_{ja} = \mu_j \text{ and } \hat{p}_{-ja} = v_a. \text{ Plug the latter into the former, we get rational expectations as in case (ii) of the proposition.}
\]

\[
\text{Therefore, only the second case in Lemma 1 applies, where } (p_j^*, p_{ja}^*) = (\mu, v_a). \text{ Thus } \hat{p}_{ja} = \mu_j \text{ and } \hat{p}_{-ja} = v_a. \text{ Plug the latter into the former, we get rational expectations as in case (ii) of the proposition.}
\]
Therefore, only the third case in Lemma 1 applies, where \((\hat{p}_{ja}, \hat{p}_{ja}) = (p^*_ja, p^*_ja) = (v_a, v_a)\). Q.E.D.

**Proof of Proposition 2**  Given the consumer beliefs of Proposition 1 and the in-app purchase prices of Lemma 1, app firms choose the app prices that maximize profits. Without loss of generality, we assume that in equilibrium \(p^*_jb \geq p^*_jb\) -- otherwise just switch subscripts \(j\) and \(-j\).

First suppose \(p^*_jb > \frac{t}{1-\beta} - v_a\). Then each firm \(j\), chooses \(p^*_jb\) to maximize profit \(\pi_j = p^*_jbD_{jb} + p^*_jaD_{ja}\)

substituting in \(p_{ja} = \hat{p}_{ja}(p_{1b}, p_{2b}) = \frac{t}{1-\beta} - p^*_jb\). Simultaneously solving the first order conditions, the equilibrium app prices are \(p^*_jb = t\). In turn, in-app purchase prices are \(p^*_ja = \frac{t\beta}{1-\beta}\) and profits are \(\pi^*_j = (1-\gamma)(1+\frac{\alpha\beta}{1-\beta})\frac{t}{2}\). Notice that, for these prices to constitute an equilibrium, we need \(p^*_jb > \frac{t}{1-\beta} - v_a\) and in-app purchase prices less than \(v_a\), both of which satisfied if and only if \(\beta < \frac{v_a}{t + v_a}\).

For the above solution to qualify as an equilibrium, it must be that firm \(-j\) cannot profitably deviate by charging \(p^*_jb \leq \frac{t}{1-\beta} - v_a\). We prove by contradiction. Given \(p^*_jb = t\) and suppose firm \(-j\) deviates and chooses \(p^*_jb \leq \frac{t}{1-\beta} - v_a\). Then upon observing both app prices, the subgame equilibrium in Stage 2 satisfies \(p^*_ja = \hat{p}_{ja} = v_a\) and \(p^*_ja = \hat{p}_{ja}(p_{1b}, p_{2b}) = \frac{t + p^*_jb + v_a}{2-\beta} - p^*_jb\). Plugging these results into profit function (1), firm \(-j\)'s Stage 1 profit from this deviation is a quadratic and concave function of \(p^*_jb\) with maximal point \(Z = \frac{(4 - \alpha - 2\beta + \alpha\beta)t - \alpha(1-\beta)v_a}{2(2 - \alpha - \beta)}\). Because \(Z > \frac{t}{1-\beta} - v_a\), the best deviation price firm \(-j\) can charge is \(p^D_{jb} = \frac{t}{1-\beta} - v_a\). This results in profit \((1-\gamma)(1-\gamma)(2\beta t - (1-\beta)v_a)v_a)(t + \gamma^2 t\), which is strictly less than \(\pi^*_j = (1-\gamma)(1+\frac{\alpha\beta}{1-\beta})\frac{t}{2}\) for any \(\beta < \frac{v_a}{t + v_a}\). Thus unilateral deviation is not profitable for firm \(-j\).
Next suppose \( p_{-jb} \leq \frac{t}{1 - \beta} - v_a \) and \( p_{jb} > \frac{t + p_{-jb} - v_a(1 - \beta)}{2 - \beta} \). Then we substitute

\[
p_{ja} = \hat{p}_{ja}(p_{ib}, p_{2b}) = \frac{t + p_{-jb} + v_a}{2 - \beta} - p_{jb} \quad \text{and} \quad p_{-ja} = \hat{p}_{-ja}(p_{ib}, p_{2b}) = v_a
\]

into the profit expressions. The unique solution to the first order conditions are

\[
p_{jb} = \frac{(6 - \alpha - 3 \beta)t - 2\alpha(1 - \beta)v_a}{3(2 - \beta) - \alpha(2 + \beta)} \quad \text{and} \quad p_{-jb} = \frac{(6 - 3 \alpha \beta)t - 4\alpha(1 - \beta)v_a}{3(2 - \beta) - \alpha(2 + \beta)}.
\]

