Statistics 583, Problem Set 4

Wellner; 4/21/98

Reading: Chapter 6, section 6.3; Ferguson MS, sections 5.6, 5.7, and 5.9; Lehmann, TSH, Chapter 6, pages 282 - 357.
Due: Wednesday, April 29, 1998

1. Due two weeks from today, May 6: Tentative outlines for your talk and written projects.

2. Ferguson, MS, problem 5.7.7, page 257.

3. A. Suppose that $V_{(1)} \leq \ldots \leq V_{(N)}$ are the order statistics of a sample of iid exponential(1) random variables. Show that

$$(V_{(1)}, \ldots, V_{(N)}) =_d (S_1, \ldots, S_N)$$

where

$$S_i \equiv \sum_{j=1}^{i} \frac{Z_j}{N-j+1}$$

and $Z_1, Z_2, \ldots$ are i.i.d. exponential(1).
B. Let $a_N(i) \equiv -\log(1 - \frac{i}{N+1})$, $i = 1, \ldots,^N$, and consider the "log - rank" statistic $S_N \equiv \sum_{j=1}^{n} a_N(Q_j)$ for testing $H : F = G$ versus $K : F <_s G$.
(i). Compute ES_N and $Var(S_N)$ under the null hypothesis H.
(ii). Can you normalize ES_N and $Var(S_N)$ so that they converge to constants? If so, find the constants.
(iii). Use the Wald - Wolfowitz - Noether finite sampling limit CLT to prove that S_N is asymptotically normal under the null hypothesis H.

4. **Optional bonus problem:** Ferguson, MS, problem 5.9.6, page 273.