gives several equivalent ways to describe the knowledge state of a person. The states are partially ordered, but there are many pairs of states—in fact, most of them lie in a "real-world" knowledge space—which are incomparable. Thus, a numerical value can never reflect this situation regardless of the possible sophistication involved in its definition and determination.

At present the main applications of this theory are to fields where the complete knowledge is sufficiently structured or is in some parts hierarchically organized. In these cases it is far better than psychometrically constructed tests. What about the numerous tests of differential psychology such as intelligence tests, etc.? Will there be a time when they will be formulated in the spirit of knowledge spaces? In the opinion of this reviewer, this is an aim worth pursuing.

7. Summary. There are many more aspects to the theory which we could not mention in this review. The book deals with the construction of knowledge spaces and learning spaces, the problem of combining smaller spaces into a larger one, and with Galois connections. The final chapter lists and describes a few open problems. Thus, the creative mathematician will find material capable of entertaining him or her for some time. The practitioner may be interested in applications. The penultimate chapter gives an extensive overview of the ALEKS system, a learning space built for the mathematics curriculum "Beginning Algebra." In particular, the assessment algorithm is described as the way to monitor learning success. Various other applications with educational and pedagogical impact are provided by the Graz school; cf. [4] among others. This group utilizes a somewhat different approach, called "competence-based knowledge space theory." There are still other ways to get into knowledge spaces; the interested reader might consult [3] and [5].

In summary, there is no doubt that reading and working with this book will be rewarding for the mathematician and useful for scientists from very different areas. In many aspects it has the potential to serve as a guideline to a new and theoretically better founded form of psychometry.

REFERENCES

[1] J.-P. Doignon and J.-C. Falmagne, *Knowledge Spaces*, Springer, New York, 1999.

[2] D. Eppstein, J.-C. Falmagne, and S. Ovchinnikov, *Media Theory*, Springer, New York, 2008.

[3] J. Heller and C. Repitsch, *Distributed skill functions and the meshing of knowledge structures*, J. Math. Psychol., 52 (2008), pp. 147–157.

[4] J. Heller, C. Steiner, C. Hockemeyer, and D. Albert, *Competence-based knowledge structures for personalised learning*, Internat. J. E-Learning, 5 (2006), pp. 75–88.

[5] R. Suck, *Knowledge spaces regarded as set representations of skill structures*, in Descriptive and Normative Approaches to Human Behavior, E. Dzhafarov and L. Perry, eds., World Scientific, River Edge, NJ, 2011, pp. 249–270.

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Probability Inequalities. *By Z. Lin and Z. Bai.* **Springer, New York, 2011. $109.00. xii+181 pp., hardcover. ISBN 978-3-642-05260-6.**

This is a relatively short book of 181 pages organized in 12 chapters with the avowed aim of introducing beginning research workers in probability and statistics to "frequently used" inequalities. As the authors explain:

> The aim of this book is to help beginners.... We provide a place to find the most frequently used inequalities, their proofs (if not too lengthy) and some references.

Of course any attempt to write a book on "Inequalities"—whether in the area of probability or some other area of mathematics—invites a comparison with the grandfather of such books, namely, [4, 5], whose first edition appeared in 1934. As is generally appreciated, this is a very high bar! But because inequalities play such an important role in research in many areas of probability and statistics, it is certainly worth trying to bring together those which are indeed "frequently used," and I applaud the authors for their efforts.

The book contains nearly 200 different inequalities, with 26 inequalities in Chapter 9 ("Moment Estimates of (Maximum of) Sums of Independent Random Variables"), 24 inequalities in Chapter 6 ("Bounds on Probabilities in Terms of Moments"), and 23 inequalities in Chapter 5 ("Probability Inequalities of Random Variables"). Included are many of my favorite inequalities, including those of Anderson, Berman, Birnbaum-Marshall, Borell, Burkholder, Cauchy(–Schwarz), Doob, Doukhan, Efron–Stein, Erhard, Esseen, Fernique, Freedman, Gordon, Hoffmann–Jørgensen, Jensen, Khintchine, Kimball, Kolmogorov, Lenglart, Lévy, Minkowski, Mogulskii, Peligrad, Petrov, Prohorov, Rosenthal, Shao, Slepian, Stein, Tong, and Wilks.

A short book of this sort cannot be completely comprehensive; the authors have chosen their favorites. Among the types of inequalities not included here are Orlicz norm inequalities for maxima (see, e.g., [1]); majorization inequalities (e.g., [10] or [9]); concentration of measure inequalities, which are becoming more frequently used in empirical process theory and problems involving high-dimensional statistics (see, e.g., [8, 11]); coupling and transportation inequalities; inequalities related to characterizations (see, e.g., [14]); and log-Sobolev inequalities (see, e.g., [8, 11]).

Are the authors successful in achieving their goal? The answer is "partly" for this reader. The book suffers from several difficulties, including the following:

(a) It lacks both an author and a subject index, and this, in combination with the divided reference lists at the ends of chapters, makes it somewhat hard to use.

(b) While some inequalities are clearly named and referenced, others are much more obscure: for example, how will the beginner know that the inequalities appearing in section 9.11 on pages 114 and 115 are due to Hoffmann–Jørgensen? The proofs given involve the probability inequalities (attributed to "Hoffmann–Jørgensen") in section 5.10, pages 47–48, but it would have been nice to name the inequalities in section 9.11 appropriately. Another example is the inequality in 6.2.b on page 53; this is often called the "Paley–Zygmund inequality" (see, e.g., [6, p. 40], [7], [3], or [1, p. 119]), but this name is not indicated here, nor is a reference given. Similarly, references are lacking for the "Mills' ratio"-type inequalities for the normal tail probability treated in sections 2.1.b and 2.1.c, pages 10–11 (see, e.g., [2], [12, p. 349], and [13]).

