Statistics 583, Problem Set 7

Wellner; 5/13/98

Reading: Chapter 8, sections 8.1- 8.4,
Efron and Tibshirani, *An Introduction to the Bootstrap*, Chapters 7 and 21.

Due: Wednesday, May 20, 1998

Reminder: Project talks begin on Friday, 5/22.
Written projects (done in TeX or LaTeX) are due on Friday, June 5.

1. The expression for the jackknife variance estimator for the median, (2.2) on page 10 in chapter 8 was derived under the assumption $n = 2m$ and that $T(\mathbb{F}_n) = X_{(m)}$ if $n = 2m - 1$, $T(\mathbb{F}_n) = (X_{(m)} + X_{(m+1)})/2$ if $n = 2m$.
   A. Derive the first equality in (2.2), page 10, using this definition of the sample median.
   B. Derive versions of (2.2) using $T(F) = F^{-1}(1/2)$ (strictly). Does the asymptotic result in (2.2) still hold?

2. Suppose that $T(F) = Var_F(X)$ so that $T_n \equiv T(\mathbb{F}_n) = n^{-1}\sum_{i=1}^n (X_i - \overline{X})^2$. Show that the jackknife estimate of the variance $\sigma_n^2(F) \equiv Var_F(T_n)$ is

$$\widehat{Var} = \frac{n^2}{(n-1)^3}(\widehat{\mu}_4 - \widehat{\mu}_2^2)$$

   where $\widehat{\mu}_k \equiv n^{-1}\sum_{i=1}^n (X_i - \overline{X})^k$ for $k = 1, 2, \ldots$. Hence, assuming that $EX^4 < \infty$, the jackknife estimate of variance is consistent for this T:

$$n\widehat{Var} \rightarrow_p \mu_4 - \mu_2^2 = \mu_2^2\{2 + \frac{\mu^4}{\mu_2^2} - 3\} = T_2(F)(2 + \gamma_2)\,.$$

3. **Optional bonus problem:** Problem 9.6, Efron and Tibshirani, page 122. (You may assume that the design matrix C contains a column of 1's – so that the model includes an intercept term.