To ensure \( p_{jb} > \frac{t + p_{-jb} - v_a(1 - \beta)}{2 - \beta} \), we then need \( v_a > \frac{\beta(6 - \alpha - 3 \beta)t}{3(1 - \alpha)(2 - \beta)(1 - \beta)} \). To ensure \( p_{-jb} \leq \frac{t}{1 - \beta} - v_a \), we need \( v_a \leq \frac{(3 - \alpha)(2 - \beta)(1 - \beta)}{2 - \beta} \).

However, the above two conditions on \( v_a \) cannot be both satisfied, and thus a contradiction.

Next suppose \( p_{jb} \leq \frac{t + p_{-jb} - v_a(1 - \beta)}{2 - \beta} \). Then we substitute

\[
p_{ja} = \hat{p}_{ja}(p_{ib}, p_{2b}) = p_{-ja} = \hat{p}_{-ja}(p_{ib}, p_{2b}) = v_a
\]

into the profit expressions. We consider two cases: \( t > \alpha v_a \) and \( t \leq \alpha v_a \). First consider \( t > \alpha v_a \). The first order conditions are uniquely satisfied at \( p_{jb}^* = t - \alpha v_a \). This solution satisfies \( p_{jb}^* \leq \frac{t + p_{jb}^* - v_a(1 - \beta)}{2 - \beta} \) if and only if \( \beta \geq \frac{(1 - \alpha)v_a}{t + (1 - \alpha)v_a} \). In turn, \( p_{ja}^* = v_a \) and \( \pi_j^* = (1 - \gamma)t / 2 \).

For the above solution to qualify as an equilibrium, it must be that firm \( j \) cannot profitably deviate by charging \( p_{jb} > \frac{t + p_{-jb} - v_a(1 - \beta)}{2 - \beta} \). Given \( p_{-jb}^* = t - \alpha v_a \) and \( p_{ja}^* = v_a \), firm \( j \)'s profit from this deviation (in which the rational expectations for firm \( j \)'s in-app purchase price become

\[
p_{ja} = \hat{p}_{ja}(p_{ib}, p_{2b}) = \frac{t + p_{-jb} + v_a}{2 - \beta} - p_{jb}
\]

is maximized at \( p_{jb}^D = t - \alpha v_a / 2 \). The deviation satisfies condition \( p_{jb} > \frac{t + p_{-jb} - v_a(1 - \beta)}{2 - \beta} \) if and only if \( \beta < \frac{2v_a}{2t + (2 - \alpha)v_a} \). Firm \( j \)'s profit from this deviation \( \pi^D \) is less than the profit from charging \( p_{jb}^* = t - \alpha v_a \) if and only if

\[
\pi^D - \pi^* = (1 - \gamma)\frac{\alpha(-4t^2 \beta^2 + (1 - \alpha)(-v_a)(4t \beta^2 - (4 - \beta(4 - \alpha \beta))v_a))}{8t(2 - \beta)^3} < 0;
\]

in other words
\[ \beta > 1 / \left( \frac{1 + \sqrt{(1 - \alpha)}}{2} + \frac{t}{v_a \sqrt{1 - \alpha}} \right) \]. Notice that the right-hand-side of this inequality is larger than 

\[ \frac{(1 - \alpha)v_a}{\alpha v_a} \cdot \]

Next consider \( t \leq \alpha v_a \). The firm profits are always decreasing in \( p_{jb} \), thus \( p_{jb}^* = 0 \). This solution satisfies the condition from Proposition 1 (iii) requiring 

\[ p_{jb}^* \leq \frac{t + p_{jb}^* - v_a (1 - \beta)}{2 - \beta} \] if and only if \( \beta \geq \frac{v_a - t}{v_a} \).

In turn, \( p_{ja}^* = v_a \) and \( \pi_j^* = (1 - \gamma)\alpha v_a / 2 \).