(c) Another possible difficulty for true beginners is that the proofs sometimes involve rather inexplicit references to (generalizations of) results stated later in the book. One example of this occurs in the proof of "Wilk's inequality" on page 50 via "Hölder's inequality." The form of Hölder's inequality stated here involves a generalization to $k \geq 2$ functions (see, e.g., [5, sections 2.7 and 6.9]) of the more usual form for two functions given later in the book on page 85, or a proof by induction. This will not be difficult for the expert, but it could easily be a hurdle for the beginner. The authors could have helped a bit by providing an explicit forward reference and perhaps a further hint.

(d) The book suffers from a large number of annoying minor typographical errors (e.g., "Berkholder" rather than "Burkholder" on page 108; and "Bnyakovski" should perhaps be "Bunyakovsky" on page v (see, e.g., [15, p. 10])), and many misplaced accent marks. Difficulty getting these right in a couple of my own books causes me to be sympathetic, but some careful editing could have prevented these troubles.

I would heartily recommend this book to beginning researchers wanting to find "frequently used" inequalities and learn some of the standard arguments. I would not recommend it for more experienced research workers. It seems to me that "the" book about inequalities in the area of probability that will stand the test of time (and comparison to [4]) is yet to be written.

REFERENCES

[1] V. H. de la Peña and E. Giné, *Decoupling: From Dependence to Independence*, Probab. Appl. (N.Y.), Springer-Verlag, New York, 1999.

[2] L. DUEMBGEN, *Bounding Standard Gaussian Tail Probabilities*, Tech. Report 76, University of Bern, Institute of Mathematics, 2010.

[3] E. GINÉ, F. GÖTZE, AND D. M. MASON, *When is the Student t-statistic asymptotically standard normal?*, Ann. Probab., 25 (1997), pp. 1514–1531.

[4] G. H. HARDY, J. E. LITTLEWOOD, AND G. PÓLYA, *Inequalities*, 2nd ed., Cambridge University Press, Cambridge, UK, 1952.

[5] G. H. HARDY, J. E. LITTLEWOOD, AND G. PÓLYA, *Inequalities*, Cambridge Mathematical Library, Cambridge University Press, Cambridge, UK, 1988. Reprint of the 1952 edition.

[6] O. KALLENBERG, *Foundations of Modern Probability*, Probab. Appl. (N.Y.), Springer-Verlag, New York, 1997.

[7] O. KALLENBERG, *Foundations of Modern Probability*, 2nd ed., Probab. Appl. (N.Y.), Springer-Verlag, New York, 2002.

[8] M. LEDOUX, *The Concentration of Measure Phenomenon*, Math. Surveys. Monogr. 89, AMS, Providence, RI, 2001.

[9] A. W. MARSHALL AND I. OLKIN, *Inequalities: Theory of Majorization and Its Applications*, Math. Sci. Engrg. 143, Academic Press, New York, 1979.

[10] A. W. MARSHALL, I. OLKIN, AND B. C. ARNOLD, *Inequalities: Theory of Majorization and Its Applications*, 2nd ed., Springer Ser. Statist., Springer, New York, 2011.

[11] P. MASSART, *Concentration Inequalities and Model Selection*, Lecture Notes in Math. 1896, Springer, Berlin, 2007. Lectures from the 33rd Summer School on Probability Theory held in Saint-Flour, 2003.

[12] P. MÖRTERS AND Y. PERES, *Brownian Motion*, Camb. Ser. Stat. Probab. Math., Cambridge University Press, Cambridge, UK, 2010.

[13] I. PINELIS, *Monotonicity properties of the relative error of a Padé approximation for Mills' ratio*, JIPAM J. Inequal. Pure Appl. Math., 3 (2002), article 20.

[14] B. SIMON, *Convexity: An Analytic Viewpoint*, Cambridge Tracts in Math. 187, Cambridge University Press, Cambridge, UK, 2011.

[15] J. M. STEELE, *The Cauchy-Schwarz Master Class: An Introduction to the Art of Mathematical Inequalities*, MAA Problem Books Series, Mathematical Association of America, Washington, D.C., 2004.

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Probability Measures on Semigroups: Convolution Products, Random Walks and Random Matrices. Second Edition.
By G. Högnäs and A. Mukherjea. Springer, New York, 2011. $124.00. xii+430 pp., hardcover. ISBN 978-0-387-77547-0.

The authors of this book intend it to present research that has as its "ultimate goal... to describe the long-term behaviour of random transformations of some set." The preface to the first edition, published in 1995, is reprinted here and explains the significance of this research in terms of its applications to Markov processes, coding theory, and number theory. Högnäs and Mukherjea have made substantial contributions to the subject themselves and present their work and that of others in a unified way. A general framework is developed through the first three chapters of the book and then applied in the last chapter to the specific case of products of random matrices with nonnegative entries.

The study begins in complete generality, with no restriction being imposed on the set and its transformations initially. In this generality, all that is known about the set from which the transformations are randomly chosen is that it is a semigroup under composition, and the first chapter covers aspects of the theory of semigroups needed for subsequent chapters. These aspects are (1) abstract semigroups culminating with the theorem of Rees and Suschkewitsch that every completely simple semigroup is isomorphic to a Rees product; (2) topological semigroups and in particular compact or locally compact ones; and (3) semigroups of matrices, including infinite matrices.

Probability measures on the semigroups and their convolution products express the random choice of successive transformations, and "long-term behaviour" means the convergence of iterated convolution prod-