

### Statistics 583, Problem Set 3

Wellner; 4/14/98

**Reading:** Chapter 6, section 6.3; Ferguson MS, sections 5.6, 5.7, and 5.9; Lehmann, TSH, Chapter 6, pages 282 - 357.

**Due:** Wednesday, April 22, 1998

1. Due three weeks from today, May 6: Tentative outlines for your talk and written projects. (Get started on the projects soon!)
2. Problem 5.6.7, Ferguson, MS, page 248, part (a).
3. Suppose that an urn contains  $N$  balls with the numbers  $z_i = i, i = 1, \dots, N$ , and we sample  $n < N$  balls from this urn. Let  $\bar{Y}_n = n^{-1} \sum_1^n Y_i$  denote the sample mean of the sampled balls.
  - A. Calculate the mean  $\mu_N = E(\bar{Y}_n)$  and variance  $\sigma_N^2 = Var(\bar{Y}_n)$  of  $\bar{Y}_n$ .
  - B. Use the Wald - Wolfowitz - Noether - Hajek finite-sampling CLT to prove that  $(\bar{Y}_n - \mu_N)/\sigma_N \rightarrow_d N(0, 1)$ .
  - C. What classical two-sample rank statistic is  $\bar{Y}_n$  equivalent to under the null hypothesis (of all of  $X_1, \dots, X_m, Y_1, \dots, Y_n$  equal in distribution with a common continuous d.f.  $F$ )?
  - D. If  $F(x) = (1 + e^{-x})^{-1}$  for  $x \in R$ , compute

$$a_N(i) \equiv -\frac{f'}{f} \left( F^{-1} \left( \frac{i}{N+1} \right) \right)$$

and show that the  $a_N(i)$ 's are related simply to the  $z_i = i$ 's in the urn. (Hint: recall our computation of  $-f'/f$  for the logistic distribution from 581.)

4. **Optional bonus problem:** Problem 5.6.7, Ferguson, MS, page 248, part (b).