For the above solution to qualify as an equilibrium, it must be that firm \( j \) cannot profitably deviate by charging 

\[ p_{jb} > \frac{t + p_{jb} - v_a (1 - \beta)}{2 - \beta} \]. Given \( p_{jb}^* = 0 \) and \( p_{ja}^* = v_a \), firm \( j \)'s profit from this deviation (in which the rational expectations for firm \( j \)'s in-app purchase price become 

\[ p_{ja} = \hat{p}_{ja}(p_{jb}, p_{ja}) = \frac{t + p_{jb} + v_a}{2} - p_{jb} \] is maximized at \( p_{jb}^0 = t / 2 \). The deviation satisfies the condition 

\[ p_{jb} > \frac{t + p_{jb} - v_a (1 - \beta)}{2 - \beta} \] if and only if \( \beta < \frac{2v_a}{t + 2v_a} \). Firm \( j \)'s profit from this deviation \( \pi_j^0 \) is less than the profit from charging 

\[ p_{jb}^* = 0 \] if and only if 

\[ \pi_j^0 - \pi_j^* = (1 - \gamma) \frac{t^2 (4 - \beta (4 - (1 - \alpha) \beta)) - 4t \alpha (2 - (2 - \beta) \beta) v_a + 4 \alpha (1 - \beta) v_a^2}{8t (2 - \beta)^2} < 0 \]; in other words 

\[ \beta > 2 \left( \frac{\alpha (t + v_a)}{\sqrt{\alpha (t^2 + \alpha v_a (v_a - 2t))}} \right) \]. Notice that the right-hand-side of this inequality is larger than \( \frac{v_a - t}{v_a} \).

Denote \( \bar{\beta} = \begin{cases} 
1 / \left( \frac{1 + \sqrt{(1 - \alpha)}}{2} + \frac{t}{v_a \sqrt{1 - \alpha}} \right) & \text{if } t > \alpha v_a \\
2 / \left( \frac{\alpha (t + v_a)}{\sqrt{\alpha (t^2 + \alpha v_a (v_a - 2t))}} \right) & \text{if } t < \alpha v_a 
\end{cases} \). Note that \( \bar{\beta} < \frac{v_a}{t + v_a} \) always holds by assumption that \( t > v_a / 2 \). We may now conclude the following. If \( \beta < \bar{\beta} \), then there is a unique equilibrium where the in-app purchase prices are below \( v_a \). If \( \beta > \frac{v_a}{t + v_a} \), then there is a unique
equilibrium where the in-app purchase prices are set at $v_a$. If $\bar{\beta} \leq \beta \leq \frac{v_a}{t + v_a}$, both equilibria are possible though the former equilibrium Pareto dominates the latter.

We must also demonstrate that neither firm can profitably deviate from the interior in-app pricing equilibrium in Stage 2 by purposefully abandoning informed consumers. Without loss of generality suppose it is firm 1 who chooses to only sell to app-only and uninformed consumers. In this case, firm 1 sells $\gamma_{1b} = (1 - \alpha)\theta_b + \alpha \beta \theta_u$ and $\gamma_{1a} = \alpha \beta \theta_u$. Given the proposed equilibrium beliefs and Stage 1 prices, firm 1’s profit from abandoning informed consumers is equal to $\pi_1 = \frac{1}{2}(1 - \gamma)\left(t(1 - \alpha(1 - \beta)) + \alpha \beta p_{ia}\right)$. Since this profit is strictly increasing in $p_{ia}$, the in-app purchase price will be set to the maximum (i.e., $p_{ia} = v_a$). The profit from this deviation is equal to $\pi_1^* = \frac{1}{2}(1 - \gamma)\left(t(1 - \alpha(1 - \beta)) + \alpha \beta v_a\right)$ which is less than $\frac{(1 - \alpha)v_a}{t + (1 - \alpha)v_a}$ if and only if $\frac{(1 - \gamma)\left(t(1 - \alpha(1 - \beta)) + \alpha \beta \frac{v_a}{1 - \beta}\right)}{2} < t\left(\frac{1}{\beta} + \frac{\beta}{1 - \beta}\right)$. Therefore, $v_a < t\left(\frac{1}{\beta} + \frac{\beta}{1 - \beta}\right)$ is a sufficient condition for neither firm to abandon informed consumers and to preserve our proposed interior equilibrium. The RHS of the inequality is minimized at $\beta = \frac{1}{2}$ and thus $t > \frac{v_a}{2}$ is a sufficient condition. Q.E.D.

**Proof of Proposition 3** From Proposition 2(i), the platform’s profit is strictly increasing in $\beta$ for any $\beta < \beta_1$. The platform’s profit described in Proposition 2(i) is strictly greater than the profit described in Proposition 2(ii), which is the unique outcome if $\beta > \beta_1$. Therefore, the platform optimally sets $\beta = \frac{v_a}{t + v_a}$. Q.E.D.

**Proof of Proposition 4** First consider the case of $t > \alpha v_a$. The effect on total welfare follows from Proposition 2. From part (i) of Proposition 2, the combined prices paid by in-app consumers are $p_{j\beta} + p_{j\mu} = t + \frac{t\beta}{1 - \beta}$. From part (ii) of Proposition 2, the combined prices paid by in-app consumers are $p_{j\beta} + p_{j\mu} = t + (1 - \alpha)v_a$. The former is greater than the latter if and only if $\beta > \frac{(1 - \alpha)v_a}{t + (1 - \alpha)v_a}$, which is less than $\beta_1$. 35
Next consider the case of \( t < \alpha v_a \). For the total consumer welfare under \( \beta > \beta_1 \) to be greater than that under \( 0 < \beta < \beta_1 \), we need the total firm and platform profit under \( \beta > \beta_1 \), \( \alpha v_a \), to be smaller than the total firm and platform profit under \( 0 < \beta < \beta_1 \), \( t(1 + \frac{\alpha \beta}{1 - \beta}) \). This is true if \( \beta > \frac{\alpha v_a - t}{\alpha v_a - (1 - \alpha)t} \) which is less than \( \beta_1 \). From part (i) of Proposition 2, the combined prices paid by in-app consumers are \( p_{jb} + p_{ja} = t + \frac{t \beta}{1 - \beta} \). From part (ii) of Proposition 2, the combined prices paid by in-app consumers are \( p_{jb} + p_{ja} = v_a \). The former is greater than the latter if and only if \( \beta > 1 - t \frac{1}{v_a} \) which is less than \( \beta_1 \). \( Q.E.D. \)

**Proof of Proposition 5** Given belief structure \( \hat{p}_{jb}(p_{jb}) = Ap_{jb} + H \) for \( j = 1, 2 \), under simultaneous pricing the equilibrium prices are simultaneous solutions to \( \frac{\partial \pi_j}{\partial \hat{p}_{ja}} = 0 \), \( \frac{\partial \pi_j}{\partial \hat{p}_{jb}} = 0 \), \( \frac{\partial \pi_j}{\partial \hat{p}_{ja}} = 0 \), and \( \frac{\partial \pi_j}{\partial \hat{p}_{jb}} = 0 \), which yields \( p_{jb} = \frac{(1 - \alpha - \beta - A\alpha \beta)t}{(1 - \alpha)(1 - \beta)} \) and \( p_{ja} = \frac{(1 + A\alpha)\beta t}{(1 - \alpha)(1 - \beta)} \). Consistent belief requires that \( \hat{p}_{jb}(p_{jb}) = p_{ja} \). Plugging the above solutions of prices into the consistent belief requirement, we get \( H = \frac{(1 + A)(1 + A\alpha)\beta - A(1 - \alpha)t}{(1 - \alpha)(1 - \beta)} \).

We next check second-order conditions. \( \frac{\partial^2 \pi_j}{\partial \hat{p}_{ia}^2} = -(1 + A\alpha \beta) / t < 0 \) for any \( A > -1 / (\alpha \beta) \). \( \frac{\partial^2 \pi_j}{\partial \hat{p}_{ia} \hat{p}_{ja}} = -\alpha(1 - \beta) / t < 0 \). \( \frac{\partial^2 \pi_j}{\partial \hat{p}_{jb}^2} = \frac{\partial^2 \pi_j}{\partial \hat{p}_{ja}^2} = \frac{\partial^2 \pi_j}{\partial \hat{p}_{ja} \hat{p}_{ja}} = \frac{\partial^2 \pi_j}{\partial \hat{p}_{ja} \hat{p}_{ia}} = 0 \), which is positive for any \( A \in (-1 - \frac{2\sqrt{\alpha(1 - \alpha)(1 - \beta)}}{\alpha \beta}, -1 + \frac{2\sqrt{\alpha(1 - \alpha)(1 - \beta)}}{\alpha \beta}) \). Also notice that \( -1 - \frac{2\sqrt{\alpha(1 - \alpha)(1 - \beta)}}{\alpha \beta} > -1 / (\alpha \beta) \). Therefore \( A \in (-1 - \frac{2\sqrt{\alpha(1 - \alpha)(1 - \beta)}}{\alpha \beta}, -1 + \frac{2\sqrt{\alpha(1 - \alpha)(1 - \beta)}}{\alpha \beta}) \) ensures that all second-order conditions are satisfied. \( Q.E.D. \)
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